

BERYLLIUM IN THE GALACTIC HALO: SURFACE ABUNDANCES FROM STANDARD, DIFFUSIVE, AND ROTATIONAL STELLAR EVOLUTION, AND IMPLICATIONS

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ABSTRACT

The recently observed upper limits to the beryllium abundances in Population II stars are much lower than Population I detections. We show that this difference reflects an intrinsic difference in the initial abundances and is not caused by different degrees of depletion driven by stellar evolution processes from similar initial abundances. We have constructed evolutionary sequences of models from the early pre-main sequence to beyond the turnoff that correspond to halo dwarfs with $[\text{Fe}/\text{H}] = -1.3, -2.3, \text{ and } -3.3$. We have considered standard, diffusive, and rotational mechanisms to estimate a maximal possible beryllium depletion. We find that halo star models in the range $6000 \geq T_{\text{eff}} \geq 5000$ K might be rotationally depleted by a factor of 1.5–2, and that the total depletion should be no more than (conservatively) a factor of 3. Implications for cosmology, cosmic-ray theory, and Galactic chemical evolution are discussed.

Subject headings: cosmology — early universe — stars: abundances — stars: interiors — stars: Population II — stars: rotation

1. INTRODUCTION

Knowledge of the primordial beryllium abundance constrains big bang cosmology; the beryllium abundance as a function of time provides an important test of cosmic-ray theory and Galactic chemical evolution. So far, models of beryllium production during big bang nucleosynthesis (BBN) have yielded much lower Be^2 abundances than those observed in Population I stars. Thus, according to these scenarios it is expected that the Be abundance has evolved. Ryan *et al.* (1990) have recently placed stringent upper limits on the Be abundance in old halo stars. *We use stellar evolutionary models in this Letter to interpret the observations as providing compelling evidence that the beryllium abundance has evolved since the inception of our Galaxy.*

The light elements Li, Be, and B can be synthesized during big bang nucleosynthesis, and their absolute and relative primordial abundances restrict the classes of BBN models allowed. Standard BBN predicts a smaller primordial Be abundance than can currently be observed. However, Witten (1984) proposed that a quantum chromodynamic (or quark-hadron) phase transition in the early universe may have produced density inhomogeneities. Subsequent inhomogeneous nucleosynthesis can perhaps produce an observable Be abundance (e.g., Boyd and Kajino 1989). Primordial beryllium is best sought for in the oldest known stars, the halo stars of our Galaxy, in which possible enrichment of beryllium from other sources (e.g., cosmic rays) is minimal.

The light elements Li, Be, and B are difficult to create in stars; nevertheless, mechanisms that might potentially enrich their abundances during the lifetime of the Galaxy exist. In particular, it has long been thought that interactions between cosmic rays and elements in the interstellar medium (such as He, C, N, and O) could produce the stable isotopes ^6Li , ^7Li , Be, and $^{10,11}\text{B}$ (e.g., Walker, Mathews, and Viola 1985). Cosmic rays have been widely accepted as the primary source of ^6Li ,

Be, and, ^{10}B because of the agreement between the predicted ratios and meteoric data. Stellar data can also be used to test cosmic-ray theory. In fact, observations of the relative abundances of Be and B in a few Population I stars (e.g., Boesgaard and Heacox 1978) have supported a cosmic-ray origin for Be and B. However, the observational uncertainties were as large as factors of 2–3, it was not possible to separate the $^{10,11}\text{B}$ isotopes, and ^6Li was not observed. Thus, while encouraging, *these tests of cosmic-ray theory cannot be regarded as definitive.* (See also Griffin and Griffin 1985.)

Furthermore, overabundances of meteoric ^{11}B , and especially ^7Li , imply the existence of other sources for these isotopes; other sources have also been proposed for ^6Li , Be, and ^{10}B (for a review see § 7.2 in Deliyannis 1990). Most importantly, currently there is no information on whether Be and $^{10,11}\text{B}$ have evolved throughout Galactic history with the predicted cosmic-ray ratios. Therefore, new observations of higher resolution (which are now possible) are required to test thoroughly the predictions of cosmic-ray theory (see also § IV), and the possibility must be considered that Be and $^{10,11}\text{B}$ (and therefore ^6Li) might have significant sources other than cosmic rays.

Beryllium may also act as a discriminant between models of chemical evolution of the Galaxy (e.g., discussion in Reeves and Meyer 1978). It is clear that to test Galactic chemical evolution and cosmic-ray theory effectively, observations of beryllium in stars with a spectrum of ages and running through the full gamut of observed metallicities are necessary. The metal-poor stars are especially important for cosmology.

However, the surface light element abundances can be particularly sensitive to interior depletion and/or enrichment mechanisms. Therefore, because it is the *initial* stellar abundances that are of interest, *it is imperative to consider the evolution of light element surface abundances following the formation of the stars.* (Note that some Population I F stars are severely Be-depleted; see § II). To establish that the Be abundance has evolved as a function of time, it is necessary to show that the difference between the Population I and Population II Be abundances is not caused by different degrees of depletion from

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² “Be” shall refer to the only stable isotope, ^9Be .

similar initial abundances. We now have the stellar evolutionary tools to begin making a quantitative analysis. In addition to investigating the manner in which standard models might affect surface light element abundance evolution, we address possible effects of stellar rotation (and in particular of rotationally induced mixing), and of microscopic diffusion.

II. CURRENT STATUS OF THE OBSERVATIONS

We begin by summarizing the status of Population I Be abundances, and then look at the Population II case. For the Sun, we have both the meteoric abundance (which measures the initial solar nebula value of 4.5 Gyr ago) and the current solar photospheric abundance. $[\text{Be}]_{\text{meteoric}}^3$ is consistently determined from different methods as 1.42 ± 0.04 (2 σ); $[\text{Be}]_{\odot} = 1.15 \pm 0.1$ (see discussion in Anders and Grevesse 1989, Chmielewski, Müller, and Brault 1975, and references therein). Be abundances in the Hyades dwarfs range from slightly below solar to as little as about \sim one-fifth solar in some F stars, with an observational uncertainty as large as 0.3–0.5 dex in $[\text{Be}]$ (e.g., Boesgaard and Budge 1989). Many of the observed abundances in Population I field stars (e.g., Boesgaard 1976) cluster around the solar value, with a few field stars as high as the meteoric value. There also exists a population of *very* Be-deficient field F stars. Such a possible depletion pattern, *which is consistent with rotational models* (Deliyannis and Pinsonneault 1990), is much more subtle than the observed Li depletion patterns. How do Be abundances in disk stars compare with those in halo stars?

Observations of Be in metal-poor stars were pioneered by Molaro and Beckman (1984) and Molaro, Beckman, and Castelli (1984), who used *IUE* spectra to obtain relatively low upper limits ($[\text{Be}] \sim +0.5$). Rebolo *et al.* (1988) obtained similar upper limits for other very metal-poor stars and detections (of order $[\text{Be}] = 0.0$ – 0.4) in some moderately metal-poor stars. Note, however, that the uncertainties inherent in *IUE* spectra are large. Furthermore, using spectra of higher quality than is possible to obtain from *IUE*, Budge, Boesgaard, and Varsik (1988) claimed a higher abundance ($[\text{Be}] = 0.85 = \frac{1}{2}$ solar; Boesgaard 1990) in one common star, HD 76932, which is indistinguishable from Population I abundances in spite of the smaller uncertainty (0.3–0.5 dex) relative to *IUE*.

Nevertheless, the high-resolution work of Ryan *et al.* (1990) yields upper limits of $[\text{Be}] < +0.1, -0.2, -0.3$, and -1.2 in HD 134439, HD 74000, HD 19445, and HD 140283, respectively. These stars were all classified⁴ as bona fide halo dwarfs by Deliyannis, Demarque, and Kawaler (1990, hereafter DDK). Clearly, these are lower than observations of Be in Population I stars that have formed at or near the present epoch.

III. BERYLLIUM DEPLETION IN STANDARD, DIFFUSIVE, AND ROTATIONAL HALO STAR MODELS

Lithium and beryllium burn at moderate temperatures and densities in stellar interiors. It has been known for some time

that the surface abundances of these elements can be depleted in standard stellar models when the base of the surface convection zone is deep enough to be sufficiently hot (e.g., Bodenheimer 1966), and dense (DDK).

Any additional mixing in the radiative envelope can also transport depleted material from the interior to the base of the convection zone, causing a reduction in the surface abundance. There is clear observational evidence for progressive surface lithium depletion in low-mass solar metallicity stars (Hobbs and Pilachowski 1988; see also DDK for a review and analysis of lithium observations in the metal-poor stars) beyond that predicted by standard models (Pinsonneault, Kawaler, and Demarque 1990 hereafter PKD). Rotationally induced mixing is a plausible explanation for the observed depletion patterns (PKD). Diffusive mechanisms acting at the base of the convection zone can lower or raise the surface Li and Be abundances (Michaud, Fontaine, and Beaudet 1984; DDK).

We will therefore examine beryllium depletion in the halo star context for three classes of models: (1) standard stellar models, (2) diffusive models, and (3) rotational models. We note that all three classes have been able to reproduce the pattern of Li abundances observed in extreme halo dwarfs, but each has a different absolute depletion (DDK; Deliyannis 1990; Pinsonneault, Deliyannis, and Demarque 1990, hereafter PDD). We will use these models to determine the maximum possible theoretical correction that should be applied to Be observations before these observations are interpreted.

There is a large body of work on Li in the literature; it is thus convenient to compare the evolution of Be to that of Li. *Theory predicts that although the surface beryllium abundance in stellar models will be less affected by either mixing or diffusion than lithium, it can nevertheless be measurably depleted.* Because Be burns at a higher temperature than Li, it is preserved to a deeper mass fraction (roughly twice as great). For standard models, this implies that affecting the Be surface abundance requires a deeper convection zone. In rotational models, this implies that it is more difficult to transport Be-depleted material to the surface. Be also diffuses less, and the downward diffusion is overcome by upward radiative levitation at a higher temperature.

The different processes that we consider here each dominate at different evolutionary states. As a result, a conservative maximum possible depletion can be estimated simply by adding (in the log) each component. In all cases, beryllium isochrones in the age range⁵ 16–20 Gyr show that combined Be depletion is predicted to be less than 0.4 dex for halo dwarfs with $[\text{Fe}/\text{H}] \leq -1.3$ and $T_{\text{eff}} \leq 6000$ K in which Li is detected; at present, we cannot rule out that more depletion might be possible for $T_{\text{eff}} > 6000$ K. At the end of the section we will discuss possible nonlinear effects that could reduce the combined depletion factor.

For the construction of evolutionary sequences of models, we have used the Yale Stellar Evolution Code (Pinsonneault 1988). For each of three metallicities ($Z = 10^{-3}$, 10^{-4} , and 10^{-5} , corresponding to $[\text{Fe}/\text{H}] = -1.3$, -2.3 , and -3.3), a range of masses spanning the halo dwarfs has been chosen (for mass list and other stellar parameters, see PDD).

a) Standard Models

Standard stellar models are computed without treating the possible effects of rotation, diffusion, magnetic fields, and other

³ The abundance is given by number relative to hydrogen on a logarithmic scale where hydrogen is 12; we represent this by using bracket notation. Thus, for element X, $[\text{X}] \equiv 12 + \log (N_{\text{X}}/N_{\text{H}})$. We also employ the usual notation for relating abundances to solar values: $[\text{X}/\text{H}] = \log (X/\text{H})_{\star} - \log (X/\text{H})_{\odot}$.

⁴ The purpose of the classification scheme was to select only the oldest stars, whose initial Li abundance is not likely to have been contaminated by Galactic Li production, and thus might best be representative of the primordial Li abundance. Both kinematic and chemical criteria were employed in the selection process.

⁵ Note that globular cluster ages inferred from stellar evolution models currently fall in this range (e.g., Buonanno, Corsi, and Fusi-Pecchi 1989).

more complex physics. In such models, surface beryllium depletion can occur only when the base of the surface convection zone is hot enough to burn beryllium (for dwarfs), or when the surface convection zone deepens substantially and dredges up beryllium-depleted material from the interior (for post-turnoff models).

The predicted light element depletion pattern is therefore a strong function of mass because lower mass (i.e., lower T_{eff}) models have deeper convection zones. Beryllium depletion in halo dwarf models is negligible for all $T_{\text{eff}} \geq 4900$ K (Fig. 1a). Note that the Spite Li plateau extends from 6300 to 5500 K (DDK). As is the case with Li (see DDK for a discussion of Li depletion and isochrone fitting), uncertainties in the opacities, reaction rates, He abundance, and mixing-length parameter do not affect his depletion pattern.

Conversely, the subgiants can experience significant dilution (of order a factor of 10); therefore, it is important to separate the subgiants from the dwarfs when interpreting Be observations. Fortunately, subgiant models do not begin diluting until ~ 5500 K; at such low T_{eff} , subgiants can more readily be distinguished observationally from dwarfs.

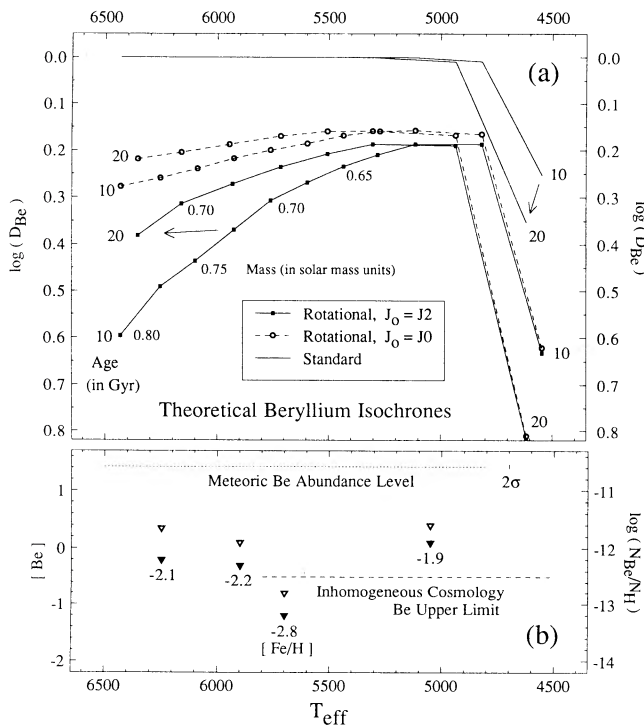


FIG. 1.—(a) Theoretical beryllium isochrones for halo dwarf model sequences with $Z = 0.0001$ (corresponding to $[\text{Fe}/\text{H}] = -2.3$) on a logarithmic depletion scale (where the depletion factor, D , is defined as the initial abundance divided by the final abundance) as a function of T_{eff} . Arrows indicate direction of evolution. Standard isochrones deplete appreciably only for the coolest models (below 4900 K). The rotational isochrones cover a range in initial angular momentum (J_0) of an order of magnitude, from $\log(J_0) = 49.2 \equiv J_0$ to $50.2 \equiv J_2$ (where J_0 is in units of $\text{g cm}^2 \text{ s}^{-1}$), which spans that of Population I stars with similar masses. (b) The halo star Be upper limits of Ryan *et al.* 1990 (solid triangles), together with a conservatively large possible upward adjustment due to combined stellar evolution effects (open triangles). Also shown are the meteoric Be abundance with its (rather small) 2σ error bar (Anders and Grevesse 1989), and the upper limit on primordial Be from inhomogeneous BBN (Boyd and Kajino 1989; Malaney and Fowler 1989). Note the difference in scale between (a) and (b): whereas (a) covers less than an order of magnitude in D_{Be} , (b) spans four orders of magnitude in the beryllium abundance.

b) With Diffusion

Microscopic diffusion relative to hydrogen (e.g. Vauclair 1983) can occur downward (gravitational settling, thermal diffusion) or upward for species that are not fully ionized (radiative levitation). In general, these mechanisms become more efficient in the outer radiative layers. However, they are rendered inefficient by turbulent motions and are thus not important in convection zones. Thus, surface abundances can be affected by diffusion out of (or into) the base of the convection zone, preferentially so for models with shallow convection zones (i.e., higher mass models, and models evolving toward the MS turnoff). In halo stars, radiative levitation is unimportant for Li (DDK); from the calculations of Michaud and Charbonneau (1990), we infer that radiative levitation of Be requires shallower convection zones than those of our models even beyond the hot edge of the Spite Li plateau. Therefore, He, Li, and Be diffuse only downward, and their diffusion isochrones exhibit increasing downward curvature for higher mass models.

Helium is abundant and diffuses relatively rapidly. Therefore, its diffusion can affect stellar structure and evolution (Michaud, Fontaine, and Charbonneau 1984; DDK). We calculate it in detail (settling, thermal, and concentration diffusion) following the formulation of Loeb and Bahcall (1990). Diffusion of trace elements Li and Be can then be adequately estimated relative to He using the time scale estimate of Michaud, Fontaine, and Beaudet (1984); Li diffuses less than He, and Be still less. For $T_{\text{eff}} < 6000$, the Be depletion is less than 0.2 dex. Higher T_{eff} imply much more Be depletion, which results in diffusion isochrones that are steeply curved.

However, the maximum amount of diffusion allowed can be constrained by the amount of curvature that the extremely flat observed halo star Li- T_{eff} relation (the Spite lithium plateau) will tolerate (DDK; Deliyannis and Demarque 1990). Note that lithium is the best element to trace possible diffusion processes (Deliyannis and Demarque 1990).

The current Li plateau tolerates no more than a 0.10 dex (or conservatively, 0.15 dex) difference in Li depletion between the hot edge (~ 6400 – 6300 K) and the middle (~ 5800 K). This implies that both our models and those of Proffitt, Michaud, and Richer (1990), which have slightly more depletion than ours, greatly overestimate the amount of diffusion. Only if the actual diffusion experienced by stars is much smaller at the edge (and correspondingly so in the middle) can the flatness constraint be met. It is probable that the Li depletion is no higher than 0.10 dex (or conservatively, 0.15 dex), which implies (in both cases) less than 0.10 dex Be depletion; we take this as an upper limit that is likely to be realistic.

The diffusion overestimate may simply be caused by an underestimate of the depth of the convection zone. The depth of the convection zone depends sensitively on the opacities, choice of mixing length, and composition, all of which are uncertain to some degree. Therefore, it is not surprising that we have overestimated the Be diffusion; among other reasons, currently available opacities are too low (Iglesias, Rogers and Wilson 1990), which results in underestimating the depth of the convection zone, and thus overestimating diffusion. It is possible that rotationally induced motions below the convection zone inhibit diffusion there (see below), or that mass loss inhibits diffusion.

c) With Rotation

Rotationally induced mixing is the single most important possible depletion mechanism for beryllium in halo dwarfs. We

follow the approach of Endal and Sofia (1981) by employing evolutionary stellar models with rotation that include angular momentum loss, transport, and the possible resulting mixing. Such models have been able to successfully account for observations of surface rotation velocities and lithium abundances in the Sun (Pinsonneault *et al.* 1989), in open cluster Population I stars of different ages (PKD), and in Population II stars (PDD). In halo star models, rotation perturbs the structure and evolution in only a very minor way (Deliyannis, Demarque, and Pinsonneault 1989); therefore, rotationally induced effects on the Li and Be abundance occur in superposition to those described for standard models.

Low-mass stars are observed to spin down and lose angular momentum from their surface. In our models, this loss generates internal angular velocity gradients that can trigger fluid dynamical instabilities. To relieve these instabilities, our models transport angular momentum from the interior to the surface. A fraction of this transport can occur by material mixing; this fraction (~ 0.03 , Pinsonneault *et al.* 1989) has been calibrated to match the solar Li depletion, and the same fraction also happens to give a depletion that is consistent with $[\text{Be}]_{\text{meteoric}} - [\text{Be}]_{\odot}$ to within the uncertainties (§ II). Rotational mixing extends from the base of the surface convection zone all the way to the core. Therefore, rotational mixing can transport beryllium-poor (and lithium-poor) material to the base of the convection zone (and thus dilute the surface abundance). Steady Li and Be depletion can thus occur simultaneously; for example, rotationally induced mixing provides an attractive explanation for the depletion of the solar Li and Be. By contrast, in standard models it is entirely possible for Be to be almost unaffected while Li is substantially depleted.

For Li isochrones that concur with the Li observations (e.g., 16–20 Gyr isochrones; see Deliyannis 1990 and PDD), the Be depletion (Fig. 1a) varies from more than 0.6 dex at 4500 K to only 0.2 dex at 4900 K, to 0.2–0.4 dex at 6400 K, depending on the initial angular momentum (J_0). Models of different J_0 experience different degrees of rotational mixing, and Be (and Li) depletion (PKD). The range in J_0 chosen here spans that appropriate for Population I stars of similar mass. For models in this mass range (PKD), the J_2 models represent a maximal possible Be depletion, applicable to *only a small fraction* of stars at the tail end of the distribution in J_0 , with most stars experiencing less depletion. Note also that uncertainties inherent to our models may be causing us to overestimate the Be depletion (see detailed discussion in Deliyannis 1990 and in PDD). Both the $Z = 10^{-3}$ and 10^{-5} isochrones are very similar to the 10^{-4} ones discussed so far (for $T_{\text{eff}} \geq 4900$ K).

d) Combined Diffusion and Rotation

Because rotational mixing occurs early and diffusion occurs later in the main-sequence lifetime of models, it is possible that both effects need to be taken into account in halo stars. We can thus estimate a conservative upper limit on the Be depletion of 0.4 dex by adding (in the log) the rotational and diffusive depletions (for T_{eff} in the range 5000–6000 K). However, rotationally induced motions and diffusive motions are not necessarily complementary. For example, when the secular shear instability is triggered, rotational mixing can wipe out diffusion-induced composition (μ -) gradients. Even more restrictive is the possibility that rotationally induced turbulence can render diffusion sufficiently inefficient, so that such gradients never even have a chance to form. Conversely, if diffusion creates a sufficiently steep μ -gradient, subsequent rotational mixing in the

direction of the gradient is suppressed (Zahn 1987; DDK; PKD).

IV. DISCUSSION

a) Evolution of the Beryllium Abundance in Time, and Implications for Galactic Chemical Evolution

Even with our maximum combined depletion factor, the halo star beryllium abundances are substantially below the Population I values. This is especially true because rotational beryllium depletion in Population I star models is more pronounced than in Population II star models (e.g., compare PKD and PDD), which renders the difference in the *initial* stellar Be abundances even greater than the difference in observed values. As pointed out by Rebolo *et al.* (1988), such results argue against models of chemical evolution that overproduce Be in bursts of star formation in an early Galactic epoch.

b) Cosmology

BBN predictions of Be have recently been updated. Boyd and Kajino (1989) pointed out that including the ${}^7\text{Li}({}^3\text{H}, n){}^9\text{Be}$ reaction in the reaction network for BBN can raise the Be production from $[\text{Be}] = -10.7$ (which is utterly unobservable) to -3.8 in standard BBN and from -3.8 to -1 in inhomogeneous BBN. The higher end of the latter can be explored with current technology (e.g., Ryan *et al.* 1990). Inhomogeneous BBN is affected by some free parameters that are not relevant to standard BBN. Malaney and Fowler (1989) emphasized that $[\text{Be}]_{\text{inhom}}$ can be as low as $[\text{Be}]_{\text{stand}}$, depending on the range of parameter space employed. (See also Terasawa and Sato 1990.) Therefore, knowledge of $[\text{Be}]_{\text{primordial}}$ cannot by itself rule out the inhomogeneous scenario in favor of the standard one, but it can constrain the parameter space. When our conservative upward correction factor of 3 in $[\text{Be}]$ is applied to HD 140283 (Fig. 1b), most of the parameter space covered in the homogeneous BBN models of Kajino and Boyd (1990) is allowed.

If an initially inhomogeneous universe can homogenize during nucleosynthesis, then much of the Be can be destroyed (Alcock *et al.* 1990); the resulting abundances of Li, Be, and B are low, with Li as low as the halo observations, and Be lower than the upper limits. In this scenario, primordial light element abundances can constrain the character of possible homogenization. Another possibility is to consider Ω_b less than unity (which is the value often used in inhomogeneous BBN calculations); Mathews *et al.* (1990) find that low Ω_b can produce a lower (and thus more realistic) ${}^7\text{Li}$ abundance, and they claim that larger values of Ω_b are allowed in this manner than those deduced from standard BBN.

c) Cosmic-Ray Theory and BBN

We have shown that the Be abundance has evolved; this implies that nonprimordial sources do exist. It would be a triumph for cosmic-ray theory if new observations of both Be and B show (after possible correction factors due to stellar evolution) that their ratio *as a function of decreasing* $[\text{Fe}/\text{H}]$ remains as predicted from cosmic ray theory. To separate any primordial component of Be (and B) from any spallative component, it is necessary (1) to establish detections of Be for stars with $[\text{Fe}/\text{H}] \leq -1$, and (2) to push the observations to stars of still lower metallicity than those of Ryan *et al.* (1990) (for HD 140283, $[\text{Fe}/\text{H}] = -2.8$). A primordial component might be indicated if a metallicity-independent plateau of Be (and B) is found between 5000 and 6000 K. If the ratios predicted by

cosmic-ray theory are indeed borne out by observations in progressively more metal-poor stars (with relevant evolutionary corrections), then deviations from the predicted ratios for even lower $[\text{Fe}/\text{H}]$ might also be indicative of a significant primordial component.

The implications for cosmology are currently being worked out: As more reactions have been added to the network used in BBN, the predicted Be/B ratios have changed, influencing how Be/B is used to distinguish between spallative production and BBN production (Boyd and Kajino 1989; Kawano, Fowler,

and Malaney 1990). There is also the possibility that Be/B might provide constraints on Ω_b that are independent of Li arguments, e.g., a very low $\text{Be}/\text{B}_{\text{primordial}}$ might suggest that $\Omega_b \sim 1$, whereas $\text{Be}/\text{B}_{\text{primordial}} \sim 0.7$ might be indicative of $\Omega_b \sim 0.1$ (depending, also, on other parameters; Kawano, Fowler, and Malaney 1990).

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