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SPECTROSCOPIC STUDIES AND ATMOSPHERIC PARAMETERS OF PULSATING DA WHITE DWARF (ZZ CETI) STARS

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ABSTRACT

We present optical spectrophotometry of a sample of 10 ZZ Ceti stars, obtained with the aim of providing independent estimates of the atmospheric parameters of these stars. This is accomplished by a study of high Balmer lines (Hγ–H8), which permits the separation of surface gravity from effective temperature effects. The observations are analyzed in terms of new grids of synthetic spectra, which incorporate the new occupation probability formalism of Hummer and Mihalas to calculate the atomic level populations. The ZZ Ceti stars studied in this work all have temperatures between 11,320 and 13,420 K. The average surface gravity for the sample is log $g = 8.03$, with $\sigma(\log g) = 0.25$. Our analysis confirms the lower-than-average gravity of R548 (log $g = 7.80$), suspected from earlier photometry, as well as the somewhat larger-than-average gravity of G226-29 (log $g = 8.20$) which may, however, be insufficient to account for its short periods. The implications of our determinations for a few objects of particular astrophysical interest are discussed.

Subject headings: atomic processes — stars: atmospheres — stars: pulsation — stars: white dwarfs

I. INTRODUCTION

The pulsating DA white dwarfs, or ZZ Ceti stars, represent one of the most homogeneous class of variable stars (see, e.g., McGraw 1979). Several investigations over the past 10 years have shown that the existence of pulsating, DA white dwarfs is restricted to a rather narrow instability strip. And indeed, fast-photometric searches for pulsating stars in that class, coupled to statistical studies of the resulting frequency of variable stars, provide strong evidence that the ZZ Ceti stars represent an evolutionary phase through which most, if not all, hydrogen-atmosphere white dwarfs are expected to cool (Fontaine *et al.* 1982; Greenstein 1982). This result provides an obvious incentive to study these stars in greater detail, as seismological studies of ZZ Ceti stars (e.g., Winget 1988) might then eventually provide constraints on the properties not only of the two dozen or so known variable stars, but on those of the *whole* sample of DA stars as well. For example, the blue edge of the instability strip appears, on the basis of theoretical modeling, to be a sensitive function of both the mass of the hydrogen envelope and the efficiency of convective mixing in the latter (see, e.g., the review of Winget and Fontaine 1982). A measurement of the effective temperature of the blue edge could thus yield important constraints on the prior evolution of DA white dwarfs and on the hydrodynamic properties of their outer layers.

Historically, the first attempts at defining the boundaries of the ZZ Ceti instability strip were made through the use of optical photometry. While earlier data were restricted to broad-band colors, Strömgren colors were later measured for a large number of ZZ Ceti stars and other DA stars (Graham 1972; McGraw 1979; Wegner 1979; Koester and Weidemann 1982; Fontaine *et al.* 1985*a*), and reduced significantly the scatter in the location of variable stars in two-color diagrams. In parallel, Greenstein (1976, 1984) measured a large number of those stars with the multichannel spectrophotometer, and carried out a detailed analysis of the ZZ Ceti star sample (Greenstein 1982); these data were used as well by Weidemann and Koester (1984) in an independent reanalysis of the same data base. The boundaries of the instability strip obtained through these studies are located near 10,000–11,000 K on the red side, and near 12,000–13,000 K on the blue side. More recently, ultraviolet energy distributions obtained with the *IUE* satellite have served as the basis of a new determination of the boundaries of the instability strip (Wesemael, Lamontagne, and Fontaine 1986: Lamontagne, Wesemael, and Fontaine 1987; Lamontagne *et al.* 1989). While there are both advantages and disadvantages to using ultraviolet spectrophotometry from the *IUE Observatory* for this project, these latest determinations represent an independent measurement of quantities which provide critical tests of pulsation calculations based on the current generation of white dwarf models.

In this paper, we reconsider the problem of the boundaries of the instability strip with the use of a largely overlooked technique. Indeed, Schulz and Wegner (1981) have demonstrated that simultaneous spectroscopy of the lower and higher Balmer lines permits reasonably accurate determinations of both T_e and log g for DA stars in the ZZ Ceti range (with typical uncertainties, in their analysis, of ± 500 K and ± 0.25 dex, respectively). Our aim is thus twofold: first, to apply the spectroscopic technique of Schulz and Wegner (1981) to a sample of ZZ Ceti stars to determine the temperature boundaries of our sample, and the relative ordering of stars within it; second, to provide the first spectroscopic estimates of the surface gravity of a sample of ZZ Ceti stars. Such estimates may permit an improved understanding of the observed variation in strength of the quasi-molecular 1400 and 1600 Å features observed in ZZ Ceti stars, since current modeling of these features predicts a significant dependance of their strength on log g (Nelan and Wegner 1985). Furthermore, attention has already been drawn to candidates with possibly peculiar gravities within the ZZ Ceti sample. R548, for example, is a suspected low-gravity object (log $g \sim 7.5$; Fontaine *et al.* 1985*a* and references therein) on the basis of its Strömgren colors, while

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the interpretation of the very short periods (~109 s) of G226–29 requires a large gravity instead (log $g \sim 8.5$; Kepler, Robinson, and Nather 1983). Both of these objects are included in our sample, as well as a few other ZZ Ceti stars of particular astrophysical interest which will be discussed below.

II. SPECTROSCOPIC OBSERVATIONS

With these considerations in mind, we have gathered optical spectrophotometric observations for a sample of 10 objects among the 21 currently known ZZ Ceti stars. No specific selection criteria were applied in the choice of these objects, as this has largely remained a backup observational project. Nevertheless, we did ensure that the two peculiar objects mentioned above were included, as well as G117-B15A, the bluest known ZZ Ceti star (which defines the blue edge in most analyses performed up to now), and GD 154, its counterpart on the red edge of the instability strip. Our data come from two distinct sources: first, we have collected, over a period of several years, observations at the Steward Observatory 2.3 m telescope, most of which were obtained with the photon-counting, blue-sensitive Reticon. Despite the long interval spanned by these observations (1985–1989), the spectra were all obtained with comparable instrumental setups. In general, the combination of entrance aperture (2″ × 3″) and grating (either 600 l/mm or 832 l/mm) used afforded a spectral resolution between 3 Å and 7–9 Å, and a coverage of the 3800–4600 Å region. For the 1989 runs, the photon-counting Reticon was replaced by a blue-sensitive, Texas Instrument CCD.

Three additional program stars (L19-2, R548, and G29-38) were observed as a backup project at the Cerro Tololo 1.5 m telescope, equipped with a GE, blue-sensitive CCD chip. For that run, the combination of grating (300 l/min) and slit width (3″) gave a spectral resolution of ~9 Å, and a spectral coverage of the 3600–5900 Å range.

The 10 objects observed in this investigation are presented in Table 1, where we have included as well relevant photometric data. All objects, save G29-38, were observed long enough for the data to cover at least one pulsation cycle (see footnote 6 for a comment on the long periods reported in GD 165). The optical spectra, which form the basis of this work, are presented in Figure 1. The signal-to-noise ratio varies from ~20 in the cool, faint star GD 154 to ~70 in the best-exposed objects. Of the seven stars observed at Steward Observatory, only one has been discussed previously by us: GD 66, for which a preliminary spectroscopic analysis based on an earlier spectrum is presented by Fontaine *et al.* (1985*b*).

TABLE 1
THE SAMPLE OF ZZ CETI STARS OBSERVED

Star	Observing Date	Exposure Time (s)	$(u-b)^a$	$(b-y)^a$	$(G-R)^b$
R548	1986 Oct 25	1800	+0.723	+0.029	−0.37
GD 66	1989 Dec 7	1350	...	...	−0.34
GD 99	1987 Feb 2	1440	+0.568	+0.064	−0.36
G117-B15A	1987 Apr 1	1920	+0.626	+0.029	−0.41
GD 154	1987 Feb 2	1440	...	...	−0.29
G238-53	1989 Jan 15	1800	...	...	−0.30
GD 165[c]	1989 Jan 16	1200	+0.594	+0.065	...
L19-2	1987 Aug 14	1800	+0.567	+0.054	...
G226-29	1985 May 5	1440	+0.522	+0.069	−0.36
G29-38	1987 Aug 15	900	+0.618	+0.054	−0.37

[a] Strömgren photometry from Fontaine *et al.* 1985*a* and Wegner 1979, 1983.
[b] Multichannel colors are based on the AB79 calibration of Oke and Gunn 1983, as described by Greenstein 1984.
[c] This object has recently been identified as a ZZ Ceti star by Bergeron and McGraw 1990.

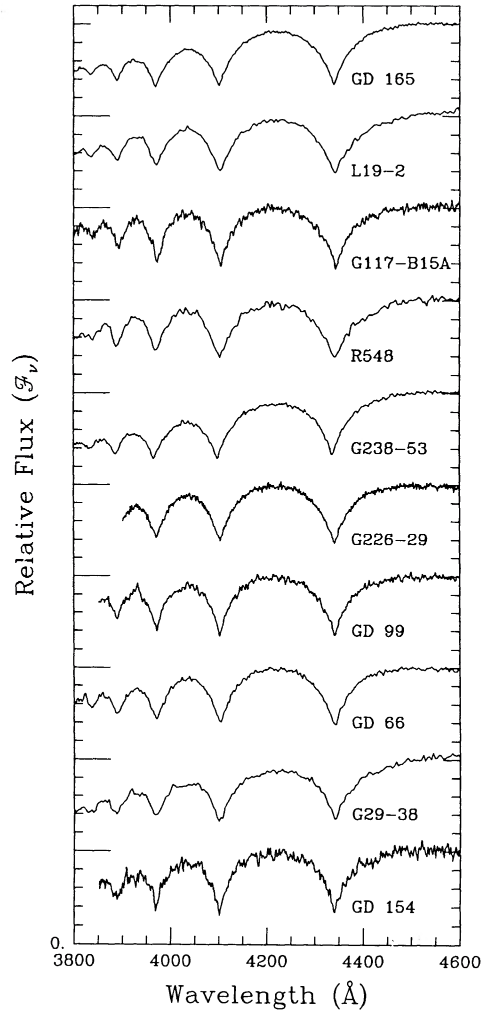

FIG. 1.—Optical spectra of the 10 ZZ Ceti stars used in our analysis. The spectral resolution varies between 3 and 9 Å. Balmer lines up to H8 or H9 are generally visible. The spectra are normalized at 4500 Å and are shifted vertically. The various zero points are indicated by long tick marks.

III. THEORETICAL FRAMEWORK

a) Model Atmosphere Calculations

Our analysis relies on a recently computed grid of LTE model atmospheres for DA white dwarf stars computed by one of us (P. B.). These models are based on now standard assumptions for ZZ Ceti stars: they are hydrogen-line blanketed, assume a pure hydrogen composition, and include convective energy transport. The efficiency of the latter is potentially an important, yet much overlooked, problem in the modeling of ZZ Ceti stars, as the pulsational instability through the κ mechanism is, of course, related to the presence of a very active convection zone in the hydrogen layer, which extends all the way up to the photosphere. In this exploratory work, we have preferred to use a standard efficiency, which is close to that used in most previous *model atmosphere* calculations, namely, that termed ML1 by Fontaine, Villeneuve, and Wilson (1981).

The use of that efficiency has allowed us to compare in a more straightforward manner our models with those used in previous studies of cool DA stars but, of course, there is no fundamental reason why ML1 should necessarily be characteristic of the atmospheres of ZZ Ceti stars. Much work remains on this particular aspect of the modeling. Further details on the construction of the LTE models, and on comparisons with earlier model grids, can be found in Bergeron (1988).

b) Calculation of Synthetic Spectra

For the analysis of the optical spectrum of ZZ Ceti stars, detailed synthetic spectra of the region longward of the Balmer jump are required. These were computed with the atmospheric structure of the LTE models, but include an improved treatment of both the line opacity and the atomic level population. For the former, we have used the unified theory of Vidal, Cooper, and Smith (1970) up to Hδ, and that of Edmonds, Schlüter, and Wells (1967) for the higher Balmer lines. The level populations, on the other hand, are calculated within the occupation probability formalism of Hummer and Mihalas (1988). While the use of this formalism in the calculation of synthetic spectra for cool white dwarfs is discussed extensively by Bergeron (1988), some salient features bear repeating.

At the high densities which characterize the photospheres of cool DA stars, two questions arise in the modeling of the optical spectra of these stars. First, how should one treat the perturbations to, and eventual disruption of, the atomic levels of the absorbing atoms caused by the neighboring particles? Second, how is the convergence of the lines near the ionization edge treated? While details provided are often sketchy, the standard procedure is to treat the level perturbation in terms of an abrupt cutoff in the number of bound levels. The quantum number of the last bound level is often estimated on the basis of geometrical arguments (i.e., maximal allowed dimension of a hydrogen atom; Wehrse 1977), or on the basis of simple pressure-ionization models (i.e., the Debye-Hückel model). The overlap of the transitions near the ionization edge, on the other hand, is generally taken into account within the framework of the Inglis-Teller calculation of the last visible transition. The opacity between the latter and the edge is then usually taken to be the bound-free opacity, either evaluated at the edge or extrapolated to *lower* frequencies. The major advantage of the new Hummer-Mihalas formalism is that it provides a self-consistent way to describe the occupation of high-lying and increasingly disrupted levels in terms of their *occupation probability*, w_{ij} ($0 \leq w_{ij} \leq 1$). The occupation probability is a smoothly varying function which can be expressed in terms of the perturbations exerted by the neighboring (charged and neutral) particles on the absorber. Furthermore, Däppen, Anderson, and Mihalas (1987) provide, within the context of this new formalism, a self-consistent way to treat the opacity associated with transitions connecting to an upper level that is sufficiently disrupted for the atom to become ionized (the so-called dissolved levels). This procedure represents a significant improvement over the cruder treatment briefly described above.

It is perhaps worth noting that the ZZ Ceti stars represent an important benchmark for the testing of the occupation probability formalism under astrophysical conditions. The modeling of ZZ Ceti stars, while not without its difficulties, is indeed exempt from certain nagging uncertainties encountered at lower temperatures: the chemical composition of ZZ Ceti stars remains well understood, and consists—to a high degree of purity—of hydrogen only. In addition, the dominant perturbation to atomic energy levels comes mostly from charged particles. This situation is in contrast to that encountered at lower effective temperatures, where the Hummer-Mihalas formalism has been applied as well. There, the atmospheres are significantly enriched with helium dredged up by convection (Bergeron *et al.* 1990) and, furthermore, neutral particles become the dominant level perturbers. These two conditions prove to be much more cumbersome to deal with in the analysis of the hydrogen line spectrum (Bergeron 1988; Bergeron, Wesemael, and Fontaine 1990).

IV. ATMOSPHERIC PARAMETERS OF THE ZZ CETI STARS

a) Fitting Technique

The original Schulz and Wegner (1981) analysis was based on photographic spectra (Wegner and Schulz 1981), and made use of equivalent widths exclusively. We go beyond their work here by using the information contained in the Balmer line profiles to derive the atmospheric parameters of ZZ Ceti stars. While the data presented in Figure 1 have been absolutely calibrated with standard stars, we do not believe that, in general, their spectrophotometric quality is sufficiently uniform to warrant attempts at matching the complete optical spectra with model calculations. Rather, we have preferred to fit individual lines. Our first step is thus to normalize the line flux, in both observed and model spectra, to a continuum set at a fixed distance from the line center. The comparison with model spectra, which are convolved with a Gaussian instrumental profile, is then carred out in terms of these line shapes only.

The fitting technique we employ here relies on the nonlinear least-squares method of Levenberg-Marquardt (Press *et al.* 1986). This very efficient technique is based on a steepest descent method which also provides standard errors (68% confidence interval) on each fitting parameter using the covariance matrix. The calculation of χ^2 in our case is carried out using the normalized line profiles as defined above. Since the uncertainty on each data point is not known in our spectra, we have performed the analysis using an constant arbitrary value of $\sigma = 1$ for each data point, and obtained the values of T_e and $\log g$ by minimizing χ^2. A new value of σ is then calculated from this solution and then used to calculate the uncertainties on each parameter. Although this approach forces the value of χ^2 to be near unity, it does provide a form of error bars, as discussed by Press *et al.* (1986).

Although the application of this fitting procedure is straightforward in the region of hot (Bergeron, Saffer, and Liebert 1990) and cool (Bergeron *et al.* 1990) DA white dwarfs, it is fraught with complications in the region of the ZZ Ceti stars. An inspection of contour plots of constant χ^2 shows that more than one minimum exist for most objects in our sample. It has long been realized (e.g., Weidemann 1971 and references therein) that the locus of ZZ Ceti stars corresponds to the region where the hydrogen lines reach their maximal strength. Figure 2 illustrates this well-known result with our current grid of models. *At a given gravity*, there is thus, in principle, the possibility of two different temperatures giving satisfactory fits, one on each side of the equivalent width peaks (see also Wesemael and Fontaine 1985). In some cases, these two minima are well separated; sometimes, it is possible to exclude the minimum which falls outside the instability strip, as given by photometric observations, but, for some other objects, both

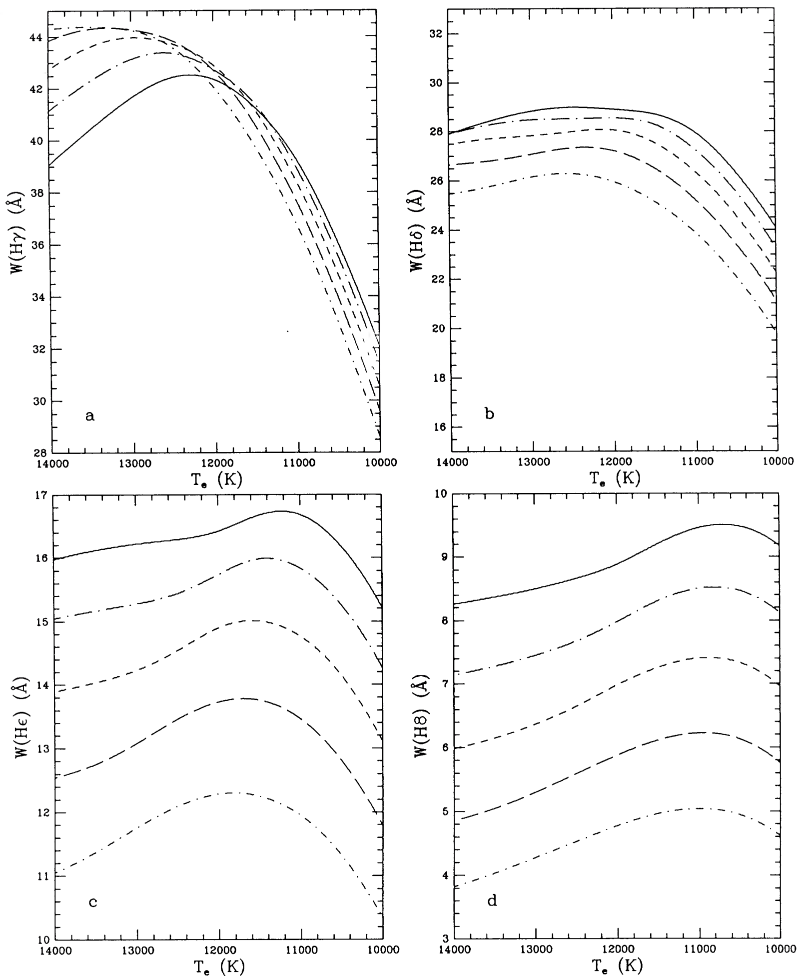

FIG. 2.—(*a*) Variation of the equivalent width of Hγ as a function of effective temperature in the range of the ZZ Ceti stars. Models at five surface gravities are shown: log g = 7.5 (*solid line*), 7.75 (*long dash–dotted line*), 8.0 (*short dashed line*), 8.25 (*long dashed line*), and 8.5 (*short dash–dotted line*). (*b*) Same as Fig. 2*a*, but for Hδ. (*c*) Same as Fig. 2*a*, but for Hϵ. (*d*) Same as Fig. 2*a*, but for H8.

solutions fall in the effective temperature domain of the ZZ Ceti stars, and the spectroscopic analysis alone is not sufficient to identify a unique solution. In some other cases, the two minima are so close to each other that they are indistinguishable; this makes the χ^2 valley relatively flat near the minimum. The corresponding uncertainties of each fitting parameter are consequently large.

On the positive side, however, Figure 2 also shows why progress *is* indeed possible with an expanded version of the Schulz-Wegner technique. Because of the crossover observed in Figure 2*a*, the low Balmer lines, Hγ in particular, are not useful gravity indicators (see also Weidemann and Koester 1980), and both Hγ and Hδ are best used as effective temperature indicators—this despite the rather slow variation of equivalent width with temperature encountered in the domain of ZZ Ceti stars. The gravity dependence of the high Balmer lines, like Hϵ and H8, whose behavior is illustrated in Figures 2*c* and 2*d*, is entirely dominated by the quenching of the upper levels and can be used effectively to determine log g. Therefore, once the effective temperature is uniquely defined, a very accurate determination of the surface gravity can be obtained.

b) *Results and Discussion of Individual Objects*

For G226-29, GD 99, and GD 154, two well-defined minima exist, one of which falls above any reasonable blue edge of the instability strip ($T_e \sim$ 15,000 K). Those objects are representative of the optimal situation, where a unique solution can be achieved without ambiguity. A unique solution can also be obtained for L19-2 and R548 although, in these cases, the two minima are so close to each other that they overlap. This results in fairly large uncertainties in the determination of the effective temperature. G117-B15A has two minima as well, one at T_e = 12,170 K, log g = 7.97, and the other one at T_e = 13,200 K, log g = 7.81. Since this star is known to be close to the blue edge of the instability strip from *all* previous analyses (whether of a photometric or spectrophotometric nature), we have chosen the hotter minimum as our adopted solution. GD 165 also possesses two minima, although the cool one is not statistically significant, and represents only a very small depression in the χ^2 valley. We have therefore adopted the hotter solution for this object as well. For G29-38, two minima are also found, one at T_e = 11,570 K, log g = 8.11 and the other one at T_e = 12,660 K, log g = 7.91 respectively. The detailed analysis of Greenstein (1988) based on multichannel and *IUE* data indicates, however, that the effective temperature of the star is close to 11,500 K, i.e. close to our cooler solution. The two remaining cases, G238-53 and GD 66, have also two solutions which both fall within the instability strip. For those stars, however, no reliable independent estimate of the effective temperature exists which could help us decide

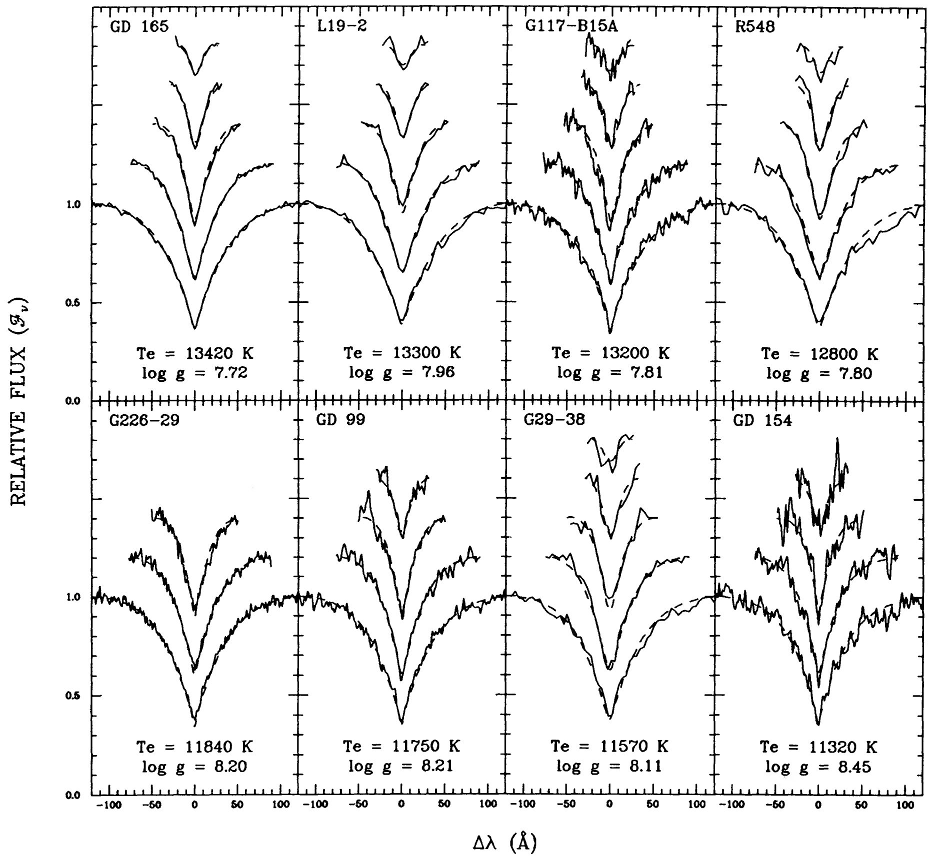

FIG. 3.—Fits to the individual Balmer lines for each of the eight objects for which a solution was obtained. The lines range, in general, from Hγ (*bottom*) to H9 (*top*), and are offset vertically from each other. For G226-29, only Hγ to Hϵ are available. The atmospheric parameters are those given in Table 2.

between these solutions, and we present and display both solutions here. The spread in T_e is of 870 K for GD 66, and of 1000 K for G238-53. Note, however, that the associated determinations of surface gravity are not devoid of interest, especially in the case of GD 66, a star which may eventually be amenable to mode identification through asteroseismology.

Results of our analysis for all ten stars are presented in Table 2 and in Figures 3 and 4. Figure 3 displays the final fits for the eight stars for which unambiguous temperature determinations were possible; Figure 4 displays the two acceptable fits

TABLE 2

ATMOSPHERIC PARAMETERS DERIVED FOR ZZ CETI STARS

Star	T_e(K)	$\sigma[T_e$(K)]	log g	σ(log g)
GD 165	13,420	230	7.72	0.03
L19-2	13,300	900	7.96	0.10
G117-B15A	13,200	460	7.81	0.06
R548	12,800	1600	7.80	0.21
G226-29	11,840	110	8.20	0.04
GD 99	11,750	120	8.21	0.04
G29-38	11,570	210	8.11	0.07
GD 154	11,320	140	8.45	0.05
G238-53[a]	13,140	340	7.67	0.05
	12,150	290	7.80	0.08
GD 66[a]	12,450	370	7.88	0.08
	11,580	120	8.04	0.04

[a] The effective temperature determination remains ambiguous for this star. Both possible solutions are given here, with their associated log g (see § IV*b*).

to both GD 66 and G238-53. Our final values, together with associated uncertainties, are all presented in Table 2. Note that the Balmer lines from Hγ to H9 are displayed in Figures 3 and 4, but that H9 is not included in our least-squares fitting technique; rather it is used, when possible, as a test of the consistency of our analysis.

Our analysis indicates that the ZZ Ceti stars studied here all have effective temperatures between 11,320 and 13,420 K. The mean value of T_e for the eight objects for which the effective temperature can be effectively constrained is 12,400 K, with a standard deviation $\sigma(T_e) = 865$ K; the mean surface gravity for the same subsample is $\log g = 8.03$, $\sigma(\log g) = 0.25$. These average values would change by less than 100 K and 0.05 dex, respectively, if the results of either fits to GD 66 and G238-53 were included as well. The resulting mean mass, based on the pure carbon evolutionary tracks calculated by Winget (1980), is $M(g) = 0.59\ M_\odot$, $\sigma[M(g)] = 0.15\ M_\odot$. This last result agrees with the mean mass determined by Weidemann and Koester (1984; $M(g) = 0.58\ M_\odot$, $\sigma[M(g)] = 0.13\ M_\odot$) from multichannel observations, reduced on the Hayes-Latham (1975) scale, of a sample of 70 DA stars. We shall compare, in § V, our determinations of effective temperatures in the instability strip with those obtained in several prior investigations.

V. COMPARISON WITH PRIOR STUDIES OF THE INSTABILITY STRIP

a) Comparison with Narrow-Band Photometry

Strömgren photometry of subsamples of ZZ Ceti stars has been carried out by McGraw (1979) and Fontaine *et al.* (1985*a*). While there seems to be little point in contrasting

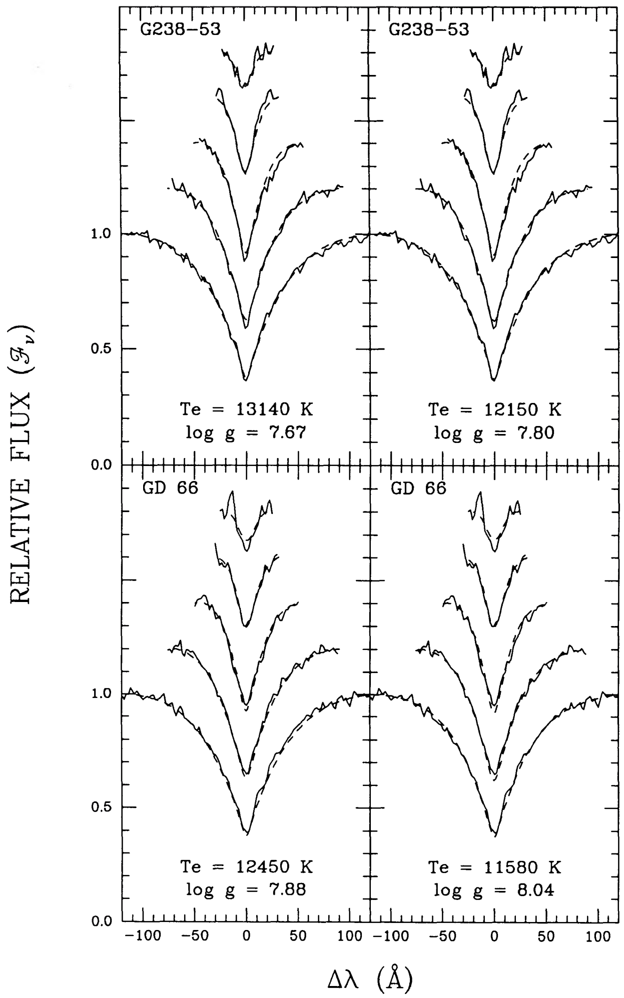

FIG. 4.—Fits to the individual Balmer lines for G238-53 and GD 66, the two stars for which two possible solutions remain. The lines range from Hγ (*bottom*) to H9 (*top*), and are offset vertically from each other. The atmospheric parameters are the two pairs given in Table 2.

effective temperatures for individual objects with those inferred from those studies, the boundaries of the instability strip provide a more suitable point of comparison. In McGraw's (1979) analysis, the strip covers the range 10,315–13,640 K. The coolest object is BPM 30551, a southern ZZ Ceti star which does not figure in our sample; the blue edge is defined by G117-B15A. Were McGraw's observations analyzed in terms of Koester's grid of model atmospheres (see Fig. 4 of Fontaine *et al.* 1985*a*), his instability strip would cover the approximate region 10,000–13,500 K, while our own model calculations would shift it to the 10,250–13,750 K interval. In the recent analysis of Fontaine *et al.* (1985*a*)—from which BPM 30551 is absent as well—the strip covers the range 11,000–13,000 K, with G117-B15A again the hottest ZZ Ceti star. The mean gravity for our sample of eight objects, $\log g = 8.03$, $\sigma(\log g) = 0.25$, is consistent with those derived in these two investigations for the individual subsamples of ZZ Ceti stars studied, namely, $\log g = 7.98$, $\sigma(\log g) = 0.17$ (McGraw 1979) and $\log g = 8.11$, $\sigma(\log g) = 0.28$ (Fontaine *et al.* 1985*a*).

b) *Comparison with Multichannel Analyses*

The most thorough studies of the atmospheric parameters of ZZ Ceti stars are based on the analysis of multichannel spectrophotometer data of Greenstein (1976, 1984). Two such analyses were performed, that of Greenstein (1982) and the reanalysis of Weidemann and Koester (1984), and we shall discuss these in turn. Our spectroscopic study has six objects in common with that of Greenstein (1982), namely, R548, GD 99, G117-B15A, GD 154, G226-29, and G29-38 (incidentally, G238-53 is listed there as well as a promising candidate for variability studies). These six objects range in effective temperature from 10,450 to 12,130 K, with an average temperature of 11,360 K, $\sigma(T_e) = 580$ K. The former values also represent the boundaries of Greenstein's instability strip, as the edges are defined by GD 154 at the cool end and by G117-B15A at the hot end. Note that there is a shift of ~ 500 K between the Oke and Gunn (1983) and Hayes and Latham (1975) calibrations, in the sense that temperatures based on the Oke-Gunn results are *cooler* than values based on the Hayes-Latham work. Were Greenstein's results brought onto the Hayes-Latham scale,[5] the six objects in common would cover the approximate range 11,000–12,600 K, with a mean temperature of 11,860 K. In our spectroscopic analysis, we find this same subsample of objects between 11,320 and 13,200 K, at an average effective temperature of 12,080 K, $\sigma(T_e) = 745$ K. While the average temperature of the variable stars common to both analyses is reassuringly consistent if the change in calibration is adopted, significant differences in individual temperatures remain: GD 154, R548, and G117-B15A are, on the average, ~ 740 K cooler in Greenstein's analysis than in ours; the difference is 1240 K if the data remain onto the Oke and Gunn scale. We note also the good agreement in the average gravity of the subsample of six objects in common: $\log g = 8.12$, $\sigma(\log g) = 0.38$, in Greenstein's analysis, and $\log g = 8.10$, $\sigma(\log g) = 0.25$, in ours.

The data base for the Weidemann and Koester (1984) analysis is identical to that of Greenstein (1982). However, their analysis differs in two important respects: first, it is not based on the original colors of the mutichannel system, but rather on improved ones based on broader and optimally defined bandpasses. Second, the derived color indices are based on the absolute flux calibration of Hayes and Latham (1975), rather than on the AB79 scale (Oke and Gunn 1983) used by Greenstein (1982). Furthermore, while we had six objects in common with Greenstein's analysis, there are only four in common with the later reanalysis, since two stars (R548 and GD 154), observed at large airmass, are excluded from the Weidemann and Koester discussion. For the remaining four objects, the multichannel analysis yields an average effective temperature of 12,260 K and gravity of $\log g = 8.19$, $\sigma(\log g) = 0.19$, while our average values for the same subsample are 12,090 K and 8.08, respectively. More interestingly, perhaps, GD 154, with an uncertain suggested temperature near 12,000 K, is 680 hotter than in our analysis and falls in the middle of their instability strip with half a dozen of ZZ Ceti stars cooler than it. Their coolest variable, the faint ZZ Ceti star G255-2, is assigned an uncertain temperature of 10,050 K. If that star is considered, it defines the red edge, 1270 K cooler than ours; if it is not, their red edge is near 11,200 K—consistent with ours *although based on different objects*. The blue edge, set by G117-B15A, is at 13,010 K in their analysis. In our work, it is set by

[5] Weidemann and Koester (1984) compare, for their sample of DA stars, the mean mass based on the determination of log *g* with that based on the radius determination (from the stellar parallax). They conclude that the Hayes and Latham (1975) calibration provides a better internal consistency between these averages than does the Oke and Gunn (1983) scale.

GD 165 (as well as by two other stars, including G117-B15A, of comparable temperature) at 13,420 K.

c) Comparison with Ultraviolet Spectrophotometry

Wesemael, Lamontagne, and Fontaine (1986) discuss ultraviolet spectrophotometric observations in the 1200–1900 Å range of a subsample of seven ZZ Ceti stars. Three additional objects are presented by Lamontagne, Wesemael, and Fontaine (1987), and at least one more ZZ Ceti star has been observed with the *IUE* since. Six objects (R548, G117-B15A, L19-2, G226-29, G29-38, and GD 154) are in common with the initial investigation of Wesemael *et al.* The latter presents and discusses two different temperature scales for ZZ Ceti stars, based on two different grids of model atmosphere calculations, namely those of Koester *et al.* (1985) and of Nelan and Wegner (1985). The difference between these scales amounts to a systematic shift of $\sim$500 K toward lower temperatures for the analysis based on the Nelan and Wegner models as compared to that based on the Koester *et al.* grid. A recent reinvestigation of this problem by Lamontagne *et al.* (1989), which makes use of more recent models calculated by Nelan and Wegner, suggests that the agreement between the scales is now much improved, and their fit to G117-B15A shows the two temperature scales to be consistent within $\sim$100 K, at least near the blue edge. Consequently, we will only discuss here published results based on the Koester *et al.* scale, which is consistent with the *new* Nelan-Wegner scale.

For the six objects mentioned above, the average effective temperature based on the ultraviolet energy distribution and the Koester *et al.* grid is 12,430 K, while that based on our spectroscopic analysis is 12,340 K. The ultraviolet instability strip extends from 11,740 to 13,000 K, while in the present investigation it is found to range from 11,320 to 13,420 K. Thus the width of the instability strip in our analysis ($\sim$2100 K) is larger than that obtained from ultraviolet spectrophotometry ($\sim$1260 K). This is due mostly to the fact that the three cooler stars in the subsample in common are assigned lower temperatures from optical spectroscopy than from ultraviolet spectrophotometry. The remaining objects are assigned higher, or comparable, effective temperatures here.

d) A Comment on Location of the Blue Edge

There are, to our knowledge, five determinations of the location of the blue edge of the instability strip in the literature, all based on G117-B15A. We repeat here these values: 13,640 K (McGraw 1979); 13,000 K (Fontaine *et al.* 1985*a*); 12,130 K (Greenstein 1982); 13,010 K (Weidemann and Koester 1984); and 13,000 K (Wesemael, Lamontagne, and Fontaine 1986, with the Koster *et al.* temperature scale). Our latest determination is based on a recently discovered Ceti star, GD 165, which sets the blue edge at 13,420 K.[6] However, two other objects, L19-2 and G117-B15A, have effective temperatures comparable, within the quoted errors, to that of GD 165. Note, furthermore, that our blue edge is independent of the argument used in § IV*b* to choose among the two possible solutions for G117-B15A. Thus, while each one of these studies clearly suffers from its own uncertainties, we believe it significant that most of these analyses—based on distinct observational material often interpreted in terms of independent model atmosphere calculations—place the blue edge at or above 13,000 K. We believe that this result must be considered a serious constraint on the modeling of the pulsation properties of ZZ Ceti stars.

It is perhaps worth repeating once more that there is no fundamental problem, in principle, in accommodating such a hot blue edge in current pulsation calculations. The location of the blue edge is related to the depth of the hydrogen convection zone, which itself is a function of the efficiency of convective energy transport in the envelope (Winget *et al.* 1982; Fontaine, Tassoul, and Wesemael 1984; Tassoul, Fontaine, and Winget 1990). The more efficient the convection, the hotter the blue edge. There is, of course, a limit to the efficiency of convective energy transport: it is reached when the temperature stratification in the convection zone becomes adiabatic. This specific case was considered by Tassoul, Fontaine, and Winget (1990), who derive a limiting effective temperature for the blue edge of 14,000 K. As long as the observed blue edge is below that value, as is observed, there is no fundamental discrepancy between theory and observations, at least within the given set of constitutive physics used by Tassoul, Fontaine, and Winget (1990).

In this respect, the recent theoretical analysis of the pulsation properties of ZZ Ceti stars of Cox *et al.* (1987) is of particular interest. The blue edge they find, between 11,000 and 11,500 K, matches our red edge (11,320 K) better than our blue edge (13,420 K). Indeed, the only determination consistent with the Cox *et al.* result is that of Greenstein (1982).[7] According to these authors, no conceivable change in the constitutive physics or convective efficiency can bring their blue edge near 13,000 K. While this is not the proper forum to discuss these discrepancies in detail, these determinations suggest—at the very least—that the Cox *et al.* results appear at odds with the bulk of the observational results currently available on the location of the blue edge of the instability strip.

VI. SOME OBJECTS OF PARTICULAR ASTROPHYSICAL INTEREST

a) GD 66

Our analysis of the complete optical line spectrum of GD 66 can be contrasted with the much more limited study of Fontaine *et al.* (1985*b*), which concentrated on the Hδ line exclusively. The results of the two-dimensional fit (T_e and log g) of Fontaine *et al.* are $T_e = 10{,}800 \pm 400$ K, and $\log g = 7.7^{+0.4}_{-0.2}$. While the effective temperature of this star is not well constrained in our analysis, the values we obtain are a full 780–1650 K hotter than that preliminary value—a disquieting result given the rather narrow instability strip for ZZ Ceti stars. Of course, there are substantial differences between the two analyses: again, our prior effort dealt with Hδ only, limited material to determine both T_e and log g. Also noteworthy is the

[6] The high temperature derived here, coupled to the reported long periods (as long as 1820 s) of GD 165, initially appeared to be in conflict with the period-effective temperature correlation (Winget 1981; Winget *et al.* 1982; Tassoul, Fontaine, and Winget 1990), which suggests that the hottest ZZ Ceti stars should have the shortest periods. However, subsequent observations of GD 165 have, up to now, failed to confirm the long periods initially reported. This star is an ideal candidate for further monitoring through continuous coverage with the Whole Earth Telescope (Nather 1989). If the very long periods are confirmed, the potential of this latest discovery in challenging our understanding of nonradial pulsations in ZZ Ceti stars appears extremely promising.

[7] Note that the blue edge quoted by Cox *et al.* (11,700 K) is in fact the *coolest* value obtained in Greenstein's (1982) analysis, namely, that based on $G'-R$ only. The value we quote in this paper, 12,130 K, is the *mean* value obtained by Greenstein on the basis of several temperature-sensitive color indices. Thus, even Greenstein's cool blue edge may be significantly hotter than that obtained by Cox *et al.*

fact that our earlier analysis was based on synthetic spectra which did *not* incorporate the new formalism of Hummer and Mihalas (1988), used here, for the computation of the atomic populations. A similar situation is encountered in the analysis of G226-29 presented by Daou *et al.* (1989), where a preliminary value of 11,000 K was fitted on the basis of the older prescription (see § III*b*), and where the importance of the adopted formalism for fits to ZZ Ceti stars was first suspected. Our favored value for G226-29 is now 11,840 K. We shall address elsewhere the differences between these formalisms for the computation of synthetic optical spectra of cool DA stars (Bergeron, Wesemael, and Fontaine 1990). Note, however, that—irrespective of the adopted solution for GD 66—our results confirm the essentially normal gravity of that star, and strengthen the conclusions of Fontaine *et al.* concerning the tentative mode identification questioned in that paper.

b) Low-Gravity Star R548

R548 (ZZ Ceti), the prototype of the variable DA stars, has long been suspected of being a low-gravity object. McGraw (1979) assigned $\log g = 7.77 \pm 0.02$ on the basis of Strömgren photometry, while Weidemann and Koester (1980) obtained $\log g = 7.75 \pm 0.15$ on the basis of broad-band and narrow-band photometry. On the basis of the multichannel data, Greenstein (1982) assigned $\log g = 7.8$, while Weidemann and Koester (1984) obtained an extremely low gravity ($\log g = 7.11$), which they discarded because of the large zenith distance during that particular observation. More recently, the Strömgren photometry of Fontaine *et al.* (1985*a*) places this object near $\log g = 7.5$. Our spectroscopic result is consistent with these previous determinations, as it also yields a somewhat lower than average value for this object ($\log g = 7.80$, $M \sim 0.46\ M_\odot$). However, GD 165 ($M \sim 0.41\ M_\odot$), and not R548, is the lowest-gravity ZZ Ceti star in our sample.

c) Short-Period Star G226-29

G226-29 is a bright ZZ Ceti star, which was discovered by Fontaine and McGraw. Its light curve has been thoroughly studied, and deciphered, by Kepler, Robinson, and Nather (1983). It consists of three closely spaced pulsations, with periods near 109.3 s. The Kepler *et al.* interpretation of the observed triplet in the power spectrum in terms of rotational splitting[8] suggests that the observed, rotationally split mode could be one with $l = 1$. In this case, however, the very short period can only be accommodated if the stellar mass is high, larger than 1 $M_\odot$—or equivalently $\log g \gtrsim 8.65$ (Lamb and Van Horn 1975). Indeed, Greenstein (1982), on the basis of his multichannel data, assigns it $\log g = 8.6$, while Weidemann and Koester (1984) find $\log g = 8.39$, in what appears to be a less than optimal fit. The Strömgren photometry of Fontaine *et al.* (1985*a*) also puts it at high gravity, near $\log g \sim 8.4$ in their Figure 4. Although our spectroscopic analysis may not be incompatible with these results if all the uncertainties are stretched in the right direction, our gravity for this object ($\log g = 8.20$) clearly appears less anomalous than those derived from (spectro)photometric data. And indeed, G226-29 in not even the most massive star in our sample, this distinction befalling GD 154 ($\log g = 8.45$).

More critically, the surface gravity we determine appears insufficient to satisfy the constraint imposed by the short periods. There may be a few possibilities—all offered with some trepidation—out of this quandary. The first that should be considered is that the gravity determined here is too low and that the true surface gravity is closer to the value inferred from (spectro)photometric analyses. This corresponds to the situation envisioned by Kepler *et al.* and pulsation calculations show, in that case, that the short periods of G226-29 (assumed to belong to $l = 1$ modes) would be consistent with hydrogen envelope masses of the order of 10^{-8}–$10^{-7} M_*$, i.e., masses consistent with those envisioned by Winget (1981) and Winget *et al.* (1982). The possibility of a high gravity is resurrected here, because G226-29 is the only star in our sample for which Balmer lines above Hϵ were not available (see Fig. 1). Thus our gravity determination relies exclusively on the latter. To test the sensitivity of our surface gravity determination to the absence of H8 and higher lines, we have redetermined $\log g$ for a subsample of five of our objects by using only the lines from Hγ to Hϵ. The largest deviation in $\log g$ observed, when compared to the values of Table 2, is of 0.15 dex, while the root-mean-square variation for the subsample of five stars is 0.11 dex. Thus a surface gravity for G226-29 as large as, say, $\log g \sim 8.6$ appears rather unlikely, even if our analysis underestimates the true $\log g$ of that star.

Because the period associated with a given mode shortens as the stellar mass or the hydrogen envelope mass increases, a match to the observed short periods would require that the hydrogen envelope be relatively massive if the stellar mass is approximately normal, as we find here. However, if the mass inferred from our gravity determination ($\sim 0.69\ M_\odot$) is the correct one, and if $l = 1$, the observed periods cannot be reproduced even for a hydrogen envelope as massive as $\sim 10^{-4} M_*$. For example, a 0.6 $M_\odot$ model from the sequences of Brassard *et al.* (1990*a*) with a fractional hydrogen envelope mass of 10^{-4} has a period for the $l = 1$, $k = 1$ mode of 121 s, significantly larger than the observed 109 s.

Of course, the difficulties encountered in matching the periods would be considerably eased if one were allowed to consider $l = 2$ or $l = 3$ modes, as the periods decrease for increasing values of l in a given model. But such an identification must also be able to reproduce the relative amplitudes of the $m = 0$ and $m = \pm 1$ peaks in the power spectrum of Kepler *et al.*, as well as account for the presumed low amplitudes of the $m = \pm 2$ or $m = \pm 2$, 3 peaks, which are not detected. The strength of these peaks depends principally of the inclination of the stellar rotation axis with respect to the line of sight. Unfortunately, the results of Pesnell (1985; see his Figs. 3 and 4) and Brassard, Wesemael, and Fontaine (1990) suggest that it may not be possible to find an observing angle which would account simultaneously for the relative amplitudes of the $m = 0$ and $m = \pm 1$ peaks, as well as yield sufficiently small amplitudes for the additional, unseen components.

The failure to account naturally for the very short periods observed in G226-29 becomes even more acute if one requires that the observed modes be *trapped* modes (Winget, Van Horn, and Hansen 1981; Brassard *et al.* 1990*b*). Indeed, the preliminary survey of Brassard *et al.* (1990*a*, *b*) suggests that trapped modes in ZZ Ceti models with thin hydrogen layers all have periods that are longer than those observed, while the trapping in massive hydrogen layers is not as effective. Clearly, the problem posed by these very short periods in what appears to be an essentially normal ZZ Ceti star is as vexing as ever.

[8] Note that the other possibility, that of magnetic splitting in a surface field of $\sim 10^5$ G, is considered unattractive by Jones *et al.* (1989).

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