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RAPID INFRARED PHOTOMETRY OF PULSATING Ap STARS: A MEASUREMENT OF STELLAR LIMB DARKENING[1]

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ABSTRACT

K band photometry of the rapidly oscillating Ap star HR 3831 (HD 83368) sets an upper limit on its infrared amplitude which lies significantly below the expectation from the star's optical pulsations. We demonstrate that this discrepency can be explained by weighting of the star's $(l, m) = (1, 0)$ mode by wavelength-dependent limb darkening. Our results for this cool SrCrEu star indicate that the dependence of the limb-darkening coefficients on wavelength is much steeper than for the solar atmosphere, which also implies a steeper T-τ curve. Simultaneous multibandpass photometry of HR 3831 in the visible and infrared should allow us to specify its limb darkening coefficients as a function of wavelength. This would lead to the first *empirical* T-τ relation for the atmosphere of a star other than the Sun.

Subject headings: infrared: general — photometry — stars: atmospheres — stars: individual (HR 3831) — stars: peculiar A — stars: pulsation

1. INTRODUCTION

Models of the atmospheres of Ap stars are complicated by their peculiar abundances, which introduce severe line blanketing and flux redistribution, and by the effects of magnetic pressure (e.g., Hubený 1986). Attempts to match theory to observation are further confused by the uneven surface distribution of elements, which results in spectroscopic variations as these stars rotate. For these reasons and others, the effective temperatures, surface gravities, and atmospheric structure of Ap stars are not well defined. Analyses of the eigenfrequency spectra of rapidly oscillating Ap (roAp) stars (Kurtz 1990)—which pulsate in high-order low-degree *p*-modes with periods of a few minutes and amplitudes of millimag—are already helping to define the radii, global magnetic fields, and other properties of the coolest Ap stars (e.g., Matthews, Kurtz, & Wehlau 1987; Kurtz, Shibahashi, & Goode 1990). In this *Letter*, we introduce a way in which rapid photometry of an roAp star in the visible and infrared may also define its atmospheric structure from estimates of the limb darkening at various wavelengths.

The roAp stars undergo amplitude modulation and phase shifts which are correlated with their rotation periods and show fine-splitting of their oscillation frequencies in Fourier spectra. These effects are best explained by the oblique pulsator model (Kurtz 1982), in which the pulsation is dominated by a mode of $(l, m) = (1, 0)$ aligned with the oblique magnetic field axis of the star. Two aspects of the roAp variables which have not been explained are the steep drop in oscillation amplitude with increasing wavelength observed in optical photometry (e.g., Weiss & Schneider 1984; Kurtz et al. 1989) and anomalous phase lags between the light and color curves (Kurtz 1982; Watson 1988).

Weiss et al. (1990) were the first to obtain rapid infrared photometry of an roAp star to study these amplitude and phase relationships at even longer wavelengths. Their simultaneous *K* and Walraven *VBLUW* measurements of α Cir set an upper limit of 0.49 mmag on the *K* semiamplitude, while they detected clear oscillations in *B* at 1.52 mmag. Unfortunately, the *B* amplitude on that night was too low, and the *K* upper limit too high, to establish whether the amplitude trend seen in the visible extends to the infrared. We decided to make infrared observations of several roAp stars which are known to vary with much larger optical amplitudes.

2. OBSERVATIONS AND FREQUENCY ANALYSIS

We monitored three cool Ap stars in the *K* band with the ESO 1.0 m telescope and InSb photometer during 1990 January 11–13 UT. The integration time was 1 minute, and the photometry was nondifferential to allow the rapid sampling rate. Two of the program stars—HR 3831 and HR 1217—are known roAp stars in the visible, with periods near 12 and 6 minutes, respectively; the third star—HR 3980—had shown signs of unusual *K* variability when observed by Kroll and Catalano (cf. Kroll 1987). We accumulated 10.6 hr of photometry of HR 3831, 6.3 hr of HR 1217, and 4.2 hr of HR 3980.

Fourier amplitude spectra of the entire set of rapid photometry for each of the stars are shown in Figure 1; the vertical lines indicate the known oscillation frequencies for the two pulsators. We detect no *K* oscillations at those frequencies (or any others) above the noise levels. (Note that the full vertical scale for each panel in Fig. 1 is only 2 millimag.) The lowest noise level was obtained for HR 3831, which was monitored the longest and through smaller air masses.

[1] Based on observations obtained at the European Southern Observatory, La Silla, Chile.

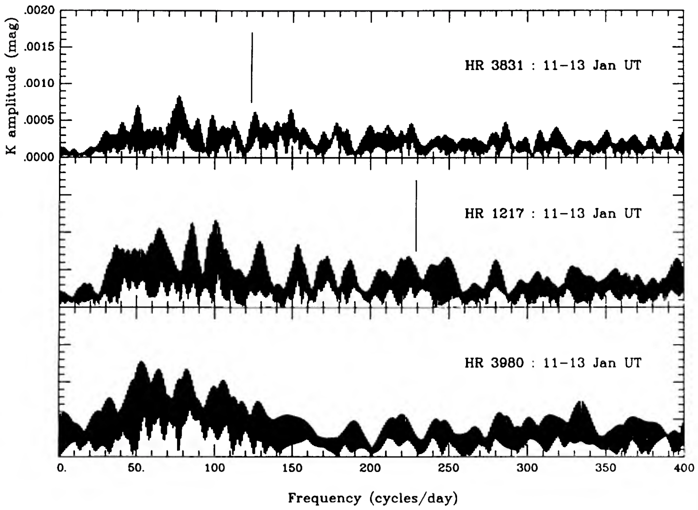

FIG. 1.—Fourier amplitude spectra of K band photometry of two known roAp variables (*upper panels*) and another cool Ap star (*bottom*). The optical oscillation frequencies of the two roAp stars are marked.

2.1. *HR 3831: A Significant Null Detection*

This roAp star has a well-defined optical oscillation with a period near 11.67 minutes ($\nu = 1.42801$ mHz $= 123.380$ cycles day^{-1}) whose amplitude and phase are modulated with the star's relatively short magnetic cycle of $2^{\mathrm{d}}.852$. Its variations—like those of other roAp stars—can be explained as an oblique $(l, m) = (1, 0)$ mode aligned with the magnetic dipole axis. The inclination ($\geq 80°$) and magnetic obliquity ($\sim 40°$) of HR 3831 are constrained by the magnetic variation curve (Thompson 1983), accurate measurements of the rotation period (Renson et al. 1984) and $v \sin i$ (Carney & Peterson 1985), and application of the oblique pulsator model to the oscillation frequency spectrum (Kurtz et al. 1990).

Kurtz & Shibahashi (1986) have published amplitude modulation curves of the B band oscillations of this star based on data obtained over 4 years apart (see their Figs. 1 and 11). These curves show no change in the modulation behavior over that time. The clear-cut modulation ephemeris allows us to specify the B oscillation amplitudes of HR 3831 during our infrared run, despite the lack of simultaneous visible photometry. We caution that, if the modulation behavior of HR 3831 has somehow changed in the time since the last observations described by Kurtz & Shibahashi (1986), then this would affect our results below. However, the available published data which now span 1980–1986 (Kurtz et al. 1990) and more recent unpublished photometry by J. M. M. support our assumption that the optical amplitude still follows the same cycle. (The other roAp star in our program, HR 1217, also follows a modulation cycle [P $= 12^{\mathrm{d}}.457$], but it undergoes beating among at least six closely spaced frequencies and exhibits other secular amplitude changes [Kurtz et al. 1989] which make its B amplitude unpredictable without simultaneous optical photometry.)

Because the pulsations of an $l = 1$ nonradial mode do not affect the area of the projected stellar disk, we attribute the light oscillations solely to changes in T_{eff} and estimate the temperature variation required to produce the observed B amplitude (assuming the star behaves as a blackbody). We can then predict the K amplitudes that should arise from the $l = 1$ mode in HR 3831 on each of the three nights. Our first night of

TABLE 1

EXPECTED TEMPERATURE AND K VARIATIONS OF HR 3831 FOR $\Delta B_{\mathrm{tot}} = 0.008 \pm 0.001$

T_{eff} (K)	ΔT (K)	ΔK_{tot} (± 0.0003)
9000............	19 ± 2	0.0030
8500............	17 ± 2	0.0029
8000............	15 ± 2	0.0028
7500............	13.5 ± 1.5	0.0027
7000............	12 ± 1	0.0026
Observed upper limit...	...	$\leq$0.0008–0.0014

data (January 11 UT) falls close to the phase of B amplitude maximum, and hence, K maximum for the same mode. Table 1 lists the values of ΔT_{eff} and full K amplitude expected for the B amplitude ($\Delta B \simeq 0.008 \pm 0.001$) on that night and a reasonable range of effective temperatures for a cool SrCrEu star. In Figure 2, we present the nightly Fourier spectra of our K photometry of HR 3831; on the first night, the predicted K semi-amplitude of ~ 0.0014 lies well above the upper limit of ~ 0.0005 set by the observations at the known oscillation frequency. Why is the actual infrared amplitude so small?

3. LIMB DARKENING AND THE PULSATION OF HR 3831

Figure 3 illustrates how limb darkening can affect the net amplitude of an $(l, m) = (1, 0)$ mode. The solid curves show the relative amplitude of temperature variation plotted as a function of latitude θ (and fixed longitude $\phi = 0°$) on the projected stellar surface, for a normal mode where

$$\Delta T/T(\theta, \phi) \propto P_l^m(\cos \theta) e^{im\phi} . \quad (1)$$

In the upper panel, the mode is seen pulsation pole–on and the pattern is symmetric about the center of the disk. In the lower panel, the pulsation pole is inclined by 50°, approximating the geometry inferred for HR 3831 at peak oscillation amplitude. (For this orientation, the plotted curve is valid only along a line of $\phi = 0°$ on the disk.)

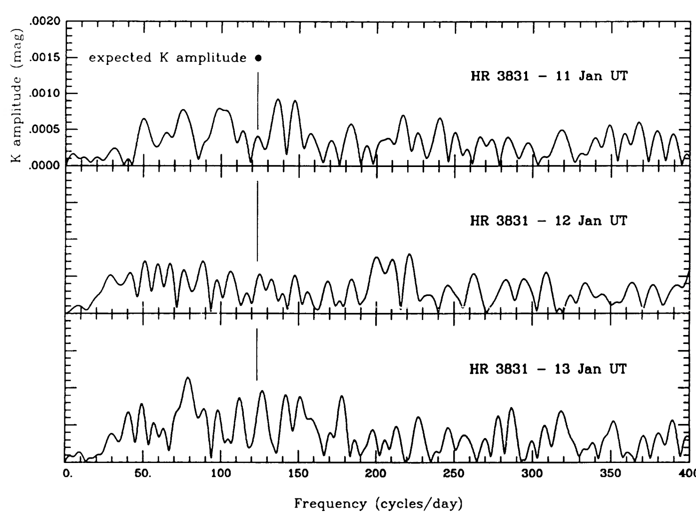

FIG. 2.—Fourier amplitude spectra of HR 3831 on each of the three nights. The known oscillation frequency is marked, as is the K amplitude we predict for the first night, when the B amplitude is near maximum. The scale is the same as Fig. 1.

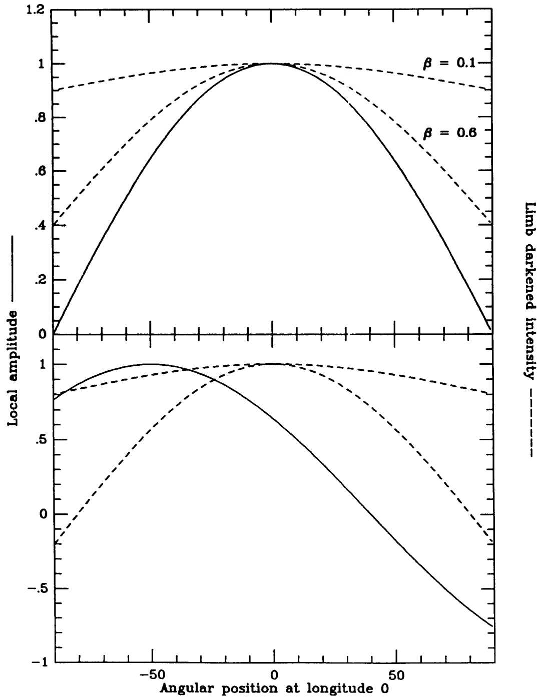

FIG. 3.—Local amplitude of temperature variations across the stellar disk (*solid curves*) of an $(l, m) = (1, 0)$ mode, normalized to standard limb-darkening curves with coefficients 0.1 and 0.6 (*dashed curves*). The upper panel shows the mode projected pole-on and is valid for any line across a disk diameter. In the lower panel, the pulsation pole is inclined by 50° and the plot applies only to longitude 0°. Note that the lower plot includes negative amplitudes where the mode undergoes a 180° phase shift.

Shown as dashed lines in both panels of Figure 3 are standard limb-darkening curves

$$I(\theta)/I_{\rm central} = 1 - \beta(1 - \cos\theta) \qquad (2)$$

with coefficients $\beta = 0.1$ and 0.6. In the pole-on case, stronger limb darkening at shorter wavelengths gives systematically lower weight to the edges of the disk pulsating at low amplitude. The situation is more complicated for the geometry of HR 3831, but the lower panel reveals how limb darkening suppresses the exposed part of the other pulsation hemisphere of the $(l, m) = (1, 0)$ mode which is pulsating in antiphase (indicated by negative amplitude in the plot).

In fact, by integrating the local pulsation amplitude of the mode across a limb-darkened stellar disk, we find that this effect significantly enhances the net amplitude at shorter wavelengths where limb darkening is more pronounced. Table 2 shows how different limb-darkening coefficients β_λ increase the ratio of $\Delta B/\Delta K$ in HR 3831 at amplitude maximum, compared to when the effects of limb darkening are ignored.

With only two bandpasses and just an upper limit on ΔK, we cannot specify β for each wavelength region. Several combinations of β_λ are consistent with the observations (see Table 2). However, our lower limit on $\Delta B/\Delta K$ indicates that the dependence of limb darkening on wavelength is steeper than for a standard atmospheric model. This is consistent with the findings of Shibahashi & Saio (1985), who suggested that the presence of oscillation frequencies in roAp stars above the critical values for the acoustic cavities of those stars could be explained by a steeper dependence of temperature on optical depth in their atmospheres. The T-τ gradient of their analytical

TABLE 2

EFFECTS OF WAVELENGTH-DEPENDENT LIMB DARKENING ON THE *B* AND *K* AMPLITUDES OF HR 3831

LIMB-DARKENING COEFFICIENTS		
(440 nm)	(220 nm)	$\Delta B/\Delta K$
0.6	0.1	6.0
0.6	0.0	6.6
0.7	0.1	7.7
0.7	0.0	8.5
0.8	0.1	11.0
0.8	0.0	12.1
No limb darkening....		2.9
Observed ratio.........		$\geq 7.6 \pm 2.6$

NOTE.—$i = 86°$, $\beta = 36°$, $T_{\rm eff} = 800 \pm 1000$ K, $l = 1, m = 0$.

"model B" compared to a standard Eddington atmosphere would produce a larger difference in the limb-darkening curves at visible and infrared wavelengths, in agreement with our results.

4. DISCUSSION AND PROSPECTS FOR THE FUTURE

Our tests confirm that limb darkening enhances the observed amplitude of an $(l, m) = (1, 0)$ mode in any orientation except equator-on, where the net amplitude is zero. Since the pulsations of most roAp stars appear to be dominated by this mode, the limb-darkening effect cannot be ignored when treating relative amplitudes in multicolor photometry.

Are there other factors which could explain the deficit in infrared amplitude? H. Shibahashi (1990, private communication) has pointed out the possibility of acoustic attenuation, which would occur if the atmospheric layer contributing most of the *K* flux were to lie above the acoustic cutoff level where the *p*-mode pulsations become evanescent. He estimates that the most severe attenuation of *K* amplitude would be no more than a factor of 2, while we find standard "solar" limb darkening will reduce the amplitude by *at least* a factor of 2. Also, if attenuation is to be important, the *K* band flux at 2.2 μm must emerge from above the acoustic barrier, which for normal A type stars lies at an optical depth of order 10^{-4}. Even allowing for the peculiarities of an Ap star, this is very high above the photosphere. Furthermore, acoustic attenuation cannot easily explain the relative amplitudes of roAp stars at optical wavelengths, where all the sources of flux should be well below the barrier.

The range of limb-darkening coefficients implied by our infrared results can qualitatively explain the *UBV* amplitudes of HR 3831 measured by Kurtz (1982). However, his published amplitudes are averages from six to eight nights of data, covering more than two modulation cycles, so we cannot properly apply our model to the amplitude ratios. The limb-darkening effect at different wavelengths may also help explain the unusual light and color phase lags reported by Kurtz (1982) for HR 3831 and other roAp stars. We intend to determine oscillation phases at different bandpasses and mode orientations in future simulations.

The preliminary reults indicate that simultaneous rapid photometry of an roAp star in many optical and infrared bandpasses (e.g., *UBVRI JHKL*) during oscillation maximum should enable us to measure limb-darkening coefficients at each wavelength and directly construct an explicit T-τ relation for the atmosphere of an Ap star in the same way as is done for the Sun.[2] HR 3831 is the best candidate because it is bright ($V = 6.2$; $K = 5.5$), it has a relatively large peak amplitude, it reaches that peak twice during its 3 day rotation cycle, and its mode geometry is one of the best constrained of any of the roAp pulsators. Even a short observing run will provide good phase coverage of its rotation period, to study how the atmospheric properties relate to variations in surface abundance. The results would have only limited relevance to normal A stars but would fill many gaps in our understanding of the upper atmospheres of magnetic chemically peculiar stars. We are currently planning a campaign of multicolor photometry of HR 3831 to apply our limb-darkened oscillation models toward a T-τ relation for this star.

[2] In principle, studies of eclipsing binary stars could also lead to empirical estimates of limb darkening; however, such efforts have not yet produced definitive results, and in any case, there are no known Ap stars in eclipsing systems.

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