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FILAMENTARY SUPERCLUSTERING IN A UNIVERSE DOMINATED BY COLD DARK MATTER

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ABSTRACT

The formation of large-scale filamentary structure in a universe dominated by cold dark matter (CDM) is examined using high-resolution N-body simulations. A measure of the filamentary nature of superclustering is provided by the relative orientations of neighboring clusters of galaxies. When all cluster pairs are considered, there is a clear tendency for the major axes of neighboring clusters to be aligned with one another over scales up to $\sim$10–15h^{-2} Mpc. When the sample is restricted to only those clusters that reside in superclusters, however, significant alignments are found for separations up to $\sim 30h^{-2}$ Mpc or more. The strength and extent of cluster alignments are found to be insensitive to the degree of biasing, while the abundance of rich clusters depends strongly on the bias factor. The cluster alignments survive even after strong nonlinear clustering has developed on small scales, and thus can provide a useful indicator of filamentary structure on very large scales today. Despite the fact that the standard CDM model has relatively little power on large scales compared with other competing models, it is found to be capable of producing large-scale alignments as strong as are observed. Other properties of superclusters in the CDM model, such as their sizes and the fraction of rich clusters which are supercluster members, also appear to be consistent with observations.

Subject headings: cosmology — dark matter — galaxies: clustering — galaxies: formation

1. INTRODUCTION

A wide variety of theories for the formation of galaxies, clusters of galaxies, and the large-scale structure of the universe has been proposed in the past few decades. Of these, models based on the assumption of a universe dominated by a (still hypothetical) class of weakly interacting nonbaryonic particles known generically as cold dark matter (hereafter CDM) have proved particularly popular and arguably most successful at accounting for many of the observed properties of galaxies and galaxy clustering (e.g., Blumenthal et al. 1984; Davis et al. 1985,; Frenk et al. 1985, 1988; Dekel and Silk 1986; White et al. 1987; White, Tully, & Davis 1988; Park 1990; Villumsen & Brainerd 1990; Weinberg & Gunn 1990). Less clear, however, is whether the CDM model can produce sufficient structure on large scales to be consistent with such observations as the strength of the cluster-cluster correlation function, the existence of superclusters and voids having dimensions of tens or hundreds of Mpc, and large-scale streaming velocities (e.g., Kirshner et al. 1981. Bahcall & Soneira 1983; Klypin & Kopylov 1983; Collins, Joseph, & Robertson 1986; de Lapparent, Geller, & Huchra 1986; Vittorio, Juszkiewicz, & Davis 1986; Tully 1986, 1987; Bertschinger & Juszkiewicz 1988; Lynden-Bell et al. 1988; Aaronson et al. 1989; Geller & Huchra 1989; Gorski et al. 1989; Groth, Juszkiewicz, & Ostriker 1989; Kaiser & Lahav 1989; Broadhurst et al. 1990; Maddox et al. 1990). This uncertainty is due in large part to the limited amount of observational data on these scales and ambiguities in the interpretation of the existing data (e.g., Sutherland 1988; Dekel et al. 1989; Postman et al. 1989; Bothun et al. 1990), which forces comparisons between models and observations to often be more of a qualitative rather than quantitative nature, with the apparent success or failure of a particular model judged largely on the basis of very subjective visual comparisons between N-body simulations and the observed galaxy distribution. Hence, before the CDM cosmogony can truly be considered a successful model for the genesis of the large-scale structure, it is essential that it be subjected to a wide variety of detailed, *quantitative* comparisons with observational data.

The standard CDM cosmology assumes that the progenitors of structures observed today were small-amplitude, adiabatic, Gaussian density fluctuations that were imprinted on the primordial dark matter distribution (perhaps originating from quantum fluctuations), and that these inhomogeneities subsequently grew by gravitational instability. The predicted shape of the CDM fluctuation spectrum varies from roughly $|\delta_k|^2 \propto k^{-3}$ or k^{-2} on galactic scales and smaller (were δ_k denotes the Fourier components of the density fluctuation field and k is spatial frequency), to $|\delta_k|^2 \propto k^{-1}$ or k^0 on the scale of clusters of galaxies, while on very large scales the spectrum retains the original Harrison-Zel'dovich form favored by inflationary models, $|\delta_k|^2 \propto k$ (Peebles 1982, 1984; Blumenthal & Primack 1983; Bond & Efstathiou 1984). The physical scaling of this spectrum is proportional to $(\Omega h^2)^{-1}$, Ω being the cosmological density parameter, and $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}. Because small-scale perturbations have higher amplitudes than those on larger scales, the evolution of structure in a CDM-dominated universe is expected to proceed hierarchically, with the nearly coeval formation of structures on subgalactic and galactic scales occurring first, followed by the later collapse of larger scale perturbations corresponding to clusters and superclusters. In order to reconcile dynamical estimates of $\Omega \approx 0.1$ generally inferred from observations with the theoretically preferred value of $\Omega = 1$, proponents of the CDM model have found it necessary to appeal to some ad hoc mechanism of "biased galaxy formation" whereby the distribution of galaxies is assumed to be more clustered than the true mass distribution (e.g., Kaiser 1984; Bardeen et al. 1986; Davis et al. 1985; Dekel & Rees 1987). This dissimilarity of the luminous and dark matter distributions is usually parameterized by the "bias factor," b, defined by

$$\sigma_{\rm gal} \equiv b\sigma_{\rm mass} , \quad (1)$$

where $\sigma_{\rm gal}$ and $\sigma_{\rm mass}$ are the rms relative fluctuations in the number of galaxies and the mass density within spheres of radius $8h^{-1}$ Mpc, for which $\sigma_{\rm gal} \approx 1$ is observed (Davis & Peebles 1983). This then provides a means of normalizing the amplitude of the initial fluctuation spectrum, with a value of $b = 1$ corresponding to a direct coupling between the galaxy and mass distributions. A difficulty of the standard CDM model with biased galaxy formation is that a large b value is required to reconcile the small values of Ω deduced from the observed galaxy distribution with the preferred Einstein–de Sitter cosmology, yet to produce sufficient large-scale velocities seems to necessitate a small value of b so that galaxies do trace the mass reasonably well on large scales. This is because in the CDM model the mass correlation function is predicted to go negative beyond a scale of $\sim 20(\Omega h^2)^{-1}$ Mpc, and thus there is relatively little power on large scales; this problem becomes severe if there is significant biasing on those scales. Davis et al. (1985) found that a value of $b = 2.5$ and $h = 0.5$ produced the best agreement between their N-body simulations and observations; however, a number of subsequent studies have suggested that smaller values of b may be more appropriate (e.g., Bardeen et al. 1986; Oemler 1987; Carlberg & Couchman 1989; Kaiser & Lahav 1989; Valls-Gabaud, Alimi, & Blanchard 1989; Evrard 1990). The desired segregation between galaxies and dark matter can alternatively be achieved in a natural way via dynamical evolutionary processes (e.g., Hoffman, Shaham, & Shaviv 1982; West & Richstone 1988; Hoffman 1989; Carlberg, Couchman, & Thomas 1990; Richstone & West 1990). Various ways of obtaining more power on large scales in the CDM model have also been discussed (Bardeen, Bond, & Efstathiou 1987; Blumenthal, Dekel, & Primack 1988).

Redshift surveys have revealed large-scale coherent features in the distribution of galaxies and clusters of galaxies which have been described as "filamentary" or "cellular" (e.g., Einasto, Joeveer, & Saar 1980; Batuski & Burns 1985), "sheetlike" (Geller & Huchra 1989), "bubble-like" (de Lapparent, Geller, & Huchra 1986), or "spongelike" in appearance (Gott, Melott, & Dickinson 1986), or some combination of these (e.g., Haynes & Giovanelli 1986; da Costa et al. 1988). Although such qualitative assessments are clearly quite subjective, they nevertheless suggest that the large-scale structure possesses a very interesting geometry which any successful cosmogonic model must be able to account for. One possible manifestation of this pattern of superclustering that is free from subjective characterizations is the observation, first noted by Binggeli (1982), that the major axes of clusters of galaxies are not randomly oriented, but rather exhibit a tendency to be aligned with other neighboring clusters over scales of several tens of Mpc. This alignment effect has been confirmed in subsequent studies by Flin (1987), Rhee & Katgert (1987), West (1989a, b), Lambas et al. (1990), and Lambas & West (1990). Such observations provide compelling evidence of a filamentary topology of superclustering and suggest a connection between the formation of clusters and the large-scale structure which may provide important clues about cosmogonical processes (see also Schombert & West 1990). Unfortunately, however, no clear observational consensus has yet emerged regarding the strength, scale, or even reality of cluster alignments, since several other studies were unable to find any evidence of such alignments (Struble & Peebles 1985; Ulmer, McMillan, & Kowalski 1989; Fong, Stevenson, & Shanks 1990), and even those studies which do claim to find evidence do not all agree on the strength of the effect or on its spatial extent. For example, West (1989b) has recently claimed that *clusters residing in superclusters* show a clear tendency to be aligned over scales of at least $30h^{-1}$ Mpc, with additional evidence suggesting that they may be aligned over even larger distances, perhaps up to $\sim 60h^{-1}$ Mpc. Binggeli's (1982) data also hinted at some cluster alignments over distances as great as $\sim 50h^{-1}$ Mpc. Rhee & Katgert (1987), on the other hand, claimed to find strong alignments of clusters occurring only for separations up to $15h^{-1}$ Mpc (although their data would seem to suggest that this effect may extend more weakly to at least $\sim 30h^{-1}$ Mpc). And Lambas et al. (1990) have recently claimed to detect relatively weak alignments up to only $15h^{-1}$ Mpc, with no alignments for larger separations. However, as emphasized by West (1989b), the fact that different studies produce such discrepant results is due at least in part to the difficulty of accurately determining the orientation of a given cluster using the projected distribution of galaxies. Since cluster position angle measurements have typical uncertainties of $\sim 30°$, position angle determinations from a single study may not always be reliable, and consequently this could easily obscure evidence of genuine cluster alignments. That West's claimed detection of cluster alignments over large scales is based on position angle data combined from several independent sources, including other studies which found no evidence of cluster alignments, suggests that there may be some validity to this argument. Alignment studies may also be affected by the relative mix of different cluster morphological types in a given sample (Schombert & West 1990).

Further evidence of a filamentary pattern in the large-scale distribution of galaxies and clusters is provided by the observation that galaxy counts in the regions surrounding rich clusters tend to be systematically higher along the direction defined by the cluster major axis and/or the position angle of the first-ranked cluster galaxy (Argyres et al. 1986; Lambas, Groth, Peebles 1988; Muriel & Lambas 1989). This angular anisotropy of the cluster-galaxy correlation function is observed to extend over scales of at least $\sim 15h^{-1}$ Mpc, beyond which it becomes undetectable. It is worth emphasizing here that cluster-cluster alignments and angular anisotropies in the cluster-galaxy correlation function are *not* merely redundant measures of the same effect, but rather are *complementary* tests for filamentariness on scales of several tens of Mpc.

Numerical simulations of cluster formation can provide a powerful means of confronting cosmogonic models with observations (see West 1990 for a review). Dekel, West, & Aarseth (1984) used N-body simulations to examine whether cluster-cluster alignments like those found by Binggeli (1982) could provide a useful probe of the primordial density fluctuation spectrum and concluded that such alignments are indeed a sensitive test of cosmogony. They found, for example, that alignments similar to those observed by Binggeli occurred in simulations with an initial fluctuation spectrum possessing a coherence length such as expected in a hot dark matter (e.g., massive neutrinos) cosmogony, while no cluster-cluster alignments were found for hierarchical clustering simulations beginning from Poisson initial conditions ($|\delta_k|^2 \propto k^0$). Similarly, West, Dekel, & Oemler (1989) and West, Weinberg, & Dekel (1990) showed that angular anisotropies in the cluster-galaxy correlation function comparable to the observed effect could be produced in N-body simulations beginning with initial conditions appropriate for hot dark matter, hierarchical clustering models with a rather flat power spectrum ($|\delta_k|^2 \propto k^{-2}$), certain types of hybrid models, and explosion scenarios. They were unable to reproduce the observed effect in simulations of hierarchical clustering with power-law spectra $|\delta_k|^2 \propto k^0$ and $|\delta_k|^2 \propto k^{-1}$, and concluded that, because the CDM fluctuation spectrum has a similar slope on cluster scales, CDM may fail to produce sufficient large-scale filamentary structure to be consistent with the observations. However, some more recent work (Melott & Shandarin 1990; Nusser & Dekel 1990; Park 1990; Weinberg & Gunn 1990) has suggested that CDM might in fact produce large-scale filamentary and sheetlike features. Bond (1987) and Bond & Szalay (1990) have also argued on the basis of Gaussian statistics and the assumption that rich clusters correspond to high peaks in the initial density field that significant alignments will inevitably occur in the CDM model over scales up to $\sim 20h^{-1}$ Mpc. However, most of these particular studies were based on either analytic calculations using linear theory, numerical codes which couple the linear Zel'dovich (1970) approximation with a viscosity approximation (e.g., Gurbatov, Saichev, & Shandarin 1989), or two-dimensional particle-mesh codes; determining whether thin filamentary features are able to survive intact once strong nonlinear clustering has developed on small scales requires a full three-dimensional N-body treatment. Villumsen & Brainerd (1990) have also argued that the appearance of large-scale coherent structures is very sensitive to the exact biasing scheme adopted. Non-Gaussian initial conditions may also lead to elongated or flattened mass distributions (e.g., Messina et al. 1990).

The ability of the CDM scenario to produce large-scale elongated superclusters is studied in this paper by examining the alignment tendencies of neighboring clusters in N-body simulations of the standard biased CDM model. Previous numerical studies of cluster formation in the CDM model by Batuski, Melott & Burns (1987) and White et al. (1987) focused primarily on the abundance and clustering properties of Abell-type clusters. Yet in spite of the present observational uncertainties, alignments of clusters of galaxies may provide a more stringent test of the CDM model. In a sense, cluster alignments provide an interesting means of simultaneously probing the initial fluctuation spectrum on more than one scale, since one is looking at the formation and orientation of clusters in relation to the formation of large scale structures such as superclusters. In view of the current observational uncertainties regarding the reality and strength of cluster alignments, the present paper can be regarded either as a direct comparison of the CDM model with the real universe or as a prediction for the CDM model which future observations will eventually support or rule out (depending on the reader's predisposition toward the existing observational evidence). The paper is organized as follows. In § 2, the N-body simulations used in this study are described. In § 3, the tendency for cluster-cluster alignments to occur in these simulations is examined, along with other properties of superclusters in the CDM model. Finally, in § 4, the results of this study are summarized and their implications discussed. A subsequent paper (West et al. 1990), will explore anisotropies in the cluster-galaxy correlation function as a probe of filamentary structure in both the CDM model and observations.

2. N-BODY SIMULATIONS

N-body simulations were performed of an $\Omega = 1$ CDM universe in a comoving cube of length $100h^{-2}$ Mpc with periodic boundary conditions using the HPM (hierarchical particle mesh) N-body code described in Villumsen (1989). The simulations had 128^3 gridcells and 64^3 particles in the top grid and they included several subgrids. The present paper describes only the results of the top grid calculations, i.e., the cluster-cluster alignments, while another paper (West et al. 1990) will describe the results of the subgrid calculations, i.e., the cluster-galaxy alignments, where higher resolution is desired for the latter case. The particle mass is $1.06 \times 10^{12}h^{-4}$ $M_\odot$ and a gridcell is $0.78h^{-2}$ Mpc across.

The initial conditions for the simulations were generated using the CDM power spectrum described in Davis et al. (1985). The particles were displaced from a uniform grid assuming a Gaussian random density field, linear theory, and the Zel'dovich approximation, The amplitude of the displacement field was fixed by setting the rms density fluctuation σ_8 in a sphere of radius $4h^{-2}$ Mpc equal to $\sigma_8 = 1/21$. This is equivalent to the standard definition of the amplitude of density fluctuations of $h = \frac{1}{2}$. The system is then evolved for 21 expansion factors until $\sigma_8 = 1$ and the positions of all particles are output for $b = 1/\sigma_8 = 2.5, 2.0, 1.5, 1.0$. The particle distribution at these different stages of the simulation are shown in Figure 1.

Clusters of galaxies were identified at each stage in the simulation using a conventional "friends-of-friends" cluster-finding algorithm to first locate all distinct groups having densities above a chosen threshold, and then selecting from these all groups whose total mass is comparable to that of rich Abell clusters. In this procedure, all particles separated by less than some specified distance are linked together, with a given value of this linking distance corresponding to a specific minimum overdensity. Clusters are composed of those particles sharing a common (either direct or indirect) linkage. A density contrast of 35 was chosen to identify clusters, since that is roughly the observed value at the Abell radius ($1.5h^{-1}$ Mpc) for a typical rich cluster. A minimum mass of 50 particles (corresponding to a total mass of $5.5 \times 10^{13}h^{-4}$ $M_\odot$) was imposed for a system to be considered comparable to an Abell cluster. Figure 2 shows particles residing in rich clusters identified at different timesteps in the simulation. The number of clusters having total masses $M_{cl} \geq 5.5 \times 10^{13}h^{-4}$ $M_\odot$ is listed in Table 1 for each bias factor, along with the average mass of the 50 richest clusters. Assuming $h \approx 0.5$, the masses of the simulated clusters identified using the above procedure are quite comparable to the masses of Abell clusters derived from observations, $\sim 10^{15}$ $M_\odot$. It is also clear from Table 1 that there is a strong evolution of the abundance of rich clusters. The observed number density of richness class $R \leq 1$ clusters in $n \approx 6 \times 10^{-6}h^3$ Mpc^{-3} (Hoessel, Gunn, & Thuan 1980), although when al-

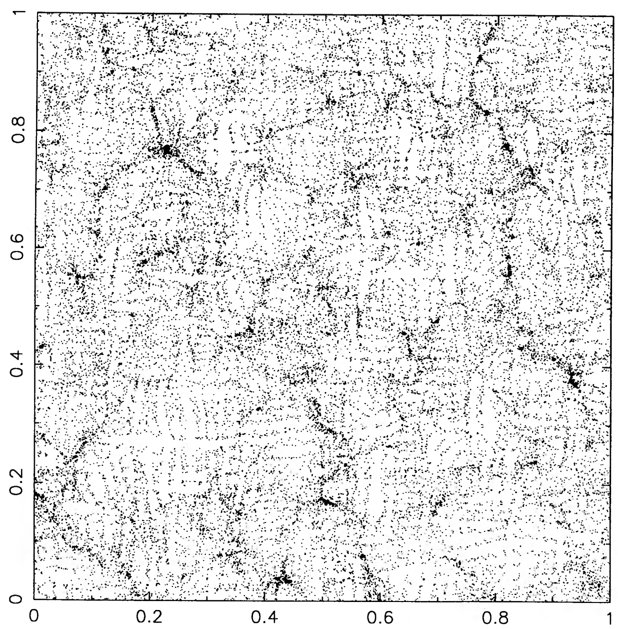

FIG. 1a

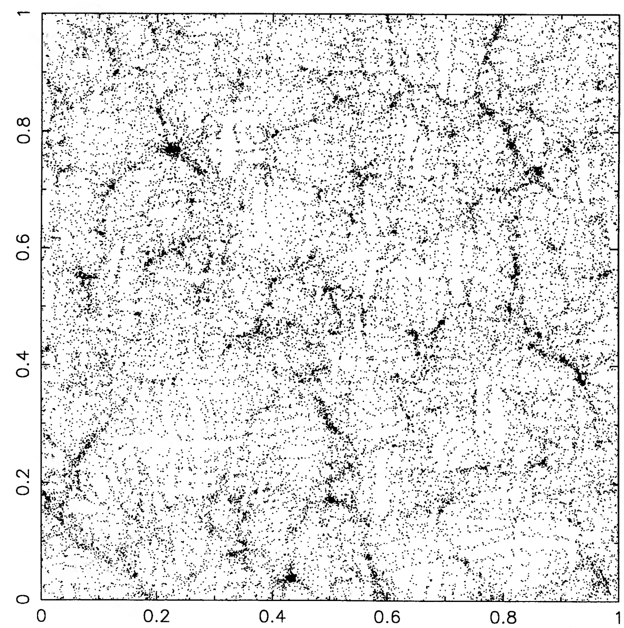

FIG. 1b

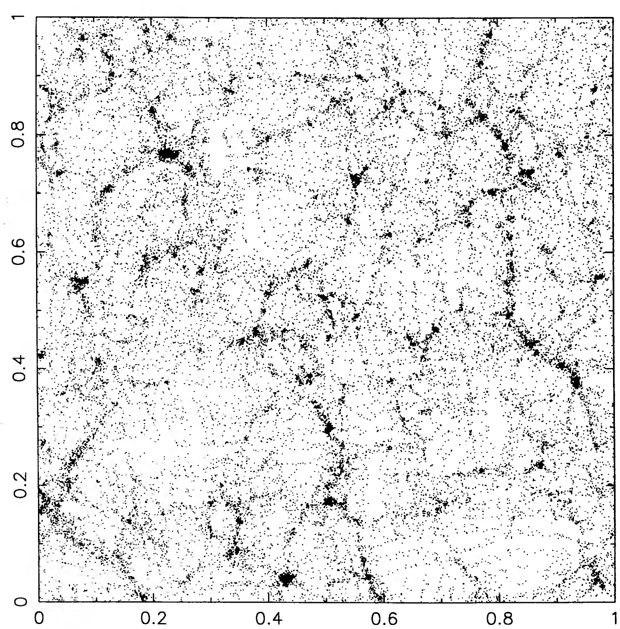

FIG. 1c

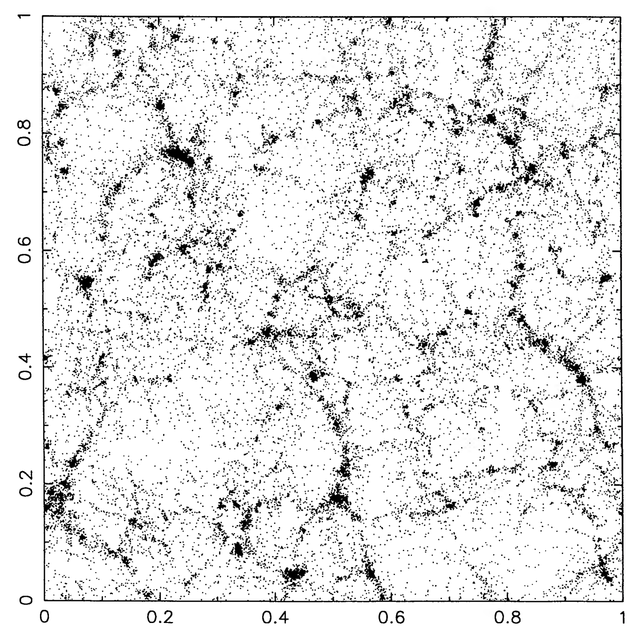

FIG. 1d

FIG. 1.—Representative slices through the simulated volume, for different bias factors, where (*a*) through (*d*) denotes $b = 2.5$, 2.0, 1.5, and 1.0, respectively. Each box is $100h^{-2}$ Mpc on a side and $12.5h^{-2}$ Mpc thick.

lowance is made for galactic obscuration a more likely value is $n \approx 10^{-5}h^3$ Mpc^{-3} (Bahcall 1988). Inclusion of the nonstatistical sample of Abell (1958) $R = 0$ clusters would further increase the total number density of rich clusters by at least a factor of 2. Thus one would expect to find anywhere from ~ 50 to 150 Abell clusters in a region comparable in size to the simulated volume. As can be seen from Table 1, best agreement with the observed number density occurs for a bias factor $b \approx 2.0$–2.5. Lower values of b lead to an overproduction of rich clusters, a result which confirms claims by White et al. (1987), Peebles, Daly, & Juszkiewicz (1989), and Evrard (1990). All of these numbers are, of course, somewhat uncertain, since neither the masses of individual clusters nor the true abundance of galaxy clusters are known very well and the identification of galaxies and clusters in the simulations is not identical to their observational definition. As will be shown, however, the results presented in the following sections are quite insensitive to these uncertainties.

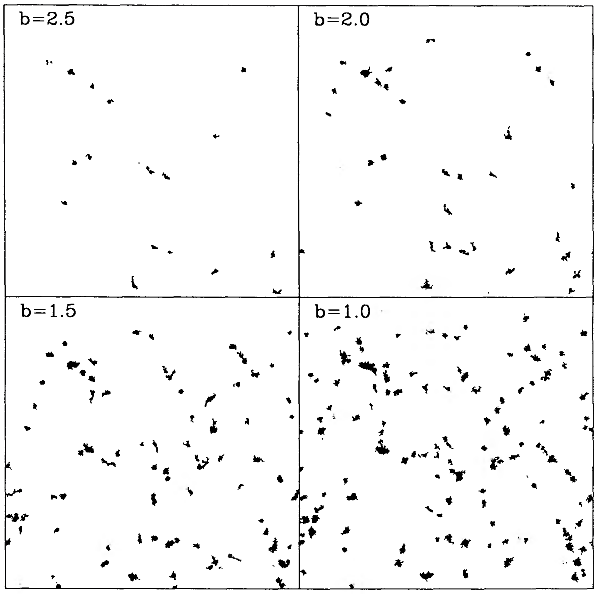

FIG. 2.—Representative clusters identified at different stages in the simulation using the cluster-finding algorithm described in the text. Each box is $100h^{-2}$ Mpc on a side, and $25h^{-2}$ Mpc in depth.

TABLE 1

CLUSTER ABUNDANCES

b	N_{cl}	$\langle M_{cl}\rangle(h^{-4}\ M_{\odot})$
2.5	59	$8.1 \pm 2.9 \times 10^{13}$
2.0	134	$1.3 \pm 0.5 \times 10^{14}$
1.5	328	$2.5 \pm 1.0 \times 10^{14}$
1.0	515	$4.2 \pm 1.7 \times 10^{14}$

3. CLUSTER-CLUSTER ALIGNMENTS

Having identified systems corresponding to rich Abell clusters in the simulated volume, the tendency for neighboring clusters to be aligned was then examined. For each cluster, its principal axes were determined from the moments of inertia of the distribution of all particles which comprise it. The relative orientations of neighboring clusters were then compared by measuring the acute angle, θ, between the major axis of a given cluster and the vector connecting its center of mass to that of another neighboring cluster. This was done both in three dimensions and in two dimensions using the projected particle distribution, as described below.

3.1. *Cluster Alignments in Three Dimensions*

Results are shown in Figure 3, where θ is the angle between the major axis of a given cluster and the direction toward another neighboring cluster, and D is their spatial separation in h^{-2} Mpc. All pairs of clusters have been included. A value of $\cos\theta = 1$ corresponds to perfect alignment in three dimensions, while if clusters are randomly oriented with respect to one another then distribution of $\cos\theta$ values will be uniform between 0 and 1. A strong tendency of clusters to be aligned over separations up to $\sim 15h^{-2}$ Mpc is clearly seen in Figure 3, for all bias factors. To assess the statistical significance of these results, the probabilities that these distributions could have been drawn from a parent population of randomly oriented clusters have been calculated using the Kolmogorov-Smirnov (hereafter K-S), χ^2, and binomial tests. Table 2 lists for different ranges of cluster separation the mean value of $\cos\theta$ and the standard error of this mean, the probability according to the K-S test that the distribution of $\cos\theta$ is consistent with a uniform distribution, the value of χ^2 under this same null hypothesis, the probability of obtaining such a value of χ^2 if the distribution of $\cos\theta$ were truly uniform, the number values of $\cos\theta$ less than the expected mean value of 0.5 and the number

TABLE 2

CLUSTER ALIGNMENTS IN THREE DIMENSIONS

$D(h^{-2}$ Mpc)	$\langle\cos\theta\rangle$	P_{KS}	χ^2	P_{χ^2}	$N_{<0.5}/N_{>0.5}$	P_{bin}
			$b = 2.5$			
$D \le 15$	0.57 ± 0.04	0.04	12.4[a]	0.19	34/42	0.41
$15 \le D \le 30$	0.53 ± 0.02	0.14	13.7	0.13	154/176	0.25
$30 \le D \le 50$	0.50 ± 0.01	0.90	9.2	0.42	703/691	0.77
			$b = 2.0$			
$D \le 15$	0.58 ± 0.02	10^{-7}	43.0	10^{-6}	114/204	$<10^{-5}$
$15 \le D \le 30$	0.50 ± 0.003	0.40	14.3	0.11	849/877	0.52
$30 \le D \le 50$	0.50 ± 0.01	0.74	10.6	0.31	3623/3727	0.23
			$b = 1.5$			
$D \le 15$	0.55 ± 0.01	10^{-13}	78.7	10^{-13}	801/1025	$<10^{-5}$
$15 \le D \le 30$	0.51 ± 0.003	0.0001	27.2	0.001	4998/5376	0.0002
$30 \le D \le 50$	0.50 ± 0.001	0.11	13.2	0.16	21984/22072	0.56
			$b = 1.0$			
$D \le 15$	0.53 ± 0.005	10^{-8}	61.1	10^{-9}	1962/2266	$<10^{-5}$
$15 \le D \le 30$	0.51 ± 0.002	0.01	14.6	0.10	12585/13019	0.007
$30 \le D \le 50$	0.50 ± 0.003	0.71	7.4	0.60	54155/54289	0.66

[a] For 9 degrees of freedom.

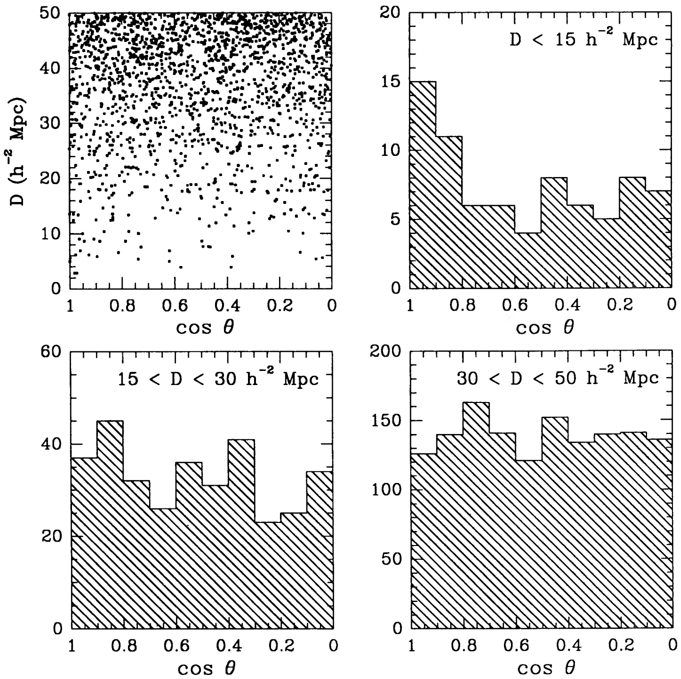

FIG. 3*a*

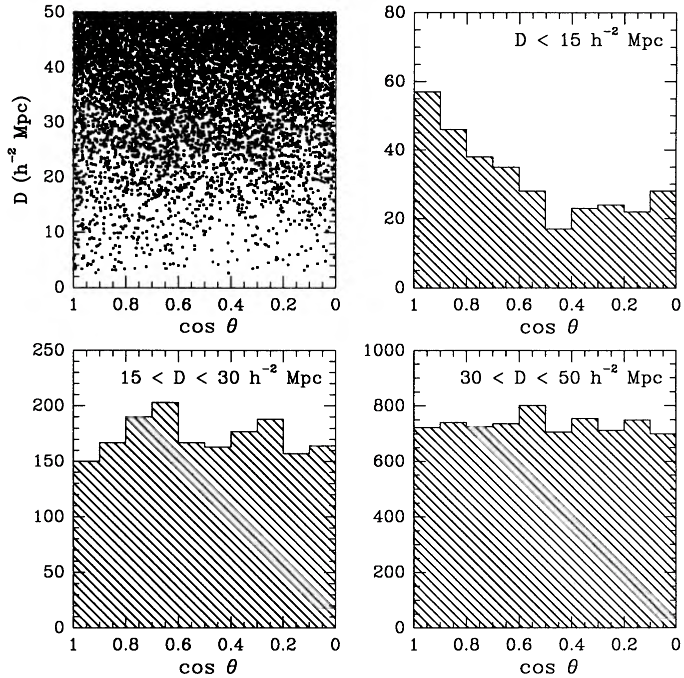

FIG. 3*b*

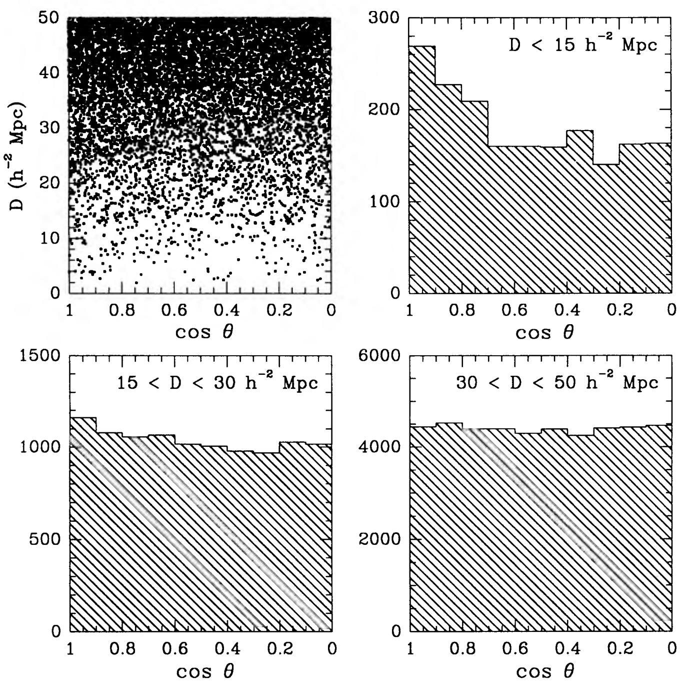

FIG. 3*c*

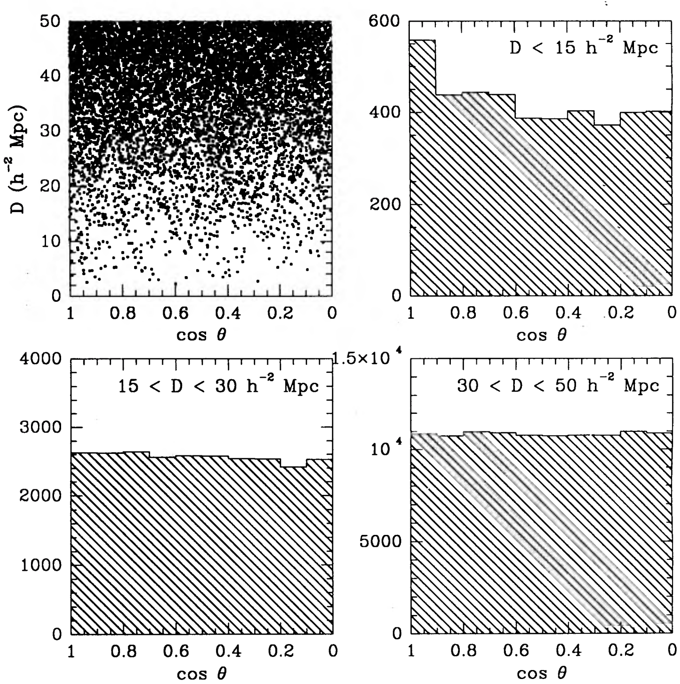

FIG. 3*d*

FIG. 3.—Alignments of clusters in three dimensions in the CDM model, where (*a*) through (*d*) corresponds to bias factors $b = 2.5$, 2.0, 1.5, and 1.0. θ is the angle between the major axis of a cluster and the vector between its center of mass and that of another neighboring cluster at a distance D away. All clusters identified by the cluster-finding algorithm at each bias factor have been used. In the upper-left panel, all points are plotted for bias factors $b = 2.5$ and 2.0, while for clarity only 20% of all points are plotted for $b = 1.5$ and 10% for $b = 1.0$. Histograms of the cos θ values are shown in the remaining panels for different ranges of cluster separations. If clusters are randomly oriented, the distribution of cos θ values should be uniform between 0 and 1, while an excess of values greater than 0.5 would indicate the presence of alignments.

TABLE 3

ALIGNMENTS OF 100 RICHEST CLUSTERS

$D(h^{-2}$ Mpc)	$\langle\cos\theta\rangle$	$P_{\rm KS}$	χ^2	P_{χ^2}	$N_{<0.5}/N_{>0.5}$	$P_{\rm bin}$
			$b = 2.5$			
$D \le 15$	0.57 ± 0.02	0.001	27.6[a]	0.001	80/106	0.07
$15 \le D \le 30$	0.52 ± 0.01	0.06	19.9	0.02	470/514	0.17
$30 \le D \le 50$	0.50 ± 0.005	0.24	10.5	0.31	2032/1996	0.58
			$b = 2.0$			
$D \le 15$	0.58 ± 0.02	0.0001	24.2	0.004	69/123	0.0001
$15 \le D \le 30$	0.51 ± 0.01	0.43	10.6	0.31	456/496	0.21
$30 \le D \le 50$	0.50 ± 0.004	0.23	10.7	0.30	2034/2116	0.21
			$b = 1.5$			
$D \le 15$	0.60 ± 0.03	0.0004	41.9	10^{-6}	54/94	0.001
$15 \le D \le 30$	0.51 ± 0.01	0.67	4.7	0.86	464/498	0.29
$30 \le D \le 50$	0.49 ± 0.005	0.36	10.7	0.30	2067/1973	0.14
			$b = 1.0$			
$D \le 15$	0.55 ± 0.02	0.04	21.0	0.01	83/113	0.04
$15 \le D \le 30$	0.52 ± 0.01	0.19	6.4	0.70	468/506	0.23
$30 \le D \le 50$	0.50 ± 0.005	0.70	9.2	0.42	2112/2078	0.61

[a] For 9 degrees of freedom.

greater than 0.5, and the probability according to the binomial test of these two values occurring for a uniform distribution (probabilities quoted for the binomial test throughout this paper are two-tailed probabilities). These results confirm the visual impression from Figure 3 that alignments of cluster major axes are present at a statistically significant level over scales of $\sim 15h^{-2}$ Mpc.

Rather than identifying as rich clusters all groups whose masses are greater than some (rather arbitrary) value, an alternative method is to instead match the observed number density of Abell clusters, without imposing an a priori lower mass cutoff. To see what effect this might have on the above results, the same procedure was repeated using only the 100 richest clusters identified by the cluster-finding algorithm. This also permits a fairer statistical comparison of cluster alignments for different bias factors by equalizing the number of clusters in each sample. These results are summarized in Table 3. As before, a statistically significant tendency for cluster alignments is found in all cases for separations up to $15h^{-2}$ Mpc.

Thus it is clear from this analysis that alignments of clusters do occur in the CDM model over scales of $\sim 15h^{-2}$ Mpc when all cluster pairs are considered. It should be emphasized, however, that these values do not necessarily represent either the maximum or characteristic size superclusters and filamentary features in the CDM–dominated universe. This is because as one considers cluster pairs with greater and greater separations, any signal of alignments generated by pairs belonging to the same supercluster or filamentary feature is inevitably diluted by the inclusion of many physically unrelated clusters which also happen to be separated by the same distance. One way around this problem is to first identify individual superclusters (using some objective criteria) and then consider alignments among only those clusters which are members of the same parent supercluster. This will be addressed in more detail below.

3.2. *Projected Alignments*

In order to better compare the N-body results with existing observational data, the tendency for neighboring clusters to appear aligned when viewed in only two dimensions was also examined. Cluster orientations were determined in this case using the projected particle distribution on the plane of the sky. Results for the 100 richest clusters identified at each bias factor are shown in Figure 4 and summarized in Table 4, where θ is now the angle between the projected position angle of a cluster's major axis and the projection of the line connecting its center to that of another neighboring cluster. As before, D is the true three-dimensional separation between each pair of clusters. Three orthogonal views of the simulated volume were used, and the results combined. In the absence of any correlation between the orientations of neighboring clusters the distribution of values of θ in this case should be uniform 0° and 90°, with a mean value of 45°. As can be seen from Figure 4 and Table 4, the results in two dimensions are quite similar to those found in three dimensions (compare Tables 3 and 4), with clear evidence of cluster alignments extending over $\sim 15h^{-2}$ Mpc. Thus alignments which are present in three dimensions in the CDM model remains clearly detectable in two dimensions as well, in spite of the inevitable loss of some information when using only the projected rather than true spatial distribution of galaxies.

The results presented here are consistent with the earlier results by Dekel (1985), who applied the analysis of Dekel, West, & Aarseth (1984) to the CDM N-body simulations of Davis et al. (1985) and found that significant alignments occur for separations up to $\sim 10h^{-2}$ Mpc. Most of the alignments signal in the present simulations indeed comes from separations $\le 10h^{-2}$ Mpc (see, for example, the top left-hand panels of Fig. 3). The present simulation is superior to the earlier one because there are *many* more simulated clusters (the simulated volume used here is a factor of 27 greater than that of the Davis

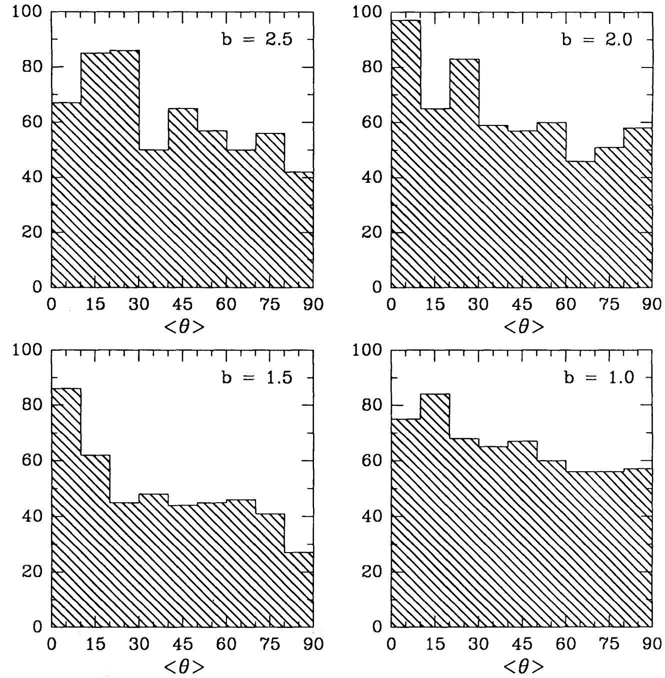

FIG. 4a

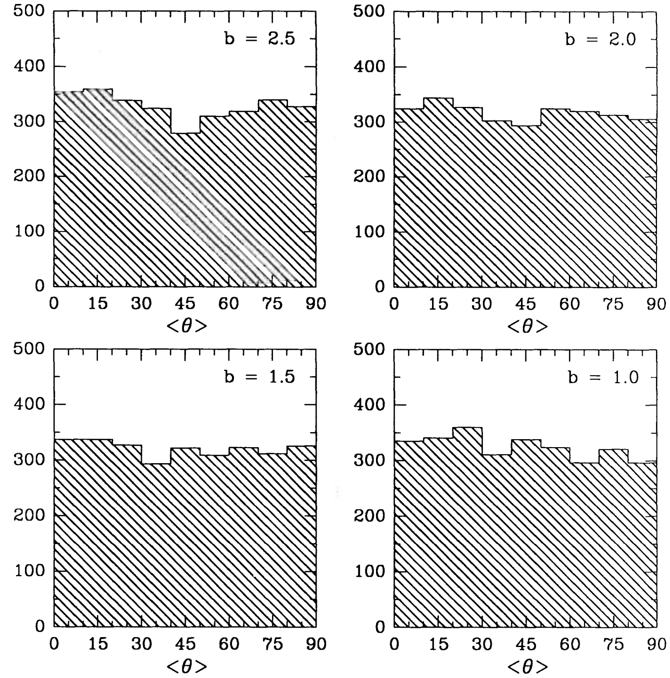

FIG. 4b

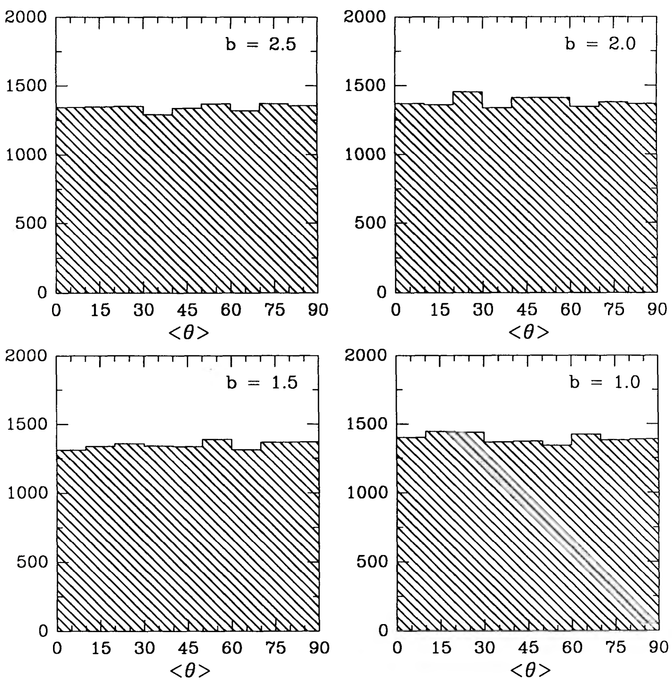

FIG. 4c

FIG. 4.—Projected alignments of the 100 richest clusters at each bias factor, for cluster separations (*a*) $D < 15h^{-2}$ Mpc, (*b*) $15 < D < 30h^{-2}$ Mpc, and (*c*) $30 < D < 50h^{-2}$ Mpc. Here θ is the angle between the orientation of the major axis of a cluster projected onto the plane of the sky and the projected direction toward another neighboring cluster. If no alignments are present, the distribution of θ values should be uniform between 0° and 90°, with a mean value $\langle\theta\rangle = 45°$.

et al. simulations), and consequently a much higher level of statistical significance can be achieved, which allows a signal of cluster alignments to be detected over larger ranges of cluster separation, while such a signal is lost in noise when the number of clusters is small.

3.3. *Projected Alignments of Clusters in Superclusters*

Following West (1989b), one can also examine cluster alignments by restricting the sample to only those clusters which reside in well-defined superclusters. As discussed earlier, this has the advantage of enhancing any signal of alignments which might be present by distilling genuine supercluster members from the general cluster populace, and consequently may provide a truer measure of the strength and scale of cluster alignments in relation to the characteristic size of superclusters and filamentary features. The same procedure used to identify superclusters in West's (1989b) study was used here by linking together the 100 richest clusters in the simulated volume using the same group-finding algorithm described in the previous section, with a value of $25h^{-1}$ Mpc chosen for the linking length (assuming $h = 0.5$). This was done independently for each time step. In this way, 25, 16, 21, and 19 superclusters containing two or more members were identified for $b = 2.5$, 2.0, 1.5, and 1.0, respectively (note that since cluster masses continue to grow with time, the 100 richest clusters at one particular time in the simulation may not necessarily be the same at another). These data are summarized in Table 5, where N_{sc} is the number of distinct superclusters identified for a given bias factor, f is the fraction of the cluster population residing within these systems, n_{max} is the number of clusters which comprise the largest supercluster, and l_{max} is the size of the longest supercluster (not necessarily the one with the most members), measured as the greatest distance between any two member clusters. For comparison, West's study identified 48 superclusters in a large sample of $R \geq 0$ Abell clusters having redshifts $z \leq 0.1$, with 65% of the clusters residing in these superclusters, the largest of which contains 13 member clusters and extends $\sim 80h^{-1}$ Mpc in length. The apparently good agreement between the properties of superclusters in the simulated volume and observed supercluster morphologies is encouraging; however, it is important to emphasize that the properties of superclusters identified in this manner are sensi-

TABLE 4

CLUSTER ALIGNMENTS IN PROJECTION

$D(h^{-2}$ Mpc)	$\langle\theta\rangle$	P_{KS}	χ^2	P_{χ^2}	$N_{<45°}/N_{>45°}$	P_{bin}
			$b = 2.5$			
$D \leq 15$	40°.4 ± 1.1	0.0001	30.4[a]	0.0002	330/228	10^{-5}
$15 \leq D \leq 30$	44°.4 ± 0.5	0.03	14.4	0.07	1510/1442	0.22
$30 \leq D \leq 50$	45°.1 ± 0.2	0.78	3.7	0.88	5987/6097	0.32
			$b = 2.0$			
$D \leq 15$	40°.2 ± 1.1	10^{-5}	32.3	0.0001	329/247	0.0006
$15 \leq D \leq 30$	44°.4 ± 0.5	0.28	5.7	0.68	1434/1422	0.84
$30 \leq D \leq 50$	44°.9 ± 0.2	0.82	7.8	0.46	6229/6221	0.95
			$b = 1.5$			
$D \leq 15$	38°.3 ± 1.2	10^{-5}	43.6	10^{-6}	258/186	0.0006
$15 \leq D \leq 30$	44°.6 ± 0.5	0.50	5.0	0.76	1456/1430	0.64
$30 \leq D \leq 50$	45°.2 ± 0.2	0.77	4.2	0.84	6034/6086	0.65
			$b = 1.0$			
$D \leq 15$	41°.8 ± 1.1	0.006	11.1	0.20	334/254	0.001
$15 \leq D \leq 30$	43°.8 ± 0.5	0.03	11.2	0.19	1537/1385	0.005
$30 \leq D \leq 50$	44°.8 ± 0.2	0.25	6.7	0.57	6346/6224	0.28

[a] For 8 degrees of freedom.

TABLE 5

SUPERCLUSTER PROPERTIES

b	N_{sc}	f	n_{max}	$l_{max}(h^{-2}$ Mpc)
2.5..........	25	0.76	9	37
2.0..........	16	0.60	11	54
1.5..........	21	0.59	8	38
1.0..........	19	0.62	11	31

tive to the true number density of rich clusters, and thus are somewhat uncertain. For instance, if all 134 clusters identified for $b = 2.0$ are used instead of restricting the sample to only the 100 richest, then in this case 24 superclusters are found, with the largest of these containing 14 clusters and the longest supercluster extending $\sim 70h^{-2}$ Mpc in length. The properties of superclusters will also depend to some extent on the choice of linking length used to identify them.

Projected alignments among these clusters were searched for

TABLE 6

PROJECTED ALIGNMENTS OF CLUSTERS IN SUPERCLUSTERS

$D(h^{-2}$ Mpc)	$\langle\theta\rangle$	P_{KS}	χ^2	P_{χ^2}	$N_{<45°}/N_{>45°}$	P_{bin}
			$b = 2.5$			
All D	40°.1 ± 1.0	10^{-6}	44.6[a]	10^{-7}	411/279	$<10^{-5}$
$D \leq 15$	39°.6 ± 1.2	0.0001	31.1	0.0001	255/165	$<10^{-5}$
$15 \leq D \leq 30$	40°.7 ± 1.7	0.008	18.1	0.02	129/94	0.03
$30 \leq D \leq 50$	42°.6 ± 3.3	0.30	12.0	0.15	28/20	0.31
			$b = 2.0$			
All D	38°.7 ± 0.9	10^{-12}	64.6	10^{-10}	521/337	$<10^{-5}$
$D \leq 15$	39°.3 ± 1.2	10^{-5}	34.7	10^{-5}	290/202	10^{-5}
$15 \leq D \leq 30$	36°.2 ± 1.6	10^{-6}	37.2	10^{-5}	158/88	$<10^{-5}$
$30 \leq D \leq 50$	38°.6 ± 2.6	0.01	14.3	0.07	67/35	0.002
			$b = 1.5$			
All D	38°.4 ± 1.2	10^{-5}	39.8	10^{-6}	253/179	0.0005
$D \leq 15$	38°.4 ± 1.4	0.0002	31.0	0.0001	195/141	0.004
$15 \leq D \leq 30$	39°.3 ± 2.9	0.07	13.1	0.11	50/34	0.10
$30 \leq D \leq 50$	33°.6 ± 7.5	0.49	4.5	0.81	8/4	0.38
			$b = 1.0$			
All D	41°.7 ± 1.0	10^{-5}	30.2	0.0002	437/337	0.0003
$D \leq 15$	41°.3 ± 1.2	0.008	11.5	0.17	264/198	0.003
$15 \leq D \leq 30$	42°.6 ± 1.6	0.0008	33.5	10^{-5}	165/135	0.10
$30 \leq D \leq 50$	35°.4 ± 7.5	0.40	4.5	0.81	8/4	0.38

[a] For 8 degrees of freedom.

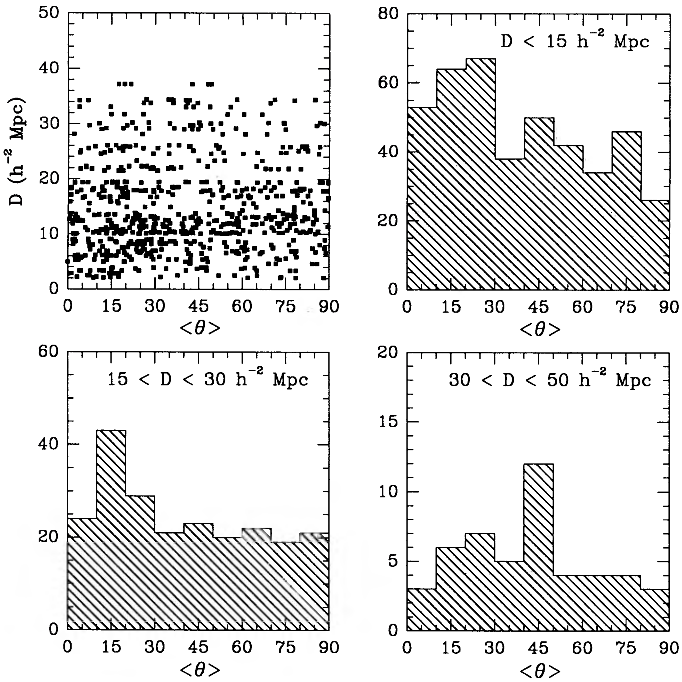

FIG. 5a

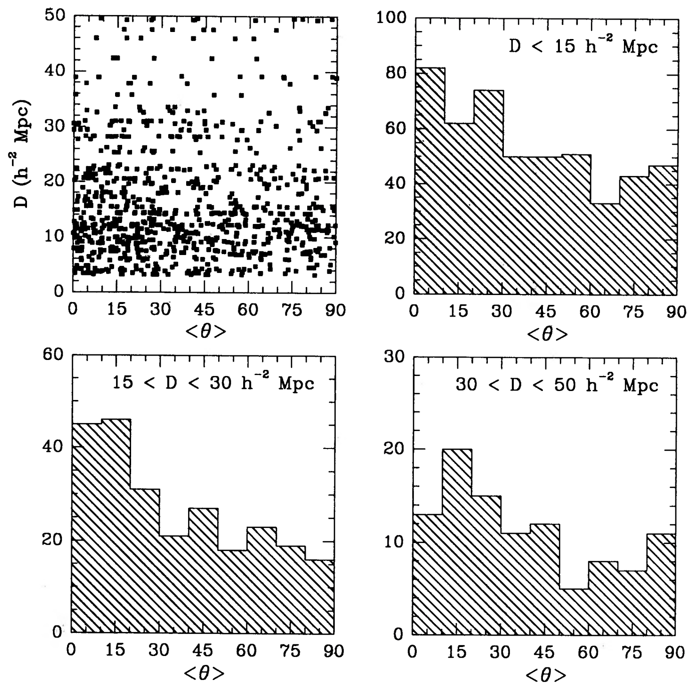

FIG. 5b

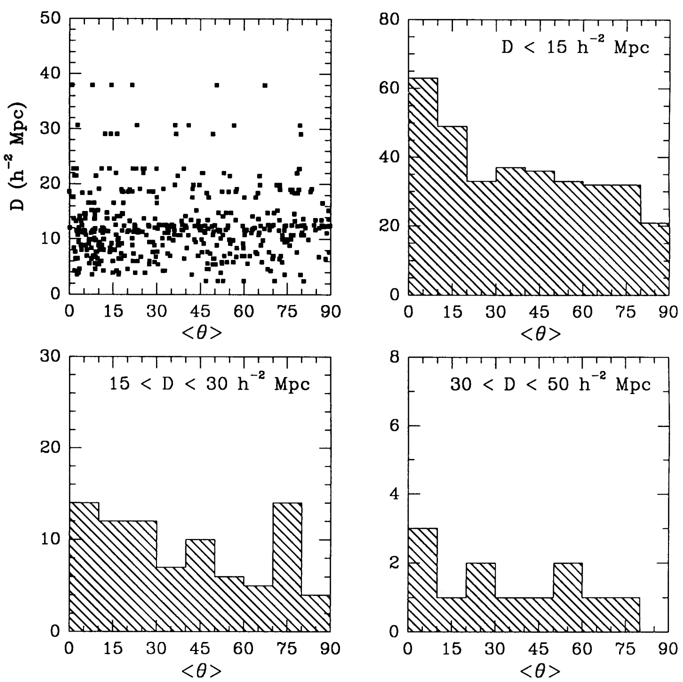

FIG. 5c

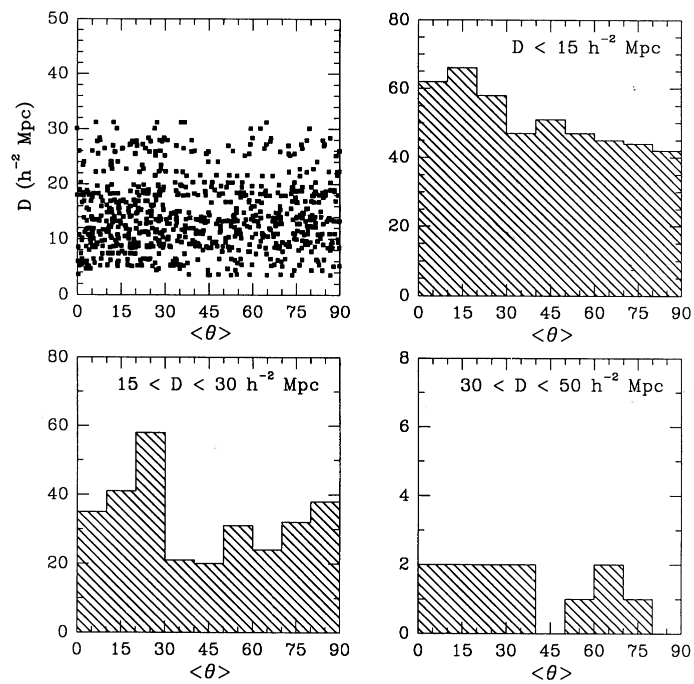

FIG. 5d

FIG. 5.—Projected alignments of those clusters which reside in superclusters, as described in the text. As usual, (*a*) through (*d*) corresponds to bias factors $b = 2.5$, 2.0, 1.5, and 1.0.

by comparing the orientation of the major axis of a given cluster with the distribution of all other clusters residing within the same supercluster. As before, three orthogonal projections of the simulated cube were used. These results are shown in Figure 5 and summarized in Table 6. A strong signal of alignments is now clearly in evidence for all separations up to $\sim 30h^{-2}$ Mpc, and in fact the alignments are much more statistically significant than those in Table 4, despite the fact that the number of cluster pairs has been greatly reduced. Thus it is clear that by restricting the analysis to only those clusters which reside in objectively defined superclusters, one finds a stronger signal of alignments over much greater scales than when all cluster pairs are included. Significant alignments are also found for cluster separations in the range $30 < D < 50h^{-2}$ Mpc for a bias factor $b = 2.0$. The fact that no statistically significant alignments beyond $30h^{-2}$ Mpc are found for other bias factors is probably due to the fact that, by chance, few of the supercluster members have separations as great as those of the largest $b = 2.0$ system.

For comparison with observations, Table 7 lists results from West's (1989b) study of cluster alignments in superclusters, which found evidence of alignments up to scales of at least $30h^{-1}$ Mpc (and perhaps up to $\sim 60h^{-1}$ Mpc). For all the statistics used, and for any reasonable value of the Hubble constant, the alignments predicted by the CDM model can successfully explain the tendency for alignment among Abell clusters in superclusters. If $h \approx 0.5$, the model in fact predicts somewhat stronger alignments than are observed.

On a related note, Schombert & West (1990) recently claimed to have detected a morphology-density relation for clusters of galaxies which is in essence a correlation between the shape of a cluster and the type of supercluster environment in which it resides. Specifically, they found that highly flattened clusters are most prevalent in regions where the number of neighboring clusters is quite high, while rounder or more irregular clusters are usually found in low-density environments. Attempts to reproduce a similar effect in the N-body simulations used here (in both three dimensions and two) failed to show any apparent correlation between cluster shape (as measured by the ratio of principal axis eigenvalues) and the local large-scale structure. However, this does not necessarily mean that the CDM model is inconsistent with these observations, since (1) cluster shapes determined from principal axes analysis are quite uncertain (see, e.g., West 1989b); (2) observed cluster morphological classifications are based on the distribution of the brightest galaxies, and their distribution may not necessarily be representative of the overall dark matter distribution (which is what is modeled in the simulations); or (3) there is always the possibility that the observed cluster morphology-density relation is not real, but rather is due to systematic errors or selection effects in the observational data, since it is based on a rather small and heterogeneous sample of clusters. This would be an interesting area for future work.

In summary, the results presented here indicate that statistically significant alignments of the major axes of neighboring clusters of galaxies are expected to occur in a universe dominated by CDM for separations up to ~ 10–$15h^{-2}$ Mpc when all cluster pairs are considered, and up to $\sim 30h^{-2}$ Mpc when only supercluster members are used. One should bear in mind, however, that it will remain a difficult observational task to detect such large-scale alignments. For instance, an important difference between the simulations described here and the real world is that the simulated clusters were identified using a cluster-finding algorithm in three dimensions, which eliminates contamination by noncluster members seen in projection, thus ensuring, among other things, that the position angles of these simulated clusters are known somewhat more precisely than those of Abell clusters. Given the level of the alignment effect found here, other sources of observational error (e.g., erroneous cluster redshifts, contamination of the Abell catalog by false clusters) might also obscure any weak tendency for alignments which might be present on large scales.

3.4. *Filaments versus Sheets*

Recent results from the CfA redshift survey by Geller & Huchra (1989), as well as the earlier study of Pisces-Perseus supercluster by Haynes & Giovanelli (1986), have suggested that the local large-scale galaxy distribution may be characterized by the presence of sheets as well as filaments. Visual comparisons between N-body simulations and these observations have generated claims that structures similar to the "Great Wall" may arise naturally in a CDM–dominated universe (e.g., White et al. 1987; Park 1990; Weinberg & Gunn 1990). However, as was emphasized earlier, such comparisons are clearly very subjective, and thus cannot really provide an exacting test for judging the success or failure of a particular cosmogonic model. Cluster alignments, on the other hand, may provide a more quantitative (and less subjective) measure of the tendency to form sheetlike structures in the CDM model. If clusters are distributed along quasi–two-dimensional surfaces such as sheet (walls), then even if the orientations of their major axes were random within these sheets, their minor axes should nevertheless exhibit some tendency to be oriented parallel to one another. If, on the other hand, only one-dimensional filaments are formed, then cluster major axes would exhibit a tendency to be aligned, while their minor axes would be randomly oriented relative to one another. To see if there is indeed such evidence of sheetlike structures in the simulations, the relative orientations of cluster minor axes were compared. These results are shown in Figure 6 for a bias factor $b = 2.0$, where ϕ is the relative angle between the minor axes of a given pair of clusters and D is their separation in h^{-2} Mpc. Clearly there is no evidence of coherent sheetlike structures occurring over any scales. Other bias factors also produce similar results. If, on the other hand, one compares the orientation of a cluster's minor axis relative to the vector joining that cluster to another neighbor (Fig. 7), then there is significant evidence of the sort of antialignments expected to be characteristic of filaments. Thus these results provide some evidence that filamentary superclusters of clusters arise more naturally in the CDM cosmogony than do walls.

4. SUMMARY

The relative orientations of neighboring clusters of galaxies in a universe dominated by CDM have been examined using N-body simulations. Alignment tests can provide an objective comparison of the CDM model with observations of the large-scale structure. A clear tendency has been found here for the

TABLE 7

OBSERVED ALIGNMENTS OF CLUSTERS IN SUPERCLUSTERS

$D(h^{-1}$ Mpc)	$\langle\theta\rangle$	P_{KS}	χ^2	P_{χ^2}	$N_{<45°}/N_{>45°}$	P_{bin}
All	40°0 ± 1.2	0.0004	37.5[a]	0.003	283/210	0.001
$D \leq 30$	39°0 ± 1.6	0.0006	37.2	0.003	166/109	0.0008
$30 \leq D \leq 60$	40°2 ± 1.9	0.04	19.1	0.32	96/79	0.20
$60 \leq D \leq 90$	45°3 ± 3.9	0.89	5.8[b]	0.67	21/22	1.0

[a] For 17 degrees of freedom.

[b] For 8 degrees of freedom.

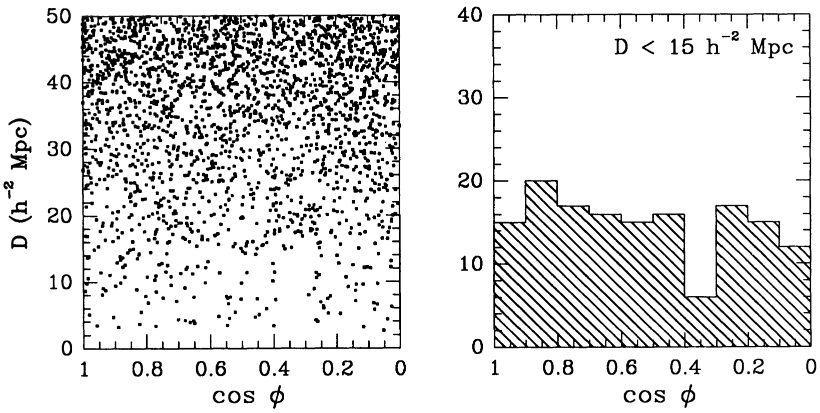

FIG. 6.—Orientations in three dimensions of the minor axes of clusters. Here ϕ is the relative angle between the minor axes of a given pair of clusters separated by distance D. Results are shown only for a bias factor $b = 2.0$, since other bias factors give very similar results.

major axes of neighboring clusters to be aligned for separations up to $\sim$10–15h^{-2} Mpc when all cluster pairs are included. When only those clusters which reside within superclusters are considered, alignments are found over even larger scales, up to $\sim 30h^{-2}$ Mpc. For an assumed value of $h = 0.5$, these results agree quite well with the observational results of West (1989b), who found a significant alignment effect among Abell clusters in superclusters over scales up to $\sim 60h^{-1}$ Mpc. The orientations of cluster minor axes also provide supporting evidence of the presence of filamentary (rather than sheetlike) features in the large-scale mass distribution. Other properties of superclusters, such as their sizes and the fraction of Abell clusters which they contain, are also found to be consistent with observations. On the basis of the present results, one would have to conclude that a CDM-dominated universe can quite successfully account for many of the observed features of the large-scale structure. A bias factor $b \geq 2.0$ is favored by these simulations, since less biasing leads to an overproduction of rich clusters, as many as 2–5 times the observed number density. However, it is important to bear in mind that CDM is not unique in its ability to produce large-scale filamentary features and cluster alignments, since other models based on hot dark matter, initial power spectra with shallow slopes, and explosion scenarios (Dekel, West, & Aarseth 1984; West, Dekel, & Oemler 1989; West, Weinberg, & Dekel 1990) all have more power on large scales and are therefore expected to produce even stronger alignments on large scales. In this sense, then, cluster alignments appear to provide only a *minimal* test which any successful model for the origin of the large-scale structure must meet, rather than a sensitive discriminant between models as was originally hoped.

Ideally, one would like to use the distribution of rich clusters themselves as delineators of the large. scale structure and any any filamentary pattern that it may possess. Unfortunately, robust statistical tests for filamentary structure have proven elusive or only marginally successful (e.g., Kuhn & Uson 1982; Moody, Turner, & Gott 1983; Vishniac 1986; Fry 1986; Bhavsar & Ling 1988). This is because of the complex nature of the problem—it is extremely difficult to develop tests which are capable of disentangling filamentary features in particular clustering in general. Furthermore, because of their relatively low space density, rich clusters are very sparse samplers of the large-scale clustering pattern and the effects of small-number statistics are severe. Thus it would seem that cluster alignments provide at present the best objective statistical evidence of the existence of coherent features in the large-scale galaxy distribution. Searches for alignments among clusters which have been discovered at high redshift (e.g., Green & Yee 1984; Gunn, Hoessel, & Oke 1986; Gunn & Dressler 1988) might also allow one to place some interesting observational constraints on the epoch of supercluster formation.

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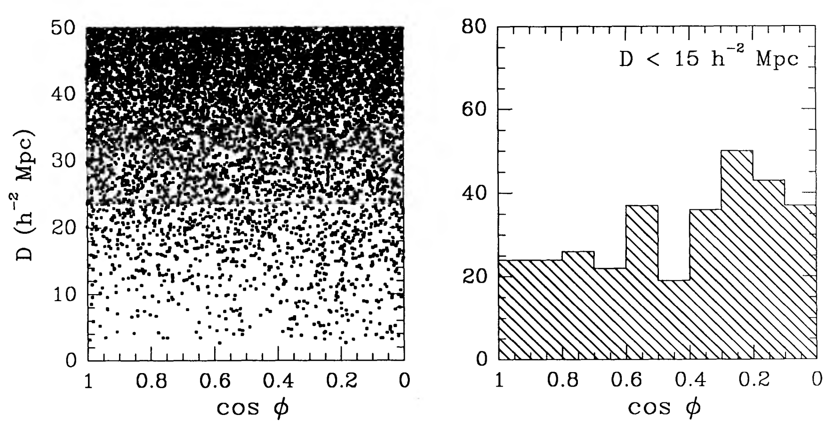

FIG. 7.—Same as Fig. 6, but where ϕ is now the angle between the minor axis of a cluster and the vector between its center of mass and that of another cluster at distance D away.

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REFERENCES

Aaronson, M., et al. 1989, ApJ, 338, 654
Abell, G. O. 1958, ApJS, 3, 211
Argyres, P. C., Groth, E. J., Peebles, P. J. E., & Struble, M. F. 1986, AJ, 91, 471
Bahcall, N. A. 1988, ARAA, 26, 631
Bahcall, N. A., & Soneira, R. M. 1983, ApJ, 270, 20
Bardeen, J. M., Bond, J. R., & Efstathiou, G. 1987, ApJ, 321, 28
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Batuski, D. J., & Burns, J. O. 1985, ApJ, 299, 5
Batuski, D. J., Melott, A. L., & Burns, J. O. 1987, 322, 48
Bertschinger, E., & Juszkiewicz, R. 1988, ApJ, 334, L59
Bhavsar, S. P., & Ling, E. N. 1988, ApJ, 331, L63
Binggeli, B. 1982, A&A, 107, 338
Blumenthal, G. R., Faber, S. M., Primack, J. R., & Rees, M. J. 1984, Nature, 311, 517
Blumenthal, G. R., Dekel, A., & Primack, J. R. 1988, ApJ, 326, 539
Blumenthal, G. R., & Primack, J. R. 1983, in Fourth Workshop on Grand Unification, ed. H. A. Weldon, P. Langacker, & P. J. Steinhardt (Boston: Birkhauser), 256
Bond, J. R. 1987, in Nearly Normal Galaxies, ed. S. Faber (NY: Springer) 388
Bond, J. R., & Efstathiou, G. 1984, ApJ, 285, L45
Bond, J. R., & Szalay, A. S. 1990, in preparation
Bothun, G. D., West, M. J., Mould, J. R., & Schommer, R. A. 1990, ApJ, 353, 344
Broadhurst, T. J., Ellis, R. S., Koo, D. C., & Szalay, A. S. 1990, Nature, 343, 726
Carlberg, R. G., & Couchman, H. M. P. 1989, ApJ, 340, 47
Carlberg, R. G., Couchman, H. M. P., & Thomas, P. A. 1990, ApJ, 352, L29
Collins, C. A., Joseph, R. D., & Robertson, N. A. 1986, Nature, 320, 506
da Costa, L. N., et al. 1988, ApJ, 327, 544
Davis, M., Efstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371
Davis, M., & Peebles, P. J. E. 1983, ApJ, 267, 465
Dekel, A. 1984, in the Eighth Johns Hopkins Workshop on Particle Theory, ed. G. Domokos & S. Koveski-Domokos (Singapore: World Publishing), 191
Dekel, A., Blumenthal, G. R., Primack, J. R., & Oliver, S. 1989, ApJ, 338, L5
Dekel, A., & Rees, M. J. 1987, Nature, 326, 455
Dekel, A., & Silk, J. 1986, ApJ, 303, 39
Dekel, A., West, M. J., & Aarseth, S. J. 1984, ApJ, 279, 1
de Lapparent, V., Geller, M. J., & Huchra, J. P. 1986, ApJ, 302, L1
Einasto, J., Joeveer, M., & Saar, E. 1980, MNRAS, 193, 353
Evrard, A. E. 1990, ApJ, 341, L71
Flin, P. 1987, MNRAS, 228, 941
Fong, R., Stevenson, P. R. F., & Shanks, T. 1990, MNRAS, 242, 146
Frenk, C. S., White, S. D. M., Davis, M., & Efstathiou, G. 1988, ApJ, 327, 507
Frenk, C. S., White, S. D. M., Efstathiou, G., & Davis, M. 1985, Nature, 317, 595
Fry, J. N. 1986, ApJ, 306, 358
Geller, M. J., & Huchra, J. P. 1989, Science, 246, 897
Gorski, K. M., Davis, M., Strauss, M. A., White, S. D. M., & Yahil, A. 1989, ApJ, 344, 1
Gott, J. R., Melott, A. L., & Dickinson, M. 1986, ApJ, 306, 341
Green, R. F., & Yee, H. K. C. 1984, ApJS, 54, 495
Groth, E., Juszkiewicz, R., & Ostriker, J. P. 1989, ApJ, 346, 558
Gunn, J. E., & Dressler, A., in Towards Understanding Galaxies at Large Redshift, ed. R. G. Kron & A. Renzini (Dordrecht: Kluwer), 227
Gunn, J. E., Hoessel, J. G., & Oke, J. B. 1986, ApJ, 306, 30
Gurbatov, S. N., Saichev, A. I., & Shandarin, S. F. 1989, MNRAS, 236, 385
Haynes, M. P., & Giovanelli, R. 1986, ApJ, 306, L55
Hoessel, J. G., Gunn, J. E., & Thuan, T. X. 1980, ApJ, 241, 486
Hoffman, Y. 1989, ApJ, 340, 69
Hoffman, Y., Shaham, J., & Shaviv, G. 1982, ApJ, 262, 413
Kaiser, N. 1984, ApJ, 284, L9
Kaiser, N., & Lahav, O. 1989, MNRAS, 237, 129
Kirshner, R. P., Oemler, A., Schechter, P. L., & Shectman, S. A. 1981, ApJ, 248, L57
Klypin, A. A., & Kopylov, A. I. 1983, Sov. Astr. Letters, 9, 41
Kuhn, J. R., and Uson, J. M. 1982, ApJ, 263, L47
Lambas, D. G., Groth, E. J., & Peebles, P. J. E. 1988, AJ, 95, 996
Lambas, D. S., Nicotra, M., Muriel, H., & Ruiz, L. 1990, AJ, in press
Lynden-Bell, D., Faber, S., Burstein, D., Dressler, A., Terlevich, R., & Wegner, G. 1988, ApJ, 326, 19
Maddox, S. J., Efstathiou, G., Sutherland, W. J., & Loveday, J. 1990, MNRAS, 242, 43P
Melott, A. L., & Shandarin, S. F. 1990, preprint
Messina, A., Moscardini, L., Lucchin, F., & Matarrese, S. 1990, preprint
Moody, J. E., Turner, E. L., & Gott, J. R. 1983, ApJ, 273, 16
Muriel, H., & Lambas, D. G. 1989, AJ, 98, 995
Nusser, A., & Dekel, A. 1990, ApJ, 362, 14
Oemler, A. 1987, in The Minnesota Lectures on Clusters of Galaxies and Large-Scale Structure, ed. J. M. Dickey (San Francisco: ASP), 19
Park, C. 1990, MNRAS, 242, 59P
Peebles, P. J. E. 1982, ApJ, 263, L1
———. 1984, ApJ, 277, 470
Peebles, P. J. E., Daly, R. A., & Juszkiewicz, R. 1989, ApJ, 347, 563
Postman, M., Spergel, D. N., Sutin, B., & Juszkiewicz, R, 1989, ApJ, 346, 588
Rhee, G. F. R. N., & Katgert, P. 1987, A&A, 183, 217
Richstone, D. O., & West, M. J. 1990, in preparation
Schombert, J. M., & West, M. J. 1990, ApJ, 363, 331
Struble, M. F., & Peebles, P. J. E. 1985, AJ, 90, 582
Sutherland, W. 1988, MNRAS, 234, 159
Tully, R. B. 1986, ApJ, 303, 25
———. 1987, ApJ, 323, 1
Ulmer, M. P., McMillan, S. L. W., & Kowalski, M. P. 1989, ApJ, 338, 711
Valls-Gabaud, D., Alimi, J. M., & Blanchard, A. 1989, Nature, 341, 215
Villumsen, J. V. 1989, ApJS, 71, 407
Villumsen, J. V., & Brainerd, T. G. 1989, preprint
Vishniac, E. 1986, in Inner Space/Outer Space, ed. E. W. Kolb, M. S. Turner, K. Olive, D. Seckel, & D. Lindley (Chicago: University of Chicago Press), 190
Vittorio, N., Juszkiewicz, R., & Davis, M. 1986, Nature, 323, 132
Weinberg, D. H., & Gunn, J. E. 1990, ApJ, 352, L25
West, M. J. 1989a, ApJ, 344, 535
———. 1989b, ApJ, 347, 610
West, M. J. 1990, in Clusters of Galaxies, ed. W. Oegerle, M. Fitchett, & L. Danly (Cambridge: Cambridge Univ. Press)
West, M. J., Dekel, A., & Oemler, A. 1989, ApJ, 336, 46
West, M. J., & Richstone, D. O. 1988, ApJ, 335, 532
West, M. J., Weinberg, D. H., & Dekel, A. 1990, ApJ, 353, 329
West, M. J., et al. 1990, in preparation
White, S. D. M., Frenk, C. S., Davis, M., & Efstathiou, G. 1987, ApJ, 313, 505
White, S. D. M., Tully, R. B., & Davis, M. 1988, ApJ, 333, L45
Zel'dovich, Ya. B. 1970, A&A, 5, 84