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STEPS TOWARD THE HUBBLE CONSTANT. IX. THE COSMIC VALUE OF H_0 FREED FROM ALL LOCAL VELOCITY ANOMALIES

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ABSTRACT

The velocity which the Virgo Cluster would have if it were freed from all local anomalies relative to the cosmological Machian frame is determined by bringing the remote velocity frame to the Virgo Cluster distance. Three independent methods are used to accomplish this: (1) distances of 17 "remote" clusters are determined *relative to Virgo* by a number of independent methods; the equation of the Hubble diagram correlation that relates redshift and the distance modulus *difference* from Virgo is then read for its velocity value at the modulus difference of $\Delta(m-M) = 0.00$; (2) a similar calculation is made using apparent magnitudes of first-ranked cluster galaxies in relatively distant clusters, and (3) the same calculation is made using distant supernovae of Type Ia. The resulting velocity of Virgo, called the "cosmic expansion velocity freed from local velocity anomalies" is $v_{\text{cosmic}}(\text{Virgo}) = 1144 \pm 18$ km s^{-1}.

The distance to the Virgo Cluster core is determined from six independent methods using recent data, giving $(m-M) = 31.70 \pm 0.09$, or $D = 21.9 \pm 0.9$ Mpc. Combining the cosmic Virgo velocity with this distance, which now is of unprecedented accuracy, gives the Hubble constant to be $H_0 = 52 \pm 2$ km s^{-1} Mpc^{-1}. If a still more accurate distance D (in Mpc) to the Virgo Cluster core becomes available, the calculated Hubble constant will be changed to $H_0 = 52(21.9/D)$ km s^{-1} Mpc^{-1}.

As a by-product (but which does not enter the method devised here to find H_0), we determine the infall velocity of the Local Group toward Virgo (the retarded expansion effect) to be 168 ± 50 km s^{-1}. A firm lower limit to H_0 can be obtained by putting this infall velocity to zero so that $v(\text{Virgo})_{\text{cosmic}} = v(\text{Virgo})_{\text{observed}} = 976 \pm 45$ km s^{-1}. For this limiting case (which is physically unreal unless the Virgo pull is zero, and therefore $\Omega = 0$) the lower limit to the global value of H_0 becomes $H_{0(\text{min})} = (45 \pm 3)(21.9/D)$ km s^{-1} Mpc^{-1}.
Subject headings: cosmology — galaxies: clustering — galaxies: distances — galaxies — redshifts

So numerous and so powerful are the causes which serve to give a false bias to the judgement that we on many occasions see wise men on the wrong as well as on right side of questions of the first magnitude. This circumstance, if duly attended to, would furnish a lesson of moderation to those who are ever so much persuaded of their being in the right on any controversy.

Alexander Hamilton (*The Federalist Papers*, No. 1)

I. INTRODUCTION

A perception is abroad that the global (cosmic) value of the Hubble constant, H_0, is unknown by a factor of 2. Various literature values remain dichotomous, generally quoted to be in the quantized ranges of 40–60 km s^{-1} and 80–100 km s^{-1} Mpc^{-1}. Concerning the reason for the disagreement, an old proposition (de Vaucouleurs 1958, 1972; de Vaucouleurs and Peters 1986) has recently been resurrected (Tully 1988) that both values are correct, the difference being due to an actual variation of H_0 with distance, with the dichotomy being replaced by a continuum variation in different distance regimes.

The first point to be made is that most astronomers agree that the *local* value of H_0 is near 50, *local* meaning distances from us to about one-half the distance to the Virgo cluster. The evidence is multiple, beginning *very* locally (Sandage 1986, 1987) from precision distances determined using Cepheid variables and high weight velocities corrected to the centroid of the Local Group.

This very local value of $H_0 = 50$ was found (Richter and Huchtmeier 1984; Huchtmeier and Richter 1986; Sandage 1988*b*, Fig. 9) also to apply over the much larger region contained in the strictly *volume-limited* 500 km s^{-1} catalog of Kraan-Korteweg and Tammann (1979). Use of this ideal sample insures that there is no Malmquist bias whatsoever in the data, and therefore that the derived mean value of H_0 *at least in this region* is correct. Of course, it is the Malmquist bias in calculations based on samples that have been mostly magnitude limited that has provided the main fulcrum upon which many of the current interpretive disagreements are balanced.

The second point to be made is that the Tully-Fisher (1977) relation that can be determined from the (essentially) magnitude-limited sample of Aaronson *et al.* (1982*a*) is *progressively displaced toward brighter absolute magnitudes* (at a given line width) as the sample is divided into bins of increasing redshift. This effect, discovered and discussed elsewhere (Kraan-Korteweg, Cameron, and Tammann 1988, hereafter KKCT; Tammann 1988, Fig. 2) was further investigated by Sandage (1988*b*, Fig. 9) where the large difference is shown again between the position of the Tully-Fisher envelope lines for the 500 km s^{-1} radio sample of Huchtmeier and Richter (1986) and that part of the Aaronson *et al.* (1982) sample that is more distant.

This progressive difference in the zero points of the TF correlation in different redshift bins was demonstrated to be due to Malmquist bias by KKCT, by Bottinelli *et al.* (1986*a*, *b*, 1987, 1988) following Teerikorpi (1987), by Sandage (1988*b*), and by Fouqué *et al.* (1990). Tully (1988), however, disagrees. He interprets the effect as being *real*, arguing that no Malmquist bias exists in his samples and that H_0 does, in fact, increase outward from the Local Group. (His conclusion depends on his assumption that the inverse TF relation is bias free—a point refuted by Teerikorpi 1990). Tully does agree that the value of H_0 is near 50 in our neighborhood but that it reaches ~ 90 (which he assumes to be the global value) at about twice the distance to the Virgo Cluster. The demonstration (Sandage 1988*a*, *b*) (1) that such an apparent increase of H with distance is artificial, (2) that it is a natural consequence of any analysis which uses magnitude-limited samples in which a fixed mean absolute magnitude is assigned to each object in the sample, and (3) that it can be directly demonstrated to be an artifact of such an analysis by adding a fainter sample to see the resulting double-valued H_0 at a given distance, was dismissed by Tully by stating that H_0 is *expected* to increase outward in the local neighborhood.

The suggestion of an increasing Hubble ratio outward had prior adherents. A variable velocity-to-distance ratio over the very large region encompassed within 20,000 km s^{-1} was proposed by Haggerty and Wertz (1971), based on a hierarchical density universe put forward by de Vaucouleurs (1970, 1971), following Charlier (1908, 1922). However, the prediction by Haggerty and Wertz was shown to be incompatible with the observed Hubble diagram that had been measured even at that time to be linear for expansion velocities as small as 1000 km s^{-1} (Sandage, Tammann, and Hardy 1972, hereafter STH; Sandage 1972*a*, Fig. 4; Sandage and Hardy 1973, hereafter SH, Fig. 7).

To be sure, Tully's (1988) proposed variation of H with distance is less extreme than that of Haggerty and Wertz. This suggested variation, similar to the early suggestion by de Vaucouleurs (1958, 1972) already mentioned, is confined to the local supercluster, but is due, nevertheless, to the same cause postulated by Haggerty and Wertz as gravitational deceleration induced by a nonuniform distribution of matter.

Although the demonstration by STH showed that no large-scale *gross* $H(r)$ dependence exists, the data available to them were not numerous enough in the velocity range from ~ 1000 km s^{-1} to 6000 km s^{-1} to map the local field precisely enough to rule out a possible more local $H(r)$ variation. Therefore, between 1972 and 1975 a study of galaxy groups, following their discovery by Humason, Mayall, and Sandage (1956, Table XI) from velocity correlations, was begun so as to fill this sparse region of the Hubble diagram to make a stronger test. The results, discussed in a series of papers (Sandage and Tammann 1975, Figs. 6–8; Sandage 1975, Figs. 4–5; Tammann 1987, Fig. 7), showed that $\langle H \rangle$, averaged over the sky and in the distance range between us and $\sim 10{,}000$ km s^{-1}, showed no evidence for a systematic variation of H_0 with distance to within our ability to measure it at the time. These studies, not discussed by Tully (1988), were replaced in his paper by his supposition that we were "deceived by the change of H_0 outward," as if the problem was new with no previous history, rather than that his proposition had been weighed in the balance between 1972 and 1987 and found wanting.

The purpose of the present paper is to circumvent these questions of the local velocity field entirely. At the moment it seems to us that the problems of reconciling the many current studies of the local deviations from an ideal Hubble flow are formidable enough to deny any firm conclusions about the global value of H_0 when the method of measurement uses local data, to which velocity corrections are applied. The uncertainties are centered on (1) selection bias and the Malmquist corrections for flux-limited and/or otherwise incomplete samples, (2) the effect of the Virgocentric perturbation which, although clearly present, is too uncertain to be of use here, and (3) the details of velocity anomalies beyond the local supercluster on the scale of ~ 500 km s^{-1} at 4000 km s^{-1} recession velocity which suggested (Shaya 1984; Sandage and Tammann 1984*a*, *b*; Tammann and Sandage 1985) *bulk motion* of the local supercluster toward the microwave background (Lucey and Carter 1988*a*, *b*), but the details of which are at present in controversy (Lynden-Bell *et al.* 1988).

In view of these apparently insurmountable uncertainties and of the irreconcilable disagreements they have engendered, we have taken another route around these multiple problems of the local velocity field. In this paper we use data for clusters and for supernovae that are so distant that the effect of velocity perturbations, even if they are real at current proposed levels (Dressler *et al.* 1987; Yahil 1988), would cause less than a 10% $\Delta v/v$ error in most *individual clusters*, and a much smaller error on the global value of H_0 when the data are averaged over the sky. Our method is to use a variety of distance indicators to clusters as well as to use supernovae of Type Ia in the redshift range from 1000 to 10,000 km s^{-1} that give distances *relative to the Virgo Cluster core*. The known *absolute* distance to the Virgo Cluster is then used to calibrate the scale.

For pedagogical reasons our actual method is to bring all the distant objects to the *Virgo Cluster distance*, obtaining thereby the value of the "unperturbed velocity of the Virgo Cluster." We then combine this "Virgo cosmic velocity" with the cluster distance to obtain H_0 (global). As a by-product we also obtain in this way a precision measure of the Local Group "infall to Virgo" (that is, the retarded expansion of the Local Group caused by the Virgo Cluster attraction for the local neighborhood) by comparing the derived "cosmic" velocity with the measured velocity.

Our present method is identical to that which had been used before to show the smallness of the velocity effect of the local supercluster on the Local Group by showing that the Virgo Cluster does not deviate much in a Hubble diagram that is made using data for "distant" first-ranked cluster galaxies (Sandage 1963 in a reply to comments by Hoyle and by Oort; SH; Tammann and Sandage 1985). In the new calculation made here we have used all the available modern data, both as to the Virgo Cluster distance and to the "reduced Virgo Cluster cosmic velocity" freed from any supposed local velocity perturbations.

In the next section we set out the data for the distances of various *clusters* of galaxies relative to Virgo. In § III we use these relative distances to calibrate the mean apparent magnitude of the first-ranked galaxies in several clusters, if each cluster were to be brought to the Virgo Cluster distance. Using this mean magnitude to the Virgo Cluster itself in the equation for the Hubble diagram of magnitude versus velocity determined from all the clusters except Virgo gives the "cosmic" velocity for Virgo. The same analysis is made in § IV for all Type I supernovae with adequate light curves using data for the six SNe Ia that have been measured in Virgo. The adopted value of $v_{\rm cosmic}$(Virgo) is set out in § V. Six independent methods are used in § VI to measure the absolute distance of the Virgo Cluster, with the results summarized in § VII. This distance, combined with the cosmic Virgo Cluster velocity from § V, leads to the global value of H_0 in § IX, comments on methods that have given an incompatibly low value for the Virgo distance and therefore an incompatibly high value of H_0 having been made in the penultimate section.

We emphasize again that the results herein are independent of all considerations of the local velocity field to far beyond twice the distance to the Virgo cluster, and therefore that this study is immune to details of that field as discussed, for instance, by Tully (1988).

II. DISTANCE OF CLUSTERS RELATIVE TO THE VIRGO CLUSTER

Distances relative to the Virgo Cluster are available for a number of clusters spread somewhat uniformly around the sky. Their more or less random distribution relative to the direction of the hot pole of the microwave background tends to minimize the effect of that dipole velocity anomaly, which is the largest velocity perturbation yet identified with confidence.

Data on relative distances for 17 of the best determined clusters are listed in Table 1. Membership assignments from the literature have been used, although some of these have evident difficulties. An example is the well-known problem of the velocity dichotomy of the Centaurus cluster (Lucey and Carter 1988*b*). A second example is the Cancer cluster which would seem to be a collection of discrete groups (Bothun *et al.* 1983). Nevertheless, many of the clusters we use here are the standard aggregates set out before in the current literature. Our sample has, then, the merit of being free of our own personal judgement because the membership, and therefore the velocity assignments, have been made by others. But we should also note that it is irrelevant for our purpose whether the clusters are dynamical units or not. All that is required here is that the "cluster" names stand for galaxy aggregates (bound or not) that are sufficiently concentrated in space (at the moment) so that a mean distance and mean recession velocity can be assigned to them with small errors, and that the brightest members of each aggregates can be identified.

The adopted corrected velocity is listed in column (2) of Table 1. This is calculated from the measured mean velocity of presumed cluster members corrected to the centroid of the Local Group using precepts from the RSA (Sandage and Tammann 1987) following Yahil, Tammann, and Sandage (1977), which is closely the same solution for the solar motion that was found later using modern data by Richter, Tammann, and Huchtmeier (1987). A further correction is made for a Virgocentric perturbation on the Local Group (retarded expansion) using the model of Kraan-Korteweg (1986) with an "infall" velocity of 220 km s^{-1} (the exact value makes very little difference here, even using infall ranges from 0 to 440 km s^{-1}).

The measured distance modulus *differences* relative to Virgo

TABLE 1

DISTANCES OF CLUSTERS RELATIVE TO THE VIRGO CLUSTER

		$(m-M)_{\rm Cluster} - (m-M)_{\rm Virgo}$				
CLUSTER (1)	v_{220} (2)	21 cm (3)	m_{10} (4)	$D_n - \sigma$ (5)	OTHERS (6)	ADOPTED (7)
UMa	1270	0.25	...	...	...	0.25
Fornax	1375	...	...	0.14	0.27[a]	0.20
Centaurus	3390	...	2.58	...	1.97[b]	2.27
Hydra I	3490	...	2.78	2.49	2.65[b]	2.65
Pegasus	3880	2.58	...	2.15	2.44[c]	2.39
Cancer	4903	3.10	...	...	...	3.10
Pisces	5114	3.05	...	2.93	...	2.99
Perseus	5470	...	3.27	3.28	...	3.28
Zw 74–23	6229	3.60	...	...	...	3.60
A1367	6644	3.50	3.97	3.65	...	3.71
A400	6988	4.05	...	...	...	4.05
Coma	7143	4.00	4.23	3.74	3.80[d]	3.80
A539	8500	4.33	...	...	...	4.33
A2634/66	8610	4.30	...	3.69	...	4.00
A1185	10470	...	5.03	...	...	5.03
A2147	10530	...	4.95	...	...	4.95
Hercules	11212	...	4.79	...	...	4.79

[a] $\Delta(m-M) = 0.39 \pm 0.10$ from six-color data of six Virgo and two Fornax SNe Ia (Leibundgut 1988, TL); 0.00 ± 0.20 from the galaxian luminosity function (Ferguson and Sandage 1988); -0.16 ± 0.16 from dE galaxies (Bothun *et al.* 1989); 0.06 ± 0.12 from globular clusters (Madejski and Bender 1989). The very small value of -0.5 ± 0.2 from globular clusters (Geisler and Forte 1990) is omitted here.

[b] $\Delta(m-M)$ from $D_n - \sigma$ relation (Lucey and Carter 1988). The value of $\Delta(m-M) = 1.41$ (Bothun *et al.* 1989) is not considered here because of possible selection effects.

[c] $\Delta(m-M)$ from SN Ia 1970J in NGC 7619 for which m_B(max) = 14.35 (TL).

[d] $\Delta(m-M) = 3.78 \pm 0.05$ from a review of several methods (Tammann 1987); 3.95 ± 0.25 from relations between color, surface brightness, radius, and Mg_2 index (de Carvalho and Djorgovski 1989); 3.89 ± 0.30 from SNe Ia (TL). We adopt a final value of 3.80.

are listed in columns (3)–(6) of Table 1. Column (3) is from the Tully-Fisher method as calibrated and discussed by KKCT based on 21 cm line width data for the spirals in the sample. We have made an upper envelope fit to the data so as to minimize selection effects. The resulting relative distances are *bracketed* by the distances derived from the same data by Aaronson *et al.* (1986) and by Bottinelli *et al.* (1988). The results of the first authors were uncorrected by them for any bias effects. Nevertheless, their samples do profit from the fact that galaxies in different clusters tend to suffer *similar* selection effects. Therefore the errors due to selection bias in the *relative* distances of Aaronson *et al.* are smaller than the errors in their absolute distances. The Aaronson *et al.* distances relative to Virgo are *smaller* than ours by 12% on average. On the other hand, Bottinelli *et al.* (1988) have applied an idealized analytical bias correction that yields relative distances *larger* than ours by 10%.

Data in column (4) are from the nuclear magnitudes of the first 10 E cluster members from Weedman (1976). The column (5) values are from the D_n-σ distance indicator for E galaxies (Faber *et al.* 1989). The values in column (6) are from various methods such as supernovae of Type Ia (Leibundgut and Tammann 1990) for Fornax and Pegasus, the surface brightness–absolute magnitude relation for dE galaxies (Ferguson and Sandage 1988; Bothun, Caldwell, and Schombert 1989) for Fornax, the luminosity peak of globular clusters (Madejski and Bender 1989) for Fornax, and new correlations involving color, surface brightness, diameter, and Mg_2 index (de Carvalho and Djorgovshi 1989) for Coma combined with a large number of independent methods also applied to Coma as discussed by Tammann (1987). The details of the column (6) entries are set out in the footnote to Table 1. The adopted mean modulus difference from columns (3)–(6) is listed in column (7).

Figure 1 shows the correlation of columns (2) and (7) of Table 1. This is the Hubble diagram plotted as the redshift versus the log of the distance ratio (expressed as the magnitude modulus difference) to Virgo. The scatter is remarkably small. It therefore makes little difference which coordinate is used as the independent variable in a least-squares regression calculation. The solution using residuals read vertically as Δ log cz has a slope of $d \log cz/d(m-M) = 0.195$. Reading the residuals horizontally using the modulus as the independent variable gives a slope of $d(m-M)/d \log cz = 5.047$. The correlation coefficient is extremely high at $r = 0.992$, indicating again the smallness of the residuals which are everywhere less than $\Delta v/v = 0.1$. Therefore, *no* velocity perturbation has been detected in this sample, in agreement with the prior results previously mentioned (Sandage and Tammann 1975; Sandage 1975).

The consequences of this lack of a perturbation signal should be explored further by enlarging the sample to include the groups used before (Sandage 1975) in the manner of Collins, Joseph, and Robertson (1986) and of James, Joseph, and Collins (1987). These authors calculate that a large velocity perturbation exists using the same sample studied by Sandage and Hardy (1973). Furthermore, the size of their signal is similar to the perturbation discussed by Lynden-Bell *et al.* (1988). However, it is important to note that Lucey and Carter (1988*a*, *b*), by adding new data to the original cluster sample, found no such signal, in agreement with the bulk model of Shaya (1984), of Lubin, Epstein, and Smoot (1983), of Fixen, Cheng, and Wilkenson (1983), and our own (Sandage and Tammann 1984*b*; Tammann and Sandage 1985).

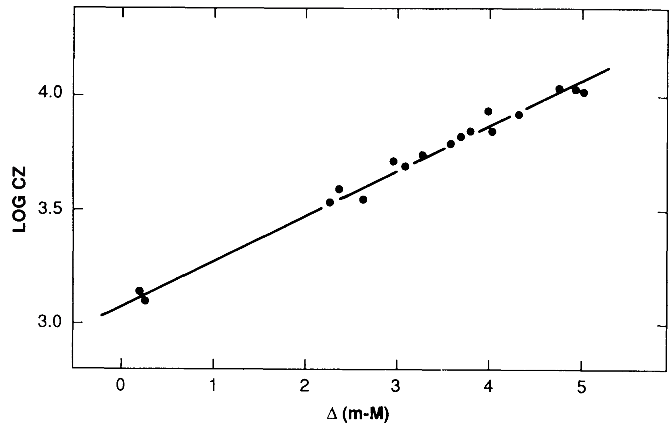

FIG. 1.—The Hubble diagram from the data in Table 1. Abscissa values are the adopted distance moduli differences of the program clusters from the Virgo Cluster. The ordinate is redshift corrected for solar motion relative to the centroid of the Local Group and for Virgocentric infall using v(infall) = 220 km s^{-1}, which, however, has an insignificant effect for "infall" assumptions between 0 and 440 km s^{-1}. The line is equation (1) of the text whose slope is 0.2, consistent with a linear velocity-distance relation.

tent with our previous analysis (Sandage and Tammann 1984*b*; Tammann and Sandage 1985).

However, because the present paper is about the global value of H_0, not about this problem of the local velocity field, we can only applaud the lack of a $\Delta v/v$ signal in Figure 1 as being ideal for our present purpose. For this reason we do not inquire further here into the irrelevant (for our purpose) independent problem of velocity deviations from the Hubble flow which so occupies the current literature, but which is absent in the sample shown in Figure 1.

The mean slope obtained by averaging the two least-squares solutions made by exchanging independent variables is 0.198 which is nearly identical to the requirement of 0.2 for a linear velocity-distance relation. Forcing the slope to be exactly 0.2 gives the equation of the regression line as

$$\log v = 0.2\Delta(m-M) + 3.072 \pm 0.008 , \qquad (1)$$

which is the equation of the line drawn in Figure 1.

It is to be noted that this equation is derived from clusters at an effective distance of $\sim$6000 km s^{-1}. At this distance even the largest velocity perturbation yet established of say 500 km s^{-1} produces a relative error of only $\Delta v/v = 0.08$ per cluster. And because the clusters in Figure 1 are spread around the sky, the actual error is much less. Assuming $\sqrt{n}$ statistics, the error in the solution due to postulated velocity anomalies, even in the worst perturbation case, would be $\Delta \log v \approx 0.009$, which is 2%, the same as the statistical error in the intercept in equation (1).

Equation (1) shows that at $\Delta(m-M) = 0.00$, the cosmic velocity, *tied to the average external frame at* $\langle v \rangle \sim 6000$ km s^{-1}, is

$$v(\mathrm{Virgo})_{\mathrm{cosmic}} = 1180 \pm 22 \text{ km s}^{-1} , \qquad (2)$$

at the *distance* of the Virgo cluster. Note that no observed velocity of Virgo has been used. Equation (2) can contain at most only the peculiar velocity that obtains for the all-sky frame at 6000 km s^{-1} relative to the true (presumably Machan) cosmological frame.

III. ROUTE TO THE VIRGO CLUSTER COSMIC VELOCITY THROUGH BRIGHTEST CLUSTER GALAXIES

We now perform the same type of analysis as in § II using the absolute magnitude of the brightest cluster galaxy as the distance indicator. It was shown elsewhere (SH) that the Hubble diagram for such galaxies, and for brightest galaxies in groups, is well defined with a slope of 0.2 when the magnitudes are corrected to a standard metric diameter, for K dimming, for galactic absorption, and for richness and cluster contrast effects. Because of some uncertainty whether the last two corrections are well defined for the sparse groups, we have not used the groups in what follows.

Data for nine clusters plus Virgo are listed in Table 2. The fully corrected apparent magnitude of the brightest galaxy in each cluster, taken from SH, is in column (2). The adopted distance modulus relative to Virgo from column (7) of Table 1 is recopied into column (3). Subtracting column (3) from column (2) reduces all column (2) values to the Virgo Cluster distance. The *mean* apparent magnitude of first-ranked cluster galaxies at the Virgo distance, found by averaging the column (4) values, is

$$\langle V_c^T \rangle = 8.39 \pm 0.10 \text{ mag} , \tag{3}$$

with a dispersion of $\sigma(M) = 0.32$ mag. This scatter is nearly the same as $\sigma(M) = 0.285$ mag of 97 first-ranked cluster galaxies about the mean Hubble line (SH) where only V_c^T magnitudes and velocities were used. The near agreement of the two dispersions speaks in favor of the validity of the *relative* cluster distances in column (7) of Table 1 used in column (3) of Table 2 which were obtained by almost entirely independent means.

The Hubble diagram of 96 first-ranked cluster galaxies shows a well-defined correlation (SH) whose equation (excluding, of course, the Virgo Cluster) is

$$\log v = 0.2V_c^T + (1.366 \pm 0.006) . \tag{4}$$

One could now insert $V_c^T = 8.21$ of the brightest Virgo Cluster galaxy (NGC 4472) to obtain a predicted Virgo velocity of 1019 km s^{-1}, but the result would be dominated by the statistical error in V_c^T of a single galaxy. Using, instead, the mean value of nine first-ranked galaxies from equation (3) gives

$$v(\text{Virgo})_{\text{cosmic}} = 1107 \pm 55 \text{ km s}^{-1} , \tag{5}$$

as the equivalent of equation (2) for the cosmic Virgo velocity freed from all local velocity perturbations. The value is derived from first-ranked cluster galaxies at an effective distance of 10,000 km s^{-1}, and hence, like equation (2), should closely approximate the Machian cosmological velocity, presumably at rest with respect to the MWB.

TABLE 2

BRIGHTEST CLUSTER GALAXIES

Cluster (1)	V_c^T (2)	$\Delta(m-M)$ (3)	$V_c^{T\,a}$ (4)
Virgo	8.21	0	8.21
Fornax	8.83	0.20	8.63
Pegasus	11.26	2.39	8.87
Perseus	11.45	3.28	8.17
Coma	11.58	3.80	7.78
Hercules	12.93	4.79	8.14
A539	12.89	4.33	8.66
A1367	12.10	3.71	8.39
A2147	13.43	4.95	8.48
A2666	12.54	4.00	8.54
Mean.			8.39 ± 0.10

[a] If at Virgo.

IV. ROUTE TO THE VIRGO CLUSTER COSMIC VELOCITY THROUGH SNe Ia

The first reliable proof that SNe Ia have a well-defined maximum absolute magnitude, and are therefore good distance indicators, was given by Kowal (1968). His Hubble diagram for SNe I shows a very small scatter. Similar conclusions were reached by Barbon, Capaccioli, and Ciatti (1975), Branch and Bettis (1978), Tammann (1978), Branch (1982), Tammann (1982), Cadonau, Sandage, and Tammann (1985), and Tammann and Sandage (1985). Kowal (1969) was also the first to show that the six type I SNe known to 1969 in Virgo Cluster galaxies had only a small range in their maximum *pg* magnitudes.

A new reduction of extant SNe light curves by Cadonau (1987), using a master shape as a template, has homogenized all previous photometry in the literature to produce a homogeneous set of maximum magnitudes (Cadonau and Leibundgut 1989). A summary of the relevant data for type Ia SNe which have adequate photometry in m_{pg} and/or in m_B is given by Tammann and Leibundgut (1990, hereafter TL) where the resulting magnitude-redshift (Hubble) diagram is used, together with a calibration of $M_B(\text{max}) = -19.79 \pm 0.12$ for SNe Ia, to derive $H_0 = 46 \pm 10$ km s^{-1} Mpc^{-1}.

We analyze these data here in a different way, analogous to the method used in the previous two sections to derive again the Virgo "cosmic velocity freed from local velocity perturbations." We consider separately the photometric data in B and in m_{pg} magnitudes.

The Hubble diagram for SNe Ia using B magnitudes shown as Figure 1 (upper part) in TL has the equation

$$\log v = 0.2m_B(\text{max}) + (0.625 \pm 0.018), \tag{6}$$

where the slope of 0.2 is assumed.

The six SNe Ia in Virgo, with accurately measured magnitudes at maximum, average

$$\langle m_B(\text{max}) \rangle = 11.91 \pm 0.07 \text{ mag} , \tag{7}$$

which, when put into equation (6), gives

$$v(\text{Virgo})_{\text{cosmic}} = 1016 \pm 55 \text{ km s}^{-1} . \tag{8}$$

A corresponding route through m_{pg} magnitudes gives

$$\log v = 0.2m_{pg}(\text{max}) + (0.681 \pm 0.029) , \tag{9}$$

for the Hubble diagram, which, with

$$\langle m_{pg}(\text{max}) \rangle = 11.84 \pm 0.15 \tag{10}$$

for the seven SNe Ia with adequate photometry in Virgo, gives

$$v(\text{Virgo})_{\text{cosmic}} = 1119 \pm 113 \text{ km s}^{-1} . \tag{11}$$

Averaging equations (8) and (11) gives our adopted answer from supernovae as

$$v(\text{Virgo})_{\text{cosmic SNe}} = 1037 \pm 45 \text{ km s}^{-1} , \tag{12}$$

as judged from field galaxies at an effective distance of $\sim$2500 km s^{-1}. These SNe Ia are moderately well spread around the sky (TL, Fig. 3).

V. THE ADOPTED VIRGO COSMIC VELOCITY AND THE INFALL VELOCITY OF THE LOCAL GROUP

Averaging equations (2), (5), and (12), with weights gives

$$\langle v(\text{Virgo})\rangle_{\text{cosmic}} = 1144 \pm 18 \text{ km s}^{-1} . \tag{13}$$

The observed mean velocity of the 6° core, reduced to the barycenter of the Local Group, is 976 ± 45 km s^{-1} (Binggeli, Tammann, and Sandage 1987). Hence, the Local Group is retarded in its expansion relative to the Virgo Cluster (the so-called "infall velocity") by 168 ± 50 km s^{-1}. This is smaller than our previously adopted value (Tammann and Sandage 1985) of 220 ± 50 km s^{-1} as a consequence of equation (13) being smaller than our previous value of 1182 ± 20 km s^{-1}, but all values are within their combined probable errors. In any case the infall value derived here is much less than the highest values determined over the years such as 470 ± 75 km s^{-1} by Tonry and Davis (1981) and 331 ± 41 km s^{-1} (Aaronson *et al.* 1982*b*). Our present value of 168 km s^{-1} now returns close to its first value of 174 ± 74 km s^{-1} determined originally by Tammann, Sandage, and Yahil (1980). Faber and Burstein (1988) argue for a still lower value of ~ 110 km s^{-1}.

VI. SIX METHODS FOR MEASURING THE DISTANCE TO THE VIRGO CLUSTER

The preponderance of evidence has forced us to conclude that the distance modulus of the Virgo Cluster core is $(m-M)_0 = 31.70 \pm 0.09$, or $D = 21.9 \pm 0.9$ Mpc. We have been persuaded to adopt this value by the results of six independent methods, each of which we assess to be of higher weight than the several at present experimental methods with which we contrast our adopted value in the penultimate section of this paper. Details of the six methods are as follows.

a) Globular Clusters

With the final discovery of the turnover in the luminosity function of globular clusters in Virgo Cluster ellipticals (van den Bergh, Pritchet, and Grillmair 1985 for M87; Harris *et al.* 1990 for three additional E galaxies as described by Harris 1988), the use of globular clusters as distance indicators moved from earlier assumptions to reality. The history of many early, generally unsatisfactory, attempts that used only the rising part of the luminosity function is reviewed by Harris (1988).

By stacking the separate luminosity functions for the four Virgo E galaxies which they observed, Harris *et al.* (in Harris 1988, Figs. 5 and 6) found an unmistakable maximum in the combined luminosity distribution at $B(\text{peak}) = 24.74 \pm 0.08$. The luminosity function for clusters in the Galaxy was then redetermined by Harris (1988, Fig. 1) using modern data by Webbink (1985), Peterson (1986), Peterson and Reed (1987), and many individual sources as well. He found the distribution to be closely Gaussian with a luminosity at the peak of the distribution of $M_B(\text{peak}) = -7.00 \pm 0.15$ mag, obtained by assuming all RR Lyrae gap positions in cluster color-magnitude diagrams have $M_V(\text{RR}) = 0.6$. This mean absolute magnitude is the same as has been obtained in a recent calibration (Sandage and Cacciari 1990) at [Fe/H] $= -1.8$ for an ensemble of RR Lyrae variables that have evolved in an average way from their initial position on the zero-age horizontal branch.

Combining the observed peak luminosity of 24.74 mag with -7.0 mag from the Harris calibration gives

$$(m-M)_{\text{Virgo}} = 31.74 \pm 0.17 \text{ mag} , \tag{14}$$

for the Virgo modulus from globular clusters.

b) Normal Novae

Normal novae had originally been found by Hubble in M87 (Bowen 1952) at the beginning of the Palomar 200 inch telescope distance scale campaign, but the discoveries were not followed up due to the limiting nature of the detection on photographic plates at the time. With the advent of CCDs, Pritchet and van den Bergh (1987) discovered and followed the light curves of nine novae in three Virgo cluster E galaxies and obtained the apparent *B* magnitude modulus difference between the Virgo Cluster core and M31 to be 6.8 ± 0.4 mag. Adopting the apparent blue modulus of M31 as $(m-M)_{AB} = 24.77 \pm 0.17$ (Sandage and Tammann 1988) gives

$$(m-M)_{AB}(\text{Virgo}) = 31.57 \pm 0.43 . \tag{15}$$

As the front absorption due to the Galaxy is assumed to be zero at the galactic latitude of the Virgo Cluster, this is also the true distance modulus.

c) Supernovae

The supernovae observed in the Virgo Cluster can be used in three independent ways to obtain the distance modulus. (1) A photometric Virgo parallax follows from the well-defined maximum apparent *B* magnitude of the six Virgo SNe Ia set out in equation (7), once the absolute magnitude at maximum for SNe Ia has been calibrated. (2) Various physical methods based on variations of the Baade-Wesselink expansion parallax procedure have been applied to several Virgo supernovae to obtain the distance directly, with no assumption required concerning standard candles. (3) The ^{56}Ni-radioactivity physical model of a carbon deflagrating white dwarf gives a theoretical prediction of the maximum absolute magnitude. A review and results from these methods is given by Branch (1985).

i) *Photometric Parallax Method*

We consider first the calibration of the absolute *B* magnitude at maximum.

1. Distances obtained for galaxies in the Cn VII group provided a calibration of SN 1937C in IC 4182 and SN 1954A in NGC 4214 (Sandage and Tammann 1982). A rediscussion of the SNe photometry gave $\langle M_B(\max)\rangle = -19.54$ for SNe Ia from these data (Tammann 1987). However, Branch (1972) had shown that the spectrum SN 1954A was abnormal for a Type Ia SN (helium lines were present), decreasing its utility as a calibrator. Considering then only SN 1937C in IC 4182 gives $M_B = -19.71 \pm 0.2$ (est) for the calibration.

Other calibrations of the maximum absolute magnitude have been set out in detail by Tammann (1982, 1987) and Leibundgut and Tammann (1990), which has already been mentioned. Omitting the details given there, the results are as follows.

2. From a new analysis of four historical SNe, Strom (1988) concluded that Tycho's SN, upon which de Vaucouleurs (1985) has based his distance scale as if it were of Type Ia, was, in fact of Type Ib. For three other historical SNe in the Galaxy—presumably of Type Ia—Strom finds $M_V(\max) = -19.7 \pm 0.5$. Cadonau, Sandage, and Tammann (1985) discussed the evidence concerning the Tycho and Kepler SNe and concluded that $M_B(\max) = -20.0 \pm 0.6$ as an uncertain calibration using only these two historical type events, assumed there to be of Type Ia. For the present paper we adopt -19.7 ± 0.6 for the value from the Galactic historical SNe, noting the 1.1 mag difference from the value used by de Vaucouleurs, accounting for much of the present controversy between the long- and the short-distance scale.

3. Graham (1987) has calculated the light curve of a SN Ia based on a carbon deflagration model. In a comparison of his model with the observed light curve of SN 1972E, he derived a distance to NGC 5253 of 3.6 Mpc, which is $(m-M)_0 = 27.90$. This value combined with $m_B(\max) = 8.42$ (Leibundgut 1988) for the SN gives $M_B(\max) = -19.48 \pm 0.40$ (est) as a calibration.

4. Much work on carbon deflagration white dwarf models has been accomplished as reviewed by Woosley and Weaver (1986). Arnett, Branch, and Wheeler (1985) have calculated bolometric peak luminosities for six SNe Ia in E galaxies as a function of the mass of ^{56}Ni produced in the explosion. Nomoto (1986) favors a value of 0.58 $M_\odot$ for this mass with a range from 0.48 to 1 solar mass. These considerations give a model calibration of $M_B(\max) = -19.5 \pm 0.5$ for SNe Ia.

Averaging the results of methods 1–4 gives $M_B(\max) = -19.59 \pm 0.24$ for the calibration. Applying this to equation (7) for the mean apparent magnitude of the six SNe Ia with adequate *B* photometry in the Virgo cluster gives

$$(m-M)_0(\text{Virgo}) = 31.50 \pm 0.25\ , \tag{16}$$

using Type Ia supernovae as a standard candle in the photometric parallax method.

ii) *Expansion Parallax Results*

In direct applications of expansion parallax methods, Branch and colleagues have determined distances to two SNe in Virgo Cluster galaxies. For SN 1981B in the Sc I galaxy NGC 4536 in the southern extension of the Virgo Cluster, Branch *et al.* (1983) found an expansion parallax modulus of $m-M = 32.2 \pm 0.6$. Even if one hesitates to assign the distance of the Southern Extension to the Virgo Cluster core, this distance determination to NGC 4536 itself gives another determination of the $M_B(\max)$ value to be -20.2 ± 0.6 mag.

For SN 1979C (of Type II) in the ScI Virgo galaxy NGC 4321 (M100) in the cluster core, Branch *et al.* (1981) obtained an expansion parallax modulus of $m-M = 31.80 \pm 0.3$. For the same SN, Bartel and colleagues (Bartel 1989) obtained a radio expansion modulus of 31.70 ± 0.5.

Averaging these three expansion parallax results gives

$$(m-M)_0(\text{expansion}) = 31.84 \pm 0.25\ , \tag{17}$$

which, when averaged with equation (15) gives a final value for the Virgo Cluster modulus from supernovae as

$$(m-M)_{\text{SN}} = 31.72 \pm 0.18\ . \tag{18}$$

d) *The D_n-σ Relation for Spiral Bulges*

The application of the D_n-σ distance indicator for E/S0 galaxies (Dressler *et al.* 1987) can also be applied to the bulges of spiral bulges (Dressler 1987). A calibration of his relation by means of M31 (appropriately corrected for absorption, see Sandage and Tammann 1988, footnote 2), M81, and the Galaxy gives a Virgo Cluster modulus (Tammann 1988) of

$$m-M = 31.85 \pm 0.19\ . \tag{19}$$

For balance we should note that Pierce (1989), using the same method, has derived $m-M = 30.8$ for the Virgo Cluster core. However, he calibrated the relation using distances to the E galaxies NGC 3377 and NGC 3379 in the Leo group determined by Ciardullo, Jacoby, and Ford (1989) from planetary nebulae—a method which has systematically given a short distance scale which we question in the penultimate section. We therefore question the zero-point calibration used by Pierce, since it will contain any systematic error that may be present in using planetary nebulae. The calibration that gives equation (19) is based on Cepheid variables alone.

e) *The* 21 cm *Line Width Method*

Much controversy surrounds results from the Tully-Fisher method, which, in different hands has produced both the long and short distance scales. The modulus values in the literature peak dichotomously around 31.7 for the long scale and 30.8 for the compressed scale. Analyses that have given the larger distance include those of Sandage and Tammann (1976, 1984*a*), Richter and Huchtmeier (1984), Huchtmeier and Richter (1986), Bottinelli *et al.* (1986*a*, *b*, 1987, 1988), and Fouqué *et al.* (1990). Studies that favor the short scale include those of de Vaucouleurs *et al.* (1981), Bothun *et al.* (1984), Aaronson and Mould (1986), Aaronson *et al.* (1986), and Pierce and Tully (1988).

In a discussion of selection effects, KKCT showed the effect on calculated distance modulus values of working with incomplete samples. Their distance modulus for Virgo progressively increased from 30.9, when only the brightest galaxies in the Virgo Cluster luminosity function were used, to 31.6 as the sample approached completeness (see their Fig. 6). From their analysis we adopt here

$$(m-M)_{\text{Virgo}} = 31.60 \pm 0.15\ , \tag{20}$$

from the Tully-Fisher method.

The importance of working with complete samples is emphasized again by Fouqué *et al.* (1990) in their independent analysis of the Virgo Cluster distance. This is the only study besides KKCT that uses a complete sample. As in KKCT, they show how a large systematic error in the derived distance arises when incomplete samples are used. The sense of the error is to derive incorrect smaller distances. The three main results in Fouqué *et al.* are (1) the cluster modulus is within 0.03 mag of equation (20) when they use the same values of the local calibrators as KKCT, (2) the intrinsic scatter of the Tully-Fisher relation is large at 0.7 mag, similar to that found by KKCT in the Virgo Cluster itself using their complete sample and by Sandage (1988*b*, Fig. 4) from field galaxies with $v > 4000$ km s^{-1}, and (3) this large dispersion is present in the S Virgo cloud alone (centered on NGC 4486, called cluster A by Binggeli, Tammann, and Sandage 1987) showing that line-of-sight effects are not the cause of an artificially large Tully-Fisher dispersion, but rather that the observed dispersion is intrinsic.

The study by Fouqué *et al.* answers the two principal objections of Burstein and Raychaudhury (1989, hereafter BR) to KKCT in their attempt to reconcile the large KKCT modulus (eq. [20] here) with the small value of 31.03 obtained by Pierce and Tully (1988). (1) BR questioned the accuracy of the magnitudes used by KKCT. Fouqué *et al.* in recovering the KKCT large modulus used new magnitudes supplied by R. Buta for more than half of their sample. The fact that the modulus obtained by them is within 0.03 mag of equation (20) when reduced to the system of local calibrators of KKCT shows that magnitude errors in KKCT do not have the effect claimed by BR. (2) As mentioned previously, line-of-sight problems were found by Fouqué *et al.* not to have the effect of artificially widening the Tully-Fisher dispersion as suggested by BR.

The agreement of the modulus by Fouqué *et al.* with equation (20) is all the more compelling because these authors have

TABLE 3

SIX METHODS FOR THE VIRGO CLUSTER DISTANCE

Method	$(m-M)_0$	Galaxy Type	Calibrators
Globular clusters	31.74 ± 0.17	E	RR Lyr
Novae	31.57 ± 0.43	E	M31
Supernovae	31.72 ± 0.18	S	...
D_n-σ relation	31.85 ± 0.19	S0, S	Galaxy, M31, M81
21 cm line widths	31.60 ± 0.15	S	13 nearby galaxies
Galaxy and M31 not to be oversized	$>31.50 \pm 0.20$	S	Galactic stars, M31
Mean.	31.70 ± 0.09	($=21.9 \pm 0.9$ Mpc)	

used a differently defined sample, independent observational data including accurate new magnitudes (Buta) that will be listed in the RC 3, and reduction procedures that are quite different from KKCT. Moreover, Fouqué *et al.* point out that the only three local calibrators used by Pierce and Tully do not have universally accepted values of the inclination-corrected line width and absorption-corrected absolute magnitude. Using their preferred parameters of the three calibrators, Fouqué *et al.* increase Pierce and Tully's Virgo modulus by 0.45 mag. From these considerations we adopt equation (20) in what follows.

f) Size of the Largest Virgo Cluster Spirals Relative to the Galaxy and to M31

In a most remarkable and largely neglected paper, van der Kruit (1986) made a convincing analysis of the scale length of the Freeman Galactic disk using the photometric data from the *Pioneer 10* spacecraft. He also showed that the radial surface brightness profiles of the seven largest Sb and five largest Sc Virgo Cluster galaxies have nearly identical exponential disk scale lengths (in arcsecs) to each other.

Postulating that these largest Virgo Cluster spirals must be at least as large (in parsecs) as the Galaxy and M31 (i.e., so that M31 and the Galaxy are not the largest galaxies in the local supercluster), he could then place a *minimum* distance to Virgo such that this condition would obtain. His Figure 8 is one of the strongest proofs available that the distance to Virgo is *at least* 20 Mpc. This, then, puts a high weight limit on the Virgo Cluster modulus to be

$$(m-M)_{\rm Virgo} > 31.5 \,, \tag{21}$$

to which we arbitrarily put an error estimate of 0.2 mag.

VII. ADOPTED VIRGO CLUSTER DISTANCE

The six nearly bias-free distance determinations set out in the last section are brought together in Table 3, the columns of which are self-explanatory. The weighted mean distance modulus from these entries is

$$\langle (m-M)_0 \rangle = 31.70 \pm 0.09 \,, \tag{22}$$

for a distance of

$$D({\rm Virgo}) = 21.9 \pm 0.9 \text{ Mpc} \,. \tag{23}$$

This value depends on distances to E/S0 as well as spiral galaxies. The six different methods for its derivation have largely independent zero-point calibrations. It carries, therefore, exceptionally high weight.

An independent review by van den Bergh (1989) with generally different methods and different prejudices gave $D({\rm Virgo}) = 20 \pm 2$ Mpc, which overlaps equation (23), which we adopt.

VIII. EXPERIMENTAL METHODS

New distance methods are continuously being proposed. It takes some time before many are well enough understood to qualify as viable contenders. Many are called—few are chosen.

Three methods that at the moment are experimental, but which have an increasingly voluminous literature are (1) the luminosity function of planetary nebulae (Jacoby 1989; Jacoby *et al.* 1989; Jacoby, Ciardullo, and Ford 1990), (2) measurement of the amplitude of the surface brightness fluctuation signal from bulges of spirals and the main parts of E galaxies (Tonry and Schneider 1988; Tonry, Ajhar, and Luppino 1989), and (3) an apparent Hβ-σ relation for H II regions (Melnick, Terlevich, and Moles 1988). Each of these methods, applied to the Virgo Cluster galaxies, suggest a distance of ~ 15 Mpc to the cluster.

In the previous sections we have set out our case using other methods which seem more persuasive that the long distance scale is correct. The preponderance of evidence, based on calibrations that are generally accepted (§§ VI*a*–VI*f*) leads to the long distance scale. If this evidence is of the high weight we suppose, then by itself it must bring into question those methods that suggest the short distance scale.

Nevertheless, simply dismissing the methods because they are contradictory (on the premise that eqs. [14]–[22] are, in fact, correct) is insufficient. Eventually an understanding must be reached why the calibration of these methods would be wrong. In the following paragraphs we make the assumption that if the weight of particular evidence is overwhelmingly great and yet if it shows a contradiction with other contrary evidence, then the contrary evidence must be false (Holmes as quoted by Doyle 1887). We must then examine the premises of the contradiction no matter how internally persuasive the contrary evidence may appear.

The following points appear to weaken the calibrations of the three aforementioned methods. Considered first the method using planetary nebulae (hereafter, PN).

It is well understood that PN are formed when intermediate mass stars (0.8 to 3 $M_\odot$) reach the tip of the asymptotic giant branch (AGB), having exhausted H and He in a CO dwarf core, and having begun thermal pulses (TP) (Weigert 1966; Schwarzschild and Härm 1967) leading to a superwind mass loss (Fusi-Pecci and Renzini 1976; Iben 1984). The envelope is expelled in the superwind. Depending on what phase in the TP the envelope is expelled, the remnant burns either H or He in a shell outside the CO core (Iben 1982, 1984; Schönberner 1987). The stellar remnant evolves toward high temperatures, heating the expelled envelope which cools by radiation, primarily in the emission lines of O and N, giving rise to the PN nebular phase.

The remnant that becomes the central star of the planetary nebula (CSPN) evolves at nearly constant bolometric luminosity from the AGB tip to the point of maximum temperature. Following Paczyński (1971), full evolutionary tracks are now available from many authors (e.g., Schönberner 1979, 1981, 1983, 1987; Kovetz and Harpaz 1981; Iben 1982, 1984; Wood and Faulkner 1986).

Both the luminosity and the evolutionary time scale of the CSPN are strong functions of the remnant's mass. The time scale varies as M^{-10}. It is this strong dependence of L and evolutionary time on mass that makes a very sharp upper bound on the PN luminosity function (Jacoby 1989).

Encouraging as these results are for use of the bright end of the PN luminosity function as a standard candle, the extraordinarily high sensitivity of the [O III] $\lambda 5007$ luminosity to the CSPN mass is its potential weakness. This sensitivity raises the possibility that the zero point of the bright end $\lambda 5007$ absolute luminosity can vary from galaxy to galaxy even though the shape of $\varphi(M)_{\rm PN}$ may remain the same (Ciardullo *et al.* 1989). Jacoby's (1989) simulations of the shape and the calibration of $\varphi(M)_{\rm PN}$ show that a change in the mean mass by only 0.03 $M_\odot$ or a change in the width of the mass *distribution* by only $\Delta\sigma(M) = 0.03$ $M_\odot$ can change the bright end limit of $\varphi(M)_{\rm PN}$ by 1 mag. Furthermore, whether the CSPN is burning H or He in its outer shell again changes the bright end luminosity also by ~ 1 mag even at fixed $\langle M \rangle_{\rm CSPN}$ and fixed $\sigma(M)$.

It is known that the CSPN have only a small spread in mass with a range between 0.55 and >0.65 $M_\odot$. The distribution shows a sharp onset at 0.55 $M_\odot$ and perhaps a long tail between 0.65 and >0.8 $M_\odot$ (Weidemann 1987*a*, Fig. 1). It is further known from the initial mass–final mass relation (Weidmann 1987*b*) that this mass range for the CSPN remnant is made from main-sequence stars whose initial mass is in the range from 1 to 3 $M_\odot$, and that the final mass varies as a function of the initial mass *and the Y and Z* values of the main-sequence progenitor star. Figure 14 of Lattanzio (1986) shows that M(final) for a given M(initial) is a sensitive function of Z, changing between 0.56 and 0.58 $M_\odot$ for M(initial) = 1.5 $M_\odot$ as Z changes by only a factor of 2 from 0.01 to 0.02 for $Y = 0.3$. Such changes in M(final) have a profound effect on $L(\lambda 5007)_{\rm PN}$ (Jacoby 1989, Figs. 6 and 7).

It can be expected that the mean evolutionary age of stars will not be precisely the same in galaxies with different star formation histories. Hence, the mean M(final) is not expected to be identical in the bulges of Sb galaxies and the spheriods of E and S0 galaxies because the mean age and the mean metallicities will be different in galaxies of different Hubble type and different absolute magnitude. Furthermore, because the metallicity of E and S0 galaxies is known to vary with distance from the center (Strom *et al.* 1976, 1978; Sandage and Visvanathan 1978; Wirth and Shaw 1983; Baum, Thomsen, and Morgan 1986) we expect the $\lambda 5007$ luminosity of the brightest PN to be a function of radial distance from the center in those galaxies with metallicity gradients.

These problems, discussed in part by Jacoby (1989) and by Ciardullo *et al.* (1988), have not yet have been solved. The "type independence" test discussed by these authors used only Leo E and S0 galaxies. This fails to address the objection of using a calibration made in the bulge of an Sb galaxy (M31) to set the zero point of $L(\lambda 5007)_{\rm PN}$ in E galaxies (in Leo and Virgo). From the above discussion this procedure is objectionable in principle.

A second potential problem with the PN method is that the bulge of M31 is filled with dust, seen well in Figures 2–4 of Ciardullo *et al.* (1988) following earlier work of Johnson and Hanna (1972), Hodge (1980), and McElroy (1983). Ciardullo *et al.* address the problem of the effect on $L(\lambda 5007)_{\rm PN}$ by excluding PN close to *visible* lanes and patches and argue that there is no residual effect. However, it is known that dust distributed within a luminous source is well hidden, remaining difficult to detect either as a color change (Collins and Code 1965, Fig. 5; Schweizer 1974) or as a patch diluted by foreground light. The difficulty is well known in the analysis of dust patches in the Milky Way using star counts where the foreground star density is high.

An indication of how serious the dust problem may be in the bulge of M31 is the observation by J. L. Tonry (1990, private communication) that he has been unable to use data taken in the bulges of M31 and M81 to calibrate his surface brightness fluctuation method of distance determination. The problem stems from the strong fluctuation signal due to dust irregularities, giving a false spatial power spectrum for the fluctuation amplitudes. The extent of the M31 problem for the $\varphi(M)_{\rm PN}$ calibration is not yet known, either because of (1) variations of the mean stellar age of the dominant population, (2) differences in the mean metallicity as a function of Hubble type, or (3) problems associated with the hidden dust content in different Hubble types from E to Im galaxies. The solution to these problems would seem to be required via a calibration route other than through M31 itself if the $\varphi(M)_{\rm PM}$ function is to be used with E and S0 galaxy types. Ideally, distances to E galaxies known by other methods should be used to calibrate the $\varphi(M)_{\rm PN}$ function.

Consider next the surface brightness fluctuation method of Tonry and Schneider (1988). Here one determines a mean absolute magnitude $\langle M \rangle$ for that particular stellar content that gives rise to the observed surface brightness. The fluctuation data are normalized to a fixed surface brightness. However, just as in the PN method we demand identical H-R diagrams among the galaxies, i.e., the mean $\langle M \rangle$ for a "representative star" must be the same from galaxy to galaxy. As in the PN case, this requires similar enough star formation histories from galaxy to galaxy.

The positions of evolving sequences in the H-R diagram (i.e., the RGB, the HB, and the AGB) are strong functions of metallicity and of ages. Again, the zero-point calibration of the fluctuation signal, which is determined by the ratio of $\langle M \rangle$ to the surface brightness, should be made empirically using E and S0 galaxies of known distance. Tonry, Ajhar, and Luppino (1989) used M32 as their calibrator and obtained $D \sim 15$ Mpc for the Virgo Cluster distance. More recently Tonry (1990) has made a semiempirical calibration of the $\langle M \rangle$/SB ratio using the computed revised Yale evolutionary tracks (Green, Demarque, and King 1987). This calibration gave an $\langle M \rangle$/SB value that requires $D \sim 22$ Mpc for Virgo (see Note added in proof). The uncertainty at the moment is, then, the zero-point calibration of the method using the unusual galaxy M32. Again, a sample of normal E galaxies at known distances would seem to be required.

Consider finally the Hβ-σ method of Melnick, Terlevich, and Moles (1988). Their sample is flux limited at 10^{-14} ergs s^{-1} cm^{-2}, not volume limited as a sample must be if the intrinsic dispersion of the derived distance indicator is to be determined. The nature of their sample is seen by plotting the log of the redshift versus the log of the absolute power (ergs s^{-1}) in Hβ that can be derived from their measured flux (in ergs s^{-1} cm^{-2})

listed in column (6) of their Table 1. (Using $H = 100$ for the Hubble constant so as to be consistent with their assumption, the absolute power is $1.08 \times 10^{44} z^2$[Hβ flux] ergs s^{-1}).

The proof that a biased sample exists is the correlation of the absolute power of their sample galaxies with redshift that exists in the MTM data. The absolute power of the galaxies in their sample *increases as the square of the distance* showing beyond doubt the existence of a strong selection bias. There is no "bias free zone" in these data as is stated by MTM. The situation is identical to the increase in the absolute radio power of the 3C radio sources with the square of the redshift found by Sandage (1972*b*, Fig. 7), which was such a clear effect there showing that a selection bias exists in the flux-limited 3C catalog. (Here the very high flux level of the 3C survey ensures that only the tip of the radio luminosity function is seen). The identical situation is evidently present with the Hβ absolute powers in the MTM sample. The correlation of Hβ power with z^2 and the range of over 10^3 in this power from the very local galaxies to those in the MTM list at $z > 0.1$ show the very wide intrinsic luminosity function, much larger than set out in Figure 5 of MTM.

The point is that a Malmquist bias exists in the MTM sample. We therefore call into question the procedure used by MTM to "correct for Malmquist bias" because the intrinsic dispersion of the distance indicator cannot be found from the available data. These authors recognize part of their problem by the final sentence of their acknowledgments.

It has not been our intention here to claim certainty in these suggestions as to where the zero points of the three new methods that give the short distance scale can be questioned. Rather, following a recommendation of the referee, our purpose in the rather long side excursion in this section has been to show several possibilities why one need not be persuaded to accept the short distance scale based on the available evidence.

IX. THE GLOBAL VALUE OF H_0

By combining the "cosmic" velocity that the Virgo Cluster core would have if it were freed from all local velocity perturbations from equation (13) with the adopted distance to the cluster from equation (23) gives the global value of the Hubble expansion rate, freed from all local influences, as

$$H_0 = 52 \pm 2 \text{ km s}^{-1} \text{ Mpc}^{-1} . \quad (24)$$

The 99% confidence range for the distance is

$$15 < D(\text{Virgo}) < 28 \text{ Mpc} , \quad (25)$$

giving a corresponding 99% confidence range for H_0 to be

$$40 < H_0 < 76 . \quad (26)$$

The unprecedented accuracy with which we now believe the distance (eq. [23]) and the "cosmic velocity" (eq. [13]) of the cluster core are known (eq. [23]) is the reason for the small quoted error (external) in equation (24). But if this distance D (in Mpc) to Virgo is eventually revised by future data, properly analyzed, the knowledge of the cosmic Virgo velocity from equation (13) will translate to a new value of H_0 as

$$H_0 = (52 \pm 2)(21.9/D) \text{ km s}^{-1} \text{ Mpc}^{-1} . \quad (27)$$

We need now to make two final points.

1. We claim that the result in equation (27) is nearly bias free. Our argument is that the *relative* cluster distances in Table 1 have been determined from data where the limiting magnitude of the cluster members *increases* with increasing distance. This has the effect that we have used galaxies in each cluster within a nearly *constant magnitude interval* below the first-ranked cluster member in applying the 21 cm line width method (KKCT). Furthermore, for the other distance indicators such as first-ranked galaxies and SNe Ia at maximum light, the intrinsic luminosity scatter ($\sigma_M < 0.3$ mag) is so small that these indicators are particularly invulnerable to selection effects. But even if this view is overly optimistic, and if, thereby, the first-ranked galaxies and SNe Ia in our sample are progressively brighter with increasing distance due to selection (Malmquist) bias, then the *true* cosmic Virgo velocity would be *smaller* than given by equation (13). In that case, H_0 would also be smaller than is set out in equation (24). A reduction in $v(\text{Virgo})_{\text{cosmic}}$ for this reason would have the further effect of reducing the Local Group Virgocentric infall velocity below the value we have determined in § V.

2. Continuing the argument one step further leads to a firm *lower limit* to H_0 using the present method. The extreme reduction of the Virgo Cluster effect on the Local Group is to put the infall (retarded recession) velocity to zero. Although physically unacceptable because the Virgo Cluster would then have *no* pull on the Local Group (the empty universe case of $\Omega = 0$), this condition does define the limit where $v(\text{Virgo})_{\text{cosmic}} = v(\text{Virgo})_{\text{observed}} = 976 \pm 45$ km s^{-1} as set out in § V. Using $D = 21.9 \pm 0.9$ Mpc from equation (23) gives, then, the lower limit on H_0 by this argument to be

$$H_{0(\text{lower limit})} = (45 \pm 3)(21.9/D) \text{ km s}^{-1} \text{ Mpc}^{-1} , \quad (28)$$

which, as with equation (27), is tied to the average cosmic frame at a distance of $\gtrsim 6000$ km s^{-1}.

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REFERENCES

Aaronson, M., *et al.* 1982*a*, *Ap. J. Suppl.*, **50**, 241.
Aaronson, M., Bothun, G., Mould, J., Huchra, J., Schommer, R. A., and Cornell, M. E. 1986, *Ap. J.*, **302**, 536.
Aaronson, M., Huchra, J., Mould, J., Schechter, P. L., and Tully, R. B. 1982*b*, *Ap. J.*, **258**, 64.
Aaronson, M., and Mould, J. 1986, *Ap. J.*, **303**, 1.
Arnett, W. D., Branch, D., and Wheeler, J. C. 1985, *Nature*, **314**, 337.
Barbon, R., Capaccioli, M., and Ciatti, F. 1975, *Astr. Ap.*, **44**, 267.
Bartel, N. 1989, in *Supernovae Shells and Their Birth Events*, ed. W. Kundt (Berlin: Springer), in press.
Baum, W. A., Thomsen, B., and Morgan, B. L. 1986, *Ap. J.*, **301**, 83.
Binggeli, B., Tammann, G. A., and Sandage, A. 1987, *A.J.*, **94**, 251.
Bothun, G. D., Aaronson, M., Schommer, B., Huchra, J., and Mould, J. 1984, *Ap. J.*, **278**, 475.
Bothun, G. D., Caldwell, N., and Schombert, J. M. 1989, *A.J.*, **98**, 1542.
Bothun, G. D., Geller, M. J., Beers, T. C., and Huchra, J. P. 1983, *Ap. J.*, **268**, 47.
Bottinelli, L., Fouqué, P., Gougenheim, L., Paturel, G., and Teerikorpi, P. 1987, *Astr. Ap.*, **181**, 1.
Bottinelli, L., Gougenheim, L., Paturel, G., and Teerikorpi, P. 1986*a*, *Astr. Ap.*, **156**, 157.
———. 1986*b*, *Astr. Ap.*, **166**, 393.
———. 1988, *Ap. J.*, **328**, 4.
Bowen, I. S. 1952, in *Ann. Rept. Director Mount Wilson and Palomar Observatories, Yearbook Carnegie Institution of Washington*, No. **51**, p. 19.
Branch, D. 1972, *Astr. Ap.*, **16**, 247.
———. 1982, *Ap. J.*, **258**, 35.
———. 1985, in *Supernovae as Distance Indicators, Physics Lecture Notes*, No. 224, ed. N. Bartel (Berlin: Springer), p. 138.
Branch, D., and Bettis, C. 1978, *A.J.*, **83**, 224.

Branch, D., Falk, S. W., McCall, M. L., Rybski, P., Uomoto, A. K., and Wills, B. J. 1981, *Ap. J.*, **244**, 780.
Branch, D., Lacy, C. H., McCall, M. L., Sutherland, P. G., Uomoto, A., Wheller, J. C., and Wills, B. J. 1983, *Ap. J.*, **270**, 123.
Burstein, D., and Raychaudhury, S. 1989, *Ap. J.*, **343**, 18.
Cadonau, R. 1987, Ph.D. thesis, University of Basel.
Cadonau, R., and Leibundgut, B. 1989, *Astr. Ap. Suppl.*, in press.
Cadonau, R., Sandage, A., and Tammann, G. A. 1985, in *Supernovae as Distance Indicators, Physics Lecture Notes*, No. 224, ed. N. Bartel (Berlin: Springer), p. 151.
Charlier, C. V. 1908, *Ark. Mat. Astr. och Fys.*, **4**, No. 24.
———. 1922, *Medd. Lunds Astr. Obs.*, No. 98.
Ciardullo, R., Jacoby, G. H., and Ford, H. C. 1989, *Ap. J.*, **344**, 715.
Ciardullo, R., Rubin, V. C., Jacoby, G. H., Ford, H. C., and Ford, W. K. 1988, *A.J.*, **95**, 438.
Ciardullo, R., Jacoby, G. H., Ford, H. C., and Neill, J. D. 1989, *Ap. J.*, **339**, 53.
Collins, C. A., Joseph, R. D., and Robertson, N. A. 1986, *Nature*, **320**, 506.
Collins, G. W., and Code, A. D. 1965, *Ap. J.*, 1965, *Ap. J.*, **142**, 1576.
de Carvalho, R. R., and Djorgovski, S. 1989, *Ap. J. (Letters)*, **341**, L37.
de Vaucouleurs, G. 1958, *A.J.*, **63**, 253.
———. 1970, *Science*, **167**, 1203.
———. 1971, *Pub. A.S.P.*, **83**, 113.
———. 1972, in *IAU Symposium 44, External Galaxies and Quasistellar Sources*, ed. D. S. Evans (Dordrecht: Reidel), p. 353.
———. 1985, *Ap. J.*, **289**, 5.
de Vaucouleurs, G., and Peters, W. L. 1986, *Ap. J.*, **303**, 19.
de Vaucouleurs, G., Peters, W. L., Bottinelli, L., Gougenheim, L., and Paturel, G. 1981, *Ap. J.*, **248**, 408.
Doyle, A. C. 1887, in *A Study in Scarlet, Beeton's Christmas Annual*, Vol. **28**, Ward, Lock, and Co. (London: New York: Melbourne). The exact quote is, "I should have more faith. I ought to know by this time that when a fact appears to be opposed to a long line of deductions it invariably proves to be capable of bearing some other interpretation."
Dressler, A. 1987, *Ap. J.*, **317**, 1.
Dressler, A., Faber, S. M., Burstein, D., Davies, R. L., Lynden-Bell, D., Terlevich, R. J., and Wegner, G. W. 1987, *Ap. J. (Letters)*, **313**, L37.
Faber, S. M., and Burstein, D. 1988, in *Large-Scale Motions in the Universe: A Vatican Study Week*, ed. V. C. Rubin and G. V. Coyne (*Pontif. Acad. Sci. Scripta Varia*, No. 76), p. 115.
Faber, S. M., Wegner, G., Burstein, D., Davies, R., Dressler, A., Lynden-Bell, D., and Terlevich, R. J. 1989, *Ap. J. Suppl.*, **69**, 763.
Ferguson, H. C., and Sandage, A. 1988, *A.J.*, **96**, 1599.
Fixen, D. J., Cheng, E. S., and Wilkenson, D. T. 1983, *Phys. Rev. Letters*, **50**, 620.
Fouqué, P., Bottinelli, L., Gougenheim, L., and Paturel, G. 1990, *Ap. J.*, **349**, 1.
Fusi-Pecci, F., and Renzini, A. 1976, *Astr. Ap.*, **46**, 447.
Geisler, D., and Forte, J. C. 1990, *Ap. J. (Letters)*, **350**, L5.
Graham, J. R. 1987, *Ap. J.*, **315**, 588.
Green, E. M., Demarque, P., and King, C. R. 1987, *The Revised Yale Isochrones and Luminosity Functions* (New Haven: Yale University Observatory).
Haggerty, M. J., and Wertz, J. R. 1971, *M.N.R.A.S.*, **155**, 495.
Harris, W. E. 1988, in *The Extra Galactic Distance Scale, A.S.P. Conf. Series No. 4*, ed. S. van den Bergh and C. J. Pritchet (Provo: Brigham Young University Press), p. 231.
Harris, W. E., Allwright, J. W. B., Pritchet, C. J., and van den Bergh, S. 1990, in preparation.
Hodge, P. M. 1980, *A.J.*, **85**, 376.
Huchtmeier, W. K., and Richter, O.-G. 1986, *Astr. Ap. Suppl.*, **63**, 323.
Humason, M. L., Mayall, N. U., and Sandage, A. 1956, *A.J.*, **61**, 97.
Iben, I. 1982, *Ap. J.*, **260**, 821.
———. 1984, *Ap. J.*, **277**, 333.
Jacoby, G. H. 1989, *Ap. J.*, **339**, 39.
Jacoby, G. H., Ciardullo, R., and Ford, H. C. 1990, *Ap. J.*, **356**, 332.
Jacoby, G. H., Ciardullo, R., Ford, H. C., and Booth, J. 1989, *Ap. J.*, **344**, 704.
James, P. A., Joseph, R. D., and Collins, C. A. 1987, *M.N.R.A.S.*, **229**, 53.
Johnson, H. M., and Hanna, M. M. 1972, *Ap. J. (Letters)*, **174**, L71.
Kovetz, A., and Harpaz, A. 1981, *Astr. Ap.*, **95**, 66.
Kowal, C. T. 1968, *A.J.*, **73**, 1021.
———. 1969, *Pub. A.S.P.*, **81**, 608.
Kraan-Korteweg, R. C. 1986, *Astr. Ap. Suppl.*, **66**, 255 (a complete galaxy listing is in Basel preprint, No. 18).
Kraan-Korteweg, R. C., Cameron, L. M., and Tammann, G. A. 1988, *Ap. J.*, **331**, 620 (KKCT).
Kraan-Korteweg, R. C., and Tammann, G. A. 1979, *Astr. Nach.*, **300**, 181.
Lattanzio, J. C. 1986, *Ap. J.*, **311**, 708.
Leibundgut, B. 1988, Ph.D. thesis, University of Basel.
Leibundgut, B., and Tammann, G. A. 1990, *Astr. Ap.*, in press.
Lubin, P. M., Epstein, G. L., and Smoot, G. F. 1983, *Phys. Rev. Letters*, **50**, 616.
Lucey, J. R., and Carter, D. 1988*a*, *M.N.R.A.S.*, **231**, 15P.
———. 1988*b*, *M.N.R.A.S.*, **235**, 1177.
Lynden-Bell, D., Faber, S., Burstein, D., Davies, R., Dressler, A., Terlevich, R., and Wegner, G. 1988, *Ap. J.*, **326**, 19.
Madejski, G. M., and Bender, R. 1989, preprint.
McElroy, D. B. 1983, *Ap. J.*, **270**, 485.
Melnick, J., Terevich, R., and Moles, M. 1988, *M.N.R.A.S.*, **235**, 297.
Nomoto, K. 1986, *Ann. NY Acad. Sci.*, **470**, 294.
Paczyński, B. 1971, *Acta Astr.*, **21**, 417.
Peterson, C. J. 1986, *Pub. A.S.P.*, **98**, 1258.
Peterson, C. J., and Reed, B. C. 1987, *Pub. A.S.P.*, **99**, 20.
Pierce, M. J. 1989, *Ap. J. (Letters)*, **344**, L57.
Pierce, M. J., and Tully, R. B. 1988, *Ap. J.*, **330**, 579.
Pritchet, C. J., and van den Bergh, S. 1987, *Ap. J.*, **318**, 507.
Richter, O.-G., and Huchtmeier, W. K. 1984, *Astr. Ap.*, **132**, 253.
Richter, O.-G., Tammann, G. A., and Huchtmeier, W. K. 1987, *Astr. Ap.*, **171**, 33.
Sandage, A. 1963, in *IAU Symposium 15, Problems of Extragalactic Research*, ed. G. C. McVittie (New York: Macmillan), p. 378.
———. 1972*a*, *Ap. J.*, **178**, 1.
———. 1972*b*, *Ap. J.*, **178**, 25.
———. 1975, *Ap. J.*, **202**, 563.
———. 1986, *Ap. J.*, **307**, 1.
———. 1987, *Ap. J.*, **317**, 557.
———. 1988*a*, *Ap. J.*, **331**, 583.
———. 1988*b*, *Ap. J.*, **331**, 605.
Sandage, A., and Cacciari, C. 1990, *Ap. J.*, **350**, 645.
Sandage, A., and Hardy, E. 1973, *Ap. J.*, **183**, 743 (SH).
Sandage, A., and Tammann, G. A. 1975, *Ap. J.*, **196**, 313.
———. 1976, *Ap. J.*, **210**, 7.
———. 1982, *Ap. J.*, **256**, 339.
———. 1984*a*, *Nature*, **307**, 326.
———. 1984*b*, in *Large-Scale Structure of the Universe, Cosmology, and Fundamental Physics*, ed. G. Setti and L. Van Hove (Garching, Geneva: ESO/CERN), p. 127.
———. 1987, *A Revised Shapley-Ames Catalog of Bright Galaxies* (2d ed.; Washington, DC: Carnegie Institution of Washington).
———. 1988, *Ap. J.*, **328**, 1.
Sandage, A., Tammann, G. A., and Hardy, E. 1972, *Ap. J.*, **172**, 253 (STH).
Sandage, A., and Visvanathan, N. 1978, *Ap. J.*, **223**, 707.
Schönberner, D. 1979, *Astr. Ap.*, **79**, 108.
———. 1981, *Astr. Ap.*, **103**, 119.
———. 1983, *Ap. J.*, **272**, 708.
———. 1986, *Astr. Ap.*, **169**, 189.
———. 1987, in *IAU Colloquium 95, The Second Conference on Faint Blue Stars*, ed. A. G. Davis Phillip, D. S. Hayes, and J. W. Liebert (Schenectady: Davis), p. 201.
Schwarzschild, M., and Härm, R. 1967, *Ap. J.*, **150**, 961.
Schweizer, F. 1974, Ph.D. thesis, University of California, Berkeley.
Shaya, E. J. 1984, *Ap. J.*, **280**, 470.
Strom, R. G. 1988, *M.N.R.A.S.*, **230**, 331.
Strom, S. E., Strom, K. M., Goad, J. W., Vrba, F. J., and Rice, W. 1976, *Ap. J.*, **204**, 684.
Strom, K. M., Strom, S. E., Wells, D. C., and Romanishin, W. 1978, *Ap. J.*, **220**, 62.
Tammann, G. A. 1978, *Mem. Soc. Astr. Italiana*, **49**, 315.
———. 1982, in *Supernovae: A Survey of Current Research, Proc. NATO Adv. Study Inst.*, ed. M. J. Rees and R. J. Stoneham (Dordrecht: Reidel), p. 371.
———. 1987, in *IAU Symposium 124, Observational Cosmology*, ed. A. Hewitt, G. R. Burbidge, and L. Fang (Dordrecht: Reidel), p. 151.
———. 1988, in *The Extragalactic Distance Scale, A.S.P. 100th Anniversary Symposium, A.S.P. Conf. Series No. 4*, ed. S. van den Bergh and C. J. Pritchet (Provo: Brigham Young University Press), p. 282.
Tammann, G. A., and Leibundgut, B. 1989, *Astr. Ap.*, submitted (TL).
Tammann, G. A., and Sandage, A. 1985, *Ap. J.*, **294**, 81.
Tammann, G. A., Sandage, A., and Yahil, A. 1980, *Phys. Scripta*, **21**, 630.
Teerikorpi, P. 1987, *Astr. Ap.*, **173**, 39.
———. 1990, *Astr. Ap.*, **234**, 1.
Tonry, J. L. 1990, private communication.
Tonry, J. L., Ajhar, E. A., and Luppino, G. A. 1989, *Ap. J. (Letters)*, **346**, L57.
Tonry, J. L., and Davis, M. 1981, *Ap. J.*, **246**, 680.
Tonry, J. L., and Schneider, D. 1988, *A.J.*, **96**, 807.
Tully, R. B. 1988, *Nature*, **334**, 209.
Tully, R. B., and Fisher, J. R. 1977, *Astr. Ap.*, **54**, 661.
van den Bergh, S. 1989, *Astr. Ap. Rev.*, **1**, 111.
van den Bergh, S., Pritchet, C., and Grillmair, C. 1985, *A.J.*, **90**, 595.
van der Kruit, P. 1986, *Astr. Ap.*, **157**, 230.
Webbink, R. F. 1985, in *IAU Symposium 113, Dynamics of Star Clusters*, ed. J. G. Goodman and P. Hut (Dordrecht: Reidel), p. 541.
Weedman, D. 1976, *Ap. J.*, **203**, 6.
Weidemann, V. 1987*a*, in *IAU Colloquium 95, The Second Conference on Faint Blue Stars*, ed. A. G. Davis Philips, D. S. Hayes, and J. W. Liebert (Schenectady: Davis), p. 19.
———. 1987*b*, *Astr. Ap.*, **188**, 74.
Weigert, A. 1966, *Zs. Ap.*, **64**, 395.
Wirth, A., and Shaw, R. 1983, *A.J.*, **88**, 171.
Wood, P. R., and Faulkner, D. J. 1986, *Ap. J.*, **307**, 659.
Woosley, S. E., and Weaver, T. A. 1986, *Ann. Rev. Astr. Ap.*, **24**, 205.
Yahil, A. 1988, in *Large-Scale Motions in the Universe: A Vatican Study Week*, ed. V. C. Rubin and G. V. Coyne (*Pontif. Acad. Sci. Scripta Varia*, No. 76), p. 219.
Yahil, A., Tammann, G. A., and Sandage, A. 1977, *Ap. J.*, **217**, 903.

Note added in proof.—In a preprint received on August 30, J. L. Tonry, E. A. Ajhar, and G. A. Luppino (1990, *Ap. J.*, submitted) discuss their Virgo Cluster distance of 21 ± 1 Mpc based on the luminosity fluctuation method (see § VIII) as they calibrate it from the Yale isochrones. But even with their distance that agrees with equation (23) here, they calculate $H_0 = 64 \pm 5$ km s^{-1} Mpc^{-1}. However, they have used the high value of 1333 ± 75 km s^{-1} for the Virgo cluster cosmological velocity. To obtain this velocity (which we have called the "cosmic expansion velocity of Virgo freed from local velocity anomalies") they adopted a model of the local velocity field that necessarily is based on distances that require uncertain Malmquist bias corrections. On the other hand, if they had used their Virgo distance with Virgo cosmological velocity of 1144 ± 18 km s^{-1} as in equation (13) here where no model of the local velocity field is required to derive it (§ I), Tonry *et al.* would have obtained $H_0 = 54.5 \pm 2$ km s^{-1} Mpc^{-1}. Hence their luminosity fluctuation method need not be taken to be in conflict with the evidence given here that $H_0 = 52 \pm 2$ km s^{-1} Mpc^{-1}.

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