# IRAS LOW-RESOLUTION SPECTRAL OBSERVATIONS OF THE 10 AND 18 MICRON SILICATE EMISSION FEATURES 

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#### Abstract

The IRAS LRS Atlas contains $8-23 \mu \mathrm{~m}$ spectra of 1816 stars with silicate emission features. In order to study the shapes of the silicate dust features, the 117 stars whose emission features appear to be optically thin and which have the best signal-to-noise ratio at the longest wavelengths were analyzed. Simple spherical dust shell models were calculated in both the optically thin and the slightly optically thick approximations. From the comparison of the predicted spectra of the dust plus stellar continuum, the emissivity function $\kappa_{\lambda}$ was derived. In the different stars, the shape of the $10 \mu \mathrm{~m}$ feature is either narrow or broad and it is peaked either at $\sim 9.7 \mu \mathrm{~m}$ or at $10 \mu \mathrm{~m}$. Either particle size effects (particles $\gtrsim 0.75 \mu \mathrm{~m}$ in radius) or optical depth effects (central optical depth $\sim 1$ ) could broaden the $10 \mu \mathrm{~m}$ feature. Chemical composition differences no doubt are also important, particularly as regards the position of the peak of the $10 \mu \mathrm{~m}$ feature. The stars with the peak at $10 \mu \mathrm{~m}$ are more closely confined to the Galactic plane than the stars with the peak at $9.7 \mu \mathrm{~m}$. The shape of the $18 \mu \mathrm{~m}$ feature is essentially the same in all stars, and can be used to extend the interstellar extinction curve past $13 \mu \mathrm{~m}$ to $22 \mu \mathrm{~m}$.


Subject headings: infrared: spectra - stars: circumstellar shells

## I. INTRODUCTION

Many late-type stars have large amounts of infrared emission in addition to the stellar continuum. In some cases, these infrared excesses are featureless, but in many they show strong broad emission features at 10 and $18 \mu \mathrm{~m}$. These features have long been identified with silicate dust (see the review of Merrill 1979). The silicate dust is thought to be in a shell surrounding a cool star that is losing mass (see the reviews of Zuckerman 1980 and Jura 1985). The shape of the $10 \mu \mathrm{~m}$ feature is always smooth and broad, although there are some differences, particularly between the stellar $10 \mu \mathrm{~m}$ feature and the $10 \mu \mathrm{~m}$ feature as it appears in emission in interstellar dust such as that found in the Trapezium in the Orion Nebula (Roche and Aitken 1984). Such smooth, broad features are not characteristic of crystalline silicates, whose exact chemical compositions are well-define by the sharp substructures in their spectra. On the other hand, broad structureless 10 and $18 \mu \mathrm{~m}$ peaks are found in both magnesium and iron silicates as long as they are amorphous in structure (Day 1979, 1981; Stephens and Russell 1979) or hydrated (Zaikowski and Knacke 1975; Knacke and Krätschmer 1980). Although the $10 \mu \mathrm{~m}$ feature can be easily observed from the ground in the $8-13 \mu \mathrm{~m}$ window, the $18 \mu \mathrm{~m}$ feature can only be studied in detail from above the ground, as was done for the 10 stars (only four of which have well-defined $18 \mu$ m emission features) observed by Forrest, McCarthy, and Houck (1979) from 16 to $39 \mu \mathrm{~m}$ from the Kuiper Airborne Observatory. Like the $10 \mu \mathrm{~m}$ feature, the $18 \mu \mathrm{~m}$ feature appears to be broad, smooth, and without substructure. Terrestrial and meteoritic silicates, even amorphous ones, show more substructure than do the stellar feature, and they peak at longer wavelengths than the $18 \mu \mathrm{~m}$ feature.

The best collection of data on the 10 and $18 \mu \mathrm{~m}$ features is the spectra taken by the low-resolution spectrometer (LRS) on $I R A S$. The LRS was a prism spectrometer working from $\sim 8$ $\mu \mathrm{m}$ to $\sim 22 \mu \mathrm{~m}$, with a resolution ranging from 20 to 60 depending on wavelength. For details, see the IRAS Explanatory Supplement 1988 (hereafter ES). Thus, this wavelength
range covers both the 10 and $18 \mu \mathrm{~m}$ features at the same time and also the connecting $13-16 \mu \mathrm{~m}$ region which has hitherto been unobtainable (because of atmospheric $\mathrm{CO}_{2}$ ).

In this paper, I analyze the stars with the best LRS spectra to determine the shapes of the 10 and $18 \mu \mathrm{~m}$ circumstellar silicate features. This information will be useful for future investigations of the chemistry of the dust grains, and may also help to define the interstellar absorption curve from 13 to $22 \mu \mathrm{~m}$. There are variations in the shapes of the emission features from star to star, and these are discussed with the use of simple spherical dust shell models. Some of this variation probably does come from real variations in the chemical composition of the dust. The effects of varying the particle sizes and the optical depths will also be discussed.

## II. BASIC ANALYSIS

The LRS Atlas (Atlas of Low Resolution IRAS Spectra 1986) contains 1816 stars in its classes 2 and 6 : stars with oxygen-rich envelopes (ES). I have searched through this atlas for stars with well-defined $10 \mu \mathrm{~m}$ features and excellent signal-to-noise ratios. The definition of "well-defined" implies that the emission feature at $10 \mu \mathrm{~m}$ is optically thin or close to thin. The 117 stars so selected are listed in Tables 1-5. The identifications are from the positional associations (ES) and Gezari et al. (1987), and the spectral types are from the General Catalogue of Variable Stars (Kukarkin et al. 1969) and supplements, Kleinmann et al. (1981), Jones et al. (1982), and Epchtein et al. (1985). There are many more similar stars in the LRS Atlas, but their spectra are all noisier in the $18 \mu \mathrm{~m}$ region and so were not considered. This list of 117 stars includes the star $\mu$ Cep (IRAS $21419+5832$ ) because it is in the LRS Atlas and because it is so well-studied in the infrared (e.g., Russell, Soifer, and Forrest 1975; Roche and Aitken 1984). In fact, its Atlas spectrum has an incorrect wavelength calibration and should not be used. The spectrum used here is an average of the correctly calibrated spectra from the LRS data base (Volk and Cohen 1989a

TABLE 1
Group 1-Stars with Narrow $10 \mu$ m Features

| Name | LRS <br> Char | Identification | Spectral <br> Classification | $T_{0}$ |
| :---: | :---: | :--- | :--- | ---: |
| $03507+1115 \ldots \ldots$ | 26 | NML Tau $=$ | IK Tau | 500 |
| $09203-5220 \ldots \ldots$ | 28 | WY Vel | M3epIb, Irr c | 600 |
| $13172+4547 \ldots \ldots$ | 29 | V CVn | M4e-M6e, SRa | 750 |
| $17579+2335 \ldots \ldots$ | 29 | WY Her | M5e, Mira | 600 |
| $18050-2213 \ldots \ldots$ | 26 | VX Sgr | M4e, SRa | 450 |
| $20000+4954 \ldots \ldots$ | 69 | Z Cyg | M5e, Mira | 487.5 |
| $21419+5832 \ldots \ldots$ | 28 | $\mu$ Cep | M2eIa, SRc | 500 |
| $22345+5809 \ldots \ldots$ | 29 | W Cep | K0epIa, SRc | 462.5 |
| $22480+6002 \ldots \ldots$ | 69 | AFGL 2968 |  | 337.5 |
| $23416+6130 \ldots \ldots$ | 69 | PZ Cas | M3, SRa | 325 |

TABLE 2
Group 2-Stars with Emission Peak $<10 \mu \mathrm{~m}$

| Name | LRS Char | Identification | Spectral Classification | $T_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $00042+4248$ | 26 | KU And |  | 400 |
| 00193-4033.... | 28 |  |  | 550 |
| $01085+3022$. | 29 | IRC + 30021 |  | 450 |
| 02351-2711..... | 29 | IRC-30032 |  | 550 |
| $04566+5606$. | 27 | TX Cam | Mira | 462.5 |
| $05151+6312$. | 29 | IRC+60154 |  | 512.5 |
| $05411+6957$. | 29 | IRC +70066 |  | 525 |
| $06300+6058 \ldots \ldots$ | 28 | IRC + 60169 |  | 400 |
| 07152-3444.. | 29 | AFGL1099 |  | 600 |
| 07308 + 3037..... | 27 | IRC+30187 |  | 475 |
| 07329-2352.. | 27 | DU Pup | Mira | 475 |
| 07536-2830..... | 28 | HU Pup |  | 375 |
| 09235-2347.. | 28 | IRC-20188 |  | 475 |
| 09429-2148..... | 28 | IRC-20197 |  | 375 |
| 10359-5955.. | 29 |  |  | 575 |
| 13442-6109..... | 27 | AFGL 4178? |  | 387.5 |
| 15226-3603... | 28 | AFGL 1771 |  | 437.5 |
| 15255 + 1944 $\ldots \ldots$ | 29 | WX Ser | M8e, Mira | 450 |
| 15568-4513... | 29 |  |  | 700 |
| 16219-5048..... | 27 |  |  | 375 |
| 16222-4738... | 27 |  |  | 325 |
| 16340-4634..... | 27 | OH $337.9+0.3$ |  | 387.5 |
| $17119+0859$. | 28 | IRC + 10322 |  | 412.5 |
| $17334+1537 .$. | 29 | MW Her | Mira | 525 |
| 17484-0800.. | 29 | IRC-10381 |  | 487.5 |
| 18009-2019... | 29 | IRC-20424 |  | 600 |
| 18018-2802..... | 28 | V1804 Sgr |  | 512.5 |
| 18076-1034..... | 28 | IRC-10401 |  | 512.5 |
| 18387-0423..... | 27 | IRC + 00363 |  | 375 |
| 18413+1354.. | 29 | IRC+10374 |  | 450 |
| 18595-3947..... | 26 | AFGL 5552 |  | 437.5 |
| 19042-4858.. | 29 | U Tel | M7e, Mira | 487.5 |
| 19059-2219..... | 28 | IRC-20540 |  | 387.5 |
| 19093-3256..... | 28 | V342 Sgr |  | 487.5 |
| 19240+3615 $\ldots \ldots$ | 29 |  |  | 400 |
| 19361-1658..... | 29 | AFGL 2425 | M | 412.5 |
| $19396+1637$. | 29 | HM Sge |  | 450 |
| $19412+0337$. | 29 | IRC 00450 |  | 550 |
| $19422+3506 \ldots \ldots$ | 28 | AFGL 2445 | M | 400 |
| 19474-0744..... | 28 | GY Aql | SR | 475 |
| 19550-0201.. | 27 | RR Aql | M6e-M7e, Mira | 550 |
| 19586+3637.... | 29 | IRC+40371 |  | 425 |
| 20042-4241... | 28 | V2234 Sgr |  | 437.5 |
| $20052+0554 \ldots \ldots$ | 29 | IRC+10451 |  | 425 |
| 20135-7152..... | 29 |  |  | 450 |
| 20440-0105..... | 27 | IRC + 00490 |  | 525 |
| 20529 + $3013 \ldots \ldots$ | 29 | UX Cyg | M4e-M6e, Mira | 600 |
| 20541-6549..... | 29 |  |  | 575 |
| 21069-3843..... | 29 | AFGL 5592 |  | 525 |
| $22233+3013 \ldots \ldots$ | 29 | RV Peg | M6e, Mira | 700 |

TABLE 3
Group 3-Stars with Emission Peak $\geq 10 \mu \mathrm{~m}$

| Name | LRS Char | Identification | Spectral Classification | $T_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $02302+4525$. | 24 | UX And | M6III, SRb | 412.5 |
| $02469+5646$ | 28 | W Per | M5v, Mira | 375 |
| $03030+5532 \ldots$ | 27 | 10 Per | M4, Irr b | 425 |
| 05090-1154.. | 22 | RX Lep | gM6, Irr b | 375 |
| 06036-2411.. | 26 | S Lep | M6, SRb | 550 |
| $06278+2729$. | 27 | DW Gem | M4, Irr b | 550 |
| 08220-0821. | 29 | FK Hya | $\mathrm{Mb}, \mathrm{Irr} \mathrm{b}$ | 625 |
| 10056-5300.. | 29 | CM Vel | M2, SRa | 550 |
| 10186-6012.. | 29 | EV Car | M4, SRb | 487.5 |
| 10226-5956.. | 29 | CK Car | M2, SRb | 550 |
| 10323-5735.. | 28 |  |  | 375 |
| 11179-6458.. | 29 | HD 98658 | M3 | 462.5 |
| 14003-7633.. | 22 | $\theta$ Aps | Mbp, SRb | 337.5 |
| $16418+5459 \ldots$ | 24 | S Dra | M6, SRb | 425 |
| 17102-1031.. | 27 | IRC-10359 |  | 412.5 |
| 17315-3414.. | 29 | AFGL 5355 |  | 525 |
| 17328-3327.. | 29 | AFGL 5357 |  | 350 |
| 17341-3453.. | 29 | AFGL 5360 |  | 512.5 |
| $17361+5746 \ldots$ | 28 | TY Dra | M8, Irr b | 512.5 |
| 17513-2313.. | 29 | V774 Sgr | Irr b | 487.5 |
| 19032-4602.. | 29 | RX Tel | M, SRa | 600 |
| $19232+5008$. | 26 | CH Cyg | M6, SRa | 1000 |
| $20194+3646 \ldots$ | 29 | BI Cyg | M4, Irr c? | 437.5 |
| 21044-1637.. | 27 | RS Cap | M4, SRb | 550 |
| $21245+6221$ | 29 | SW Cep | M6, SRb | 475 |
| $22017+2806 \ldots$ | 26 | TW Peg | M7, SR | 462.5 |
| $22097+5647$. | 29 | CU Cep | M4, Mira | 475 |
| $23138+6204$. | 27 | IRC+60393 |  | 425 |
| $23504+6043$. | 29 | TZ Cas | M2Iab, Irr c | 550 |

TABLE 4
Group 4 -Stars with Optically Thick Emission, Peak $<10 \mu \mathrm{~m}$

| Name | LRS Char | Identification | Spectral Classification | $T_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| 02522-5005 | 24 | R Hor | M7e, Mira | 650 |
| 04140-8158. | 24 | U Men | SR | 475 |
| 05052-8420. | 27 | V01835 |  | 550 |
| 05096-4834. | 25 | S Pic | M7e-M8e, Mira | 650 |
| 05098-6422. | 26 | U Dor | M7e, Mira | 487.5 |
| 05559 + 7430 | 24 | V Cam | M7e, Mira | 475 |
| $06500+0829$ | 28 | GX Mon | M9, Mira | 575 |
| 07209-2540. | 24 | VY CMa | cM3e, Irr c | 325 |
| 07304-2032.. | 29 | Z Pup | M6e-M9e, Mira | 600 |
| $16011+4722$. | 24 | X Her | M6e, SRb | 462.5 |
| 17080-3215.. | 28 | AH Sco | M3e, SRa | 450 |
| 18213+0335 . | 28 | IRC + 00349 |  | 750 |
| $18349+1023$. | 26 | V1111 Oph |  | 437.5 |
| $18409+1220$. | 28 | KX Her | Mira | 512.5 |
| $19244+1115$. | 28 | IRC+10420 | F8-G0 I | 300 |
| $20350+3741$. | 29 | IRC +40435 |  | 600 |
| $21270+7135$. | 29 | IRC+70171 |  | 550 |
| $21456+6422$. | 28 | RT Cep | M6, Mira | 550 |
| $23496+6131$. | 27 | IRC +60427 |  | 425 |

TABLE 5
Group 5-Stars with Optically Thick Emission, Peak $\geq 10 \mu \mathrm{~m}$

| Name | LRS <br> Char | Spectral <br> Identification | Classification | $T_{0}$ |
| :---: | :---: | :--- | :--- | ---: |
| $05367-3736 \ldots \ldots$ | 25 | RU Aur | M8e, Mira | 525 |
| $06363+5954 \ldots \ldots$ | 27 | U Lyn | M8e, Mira | 800 |
| $14020-3515 \ldots \ldots$ | 29 | AQ Cen | Mira | 800 |
| $15576-5400 \ldots \ldots$ | 27 | HD 143183 | M1-M2 | 487.5 |
| $17488-2800 \ldots \ldots$ | 29 | KW Sgr | Mp | 487.5 |
| $20015+3019 \ldots \ldots$ | 29 | V719 Cyg | M4, Irr b | 412.5 |
| $21389+5405 \ldots \ldots$ | 24 | RU Cyg | M7e, SRa | 475 |
| $22142-8458 \ldots \ldots$ | 23 | SAO 258927 |  | 425 |
| $22212+5542 \ldots \ldots$ | 26 | RW Cep | M0 Ia-I, Irr c | 400 |

and Simpson and Rubin 1990 describe the extraction procedure).

The LRS was calibrated by assuming that $\alpha$ Tau (spectral type K 5 III) radiates as a $10,000 \mathrm{~K}$ blackbody (ES). Volk and Cohen (1989b) showed that this calibration produced stars with continua hotter than blackbodies. I have used their revised calibration to correct the shapes of the LRS spectra.

For the basic analysis, I assumed that the dust surrounds the star in an optically thin shell, a reasonable assumption considering the selection procedure. The observed flux, then, consists of the stellar continuum plus the emission from the dust grains whose temperatures depend only on their distances from the star. For the stellar continuum, I used a blackbody of 3000 K for most stars, with the exceptions IRAS $03507+1115$ (NML Tau) and IRAS $19244+1115$ (IRC +10420 , spectral type F8-G0) which have been observed to have $T_{\text {eff }}$ of 2500 and 6000 K , respectively. Even moderate errors in this temperature assumption would have little effect on the subsequent analysis since the wavelengths of the LRS are in the RayleighJeans part of the spectrum. This fitted stellar continuum should start at a wavelength short enough that there is no additional silicate dust emission (or other excess emission). From observations of $\mu$ Cep (Russell, Soifer, and Forrest 1975) over the whole range from 2 to $14 \mu \mathrm{~m}$, it appears that the wavelength should be at least as short as $3-4 \mu \mathrm{~m}$. On the other other hand, since most of the stars are variable, one should fit the continuum at as long a wavelength as possible in the infrared, since the flux varies less at longer wavelengths (Forrest, Gillett, and Stein 1975). Wherever possible (97 out of 117) stars, I fit the continuum to the 4 or $5 \mu \mathrm{~m}$ fluxes listed in the complication of Gezari et al. (1987); less than 8\% gave unreasonable fits, probably because of the variability or the CO feature at $4.7 \mu \mathrm{~m}$. Another possibility is that the overall flux calibrations of the LRS spectra are not correct (Volk and Cohen 1989b). I will discuss the consequences of an incorrect fitted stellar continuum later. At $7.8 \mu \mathrm{~m}$, the stellar continuum defined by the 4 or $5 \mu \mathrm{~m}$ flux typically was found at about half the observed flux for LRS classes 27, 28, and 29, at $\frac{2}{3}$ the observed flux for classes 25 and 26, and at the observed flux level for classes 22-24. For the stars that are not in the Gezari et al. (1987) compilation or for the 7\% where the fitted continuum was higher than the observed or practically zero at $8 \mu \mathrm{~m}$, I fit the continuum at $7.8 \mu \mathrm{~m}$ using the above empirical result for each LRS class.

After the stellar continuum subtraction, the remaining flux is solely the emission from the dust shell. This can be written as

$$
\begin{equation*}
F_{\lambda}=\text { constant } \times \int_{R_{0}}^{R_{\max }} \kappa_{\lambda} B(T) \rho(R) 4 \pi R^{2} d R \tag{1}
\end{equation*}
$$

where $\kappa_{\lambda}$ is the emissivity of each dust grain, $B(T)$ is the blackbody function, $T$ is the dust temperature at distance $R$ from the star, and $\rho(R)$ is the dust density as a function of $R$. I will consider only a very simple model here, where there is no dust interior to $R_{0}$, where $R_{\max }$ is so large that there is no contribution from $B(T)$ at $24 \mu \mathrm{~m}$, and where $\rho$ is given by the assumption of uniform mass flow: $\rho \propto R^{-2}$. I also assume that $\kappa_{\lambda}$ is not a function of the temperature of the dust, i.e., that all the dust has the same amorphous or crystalline structure, whatever the temperature. With these assumptions, one can now write

$$
\begin{equation*}
F_{\lambda}=\text { constant } \times \kappa_{\lambda} \int_{R_{0}}^{R_{\max }} B(T) \rho_{0} R_{0}^{2} 4 \pi d R=\kappa_{\lambda} G_{\lambda} \tag{2}
\end{equation*}
$$

The dust temperature at any distance $R$ from a star of temperature $T_{*}$ and radius $R_{*}$ is given by

$$
\begin{equation*}
\int_{0}^{\infty} \frac{\pi a^{2} Q_{\mathrm{abs}} \pi B\left(T_{*}\right) 4 \pi R_{*}^{2}}{4 \pi R^{2}} d \lambda=\int_{0}^{\infty} 4 \pi a^{2} Q_{\mathrm{abs}} \pi B(T) d \lambda \tag{3}
\end{equation*}
$$

where $Q_{\text {abs }}$ is the efficiency for absorption or emission as a function of wavelength. The function $Q_{\text {abs }}(\lambda)$ for astronomical silicate given by Draine (1985) was used and $T_{*}=4000 \mathrm{~K}$. All further calculations used the normalized radius $R / R_{*}$. For example, for $T_{0}=1000 \mathrm{~K}, R_{0}=13 R_{*}$. A grid of normalized model dust shells $G_{\lambda}=F_{\lambda} / \kappa_{\lambda}$ was produced and is given in Figure 1 for temperatures $T_{0}$ at the inner radius $R_{0}$.

The advantage of this optically thin formalization is that now I can divide each observed dust spectrum (observed flux minus stellar continuum) by the model $G_{\lambda}=F_{\lambda} / \kappa_{\lambda}$ and the result is the normalized dust emissivity $\kappa_{\lambda}$. The temperature $T_{0}$ was chosen for each star so that the ratio $\kappa(\lambda=18 \mu \mathrm{~m}) /$ $\kappa(\lambda=10 \mu \mathrm{~m})$ is the same for all stars. This ratio $\kappa_{18} / \kappa_{10}$ was chosen to be 0.40 , the same ratio found by Draine and Lee (1984) for astronomical silicate, although there is no a priori reason why it should be correct. This proceedure is significantly different from that of other researchers, who either fit a blackbody to the low points at 8 and $13 \mu \mathrm{~m}$ (e.g., LittleMarenin and Little 1988) or who assume that the temperature at the inner edge of the shell $T_{0}$ is exactly the dust condensation temperature (e.g., Skinner and Whitmore 1987). The condensation temperature for silicate dust grains is in the range 1000 1500 K (Salpeter 1977). By assuming that the inner temperature can be less than the condensation temperature, I have effectively assumed that the mass loss, and hence the dust formation, is episodic at a time scale far longer than the pulsation period, and that such a dust formation episode has not happened recently for most stars. It is also possible that the silicate dust condensation temperature is colder than 1000 K (Onaka, de Jong, and Willems 1989a). The reason for this different procedure is that the purpose of this study is to investigate the amount of the dust opacity, especially at wavelengths longer than $13 \mu \mathrm{~m}$. Other authors sometimes assume the shape of the dust opacity. For example, Onaka, de Jong, and Willems


Fig. 1.-Sample dust shell models $G_{\lambda}$. The temperature $T_{0}$ is given for each model.
(1989a, 1989b) calculated very similar models with an assumed opacity from silicate and aluminum oxide dust in order to match the LRS spectra of Mira variables. Their fits from 9 to $13 \mu \mathrm{~m}$ are very good, but not as good from 13 to $23 \mu \mathrm{~m}$ because of their assumed opacity.

The temperatures $T_{0}$ are listed in Tables 1-5 and the resulting values of $\kappa_{\lambda}$ are plotted in Figures 2-7. Changing the assumed stellar continuum can change the derived value of $T_{0}$ by $\sim 0 \%-10 \%$. The effect on the derived $\kappa_{\lambda}$ is shown in Figure 2, where arbitrary amounts of stellar continuum have been subtracted from the spectrum of IRAS $19232+5008$. The changes are insignificant for $\lambda=9.5$ to $\sim 12 \mu \mathrm{~m}$ and $\lambda \gtrsim 16.5$ $\mu \mathrm{m}$, and only large at $\sim 8 \mu \mathrm{~m}$, where the stellar contribution is largest.

The largest value of $T_{0}$ is 1000 K for IRAS $19232+5008(\mathrm{CH}$ Cyg ) (Fig. 2). Since it is now thought that the maximum temperature to be found in a circumstellar dust shell is no more than 1000 K (Jura 1985), if the spherical dust shell model as given here is appropriate for CH Cyg, the ratio $\kappa_{18} / \kappa_{10}$ cannot be much larger than 0.40 . Of course, if the dust production is episodic instead of uniform, there might be much less dust at large distances from the star. Then dust shell models with approximately the same $F_{\lambda} / \kappa_{\lambda}$ as the models in Figure 1 would have much cooler temperatures $T_{0}$ at the inner radii $R_{0}$. If the ratio $\kappa_{18} / \kappa_{10}$ is as large as 0.45 , then $T_{0}$ for CH Cyg would be 1250 K for the spherical dust shell as described above, and if the ratio is 0.50 , then $T_{0}$ would be $\sim 1500 \mathrm{~K}$, which is probably too high.

Comparison of the different dust emissivities, $\kappa_{\lambda}=F_{\lambda} / G_{\lambda}$, shows that the appearances of the dust emissivities fall into five different groups. Within each group, the dust emissivities are similar to each other with the exception of the $8-9 \mu \mathrm{~m}$ region, where $\kappa_{\lambda}$ is very uncertain because of the uncertainties in the subtraction of the stellar continuum. The five groups of stars are listed in Tables 1-5 and Figures 3-7:

1. Stars with $\kappa_{\max }$ at $\lambda<10 \mu \mathrm{~m}$ and narrow $10 \mu \mathrm{~m}$ features (these include the well-studied stars $\mu$ Cep and PZ Cas (Forrest, McCarthy, and Houck 1979).
2. Stars with the $10 \mu \mathrm{~m}$ feature broader on the long wavelength side and $\lambda\left(\kappa_{\max }\right)<10 \mu$ m.
3. Stars with the $10 \mu \mathrm{~m}$ feature broader on the long wavelength side and $\lambda\left(\kappa_{\max }\right) \geq 10 \mu \mathrm{~m}$.
4. Stars with even broader $10 \mu \mathrm{~m}$ features than in Group 2 above, partially filling in the $14-16 \mu \mathrm{~m}$ region, and $\lambda\left(\kappa_{\max }\right)<10$ $\mu \mathrm{m}$.


Fig. 2.-Emissivities $\kappa_{\lambda}$ for CH Cyg with arbitrary amounts of stellar continuum subtracted. All the models had the same $T_{0}$.


Fig. 3.-Emissivities $\kappa_{\lambda}$ for Group 1. The temperature $T_{0}$ is given for each star.
5. Stars with the same broader line shape as found in Group 4 , but with $\lambda\left(\kappa_{\max }\right) \geq 10 \mu$ m like Group 3 .

The dust emissivities $\kappa_{\lambda}$ have been averaged for each group and plotted in Figure 8. In spite of the differences in the $10 \mu \mathrm{~m}$ feature, I conclude that the $18 \mu \mathrm{~m}$ features are essentially the same. This is true for high-temperature dust shells as well as low-temperature dust shells. [The fluctuations, or wiggles in the spectra, from 19 to $22 \mu \mathrm{~m}$ are probably not real but are probably due to poor cancellation of the LRS spectral response (ES), since they appear in sources with the silicate feature in both emission and absorption and in normal A-M stars without dust shells from LRS classes 18 and 19.]

## III. DISCUSSION

Simpson (1987) presented a similar analysis as in § II but using a simple $\lambda^{-1}$ emissivity (or $T \propto R^{-2 / 5}$ ). If we write that the total flux from a particle is proportional to the temperature to some power $\alpha$, then $\alpha=4$ for a blackbody and $\alpha=5$ for $\lambda^{-1}$ emissivity. For Draine's (1985) $Q_{\text {abs }}$ for a particle of radiius $a=0.1 \mu \mathrm{~m}$, we find that $\alpha$ ranges from 4.8 at $T=2000 \mathrm{~K}$ to 3.1 at $T=650 \mathrm{~K}$ to 6.1 at $T=50 \mathrm{~K}$. Thus it is not surprising that the values of $T_{0}$ in Tables $1-5$ compared to those tabulated by Simpson (1987) differ by $\leq 50 \mathrm{~K}$ for $T_{0} \lesssim 750 \mathrm{~K}$ and are greatly different ( 1000 K vs. 2000 K ) only for the hottest shells.

The chief reason that $\alpha$ falls below 4 in the temperature range $300-1300 \mathrm{~K}$ is that Draine's astronomical silicate is relatively transparent from $\sim 1$ to $8 \mu \mathrm{~m}$ compared to longer and shorter wavelengths (although not as transparent as terrestrial silicates; Draine and Lee 1984). Thus, the Wien side and the peak of the blackbody function contribute less to the total


Fig. 4.-Emissivities $\kappa_{\lambda}$ for Group 2
emission than they would for a gray body or a $\lambda^{-1}$ emissivity. There are questions as to the relative opaqueness ("dirtyness") of astronomical silicates (including interplanetary dustFrazier 1977; Murdock and Price 1985) compared to terrestrial silicates at visible and near-infrared wavelengths (e.g., Merrill 1979; Rogers et al. 1983). The problem is that if astronomical silicates are as transparent as terrestrial silicates at visible wavelengths, they will not absorb much visible light, and so will not become hot enough to radiate as much as is observed in the infrared. This problem even occurs in the 5-8 $\mu \mathrm{m}$ region, where $\mu$ Cep has excess emission (Russell, Soifer, and Forrest 1975). This excess emission cannot be due to dust if the silicates are similar to terrestrial silicates (Tsuji 1978 suggested the emission is due to $\mathrm{H}_{2} \mathrm{O}$ in the circumstellar dust
cloud), but it could be due to dust if the silicates are "dirty" enough (i.e., have a large enough imaginary part of the index of refraction $k$ ). It is certainly apparent from the figures that $\kappa_{\lambda}$ is not small at $8 \mu \mathrm{~m}$ (in spite of the uncertainty) or at $14 \mu \mathrm{~m}$, in contrast to laboratory measurements of silicates, and even Draine's (1985) astronomical silicate.

## a) Position of the $10 \mu \mathrm{~m}$ Peak

The first question is whether the different positions of the 10 $\mu \mathrm{m}$ silicate emission peak are real. Certainly subtracting a 3000 K stellar continuum shifts the apparent peak of the silicate feature to longer wavelengths, particularly if the feature is weak and the continuum strong (LRS classes 22-25). Moreover, dividing by one of the hotter model dust shells can also


Fig. 4-Continued
shift the apparent peak to longer wavelengths. However, 70\% of the stars in Groups 3 and 5 clearly show that the peak is at $10 \mu \mathrm{~m}$ or greater in the original LRS spectra. The wavelength calibration of each individual LRS spectra was determined from the edges of the filters (ES) rather than from the in-scan position of the source in the survey, which was less accurate. From perusal of the individual spectra in the LRS data base, I estimate that the wavelengths of point sources are good to about $\frac{1}{2}$ sample, or $\frac{1}{4}$ resolution element (one resolution element is $\sim 0.3 \mu \mathrm{~m}$ at $10 \mu \mathrm{~m}$ ). Moreover, each star consistently has the $10 \mu \mathrm{~m}$ feature peak at the same wavelength in all its individual spectra.

Cheeseman et al. (1989) have developed an artificial intelligence program for the automatic classification of LRS spectra. This program is also described by Goebel et al. (1989). Because they looked at all the stars in the LRS Atlas, their sample is much larger than that considered here. Their AutoClass program also finds a large class of stars with strong silicate emission features that peak at $\geq 10 \mu \mathrm{~m}$. As a class, these stars are much more confined to the Galactic plane than are the stars whose silicate features peak at less than $10 \mu \mathrm{~m}$. Thus, they conclude that the difference between the classes is real. Since the stars in Tables 1-5 were chosen on the basis of their good signal/noise, they are necessarily much brighter than average, and hence much closer. For this reason, many have Galactic latitudes $b \geq 10^{\circ}$. However, if distances above the Galactic plane are calculated for all the stars, under the assumption that asymptotic branch giants have the same absolute magnitudes at $8 \mu \mathrm{~m}$, the stars in Tables 2 and 4 have a significantly larger
dispersion about the Galactic plane than the stars in Tables 3 and 5.
b) Width of the $10 \mu \mathrm{~m}$ Emission Feature—Particle Size Effects

Forrest, Gillett, and Stein (1975) were the first to suggest that the different widths of the $10 \mu \mathrm{~m}$ feature are due to different dust particle sizes. Papoular and Pégourié (1983) published detailed computations showing this effect. Figure 9 shows normalized plots of $Q_{\text {abs }}$ ( $\kappa_{\lambda}$ as discussed here) calculated for spheres of radius $a$ using a Mie scattering program. Draine's (1985) values of the complex index of refraction for astronomical silicate were used. Because of the very sharp increase of the imaginary part of the index of refraction as $\lambda$ increases from 7.5 to $9 \mu \mathrm{~m}$, there is no broadening at the short wavelength side of the $10 \mu \mathrm{~m}$ feature for particles with $a$ up to $\sim 1 \mu \mathrm{~m}$. However, the $10 \mu \mathrm{~m}$ feature is substantially broadened on the long wavelength side as the individual particle becomes optically thick with increasing radius. Thus the idea that some dust shells contain particles larger than the canonical $0.1 \mu \mathrm{~m}$ is consistant with the LRS data.
c) Width of the $10 \mu \mathrm{~m}$ Emission Feature-Optical Depth Effects

A finite optical depth can also cause the $10 \mu \mathrm{~m}$ feature to appear broader than an optically thin feature. I will discuss this in terms of IRC + 10420 (IRAS 19244 +1115), a star in Group 4. Dyck et al. (1984), Ridgway et al. (1986), Cobb and Fix (1987), and Fix and Cobb (1988) present speckle interferometer measurements of the diameter of IRC +10420 from 2 to $10 \mu \mathrm{~m}$. The short-wavelength measurements mainly see the star and


Fig. 5.-Emissivities $\kappa_{\lambda}$ for Group 3
hot dust, but the $10 \mu \mathrm{~m}$ measurements are mainly of the dust shell. A good fit must include both the absolute flux at all wavelengths and the angular diameter. The simple spherical dust shell models cannot, in fact, fit all the interferometry data at all wavelengths, but consideration of them is very instructive.

The dust shell models were calculated for the slightly optically thick case with no interstellar absorption. In the slightly optically thick approximation, the dust in the shell absorbs the infrared radiation on its way to the telescope, but the temperature structure of the dust shell is the same as the temperatures in the optically thin case. (Under a detailed radiative transfer calculation, the dust temperatures at large radii from the star would be cooler than under the slightly optically thick approximation, but about the same at $R_{0}$.) The largest optical
depth considered here was $\sim 1.2$ between the star and the outside of the shell at the $10 \mu \mathrm{~m}$ peak. Figure 3 of Jones and Merrill (1976) plots dust temperatures for models with $\tau_{10 \mu \mathrm{~m}}=6.0$, a more extreme case than considered here. For these models, $\kappa_{\lambda}$ was the value derived for the Group 1 emission stars (Fig. 8).
For each model, I calculated the visibility amplitude at 8.7 and $9.8 \mu \mathrm{~m}$, the wavelengths of the best data of Fix and Cobb (1988). The appropriate visibility function is given by a onedimensional Fourier transform of the strip intensity distribution, since the observations were made by scanning a slit (Ridgway et al. 1986); it is necessary to include the point source contribution of the star itself in the strip intensity. The dust shell models do not have the appearance of a Gaussian (the visibility function of a Gaussian is also a Gaussian and is


FIg. 6.-Emissivities $\kappa_{\lambda}$ for Group 4
commonly fit to observed data for the estimation of a source size), but it is better described by a narrow core with a halo. The corresponding visibility function is much broader than that of the Gaussian with the same full width at half-maximum (FWHM) and it is relatively flatter, especially when an additional point source is included. Good fits to the visibility functions at all wavelengths could not be obtained unless a fairly substantial point source was added to the source profile. This point source could include emission from very hot dust in the immediate vicinity of the star, as well as the star itself. Also, the FWHM of the model had to be larger than the FWHM of the Gaussian that best fit the visibility data in order to produce enough total flux. A reasonable (but not unique) fit could be found to both the visibility data of Fix and Cobb (1988) at 8.7 $\mu \mathrm{m}$ (except for the two highest frequency points) and the IRAS data for the model with $T_{0}=350 \mathrm{~K}$ and $\mathrm{FWHM}=0.45$. However, the visibility data at $9.8 \mu \mathrm{~m}$ have the same width as at $8.7 \mu \mathrm{~m}$, and thus the larger contribution from the cool dust at large distances from the star requires a point source contribution of $\sim 33 \%$ for a good fit. The $10 \mu \mathrm{~m} \mathrm{~N}$ filter visibility amplitude measurement of Cobb and Fix (1987) is somewhat narrower, but still wider than the calculated amplitude, and thus point source contributions must be included. The star does not contribute more than $5 \%-12 \%$ of the total flux at these wavelengths if the continuum at $4.8 \mu \mathrm{~m}$ (Thomas, Robinson, and Hyland 1976) is entirely due to the star, and less if the $4.8 \mu \mathrm{~m}$ continuum includes contributions from the dust shell (Forrest, McCarthy, and Houck 1979; Ridgway et al. 1986). Ridgway et al. (1986) also had to add a $40 \%$ point source contribution to match the visibility data at $5 \mu \mathrm{~m}$. The flux scale
for IRC +10420 in the LRS Atlas is about $25 \%$ lower than the IRAS $12 \mu \mathrm{~m}$ point source flux (IRAS Point Source Catalog, Version 2 1988). If the model used the $12 \mu \mathrm{~m}$ point source catalog flux, as suggested by Volk and Cohen (1989b), instead of the lower LRS flux, an additional point source contribution of $30 \%$ would be necessary to match the $8.7 \mu \mathrm{~m}$ visibility and $50 \%$ for the $9.7 \mu \mathrm{~m}$ visibility. We note that at $20 \mu \mathrm{~m}$, the LRS flux is much smaller than the flux observed by Forrest, McCarthy, and Houck (1979).

A possible reason for the discrepancy between the observations and the model predictions is that the smooth, spherically symmetric dust shell model is too simple. Other models are discussed by Cobb and Fix (1987). For example, some dust much hotter than 350 K must be present to account for the visibility measurements at $5 \mu \mathrm{~m}$ of Ridgway et al. (1986). Also, Bowers (1984) prefers a clumpy distribution from VLA observations of OH masers.

The dust shell spectrum of IRC +10420 produced by this model is plotted in Figure 10, along with the observed LRS data. It is seen that with the addition of some optical depth, the narrow Group 1 opacity produces a $10 \mu \mathrm{~m}$ feature that is as wide as the observed Group 4 feature. I conclude that, at least in some cases, a broad $10 \mu \mathrm{~m}$ feature can be caused by optical depth effects. In other cases, where the star can be clearly seen at visible wavelengths, either the dust is not optically thick or the geometry is not represented by a uniform spherical shell.

## d) Other Variations

It is clear that there must be some variation in the chemical composition of the dust shells in order to explain the variations


Fig. 7.-Emissivities $\kappa_{\lambda}$ for Group 5
in the wavelength and width of the peak of the $10 \mu \mathrm{~m}$ feature. For example, the dust emission feature in the Trapezium of the Orion Nebula (Forrest, Gillett, and Stein 1975) has the same peak as the Group 1 feature as seen in $\mu$ Cep with no broadening on the long-wavelength side (as might be explained by particle size effects or optical depth effects), but it is broader on the short-wavelength side. (We note that, compared to the 10 $\mu$ m peak, the LRS spectrum of $\mu$ Cep has more emission at 13 $\mu \mathrm{m}$ than the spectrum of Russell et al. 1975; the short wavelength continuum is also higher.) (The $10 \mu \mathrm{~m}$ feature would be narrower on the long wavelength side if a colder dust model were used, and the $\kappa_{18} / \kappa_{10}$ ratio were less than 0.35 .) Two stars in Group 1 with this short a wavelength cut-on are NML Tau and WY Vel. A star with a much narrower feature at $10 \mu \mathrm{~m}$ is $\alpha$


Fig. 8.-The emissivities averaged for each group


Fig. 9.-Mie calculations of $Q_{\text {abs }}$ for silicate particles of radius $a$ in $\mu \mathrm{m}$. Note that the $10 \mu \mathrm{~m}$ feature is broadened at longer wavelengths as the particle size increases.

Ori (IRAS $05524+0723$ ); since there is only one member in this "group," it was not included in this study. However, the 18 $\mu \mathrm{m}$ feature does look like that of the other stars if a model with $T_{0}=450 \mathrm{~K}$ is used. With such a low temperature, it is not necessary to require a chromosphere (Skinner and Whitmore 1987) to get a reasonable flux at $20 \mu \mathrm{~m}$.

The silicate dust emission feature in the Trapezium (Forrest, Gillett, and Stein 1975) is very similar to the feature in Group 1 (the narrowest) from 9.7 to $13 \mu \mathrm{~m}$ but is broader from 8 to 9.7 $\mu \mathrm{m}$. The absorption coefficient derived from the Trapezium emission feature gives good agreement with the $10 \mu \mathrm{~m}$ absorption in protostars, molecular clouds, and $\mathrm{H}_{\text {II }}$ regions (Gillett et al. 1975; Willner 1977). However, Roche and Aitken find that the interstellar absorption toward the Galactic center (1985) and toward Wolf-Rayet stars (1984) is better given by the shape of the $\mu$ Cep feature. Although there is a lot of variation among the groups from 8 to $15 \mu \mathrm{~m}$, the $18 \mu \mathrm{~m}$ feature is very similar in all the groups. It is possible, then, that the interstellar absorption coefficient from $12-23 \mu \mathrm{~m}$ could be described by the shape of the $\kappa_{\lambda}$ derived for Group 1.

The exact chemical composition of the dust is not known. Laboratory measurements of amorphous silicates (Day 1979,


Fig. 10.-LRS spectrum of IRC +10420 . The upper solid curve is the total observed flux (scaled by $3.521 \times 10^{11} \mathrm{~W} \mathrm{~m}{ }^{-2} \mu \mathrm{~m}^{-1}$ ) and the lower solid curve is the flux of a 6000 K blackbody, fit to the measured flux of Thomas, Robinson, and Hyland (1976) at $4.8 \mu \mathrm{~m}$. The dashed line is the slightly optically thick dust shell model described in the text plus the 6000 K blackbody (the stellar contribution).
1981) show broad, smooth features that peak at $9.5-10 \mu \mathrm{~m}$ and 19-20 $\mu \mathrm{m}$. Likewise, carbonaceous chondrite meteorites and hydrated silicates (Knacke and Krätschmer 1980) have $10 \mu \mathrm{~m}$ features that look similar to the features described here, but the $18 \mu \mathrm{~m}$ feature peaks at a much longer wavelength than is observed in the circumstellar dust shells and shows considerable structure as well. Pégourié and Popular (1985) and Papoular and Pégourié (1986) discuss the formation of dust grains of these materials. Onaka, de Jong, and Willems (1989a, b) and Stencel et al. (1990) suggest that young dust shells contain aluminum oxide, which has a smooth feature at $12 \mu \mathrm{~m}$. This feature could be the cause of the apparent "broadening" of the $10 \mu \mathrm{~m}$ feature, instead of particle size or optical depth.

## IV. SUMMARY

The IRAS LRS Atlas was searched for stars with strong well-defined silicate emission features at 10 and $18 \mu \mathrm{~m}$. These spectra were divided by synthetic spectra from simple models of optically thin spherical dust shells with the emissivity function $\kappa_{\lambda}$ removed from under the integral sign. The result was the normalized emissivities $\kappa_{\lambda}$, which were plotted and compared. The $18 \mu \mathrm{~m}$ features appear basically the same in all the stars, but there are differences in the $10 \mu \mathrm{~m}$ features. The $10 \mu \mathrm{~m}$ features vary in width and the position of the peak varies from $<9.7$ to $\sim 10 \mu \mathrm{~m}$. The stars with the $10 \mu \mathrm{~m}$ peak at longer wavelength seem to be more closely confined to the Galactic plane. At the resolution of the LRS, there is never any substructure visible in either the 10 or $18 \mu \mathrm{~m}$ features that would indicate crystalline structure for the dust, regardless of the temperature of the dust in the shell.

Simple models were calculated for slightly optically thick dust shells. These models have broader $10 \mu \mathrm{~m}$ features than the optically thin models; such models can be fit to at least some of the stars whose dust shells have broad $10 \mu \mathrm{~m}$ features, although the broadening caused by a finite optical depth occurs on both the short and the long wavelength side of the feature. On the other hand, Mie calculations of the absorptivity (emissivity) $Q_{\text {abs }}$ for Draine's (1985) astronomical silicate shown broadening only on the long wavelength side of the 10 $\mu \mathrm{m}$ peak as the particle radius increases from $\sim 0.25 \mu \mathrm{~m}$ to $>1.0 \mu \mathrm{~m}$. Thus the different shapes could be explained if the different stars have a variety of particle sizes or optical depths. There must also be a variation in chemical composition as well.

The observed interstellar extinction in diffuse regions (such as is found between the Sun and the Galactic center) is welldescribed by the shape of the $10 \mu \mathrm{~m}$ feature as found in the group with the narrowest feature (Group 1). The peak is at $\sim 9.7 \mu \mathrm{~m}$. Thus, the $\kappa_{\lambda}$ for $12-23 \mu \mathrm{~m}$ for the Group 1 stars may also be a good representation for the interstellar extinction for those wavelengths, especially since the $18 \mu \mathrm{~m}$ feature does not vary even when the $10 \mu$ m feature varies.

This work was supported (in part) by the $I R A S$ General Investigator Program and by NASA Ames Research Center Interchange Grant NCC2-548. I thank P. Swann and G. Villere for their help in accessing the LRS data before publication of the Atlas, and G. Augason, M. Cohen, J. Goebel, R. Rubin, and H. Walker for helpful discussions, and P. Wesselius for providing the Gronigen LRS data base. Finally, I thank the anonymous referee for his thoughtful comments.

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