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SCALING INVARIANCE IN THE GALAXY DISTRIBUTION. II. SPATIAL AND LUMINOSITY EFFECTS

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ABSTRACT

We analyze the variation of the galaxy clustering properties when either the luminosity range, the depth or the location in space, or the morphology class of the tested subsample varies. Although we examine these variations with the two-point correlation functions, we show that the results derived from them are difficult to interpret because of normalization problems. Thus we rather concentrate on the (rescaled) void probability function, which is free of normalization effects and provides information of the high order properties of the statistics.

We show that, in the CfA catalog, bright galaxies are more clustered than faint ones at the higher order of the statistics, although they both share the same correlation functions, We compare these results with the predictions of biased galaxy formation models. We confirm that elliptical and lenticular galaxies have a stronger two-point correlation function than spirals. But we show that both morphological classes do share the same high order statistics.

We also compare the statistical properties of different samples of (bright) galaxies, with increasing depths. We confirm a variation of the correlation functions with the depth which is however completely normalization-dependent. We interpret it as due to morphological segregation, as well to the fact that different parts of the catalog are explored. Moreover, we show that all the samples (with different depths, locations, or morphological classes) do share almost indistinguishable high order statistics. This firmly confirms the existence of the scaling properties already established in the first paper in this series.

Subject headings: cosmology — galaxies: clustering

1. INTRODUCTION

The galaxy distribution has been intensively investigated during recent years, with the main purpose to draw conclusions about the theories of galaxy formation and the early universe. The most widely used indicator, namely the two-point correlation function (hereafter CF), appears, however, insufficient to discriminate between models. It seems also ill-suited to account for the many observed features of the galaxy distribution like voids, filaments, or sheets. This motivated the use of other statistical indicators, and in particular the "void probability function" (hereafter VPF). The latter has been proved to be an efficient statistical tool for the study of the galaxy distribution (White 1979; Fry 1984; Schaeffer 1984) and has been successively calculated from two-dimensional (Sharp 1981; Bouchet & Lachièze-Rey 1986) as well as three-dimensional galaxy catalogs (Maurogordato & Lachièze-Rey 1987, hereafter Paper I; Hamilton, Saslaw, & Thuan 1985). In Paper I we estimated the VPF in the Center for Astrophysics catalog (hereafter CfA) and proved the existence of a scaling invariance in the galaxy distribution, in the form predicted by the so-called hierarchical models (Schaeffer 1984, 1987b; Fry 1984, 1986). This observed scaling invariance gave us the opportunity to compare the distributions of various subsamples of the CfA catalog, with the goal of checking the luminosity, or morphology segregration, which may in principle provide crucial tests for galaxy formation models, in particular the biased formation models.

However, the CfA catalog is very inhomogeneous, reflecting the presence of the local supercluster and of various inhomogeneities. Moreover, it has been claimed by Einasto, Klypin, & Saar (1986) that the galaxy density could vary systematically with the depth of the sample under study, although the origin of this "spatial effect" remains a subject of controversy. One of the goals of this paper is to examine this effect at a deeper order, i.e., with the VPF. We also present here a new analysis of some results obtained in Paper I since they mixed different kinds of effects. Here we clearly distinguish "depth," "luminosity" and "morphological" effects.

We show first, in § 2, that the strong inhomogeneities present in the CfA catalog make the conventional analysis with correlation functions ambiguous, in the sense that they depend on the choice of a normalization. In § 3 we test the existence of a luminosity segregation with the CF and the VPF. Our results are then compared (in § 3.4) with some predictions of the models of biased galaxy formation. In § 4 we compare the distributions of the galaxies in vaious subsamples of the CfA with different depths and locations as a test for spatial effects which are claimed to be present. Finally, in § 5 we explore the effects of morphology segregation, and we determine to what extent morphology effects can be responsible for the segregations found before.

1.1. *The Catalog*

We have used the north CfA catalog, as indicated in Paper I, from which we extract absolute magnitude and volume–limited samples for our purpose. We first corrected the observed heliocentric redshifts for the solar motion, using a value of 308 km s^{-1} toward $l = 105°$, $b = -7°$ (Yahil, Tammann, & Sandage, 1977) so that $\delta v = -79 \cos l \cos b + 296 \sin l \cos b - 36 \sin b$. We also checked that no difference can be found in our results when using different correc-

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tions for solar motion. We then expressed all redshifts in a frame where Virgo is at rest, following the Aaronson et al. (1982) prescription: 250 km s^{-1} as infall velocity of the Milky Way, to which a peculiar velocity (-65, -143, 81), in their coordinate system, must be added. This results in a total velocity component of 331 km s^{-1} toward the Virgo Cluster.

For the purpose of dealing with complete subsamples only, and in order to check possible luminosity segregation effects, we define "bright" galaxies, with $M < -18.5$, and "faint" ones with $-18.5 < M < -17.5$. These words, throughout the paper, refer to these definitions. The separation between the different classes is done in coherence with the Virgo infall correction adopted.

1.2. *Virgo Infall Corrections*

Redshifts do not give exact distances: the Virgo infall corrections affect the positions of galaxies in the sample and therefore their estimated magnitudes; this in turn introduces a bias in the magnitude-selected samples. For this reason we have carefully checked the dependence of our results on the choice of a model for Virgo infall (hereafter, VI) correction. Let us point out that our results in this paper deal only with differential estimates of statistical indicators (like CF or VPF) between one subsample and the other. We show below that, although absolute estimates of these indicators depend strongly on VI corrections, the differences relevant to our analysis do not depend on them, so that our results are insensitive to Virgo infall.

To demonstrate this independence we have performed our whole analysis (including the selection of the subsamples) with different choices for the VI correction: (1) no correction at all; (2) correction of the Local Group motion: dipolar infall towards Virgo and peculiar field; and (3) corrections according to the prescriptions of Aaronson et al. (1982) adopted by Davis et al. (1988). The results presented in this paper are obtained with the second prescription.

As we show below, all results of our analysis remain, at least qualitatively, when we change the correction and quantitative variations never exceed the uncertainties. Since any realistic model would probably lead to prescriptions between those adopted here, the stability of our results seems sufficient to rule out any VI dependence. By the way it would be very difficult to understand how errors due to VI correction could conspire in order to mimic the property of scaling invariance found in this paper.

2. NORMALIZATION EFFECTS

2.1. *Fluctuations of the Density and the Luminosity Function*

We define, as indicated in Table 1, subsamples with different luminosity ranges and spatial boundaries, each of them being complete in volume and absolute magnitude. For any subsample, we refer to the "effective" density as the measured number of galaxies in it, divided by its volume: $n_{\rm eff} = N_{\rm gal}/(1.83 * D_m^3)$, for a subsample of the CfA (subtending a solid angle of 1.83 sr), containing $N_{\rm gal}$ galaxies and limited to a distance D_m.

For the bright galaxies, this effective density varies from one subsample to another. Consequently, it also differs from a universal density (for the corresponding luminosity class), if any. This reflects the well-known large scale (around $10h^{-1}$ Mpc and more) inhomogeneities (superclusters, voids, ...) of the galaxy distribution. These density fluctuations are present at the size of the CfA itself and beyond, as it appears qualitatively from the observations of superclusters or large voids. In the following, we will explore the statistical properties of these subsamples (using CF and VPF) at scales smaller than $8h^{-1}$ Mpc, searching, for instance, for a possible variation of these properties with the distance (from us), or with the size of the sample. It will be important to distinguish such possible effects from the large scale density fluctuations reported above.

Since we also intend to explore a possible luminosity segregation, we will also compare subsamples of different luminosity classes. Here we restrict the discussion to the two classes—bright and faint—defined above. As for the bright ones, the effective density of faint galaxies varies with the size or position of the subample inside the CfA catalog. More interestingly, the ratio of bright over faint also varies. This expresses the large-scale fluctuations of the luminosity function inside the CfA catalog: bright and faint galaxies do not share the same distribution. This difference motivates us to check, with other indicators, the possible existence of a luminosity segregation. We explore luminosity segregation with correlation functions in § 3.2, and also mainly with the VPF, in § 3.3. For this purpose it will be important that the searched effects (at scales less than $8h^{-1}$ Mpc) are not confused with the large-scale fluctuations of the luminosity function. It should not be forgotten that the luminosity function for a subsample (for instance limited to $30h^{-1}$ Mpc) does not coincide with the one for the whole CfA, or for the whole universe.

A simple way to quantify the density fluctuations from one sample to another involves the ratio $n_{\rm eff}/n$, where n is the density of the sample as estimated from universal galaxy density and luminosity function. This ratio is given in Table 1, where the luminosity function results from a best fit of the whole CfA catalog (accordingly with our VI correction). Its variation reflects, as we said above, the well-established large-scale density fluctuations. The fact that it does vary differently (with respect to scale) for bright and faint galaxies indicates that these luminosity classes suffer different inhomogeneities, both in level and location. This suggests strongly that faint and bright galaxies are not similarly clustered, as we test below more precisely.

2.2. *Correlation Functions*

In the BBGKY hierarchy, the density is a first order indicator; the two-point correlation function is of second order; and

TABLE 1

SUBSAMPLES

$n_{\rm eff}/n$	$D < 20$ (Mpc)	$20 < D < 25$ (Mpc)	$25 < D < 32$ (Mpc)	$32 < D < 40$ (Mpc)
$-17.5 < M < -17$	1.14	...	...	...
$-18 < M < -17.5$	1.68	0.67	...	...
$-18.5 < M < -18$	2.62	0.98	0.492	...
$M < -18.5$	3.76	1.727	1.086	0.67

the VPF deals with higher orders. It is important that effects at different order are not confused, especially for a comparison with dynamical predictions. We point out here a danger of confusing orders in statistical studies of the galaxy distribution.

The usual method to estimate the correlation functions (Groth & Peebles 1977) requires the use of a density $n_{\rm norm}$ for the normalization. The result ξ of an estimate of correlation functions depends on this choice following $1 + \xi \propto n_{\rm norm}^{-1}$ (although the dependence can be different for different prescriptions). Here we address the question how to estimate $n_{\rm norm}$ when we study a subsample of the CfA, given that its effective density may strongly differ from the universal value.

The conventional method uses a "universal" density for the normalization. Without discussing the problem of how to estimate such an universal density, it is easy to realize the following consequence: in an overdense subsample (with respect to this universal density), we will count more pairs of galaxies (at any separation) than in the random sample with universal density used for comparison. The larger overall number of pairs is not compensated for by the normalization, so that the procedure leads to a nonzero value of ξ, even in the absence of any true correlation at the corresponding scale. This leads to express large-scale density (first-order) fluctuations by small-scale correlation functions (second-order). This becomes especially delicate when the density fluctuates at scales comparable with the sample size, as is the case for the galaxy distribution. This usual procedure leads therefore to conclusions which may not correspond to the intuitive concept, as it appears forinstance in Peebles (1980). This also requires the knowledge of an universal luminosity function. Since, in the northern CfA (Davis & Huchra 1982), Schechter's (1976) law does not fit the luminosity function with the same accuracy for all luminosities (and is particularly badly known at faint luminosities, $M > -18$), this would introduce additional errors and simulate a spurious clustering.

On the other hand, these effects are separated if we use $n_{\rm eff}$ for the normalization, although the quantity $\xi_{\rm eff}$ estimated in this way does not correspond exactly to the usual definition of the correlation functions. The two normalizations lead to estimates ξ and $\xi_{\rm eff}$ in the ratio: $(1 + \xi)/(1 + \xi_{\rm eff}) = n_{\rm eff}/n$, where n is the universal density and $n_{\rm eff}$ is the effective density. Any measure of clustering properties by using ξ requires a deep discussion of these effects (see, for instance, Blanchard & Alimi 1988). We illustrate below this normalization dependence of the correlation functions, for the studies of luminosity segregation and distance effects. We haved used $n_{\rm eff}$ and $\xi_{\rm eff}$ throughout this paper, and we promote the use of the VPF, which is less sensitive to these normalization effects, as we show below.

2.3. *The Void Probability Function*

The value $P_0(V)$ of the VPF (for a definition and properties, see Paper I) depends on the density of the sample. As pointed out in Paper I, the interesting quantity is rather the normalized function $\chi = \log [P_0(V)]/nV$. We have shown in Paper I that the VPF obeys a scaling property, which is here generalized. Beside its theoretical interest (see Paper I and below), this property offers an important advantage: the use of the scaling-variable $q = nV\langle\xi\rangle$ allows to compare samples with different characteristics, even different densities. Moreover, with the prescriptions mentioned before, the function $\chi(q)$ is normalization independent (there is in fact a slight normalization dependence at large scale without consequences for our analysis).

3. LUMINOSITY SEGREGATION IN SPATIAL SLICES

3.1. *The Subsamples*

In order to establish the luminosity-segregation properties without confusion with other effects (size of the sample, or location in space), we compare different luminosity classes in the same volume limited to $25.12h^{-1}$ Mpc. We selected two subsamples in such a way that they have the same value of $n_{\rm eff}$:

CfA25-Br, with $M < -18.5$, $n_{\rm eff} = 2.47 \times 10^{-2}$,

$$n = 9 \times 10^{-3}$$

and

CfA25-F, with $-18.5 < M < -17.5$, $n_{\rm eff} = 2.47 \times 10^{-2}$,

$$n = 1.8 \times 10^{-2}$$

(the estimate n of an universal density for the subsample is given only for illustration; it derives from a best-fit luminosity function for the whole CfA, consistent with the VI model chosen). As mentioned in § 2.1, the different values of the ratio $n_{\rm eff}/n$ for the two subsamples expresses the fact that bright galaxies are much more concentrated near us than the fainter ones. Does this effect appear at higher orders?

3.2. *Correlation Functions*

Before exploring this apparent luminosity segregation with the VPF (§ 3.3), we examine the behavior of the correlation functions, following a large number of authors (Valls-Gabaud, Alimi, & Blanchard 1989; Hamilton et al. 1985; Davis et al. 1988): the estimates of $\xi_{\rm eff}$ for the two samples defined in § 3.1 almost superpose (Fig. 1). Thus we confirm that no luminosity segregation exists at this order of the galaxy statistics.

To illustrate the normalization effects discussed in § 2.2 we also calculated ξ with the usual prescription: it appears stronger, by about a factor of 2, for bright galaxies. As we discussed above, this difference may be entirely accounted for by the density fluctuations between the two samples. We turn to our main purpose, i.e., to check if a segregation is present at higher orders in the statistics.

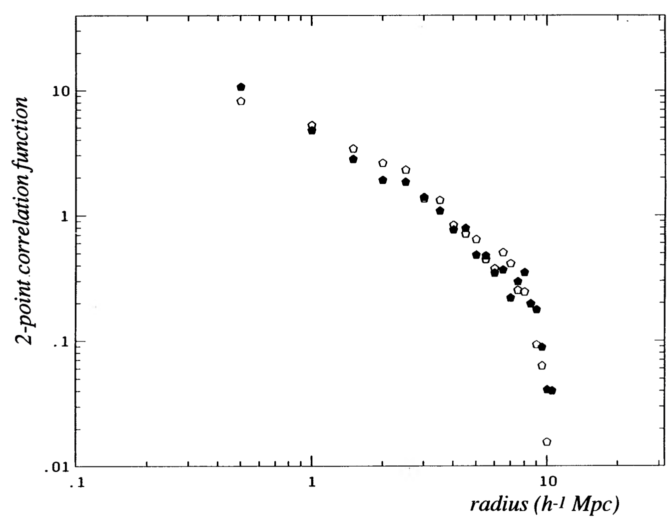

FIG. 1.—The correlation functions $\xi(r)$ vs. the radius r (in h^{-1} Mpc) in subsamples of different luminosities but with same depth $D < 25.12h^{-1}$ Mpc: filled pentagons correspond to bright galaxies with $M < -18.5$; empty pentagons to faint ones with $-18.5 < M < -17.5$.

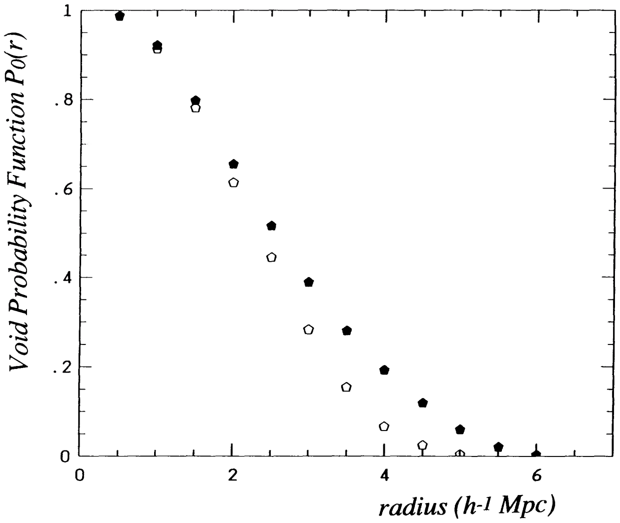

FIG. 2a

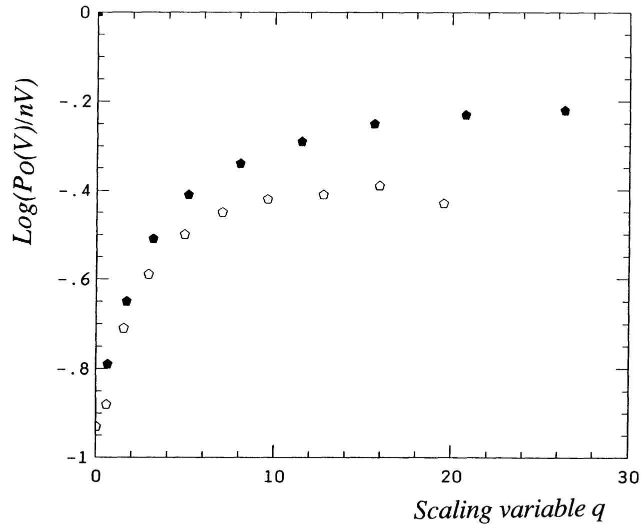

FIG. 2b

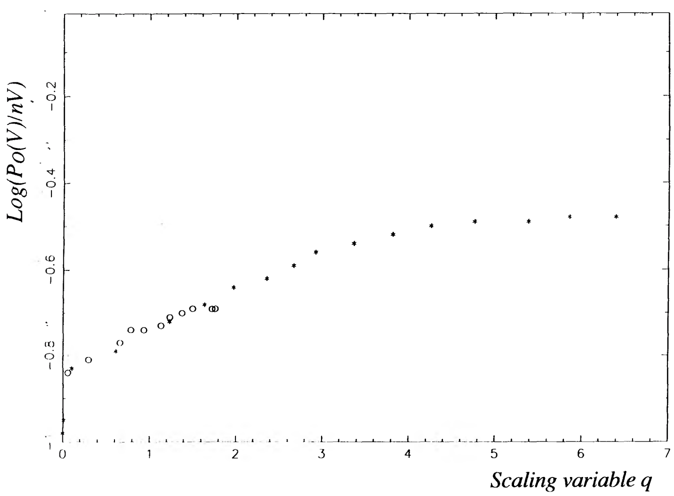

FIG. 2c

FIG. 2.—Luminosity segregation for VPF. (*a*) The void probability function $P_0(V)$ vs. the radius r of the volume tested V for two subsamples of same density and same volume but involving different luminosity classes: bright galaxies (CfA25-Br, with $M < -18.5$, $D < 25.12h^{-1}$ Mpc) (*filled pentagons*); and faint galaxies (CfA25-F with $-18.5 < M < -17.5$, $D < 25.12h^{-1}$ Mpc) (*empty pentagons*). (*b*) Variation of $\chi = \log [P_0(V)]/nV$ vs. $q = nV\langle\xi\rangle$: bright galaxies (CfA25-Br: *filled pentagons*) and faint ones (CfA25-F: *empty pentagons*). (*c*) High luminosities effects: $\chi(q)$ is plotted for two samples limited to $60h^{-1}$ Mpc: CfA60-60 with magnitudes M -19.4 (*stars*) CfA60-80 with magnitudes $M < 20$ (*circles*).

3.3. *The VPF*

Since the two samples CfA25-Br and CfA25-F have not only the same volume and shape but also the same effective density, we can compare directly their VPF to check high-order properties: brightest galaxies (Fig. 2*a*) appear more clustered than faint ones, since $P_0(V)$ is larger for CfA25-Br than for CfA25-F. This establishes the luminosity segregation at high order in a very direct way, independently of any normalization.

We have shown in Paper I that it is advantageous, for the comparison between subsamples, to use the quantity $\chi = \log [P_0(V)]/nV$ as a function of the scaling variable $q = nv\langle\xi\rangle$. In addition, this allows us to compare our results with the predictions of theoretical models. Figure 2*b* shows the function $\chi(q)$ for the subsamples of the CfA limited to $25h^{-1}$ Mpc, which again makes the luminosity segregation apparent. Following a suggestion of P. Schaeffer (private communication), q has been directly calculated from the J_3 integral to reduce the noise.

It should be noticed that the high-order segregation remains present if the universal density is used instead of $n_{\rm eff}$. We have also shown that it does not depend on the VI correction chosen (even in the limiting case of no correction at all).

3.4. *Discussion and Comparison with Models*

We have shown in § 3.1 that the distributions of bright and faint galaxies differ, at the lowest (first) order, at scales around $25h^{-1}$ Mpc, without confusion with possible depth or location effects. This is a way to express that the luminosity function of galaxies in the nearest $25h^{-1}$ Mpc strongly differs from that of the whole CfA. Given this large-scale effect, we have confirmed that the two-point CF are identical for bright and faint galaxies, but that their (normalized) VPFs differ which proves that they share different high-order properties.

We emphasize that this high-order luminosity segregation does not depend on the choice of a particular VI model, or on a particular normalization. We have also checked that it is not an effect of morphological segregation (see the last section). However, these results, established only for a local sample of the universe, cannot be claimed to be universal. Any generalization would require the examination of a similar sample at a different place in the universe, such as, for instance, in the southern hemisphere. The luminosity segregation present at high order of the statistics is of special interest since high-order effects come from nonlinear dynamics during the collapse of structures. Thus, our results may indicate that the distributions of faint and bright galaxies come from different nonlinear dynamics.

For instance, biased galaxy formation models predict clustering differences between bright and faint galaxies. Quantitative predictions have been made for instance by Schaeffer (1987a). It is difficult to apply the formulae of his theoretical analysis to limited and inhomogeneous samples. However, since Schaeffer's model (1984) reproduces quite well the low-order CF and the VPF for the CfA as a whole (cf. Paper I), we try here to check its predictions for luminosity segregation. In order to make consistent comparisons, we adopt the same luminosity function as Schaeffer (1987a), which can be fitted by a Schechter's form with $\alpha = -1.25$ and $L_0 = 3.4 \times 10^{-9} h^{-2}$ in solar units. From his model we have calculated the correlation function for a subset of galaxies with luminosity range from L_1 to L_2 and, from it, the factor between the correlation functions for two distinct luminosity classes. We choose the value $v = 0$ for the free parameter of the model, which corresponds to the best fit with the CfA data (Paper I). This allows us to predict

$$\xi(M < -18.5)/\xi(-18.5 < M < -17.5) = 4 .$$

Comparison with observational results show that this model predicts a too large overclustering for bright galaxies versus fainter ones. However, given the uncertainties of the correlation functions and their normalization dependence, we stress again that not too much importance should be given to the numbers, since the models do not try to reproduce the large-scale variation of the density and luminosity functions among subsamples, which are very important as we have shown before.

3.5. *High-Luminosity Segregation*

We also tested the clustering at higher luminosities. For this purpose, we compared two samples (limited to $40h^{-1}$ Mpc), corresponding to galaxies of absolute magnitude greater than -19.4 (CfA40-60) and greater than -20 (CfA40-80). The notations mean that the galaxies of the samples are observable up to 60 and $80h^{-1}$ Mpc respectively. We also compared the subsamples CfA60-60 and CfA60-80 (limited to $60h^{-1}$ Mpc and to magnitudes -19.4 and -20, respectively).

The second-order statistics (correlation functions or J_3 integrals) confirms the overclustering of galaxies brighter than magnitude -20 (samples CfA40-80 and CfA60-80) already found by Hamilton (1988). We found, however, that this effect, expressed by the correlation functions, depends on the VI correction chosen. We also found this overclustering above magnitude -20 with the VPF.

However, it is remarkable that no overclustering of very bright galaxies appears with the function $\chi(q)$ (Fig. 2*c*): the VPF variation is compensated by the CF variation so that the scaling is preserved. We checked that this scaling of the VPF remains true independently of the VI correction.

4. SPATIAL EFFECTS

4.1. *Subsamples*

It has been suggested, first by de Vaucouleurs, that the density of the galaxy distribution could vary with the depth of the sample. This manifestation of a scale-dependent filling factor, recently analyzed by Einasto, Klypin, & Saar (1986), has led Calzetti, Giavalisco, & Ruffin (1988) to suggest a fractal structure of the universe. In this section, we check the suggestion that the statistics of the galaxy distribution could depend on the depth of the sample under study. We will also check a possible variation with the spatial location of the sample in space. To avoid incompleteness, and mixing of effects of different natures, we concentrate on the bright galaxies only. We define the subsamples, complete in volume and absolute magnitude:

CfA20-Br with $D < 19.95h^{-1}$ Mpc, $n_{\rm eff} = 3.386 \times 10^{-2}$,

CfA25-Br with $D < 25.12h^{-1}$ Mpc, $n_{\rm eff} = 2.472 \times 10^{-2}$,

CfA32-Br with $D < 31.62h^{-1}$ Mpc, $n_{\rm eff} = 1.727 \times 10^{-2}$,

and

CfA40-Br with $D < 39.81h^{-1}$ Mpc, $n_{\rm eff} = 1.167 \times 10^{-2}$.

4.2. *Correlation Functions*

Many authors have reported important variations of the two-point correlation functions in different subsamples of the CfA, with different depths and locations. Interpreting these variations is delicate, because of the normalization dependence of the correlation function in an inhomogeneous sample, discussed, for instance, by Coleman, Pietronero, & Sanders (1988), Sutherland (1988), or Blanchard & Alimi (1988). Using the normalization with effective density, the estimated two-point CF (for bright galaxies) increases with the depth of the sample (Fig. 3*a*). The slope keeps the constant value of -1.8, and the correlation radius increases from about 2.5 to $5.5h^{-1}$ Mpc. This effect reproduces well the curves of Einasto et al. (1986) with a correlation length proportional to the depth of the sample. It cannot be due to large-scale density fluctuations since the normalization with effective density precisely allows to be independent of them, nor is it due to luminosity segregation since this analysis clearly separates luminosity and spatial effects. It is therefore a real variation of second-order properties (correlations) from one sample to the other.

In order to illustrate the influence of the normalization, we also calculated the correlation functions with the standard prescription: the effect is completely reversed as appears in Figure 3*b*: the slope steepens and the correlation length decreases from CfA20 to CfA40. Thus the variation of the correlation function with the depth of the sample is strongly normalization dependent.

4.3. "*Depth*" *or* "*Location*" *Effect*

Given the proposed interpretations of these variations, it is interesting to check if this is really a systematic effect with the depth of the sample. If not, it may simply result from the different spatial locations of the subsamples inside the CfA. In order to tentatively discriminate between these two possibilities, we defined an additional subsample, complementary to CfA25-Br, called CfA25/40-Br, delimited by $25.12h^{-1}$ Mpc $< D < 39.81h^{-1}$ Mpc and $M < -18.5$. Our results (Fig. 4) show strong differences in the estimated correlation functions. This seems to imply that "location" effects play a very important role and confirms the difficulty with the reported interpretation in terms of a fractal structure of the galaxy distribution. Similar conclusions are reached with other subsamples of the CfA. Part of the effect shown in Figure 4*b* may also be explained by morphological segregation: CfA25/40-Br contains a smaller proportion of elliptical and lenticular (hereafter E + S0) than CfA25-Br (see the last section).

4.4. *VPF*

In order to have a clearer view it seems appropriate to turn towards the (rescaled) VPF, which is less dependent on normalization. We calculated the function $\chi(q)$ for the subsamples with increasing depth: it is remarkable that the different curves

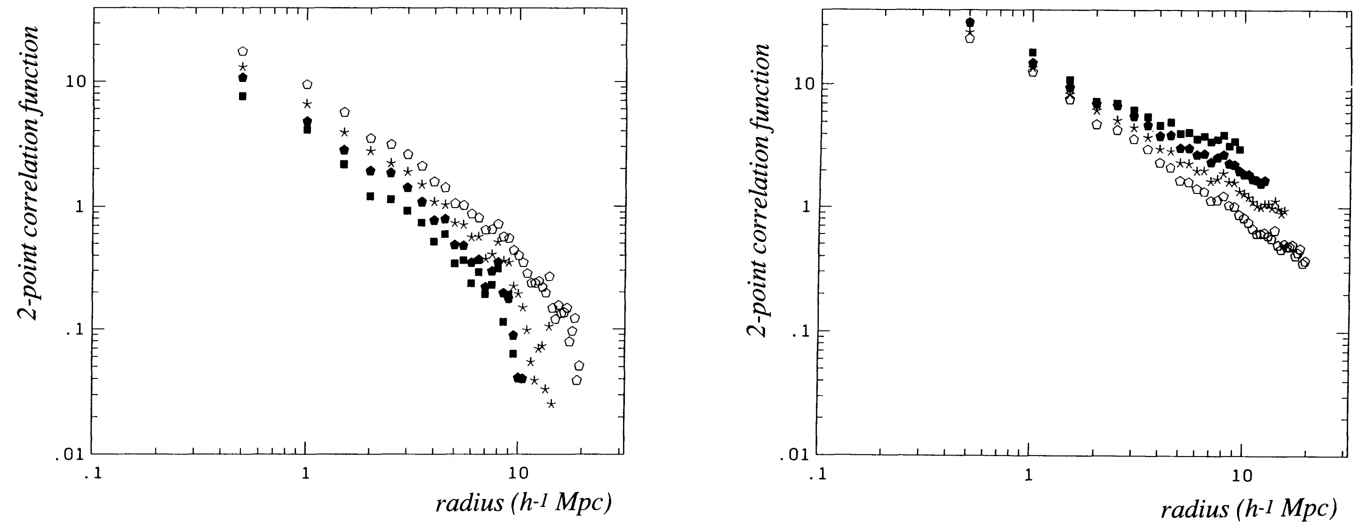

FIG. 3.—Correlation functions for four subsamples of bright galaxies ($M < -18.5$) of varying depth: CfA20-Br with $D < 19.95h^{-1}$ Mpc (*filled squares*); CfA25-Br with $D < 25.12h^{-1}$ Mpc (*filled pentagons*); CfA32-Br with $D < 31.62h^{-1}$ Mpc (*stars*); CfA40-Br with $D < 39.81h^{-1}$ Mpc (*empty pentagons*). The normalization uses the effective density of the subsample.

for $\chi(q)$ (Fig. 5) superpose quite well, although the samples under study have very different values of density, and CF.

These results are a strong confirmation of the scaling properties established in Paper I: despite their very different low-order statistical properties (density and CF), the samples analyzed all share a unique high-order statistical distribution. The scaling relation (see Paper I) holds true very generally, as far as we can judge from a limited catalog: the function $\chi(q)$ appears universal, independent of the position in space, of the shape and volume of the catalog, or of its effective density (although a given value of q corresponds to different scales in the distinct samples). This effect was predicted by the so-called hierarchical models (Balian & Schaeffer 1988 and references therein; Fry 1986), and thus our results may be seen as clues in favor of this kind of models.

4.5. *Discussion*

Since the distances and luminosities of the galaxies under study are estimated from redshift measurements, it may be suspected that the Virgo infall model, used to correct those, influences our results. As described above, we have pursued our whole analysis with different prescriptions for VI corrections: our results remain very stable. Thus we are confident that our conclusions are insensitive to this correction.

The presence of some kind of hierarchical structure in the matter distribution (Einasto et al. 1986); or Calzetti *et al.* 1988)

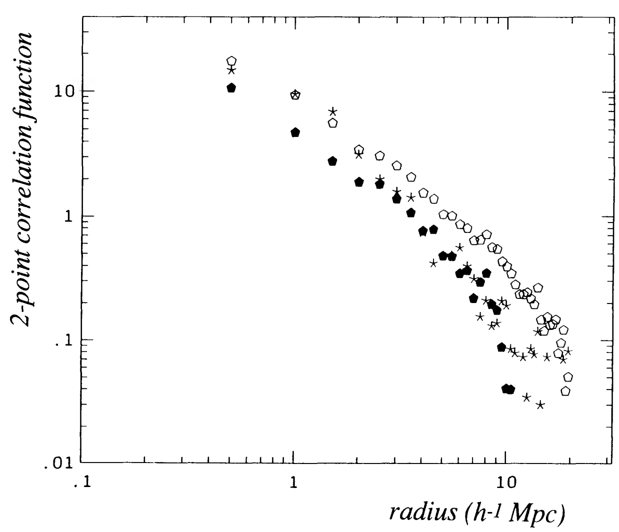

FIG. 4*a*

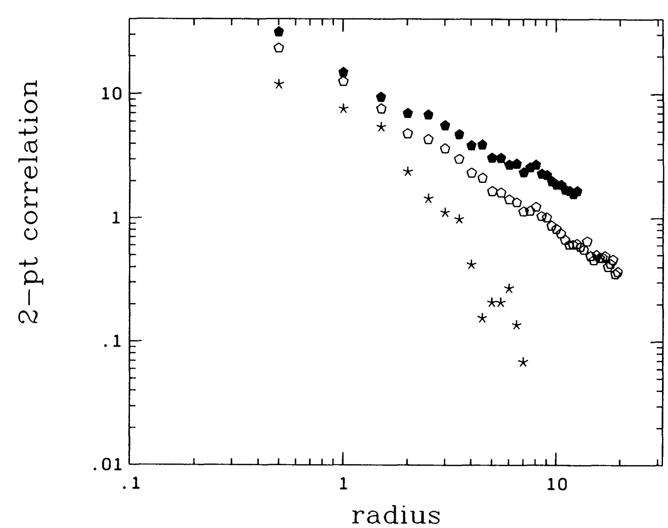

FIG. 4*b*

FIG. 4.—Correlation functions for three subsamples of bright galaxies $M < -18.5$ of varying localization (4*a* for effective normalization; 4*b* for universal normalization): CfA25-Br with $D < 25.12h^{-1}$ Mpc (*filled pentagons*), CfA25/40-Br with $25.12h^{-1}$ Mpc $< D < 38.81h^{-1}$ Mpc (*stars*); and CfA40-Br with $D < 38.81h^{-1}$ Mpc (*empty pentagons*).

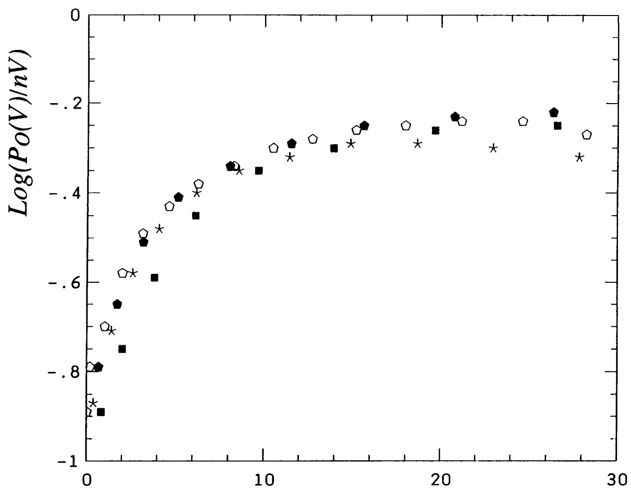

FIG. 5.—The function $\chi(q)$ for four subsamples of bright galaxies of varying depth: CfA20-Br with $D < 19.95h^{-1}$ Mpc (*filled squares*), CfA25-Br with $D < 25.12h^{-1}$ Mpc (*filled pentagons*), CfA32-Br with $D < 31.62h^{-1}$ Mpc (*stars*), and CfA40-Br with $D < 39.81h^{-1}$ Mpc (*empty pentagons*).

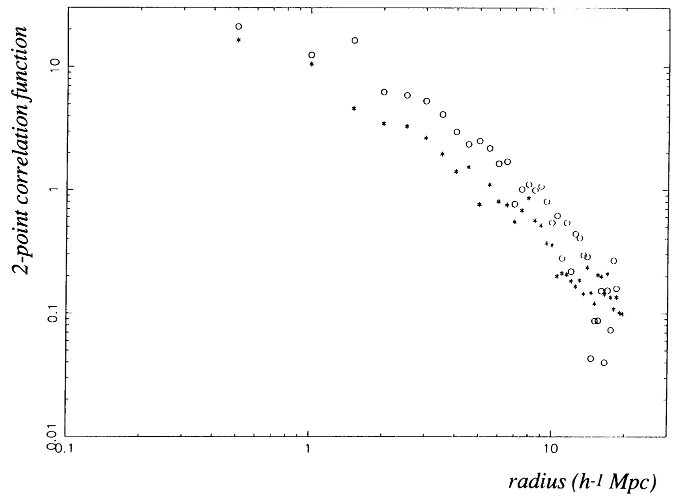

FIG. 6.—Correlation functions for E and S0 galaxies (*circles*) and Sp galaxies (*stars*) in CfA40-Br.

would imply that the reported effect is due to a systematic variation with the depth. Jones et al. (1988), and Balian & Schaeffer (1988) have shown that this interpretation hardly holds. Our results lead to the same conclusion. The variation found with CF rather mixes location and morphology effects. The fact that the high-order statistics [the function $\chi(q)$] is independent of depth also strengthens these conclusions. More simply, the different parts of the CfA explored are subject to positive or negative density fluctuations.

On the other hand, these results confirm and extend the existence of the scaling invariance already found in Paper I: the distribution of bright galaxies, although strongly fluctuating with volume or location (or both), is effectively described by a single function $\chi(q)$. This helps to demonstrate the interest of $\chi(q)$ as a very reliable indicator even in the presence of strong density fluctuations. It would be very interesting to link this function with other types of analysis, such as those based on a fractal or multifractal dimensions, and to predict its value from dynamical processes involved in the galaxy formation models.

5. MORPHOLOGICAL SEGREGATION

5.1. *Correlation Functions*

To test morphology segregation, we work in a sample of fixed depth and luminosity class. This avoids the possible influence of the different luminosity functions (Tammann, Yahil, & Sandage 1970). We divide CfA40-Br into two morphological subsamples: CfA40-E-S0 with 104 elliptical and lenticular galaxies, $n = 0.0266$, and CfA40-Sp with 253 spiral galaxies, $n = 0.0648$.

In Figure 6, the correlation functions for both samples are compared. Elliptical galaxies are more correlated than spiral, with a little steeper slope and a bigger correlation length. The best fit gives, for CfA40-E-S0 and CfA40-Sp respectively, slopes of 1.8 and 1.6 and amplitudes of 38 and 13. The previous three-dimensional estimates of Davis & Geller (1976), by deprojection of their two-dimensional analysis of the Nilson catalog, gave amplitudes of 55 for elliptical-elliptical (E-E), 36 for lenticular-lenticular S0-S0), and 12 for spiral-spiral (Sp–Sp) correlations, which agrees with our values. We could not, however, detect a strong slope variation with morphological type, although their predictions inferred a decrease from 2.25 to 2.07, and 1.73 from E-E to S0-S0 and Sp-Sp galaxies, nor does this effect on the slope seem present in the first results obtained by L. N. da Costa and P. Pellegrini (private communication) on the Southern Sky Redshift Survey.

5.2. *Void Probability Function*

The calculation of VPF shows that both families are well represented by the same curve with a universal function $\chi(q)$, independent of the morphological type (Fig. 7), with the same conclusions than the previous section: the differences in clustering between galaxies of different morphological types are confined to the lowest orders of the statistics.

5.3. *Morphological Influence on Previous Results*

After having shown the strong dependence of low-order clustering on morphological types, it is important to check if some effects measured before were not due to this dependence.

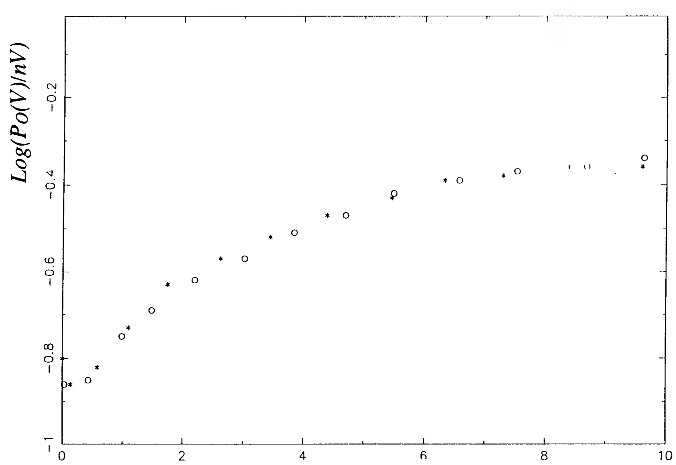

FIG. 7.—Scaling between E and S0 galaxies (*circles*) and Sp galaxies (*stars*) in CfA40-Br.

TABLE 2

PERCENTAGE OF E + S0 GALAXIES IN SAMPLES STUDIED

Sample	N_{EL+S0}	N_{Sp}	N_{TOTAL}	$r = N_{EL+S0}/N_{TOTAL}$
CfA25-Br	90	142	239	37
CfA25-F	80	138	239	33.5
CfA32-Br	101	215	333	30
CfA40-Br	127	284	449	28
CfA2040-Br	71	179	285	25
CfA2540-Br	56	142	210	26.7

For this purpose, we compare the percentage r of E + S0 in the different samples studied (Table 2).

The fainter subsample (CfA25-F) contains roughly the same proportions of elliptical and lenticular (34%) as the brighter one, CfA25-Br (37%). The underclustering of faint galaxies is therefore not due to morphological segregation. This was to be expected, since $\chi(q)$ differs from bright to faint galaxies, although it does not differ from E-S0 to S0, so that the two kinds of differences in clustering are not of the same order. On the other hand, the proportion of elliptical and lenticular galaxies shows a systematic variation from 0.37 to 0.28 when the volume of the sample is increased from 25 to $40h^{-1}$ Mpc. ξ with universal normalization (for bright galaxies) increases from CfA25/40-Br and CfA20/40Br, CfA40-Br, to CfA25-Br and CfA20-Br (Figs. 4b and 3b), corresponding also to the proportion of ellipticals might then be partially responsible for the volume or location effects found.

6. DISCUSSION AND CONCLUSION

We have defined different samples of the CfA catalog in order to check separately whether the clustering properties of galaxies depend upon the mean luminosity, morphology, depth, or spatial location of the sample. As a first conclusion, we confirm previous results obtained with the correlation functions, but we show that they are very dependent on the normalization chosen. Definite conclusions cannot therefore be derived from their use without deeper discussion. On the other hand, we emphasize the use of the VPF, which is almost normalization-independent and gives useful information concerning the high order of the statistics.

We first compared the distributions of bright and faint galaxies. The differences in their density fluctuations suggested that they have different statistics, but we confirm the absence of luminosity segregation with correlation functions at intermediate luminosities. On the other hand, the VPF analysis shows an important overclustering of bright galaxies at the higher order in the hierarchy of correlation functions.

We compared also subsamples of the CfA of different depths but involving the same luminosity class. We confirmed strong differences in their correlation functions, which depend on the choice of the normalization. We interpret them as due in large part to the difference in densities between these samples. We also showed an influence of the morphology segregation and of the fact that we explore different regions of the catalog. On the other hand, we have shown that these samples share the same function $\chi(q)$ (the scaled VPF), independently of any normalization problem: no difference in the statistical properties seems to exist at an order greater than the two-point correlation functions although the volumes, locations, or morphological classes of the samples strongly differ. This confirms the scaling law already discovered in Paper I.

Finally, we test samples of different morphological classes (E + S0 vs. Sp). We confirm the well-known result that correlation functions for E + S0 are stronger than for Sp, but we show that these different morphological classes also do share the same high order statistics, which also confirms the scaling.

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