THE ASTROPHYSICAL JOURNAL, 364:35–64, 1990 November 20

CLUES ON THE HOT STAR CONTENT AND THE ULTRAVIOLET OUTPUT OF ELLIPTICAL GALAXIES

LAURA GREGGIO AND ALVIO RENZINI[1]
Dipartimento di Astronomia, Università di Bologna
Received 1989 November 27; accepted 1990 March 21

ABSTRACT

The relatively strong ultraviolet radiation flux emitted by elliptical galaxies shortward of ~2000 Å is certainly due to the presence of hot stars. Yet, so far it has not been firmly established whether such hot stars are young, massive objects being currently formed in these galaxies, or whether old, low-mass stars suffice to produce the observed UV radiation, as well as the observed correlation with the galaxy metallicity. In this paper we investigate in great detail under which conditions the old-star option can give acceptable solutions. Several kinds of hot star candidates have been considered, including post–red giants, hot horizontal-branch stars and their AGB–manqué progeny, post–early–AGB stars, and post–AGB stars of the helium- and hydrogen-burning varieties. Among binary star candidates we have considered post–red giants produced by a Roche-lobe overflow, and accreting white dwarfs. We present numerical simulations showing how sensitive is the production of the various kinds of hot stars to small differences in the assumptions concerning the helium enrichment and the mass-loss trends with increasing metallicity. These numerical simulations and additional arguments tend to disfavor classical post–AGB stars as the most important UV producers, at least among the galaxies with the strongest UV upturns. Also binary stars seems insufficient, unless stable mass transfer onto white dwarf primaries will turn out to be possible for a wider range of initial binary parameters than generally accepted. Rather, this study tends to favor hot HB stars, AGB–manqué, and post–EAGB stars, whose production, however, is extremely sensitive to ill-known parameters. For this reason we conclude that stellar evolution theory alone cannot currently solve the problem of the origin of the UV output of elliptical galaxies. We therefore list in detail for each of all the above candidates the major observable characteristics which can help future space observations to discriminate among the various options, with particular reference to resolvable stellar systems such as galactic globular clusters, M32, and the bulges of the Milky Way and M31. Finally, we also discuss the predicted evolution with lookback time (redshift) and argue that the UV upturn of ellipticals is likely to be the most rapidly evolving feature in the spectrum of these galaxies, possibly allowing the detection of evolutionary effects already at rather modest redshifts ($z \lesssim 0.2$).

Subject headings: galaxies: evolution — galaxies: stellar content — stars: evolution — ultraviolet: general

I. INTRODUCTION

Elliptical galaxies emit a significant flux of radiation at wavelengths below ~2000 Å, with the flux increasing for decreasing wavelength (e.g., Code and Welch 1979; Bertola *et al.* 1980; Perola and Tarenghi 1980; Oke, Bertola, and Capaccioli 1981; Bertola, Capaccioli, and Oke 1982; O'Connell, Thuan, and Puschell 1986; Bertola *et al.* 1986; Burstein *et al.* 1988*a*, hereafter B3FL). For this reason, the corresponding part of the galaxy's integrated spectrum is often referred to as the *UV rising branch.* The identification of the kind of stars which are responsible for the UV rising branch of elliptical galaxies is a matter of considerable astrophysical interest, which impacts on issues such as the formation and evolution of these galaxies, and the ionization of the interstellar and intergalactic medium at the various cosmological epochs.

Several hypotheses for the origin of the UV light have been formulated so far (for a critical review see Renzini and Buzzoni 1986, hereafter RB). They logically split into two competing categories: (1) young, massive stars which would be currently forming in these galaxies and (2) old, low-mass stars in various advanced evolutionary stages. The latter category includes (1) hot horizontal branch (HB) stars, burning helium in their core, and their shell helium-burning progeny, (2) post–asymptotic giant branch stars (P–AGB) like planetary nebula nuclei, burning either hydrogen or helium in a shell, and (3) various kinds of binary systems in which either one component is close to becoming a white dwarf (WD), or a WD component is accreting *and* burning hydrogen (helium) fuel from a nondegenerate companion.

The young star versus old star dilemma directly connects with one issue which is currently matter of debate: whether ellipticals are currently star-forming galaxies or whether they can be regarded as passively evolving stellar populations (see O'Connell 1986; Renzini 1986). In the former case the spectral evolution of each elliptical would be dominated by its individual, detailed history of star formation, which is hardly predictable. In the latter case, though, this limitation does not apply, thus allowing the potential use of these galaxies as *cosmological clocks* for the estimation of, e.g., the *age* of high-redshift galaxies, or their lookback time. Indeed, only the passive evolution of a stellar population is subject to laws that—at least in principle—can be rigorously formulated, while star formation processes rather resemble meteorological events, due to the prominent role played by chaotic hydrodynamics. Deciding the issue of the nature of hot stars in ellipticals should therefore tell us if population synthesis studies will be able to offer a relatively straightforward cosmological tool, free from the ambiguities that are introduced when the spectrophotometric

[1] Also European Southern Observatory

evolution is dominated by specific, but unpredictable star formation histories.

In this paper we try to extract from the stellar evolution theory as much insight as currently feasible on the nature and characteristics of the hot stars that can be present in old stellar populations, thereby potentially affecting their ultraviolet output. Rather than claiming yet another solution to the problem of the UV radiation of elliptical galaxies, we shall attempt to make clear what uncertainties have so far prevented a satisfactory solution of the problem, and what kind of future observations may help in discriminating among the various candidates. In doing so we take advantage of this opportunity to exemplify a partly original methodological approach to single and binary star evolution, in which each question is analyzed in the broader framework of stellar population studies.

In § II we briefly review the most significant characteristics of the observed UV upturn in elliptical (and S0) galaxies, we discuss the critical question of the metallicity spread within these galaxies, and we constrain our theoretical strategy to be germane to the kind of data which are actually available. Then, in §§ III and IV we discuss in full detail the problems and complexities which are inherent to the single old-star candidates, while in § V we display various unified scenarios for these candidates and discuss possible ways of generating the observed UV-metallicity correlation. We also present the expected trends with lookback time. Binary candidates are then presented in § VI. In § VII all the various candidates are re-examined in the light of the available information concerning old stellar populations which have been resolved into individual stars: i.e., galactic globular clusters, the bulge of the Milky Way and of M31. Future ground-based and space observations which can help to unambiguously identify the nature of the hot stars in ellipticals are also briefly discussed. Our main conclusions are then summarized in § VIII.

II. OBSERVABLES AND MODELS: A PROPORTIONATE EFFORT

a) *Evidence*

Much of the available observational evidence is presented and discussed in the thorough study of B3FL. The main points are the following:

1. All studied ellipticals have a detectable flux shortward of ~2000 Å.

2. The slope of the UV rising branch is very close to the Rayleigh-Jeans side of a blackbody, i.e., $F_\lambda \propto \lambda^{-4}$, or slightly flatter, implying temperatures in excess of 25,000–30,000 K for the hot star component.

3. Given the poor signal to-noise of the *IUE* data, no specific spectral features have been recognized yet.

4. There are very large galaxy to galaxy differences in the level of the UV rising branch. Normalizing the spectra to the V luminosity (at $\lambda = 5500$ Å), the flux at 1500 Å varies by almost one order of magnitude.

5. The strength of the UV rising branch, as measured by the $(1550 - V)$ color, correlates most tightly with the metallicity-related Mg_2 index (the UV-Mg_2 correlation was first discovered by Faber 1983). Moreover, those few galaxies which show clear evidence of ongoing star formation do *not* follow this correlation (e.g., NGC 205). For a possible connection with other morphological properties of early-type galaxies see Longo *et al.* (1989).

Faber (1983) lists six arguments which militate against the young-star option. These include some which actually apply to the bulge of M31, rather than to ellipticals themselves. Clearly the UV rising branch of the M31 bulge cannot be due to young massive stars, otherwise they would have been individually detected, e.g., such as in NGC 205. While the case of M31 demonstrates that old populations are indeed able to generate very hot stars, the analogy with ellipticals cannot be pushed so far as to firmly conclude against a young star contribution in these other galaxies. Other arguments of Faber include the apparent similarity between the UV and the optical light distribution, favoring old candidates against young stars, which one would expect to be patchily distributed and perhaps strongly concentrated toward the nucleus. Then Faber mentions type I supernovae in ellipticals, which may well have old precursors. Actually, it is the absence of type II supernovae (Tammann 1982) which argues against the massive stars hypothesis, unless massive super metal-rich stars produce preferentially dim supernovae which would have passed undetected: a rather *ad hoc* way out. Finally, the strongest argument is provided by the UV-metallicity correlation, "as there is no known reason why star-formation rate should correlate closely with composition" (Faber 1983). Yet, at least in principle the connection could be indirect in nature. It is known, in fact, that the Mg_2 index correlates with the optical luminosity, or with other quantities which in turn correlate with the optical luminosity, like the central velocity dispersion (see B3FL). Hence, there could be a direct physical link of star formation with the depth of the potential well of the galaxy, and therefore with its luminosity. This is not physically unconceivable, as hydrodynamical models show that cooling flows may be present only in some among the most luminous galaxies, while, in general, winds or subsonic outflows prevail among ellipticals (D'Ercole *et al.* 1989). In principle, star formation is therefore allowed (but by no means proven!) among the most luminous galaxies, which in turn are also the most strong-lined ones, while it is very unlikely in fainter galaxies. However, B3FL argue that the $(1550 - V)$ color correlates more tightly with Mg_2 than it does with optical luminosity itself, an argument which favors the old star candidates. We are largely sympathetic with this conclusion, but remain with the impression that no really conclusive argument has yet emerged either in favor or against the young star hypothesis; we shall return to this point in § VII*e*. The only firm point at this stage is that an old star contribution must be present; whether it is sufficient to explain all the UV rising branches is still an open question.

The interpretation of the UV-metallicity connection will play a central role in this paper, and it is worth analyzing it in some detail. It involves three observables, namely the flux at 1550 Å and at V, and the strength of the Mg_2 blend. As pointed out by B3FL the relation of the Mg_2 index with either [Mg/H], [Fe/H], or Z remains very uncertain. On physical grounds we may expect a nonlinear relationship, as the formation of MgH molecules is sensitive to temperature, and therefore to the location of the red giant branch (RGB) in the H-R diagram (see Mould 1978), which in turn depends on metallicity. As an additional complication, there is no guarantee that in elliptical galaxies Fe and Mg should maintain solar proportions. An increase only in [Fe/H] would shift the RGB to lower temperatures, thereby favoring MgH formation and increasing the Mg_2 index. An increase in [Mg/H] will clearly lead to a stronger Mg_2 index, via both the direct abundance effect, and the associated shift of the RGB to lower effective temperatures (see Renzini 1977). Moreover, this shift should cause a reduction of iron ionization in red giant atmospheres, and might then lead to a decrease of the iron index ⟨Fe⟩, which mostly results from the blend of many Fe II lines. Hence, we may expect the Mg_2 index to increase with increasing [Mg/H] and [Fe/H], while the iron index may decrease with increasing [Mg/H]. Moreover, the same kind of feedback is likely to cause a reduced sensitivity of the iron index to [Fe/H]. The calculation of these indices for synthetic populations should help to quantify these expected trends, and, incidentally, may help to understand why in ellipticals the ⟨Fe⟩ index spans a much narrower range compared to the Mg_2 index (Burstein *et al.* 1984) or why it exhibits a flatter gradient within individual galaxies (Efstathiou and Gorgas 1985; Davies and Sadler 1987; Peletier, Valentijn, and Jameson 1990). In conclusion, the lack of a usable relation between the Mg_2 index and $\langle Z \rangle$ prevents a fully quantitative understanding of the observed UV-Mg_2 correlation, and therefore our discussion will forcefully remain, to some extent, of semiquantitative nature.

b) Metallicity Spreads and the Quest of M32

The interpretation of the observed trend of the $(1550 - V)$ color with metallicity also suffers from poor knowledge of the actual metallicity distribution within individual galaxies, while the UV upturn could be mostly produced by stars in the metal-rich tail of the distribution, rather than by those with $Z \simeq \langle Z \rangle$. We believe that this is a crucial aspect of the problem, worth discussing in some detail. The existence of a sizable metallicity spread within elliptical galaxies is a natural result in chemical evolution models (e.g., Arimoto and Yoshii 1987), with the abundance spread reaching up to one to two orders of magnitude, depending on the total luminosity (mass) of the galaxy. However, the presence of a significant metallicity spread has been regarded as an unwelcome complication in population synthesis studies, which rather tend to prefer large spreads in stellar ages (e.g., Pickles 1987). Typical in this connection is the controversial case offered by the nearby dwarf elliptical M32, which has come to play the role of a *template* in population synthesis studies. O'Connell (1986, and references therein) argued for a solar metallicity and virtually no dispersion, and on these premises obtained a typical age 4–6 Gyr for the last major episode of star formation in this dwarf galaxy. On the other hand, Renzini (1986) argued that a completely old stellar population could not be excluded, and adopting a half solar average metallicity got ~ 14 Gyr (i.e., comparable to that of galactic globular clusters), a conclusion which would be further strengthened if a sizable metallicity spread is present. O'Connell (1988) has subsequently pointed out that blanketing effects were neglected in Renzini's temperature-color transformations, but a quantitative exploration of the objection shows that the effect is only marginal, and that an old age (~ 12 Gyr) is obtained even when blanketing effects are properly taken into account (see also Bertelli, Chiosi, and Bertola 1989). So the question of the age distribution of M32 stars still remains, the crucial point being the preliminary determination of the actual metallicity distribution within this galaxy. More recently, Boulade, Rose, and Vigroux (1988) have applied spectral synthesis techniques, and they concluded that the metallicity of M32 is just about solar ([Fe/H] ~ -0.1) with virtually no dispersion about the mean. They then inferred that an intermediate age component is inescapably required to fit their spectra.

The case of M32 has recently been addressed using a different population synthesis approach. Rather than using a stellar data base, Bica, Alloin, and Schmidt (1990) use empirical integrated spectra of star clusters as the building blocks for their spectral synthesis of composite populations. They maintain that this automatically ensures the *Natural* proportions among the various stellar components for each age-metallicity bin, thus allowing a formulation of the synthesis problem which is free from any necessity of theoretically motivated *astrophysical constraints.* Applying this method to M32, Bica *et al.* conclude favoring a solution with virtually no metallicity dispersion ($\sim$95% of the light from stars with $[Z/Z_\odot] = -0.25 \pm 0.25$) and with a major intermediate-age component ($\sim$30% of the light from $\sim$5 Gyr old stars).

All these claims have been seriously challenged by the subsequent observation of individual stars in M32. Indeed, Davidge and Jones (1989) and Freedman (1989) have independently obtained deep CCD color-magnitude diagrams showing that the upper red giant branch of M32 spans a considerable color range, a clear evidence of the presence of a metallicity spread. According to Freedman, M32 stars in the studied field range from "metallicities comparable to M92 up to metallicities exceeding that of 47 Tuc" (i.e., from $[M/H] \simeq -2$ to more than -0.7). She further estimates the average metallicity of the stars in her sample to be $\langle [M/H] \rangle = -0.5$, while the mode of the distribution is at $[M/H] = -0.7$ and the 1 σ range is 0.6 dex. Incompleteness in the V band photometry may have caused an underestimate of the average metallicity, but in this case the spread would be even larger. We find very hard to reconcile this important result with those population synthesis studies which have indicated a solar metallicity with no dispersion. The only surviving concern is the fact that Freedman's stellar photometry is restricted to a field 2′ south of the nucleus, where instead the spectroscopic studies have been conducted. Still, the absence of a $(B-V)$ color gradient in M32 (Sharov and Lyuti 1983, and references therein) argues in favor of the studied field being representative of the whole galaxy, unless a metallicity gradient toward the center is closely balanced by an opposite gradient in the average age, so as to maintain $(B-V)$ constant. We consider rather speculative this last chance, and interpret Freedman's result in support of the notion that this galaxy contains stars in a wide metallicity range, and that its average metal content can well be 2–3 times below solar. If so, an old age for M32 stars naturally follows.

The direct stellar photometry of M32 allows to address further the problem of the age of stars in this system, as the possible presence of bright AGB stars might be interpreted as evidence for an intermediate-age population. Freedman offers two possible interpretations of the color-magnitude diagram. For a distance modulus of 24.0 mag the brightest stars in M32 would perfectly match the RGB tips of galactic globulars in the same metallicity range, and we are pleased to add that this analogy extends to the brightest objects at larger metallicity, which would then perfectly overlap with the long period variables found in metal-rich galactic globulars (see Freedman's Fig. 9*a* with Fig. 1 in Frogel and Elias 1988). The other interpretation is based on an assumed modulus of 24.4 mag, and therefore the red giants in the M32 field now overshoot by 0.4 mag the RGB tip luminosities. Freedman clearly disfavors this interpretation, which would have required a hypothetical burst of star formation $\sim$5 Gyr ago to have generated stars in a very wide range of metallicities (from a very poorly mixed interstellar medium!). We fully concur that this sounds astrophysically very unplausible, and further argue that the fair continuity of the luminosity function across the presumed RGB tip can hardly be reconciled with the intermediate-age hypothesis.

In any case, the presence of a sizable metallicity spread is well documented in M32, and the substantial failure of current population synthesis methods in deriving the metallicity distribution of composite stellar systems emphatically calls for their critical reconsideration (see Tinsley 1980; RB; and Renzini 1989*b*, for early warnings in this direction). Severe incompleteness of the stellar libraries implemented in current synthesis methods is probably to be blamed for this (see RB's § 3.4). Concerning the cluster data base method, although certainly superior in principle, this approach may in practice be plagued by at least three limitations. First, we would like to have a uniform coverage of the age-metallicity plane with template clusters, but the two sequences of galactic and Magellanic Cloud globulars cover only two rather narrow strips in this plane, and we run out of templates particularly at the high-metallicity end (Renzini 1987). The second problem concerns sample size, i.e., the cluster templates are typically $\sim 10^5\ L_\odot$ objects, which inevitably fall short of stars in rapid evolutionary phases, such as the AGB and the P–AGB. Notice that AGB and P–AGB stars are, respectively, the coolest and the hottest objects in a population, and therefore the *wings* of the spectral energy distribution (SED) of our cluster templates are expected to be dominated by small number effects. This will affect mainly the UV and the near infrared; we shall return to this problem in the next subsection. Third, the stellar populations of nature may not just be a biparametric family, with only age and overall metallicity changing from one to another. The *second parameter* syndrome is a well-known property of galactic globular clusters, and it has been claimed that the globulars in just the next spiral (M31) may already show different *family traits* (Burstein *et al.* 1984). Therefore, some combination of these limitations may be responsible for the failure of the cluster-library method applied to M32, although a long known shortcoming can also play a rôle, i.e., the lack of uniqueness which often undermines empirical population syntheses, with astrophysically vastly different solutions receiving nearly identical merit by a blind optimization method (see Table 4 in Bica *et al.* 1990). This may result from Bica *et al.* having worked on a limited part of the spectrum, leaving aside both the wings and the main body of the SED in the near IR, and fitting just a set of nine spectral indices in the optical.

In conclusion, for what is of direct concern for this paper, it appears most plausible that each elliptical galaxy contains stars in a rather wide range of metallicities, with the bulk of stars from, say 1/10 solar or less, to several times solar, not unlike what is acutally observed in the bulge of the Milky Way and M31, and suggested by the halo of M31 (e.g., Whitford and Rich 1983; Mould 1986*a*, *b*; Rich 1988; Terndrup 1988; Rich *et al.* 1989). We shall constantly keep this in mind while dealing with the UV output of elliptical galaxies, and will correspondingly attempt to develop all *metallicities* scenarios for each hot star candidate.

c) Stellar Ingredients and Some of Their Limitations

Superficially, it may appear that the goal of our theoretical effort should consist in producing model SEDs which nicely match the observed UV rising branches. On the contrary, we maintain that times are far from being ripe for this kind of approach, and we prefer a less ambitious procedure. This attitude follows from two considerations: (1) On the observational side, the accessible spectral range is currently restricted longward of ~ 1200 Å, while the Lyman continuum remains unexplored. Besides, the poor signal to noise of the available spectra does not allow to identify any particular feature. (2) On the theoretical side, for whatever stellar candidate, the actual temperature distribution (which in turn determines the slope of the UV rising branch) is sufficiently uncertain that plausible ad hoc assumptions can always be made to force the synthetic slope to match the observed one. This is very clearly demonstrated by the synthetic SEDs constructed, e.g., by Nesci and Perola (1985) or Mochkovitch (1986), which show that by properly tuning just a few of the many free parameters every kind of hot star candidate can *successfully* fit the observations. The lesson to draw from numerical experiments of this kind is indeed that the spectral synthesis approach has so far proven unable to discriminate among the various candidate hot star components. This limitation follows from the great theoretical uncertainties affecting the temperature distribution to be expected in the case of each candidate, which are worth mentioning here in some detail.

We preliminarily note that common to every candidate is the uncertainty concerning the evolutionary behaviors of super–metal-rich stars, in whatever mass range or evolutionary phase. This follows from our current difficulty of studying sufficiently large samples of such stars. Passing now to discuss the individual candidates, we start with the case of massive stars which will not be further examined in this paper, but which is nevertheless worth mentioning. Not unexpectedly, in population synthesis experiments the slope of the UV rising branch turns out to be very sensitive to the adopted slope of the initial mass function, which remains an adjustable parameter in every young star model (Nesci and Perola 1985). Moreover, generally only the core hydrogen-burning phase of massive stars has been taken into account (e.g., Bruzual 1983; Nesci and Perola 1985), thereby neglecting the helium-burning phase during part of which these stars disguise themselves as A- and B-type supergiants. Unfortunately, these warm supergiants would contribute most significantly just around the 2000 Å minimum, and will tend to *fill* it, thus leading to a *flatter* SED. In this respect, we notice that the presence of AB supergiants is well documented in the Milky Way and in the Magellanic Clouds (Humphreys and McElroy 1984), but the interpretation of their number and temperature distribution is still a matter of some theoretical embarrassment (Bertelli, Bressan, and Chiosi 1984; Chiosi and Maeder 1986). Therefore, even the use of theoretical stellar evolutionary tracks which do include the core helium-burning stage (e.g., Guiderdoni and Rocca-Volmerange 1987; Rocca-Volmerange 1989) can result in an underestimate of the actual number of B- and A-type supergiants relative to their O-type MS precursors, thus leading to an overestimate of the spectral slope in the wavelength range of the observed UV rising branch. In the attempt to avoid these theoretical uncertainties, Bica and Alloin (1988) apply the cluster database method to the interpretation of the UV upturn in giant elliptical galaxies and conclude that only young, massive stars can explain it. This offers yet another opportunity of realizing a potential drawback of the method. The available data base lacks in fact any UV information on metal-rich cluster templates ([M/H] $\gtrsim 0.0$), and Bica and Alloin adopt for them the same UV spectrum of the clusters in the next, less metal rich group ([M/H] $\simeq -0.7$). Since these latter clusters are all very weak UV emitters, their assumption *a priori* excludes the possibility that the UV upturn is produced by an old, metal-rich population. In the whole data base the only cluster with a well-developed UV rising branch is the young LMC cluster NGC 2004, and therefore every *optimization* program will inevitably single out the only UV-bright cluster in the whole library, forcing the conclusion that elliptical galaxies are currently forming massive stars. It is worth realizing, however, that this result may just be the inescapable consequence of the incompleteness of the data base.

In the case of HB stars, it is well known that their effective temperature is extremely sensitive to very many parameters (such as mass-loss efficiency, age, helium abundance, metallicity, [CNO/Fe], ...) which are all virtually unknown for the stars in elliptical galaxies. It is therefore no surprise if HB stars with the properly tuned temperature distribution can fit the observed UV rising branch. The real problem is to independently justify why such distribution should be precisely the one required to fit the observed rising branch. The next section will further clarify several aspects which are specific to the HB candidate.

Quite similar arguments apply to P–AGB candidates, for which a set of four evolutionary tracks by Schönberner (1983) has been generally used. Besides the problems discussed in § IV, we would like to mention additional complications which are worth keeping in mind. It is well known that the evolutionary rates along the P–AGB tracks, and then the corresponding temperature distribution, are primarily controlled by the mass of the thin hydrogen-rich envelope which has survived the heavy AGB losses, and by the rate at which it is further reduced (Paczyński 1971*b*; Schönberner 1983; Iben 1984). The mentioned P–AGB models have been computed under very specific assumptions concerning the hydrogen-rich envelope, such as that the planetary nebula ejection at the tip of the AGB leaves an envelope which is in thermal equilibrium at the effective temperature $\log T_e = 3.7$, and that mass loss is practically unimportant during the whole P–AGB phase. Such assumptions are fairly reasonable insofar as one is primarily interested in a first exploration of the diversities of the P–AGB evolution, but are serious limitations if what matters is the actual temperature distribution of P–AGB stars in a realistic population. For example, the former assumption is equivalent to maintaining that for an AGB star with a core mass of 0.6 $M_\odot$ the AGB mass loss ceases when the envelope mass has decreased to $\sim 10^{-3}$ $M_\odot$. On the other hand, if one rather assumes a residual envelope mass of 6×10^{-4} $M_\odot$ then the P–AGB model would zip in a thermal time scale all across the H-R diagram up to an effective temperature of $\sim$40,000 K, and the time spent in the range between 20,000 and 40,000 K would drop by orders of magnitude. Moreover, mass-loss during the P–AGB phase itself can appreciably affect the evolutionary rate, and correspondingly the temperature distribution of the P–AGB component of a stellar population. In the above example of a 0.6 $M_\odot$ P–AGB star, a mass-loss rate of just $\sim 10^{-7}$ $M_\odot$ yr^{-1} would completely dominate the evolution, drastically reducing the P–AGB lifetime. These simple examples illustrate how subtle differences in the assumptions concerning apparently unimportant details of the P–AGB evolution can have dramatic consequences on the very basic ingredients used in spectral synthesis experiments (Renzini 1983, 1989*a*).

Finally, our understanding of binaries to be discussed in § V is in an even more rudimentary stage of its development compared to that of P–AGB stars. While some classes of low-mass binaries are certainly able to produce hot objects which may contribute to the UV rising branch of ellipticals, we are still far from being able to theoretically predict their detailed temperature distribution. Any assumed distribution would therefore be almost completely arbitrary at the present stage.

d) An Alternative Approach

Given the observational indeterminations and the theoretical ambiguities so far discussed one might suspect the whole attempt of understanding the UV rising branch of ellipticals to be a rather futile exercise at this time. We are not so pessimistic, but we prefer to clearly spell out our assumptions, procedures, and goals, and we do it in this subsection.

First of all, we will not rely on synthetic SEDs, as they would be totally dependent on the arbitrary assumptions concerning the temperature distributions of the various candidates. We rather concentrate on a systematic use of the so-called *fuel consumption theorem* (Renzini 1981*b*, 1989*b*; RB), which connects the relative bolometric contribution of stars in any evolutionary phase to the amount of fuel that each evolving star burns during that phase:

$$\frac{L_j}{L_T} \simeq 9.75 \times 10^{10} B(t) F_j[M_{\rm TO}(t)] \; . \qquad (1)$$

Here L_T and L_j are, respectively, the total bolometric luminosity of a stellar population, and the part of it which is provided only by the stars in the generic phase "j". The function $B(t)$ is the specific evolutionary flux of the population (i.e., the death rate per unit luminosity), F_j is the amount of fuel burned during the phase "j" by stars of initial mass $M_{\rm TO}$, which in turn is the mass of stars at the main-sequence turnoff when t is the age of the population. This relation applies to *simple* stellar populations, i.e., to assemblies of coeval, initially chemically homogeneous, single stars. For old populations ($t \gtrsim 10^{10}$ yr) one has within a $\sim$10% accuracy $B(t) \simeq 2.2 \times 10^{-11}$ stars $L_\odot^{-1}$ yr^{-1}, practically independent of composition. The fuel consumptions

$$F_j = \Delta M_j^{\rm H} + 0.1 \Delta M_j^{\rm He} \qquad (2)$$

are given in terms of the corresponding amount of burned hydrogen and helium and are measured in $M_\odot$ units. Note that the 0.1 coefficient comes from the fact that by burning helium one gets $\sim$10% of the energy liberated by burning the same amount of hydrogen. Therefore, for old populations equation (1) simply becomes

$$\frac{L_j}{L_T} \simeq 2 \times F_j[M_{\rm TO}(t)] \; . \qquad (3)$$

We now define $L^{\rm UV}$ to be the total bolometric luminosity of all the hot stars contributing to the UV rising branch of a given stellar population. In a single-metallicity population we then have $L^{\rm UV}/L_T \simeq 2 \times \sum_j F_j^{\rm UV} = 2 \times F^{\rm UV}(Z)$, where the summation over the fuel consumptions extends to all the evolutionary stages which contribute to the UV rising branch, i.e., with j = hot HB, P–AGB, etc. This can be easily generalized to the case of a single-age composite population characterized by a metallicity distribution function $\Phi(Z)$, more akin to real elliptical galaxies, for which one derives:

$$\frac{L^{\rm UV}}{L_T} \simeq 2 \int_0^{Z^{\max}} dZ \; \Phi(Z) F^{\rm UV}(Z, t) = 2\langle F^{\rm UV} \rangle_Z \; , \qquad (4)$$

where $\langle F^{UV} \rangle z$ is then the metallicity-averaged fuel consumption of the UV producers. This is the basic equation that we apply to investigate the UV output of elliptical galaxies.

Having set our tools, we now *assume* that the (1550 $-V$)–Mg_2 correlation is generated by L^{UV}/L_T being an increasing function of Z, or, equivalently, that the *hot star* fuel consumption $F^{UV}(Z)$ is an increasing function of Z. If so, galaxies characterized by a more populated high-metallicity wing of the $\Phi(Z)$ distribution would be richer in powerful UV producers, thus exhibiting a stronger UV rising branch.

We are fully aware that a F^{UV} increasing with Z may not be the only way of producing the desired correlation. For example, if the average temperature of the hot star component is a decreasing function of Z, then the 1550 Å flux would increase even if F^{UV} (i.e., L^{UV}/L_T) remains the same (for example, for fixed bolometric L^{UV}, the flux at 1550 Å will be larger in a population of hot stars at, say 30,000 K compared to the case of stars at 60,000 K, as in both cases the UV rising branch is still on the Rayleigh-Jeans side of the Planck function). We also note that a decreasing temperature for the hot star component as Z increases would just be in the same direction typical of most other evolutionary phases (e.g., MS, RGB, AGB), contrary to the widespread perception according to which the $(1550-V)$–Mg_2 correlation would contradict this trend. However, we here assume that a systematic trend with Z of the average temperature of the hot star component may help establishing the correlation, but that it is not its primary cause. In the following sections we will proceed to explore under what circumstances the fuel consumption F_j^{UV} of each hot star candidate may indeed increase with Z.

Before passing to examine the individual candidates, an estimate is necessary of the range covered by the hot-component relative contributions L^{UV}/L_T within the B3FL sample of elliptical galaxies. This requires an integration of the flux over all wavelengths (including the infrared), and an assumption about the typical temperature of the hot component. In the case of NGC 4649 this gives $L^{UV}/L_T = 0.014$, 0.021, and 0.057, respectively for $T = 20{,}000$, 40,000, and 70,000 K (see RB's Fig. 15). We correspondingly adopt $L^{UV}/L_T \simeq 0.02$ for this galaxy, which is one of those with the strongest UV rising branch (see B3FL Fig. 2). Other galaxies are generally fainter in the UV, with the flux down by a factor of ~ 10 in the faintest galaxies (e.g., M32). We conclude that L^{UV}/L_T is likely to range from ~ 0.002 for *red* galaxies $(1550-V \simeq 4.5)$, up to ~ 0.02 in *blue* galaxies $(1550-V \simeq 2.0)$. Equation (4) then indicates that the metallicity-averaged fuel consumption $\langle F^{UV} \rangle z$ must correspondingly range from $\sim 0.001\ M_\odot$ up to $\sim 0.01\ M_\odot$ of hydrogen, or, equivalently, a 10 times larger amount of helium. Ultimately, it is basically the astrophysical likelihood of this trend that we explore in the present paper.

e) Number-Luminosity Connection

In § II*c* we have stressed that when dealing with the contribution of stars of a certain kind to the total light of a stellar population what matters is how much fuel they consume. On the other hand, it is worth appreciating that when actually individually searching for such stars what matters is their *number*, and therefore the *durations* t_j's of the corresponding evolutionary stages. In a simple (i.e., chemically homogeneous) population the connection between the number of stars in any post–MS evolutionary stage and the corresponding duration is given by

$$N_j = B(t) L_T t_j \,, \qquad (5)$$

where L_T is in $L_\odot$ units and t_j is in years (Renzini 1981*b*; RB). In a real galaxy characterized by a (luminosity-weighted) metallicity distribution $\Phi(Z)$ the same relation is given by

$$N_j \simeq B(t) L_T \int_0^{Z^{max}} dZ \Phi(Z) t_j(Z) = B(t) L_T \langle t_j \rangle_Z \,, \qquad (6)$$

where we have used the fact that the specific evolutionary flux $B(t)$ is practically independent of metallicity. More often, rather than the total bolometric luminosity L_T, what is immediately available is the integrated magnitude in some optical color. In order to apply equation (5) we need a *bolometric correction* for the whole population. For example, given the absolute magnitude M_V and color $B-V$ of a galaxy (or of a part of it), we define

$$L_V = 10^{-0.4(M_V - M_{V,\odot})} \,, \qquad (7)$$

$$L_B = 10^{-0.4(M_B - M_{B,\odot})} = 10^{-0.4[(B-V)-(B-V)_\odot]} L_V \,. \qquad (8)$$

With a little algebra we obtain

$$L_T = 10^{-0.4(BC - BC_\odot)} L_V = 10^{-0.4[BC - BC_\odot - (B-V) + (B-V)_\odot]} L_B \,, \qquad (9)$$

where BC and $BC_\odot = -0.12$ are the bolometric corrections (to the V magnitude) for, respectively, the whole population and for the Sun; we further adopt $(B-V)_\odot = 0.6$. Bolometric corrections for stellar populations and galaxies have been estimated by A. Buzzoni (1988, private communication). He obtains $BC \simeq -0.48$ for the cluster M3, or $L_T \simeq 1.4 L_V \simeq 1.51 L_B$, having adopted $(B-V)_0 = 0.68$ for this fairly low metallicity cluster. In the case of a solar metallicity synthetic population (with $t = 15$ Gyr) Buzzoni obtains $BC = -0.6$, and when integrating the actual SED of individual galaxies—from the UV to the IR—he gets $BC = -0.8$ for NGC 4486, 3379, and 4472, and $BC = -0.9$ for NGC 4649. For a *typical* elliptical galaxy we correspondingly adopt $BC = -0.8$ and $(B-V)_0 = 0.87$, and therefore:

$$L_T \simeq 1.9 L_V \simeq 2.4 L_B \,. \qquad (10)$$

We note that the determination of these coefficients involves several nontrivial manipulations, and that the result is somewhat sensitive to the colors and metallicity of the involved stellar populations. We also take the opportunity to remark an inconsistency between the coefficients of L_V and L_B in a previous report (Renzini 1989*b*). These luminosity/number conversions will be used in § VII, when dealing with possible observational strategies to search for hot star candidates in various astrophysical environments.

III. HORIZONTAL BRANCH AND POST–RGB CANDIDATES

Galactic globular clusters with intermediate to low metallicities ([Fe/H] $\lesssim -1.4$) often contain fairly hot HB stars, and therefore HB stars have soon been regarded as plausible candidates for the production of the UV rising branch (e.g., Code and Welch 1979; Gunn, Stryker, and Tinsley 1981). However, in these systems the HB tends to become very red with increasing metallicity, i.e., the tendency is opposite to that required by the UV-Mg_2 correlation. On the other hand, elliptical galaxies are thought to be dominated by fairly metal rich—or even super metal-rich—populations ([Fe/H] $\gtrsim 0.0$), for which knowledge of their HB morphology is lacking. Could the UV-Mg_2 correlation arise from the HB turning to hotter and hotter temperatures for the metallicity increasing beyond solar? This is what we explore in this section. Indeed, the HB morphology versus metallicity relation is so sensitive to many parameters that a nonmonotonic behavior in the relevant range of Z certainly cannot be excluded (Renzini 1981*b*; RB). Yet, the most metal-rich globular with an available color-magnitude diagram (NGC 6553, with [Fe/H] $\simeq$ 0.0–0.3) still exhibits a *stubby red* HB (Barbuy, Bica, and Ortolani 1989). Since the spectrum of this cluster shows metallic absorption features of similar strength to those in giant ellipticals (Bica and Alloin 1987), one may be tempted to conclude that hot HB stars cannot be viable candidates for the UV rising branch. We believe that this is a premature conclusion, as hot HB stars could in principle be produced by the most metal-rich tail of the wide metallicity distribution one expects to be present in ellipticals (see § II), i.e., by stars more metal-rich than NGC 6553 itself. Indeed, the identity of some galaxy spectral features with those of a single-metallicity population (such as metal-rich galactic globulars) does not ensure the identity of the SED in other parts of the spectrum, particularly in its wings (see Renzini 1986).

a) Synthetic RGB Evolution

At least two poorly known trends control the behavior of the HB morphology as a function of the metal content Z: the Z-dependence of the rate of mass-loss during the RGB phase, and the helium-metallicity relation ($\Delta Y/\Delta Z$).[2] To explore this connection we have undertaken the following instructive, albeit laborious experiment. To describe mass-loss along the RGB we adopt a slightly modified version of the standard rate (Reimers 1975; Fusi Pecci and Renzini 1976):

$$\dot{M} = 4 \times 10^{-13}\eta\left(1 + \frac{Z}{Z_{\rm crit}}\right)\frac{L}{gR}\,(M_\odot\ {\rm yr}^{-1})\,, \qquad (11)$$

where we have introduced the factor $(1 + Z/Z_{\rm crit})$ so as to mimic a direct metallicity dependence, with $\dot{M}$ increasing with Z by an amount which, by definition, reaches a factor of 2 at $Z = Z_{\rm crit}$. We do not pretend to have a specific physical or astrophysical justification for equation (11), although a trend of this kind could in principle be related to, e.g., a mass-loss enhancement due to grain formation. We just note that for $Z \ll Z_{\rm crit}$ the standard value is recovered, and as we are interested in fairly high values of $Z_{\rm crit}$ ($\gtrsim Z_\odot$), this assumption has no influence on the HB morphology for metallicities in the range covered by galactic globulars. Even for $Z \gtrsim Z_\odot$ the implied increase of $\dot{M}$ over standard values is very modest, well within the present observational uncertainties, and yet we are going to show that such modest increase can dramatically enhance the strength of the UV rising branch.

Given the current uncertainties affecting the ($\Delta Y/\Delta Z$) ratio (Kunth 1983; Maeder 1983; Peimbert 1983; Pagel 1989; Steigman, Gallagher, and Schramm 1989), we just treat it as a parameter, and for the helium enrichment relative to metals we assume

$$Y(Z) = Y_p + \frac{\Delta Y}{\Delta Z}\,Z\,, \qquad (12)$$

with $Y_p = 0.24$ for the primordial helium. Values of the parameter $\Delta Y/\Delta Z$ between 0 and 3 are explored.

We then attempt a *reasonable* extrapolation of the HB morphology as a function of metallicity, and follow the approach described in Fusi Pecci and Renzini (1976) and Renzini (1977). The HB morphology (i.e., the temperature distribution of HB stars) is a function of at least four quantities, namely, the core mass $M_{\rm H}^{\rm HB}$ and total mass $M_{\rm HB}$ at the beginning of the HB phase, and the composition parameters Y and Z. In turn, the total mass $M_{\rm HB}$ is a function of the initial mass M_i (and then of age), taking in due account the mass-loss during the previous RGB phase. We approximate M_i with the mass at the base of the RGB $M_{\rm RG}(Y, Z, t)$ as given by equation (A1), and the integration of the equation $dM = \dot{M}dt$ then gives the mass of evolving stars as a function of L while they climb along the RGB:

$$M^{2.43} = M_{\rm RG}^{2.43} - 2.43 \times g(Y, Z, \alpha, \gamma)\eta\left(1 + \frac{Z}{Z_{\rm crit}}\right)L^\gamma\,, \qquad (13)$$

where the functions g and γ are given in the Appendix and where α is the mixing length parameter. The growth of the hydrogen-exhausted core along the RGB is followed via a composition-dependent core mass-luminosity relation $M_{\rm H}(Y, Z, L)$, as given by equation (A3). The RGB evolution is terminated by the core helium flash, which takes place at a luminosity $L_{\rm fl}(Y, Z, M)$ given by equation (A2). For $L = L_{\rm fl}$ equations (13) and (A3) then give, respectively, the stellar mass and the core mass at the tip of the RGB, which are also the total mass and the core mass at the beginning of the HB phase ($M_{\rm HB}$ and $M_{\rm H}^{\rm HB}$).

We first proceed to the calibration of the mass-loss parameter η. For all our models we adopt an age $t = 15$ Gyr, while the effects of changing the age are discussed in § V*b*. Having fixed t, for any adopted value of η and α (but note that what matters is the product $\eta\alpha^{0.4}$), from the above equations we then obtain $M_{\rm RG}$, $M_{\rm H}^{\rm HB}$, and $M_{\rm HB}$ (or $M_{\rm P\text{–}RGB}$) as a function of Z. Now, in order to get an idea on the resulting HB morphology, we compare $M_{\rm HB}$ to the expected mass of RR Lyrae stars $M_{\rm RR}(Y, Z)$ given by equation (A4). For $M_{\rm HB} > M_{\rm RR}$ all or most of the HB phase is spent at effective temperatures lower than those of RR Lyrae variables, i.e., one has red HBs and the corresponding HB contribution to the UV rising branch is negligible. An interesting contribution may instead arise when $M_{\rm HB} < M_{\rm RR}$, because in this case hot HB stars can be produced: The lower the HB mass, the higher the effective temperature. The calibration of η is then obtained by imposing the condition $M_{\rm HB} = M_{\rm RR}$ for $Z = 0.001$, $t = 15$ Gyr and $\alpha = 1.5$, so as to approximately reproduce the observed HB morphology of Galactic globular clusters of intermediate metallicity, which usually are very rich in RR Lyrae stars (e.g., M3). This gives $\eta = 0.36$, but we note that none of our conclusions is seriously dependent upon the particular age chosen. Had we adopted a lower age, this procedure would have just given a slightly larger value of η.

Having fixed η, Figure 1 shows all the relevant masses $M_{\rm RG}$, $M_{\rm RR}$, $M_{\rm HB}$, and $M_{\rm H}^{\rm HB}$ as a function of metallicity Z. Each panel refers to a different choice for the helium-enrichment parameter $\Delta Y/\Delta Z$ in equation (12). Since for a given age $M_{\rm RG}$ increases with increasing metallicity and decreases with increasing helium, the function $M_{\rm RG}(Z)$ is monotonic for $\Delta Y/\Delta Z = 0$, but shows a more and more pronounced high-Z drop when $\Delta Y/\Delta Z$ grows beyond unity. Therefore, for large values of $\Delta Y/\Delta Z$ less mass needs to be lost during the RGB phase in order to produce hot HB stars. The RR Lyrae mass $M_{\rm RR}$ is a monotonically decreasing function of Z, whatever value of $\Delta Y/\Delta Z$ is adopted, but it tends to increase with increasing

[2] Here we ignore the effects of possible trends of [CNO/Fe] with [Fe/H].

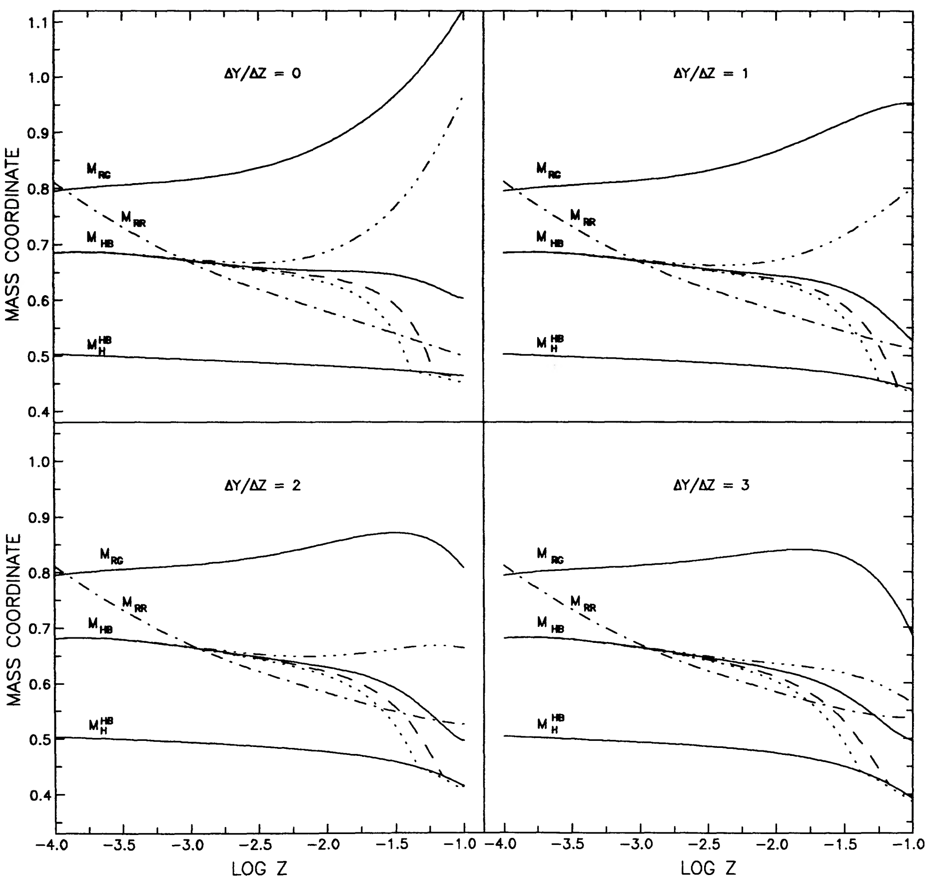

FIG. 1.—Several relevant mass values (in $M_\odot$ units) as functions of metallicity, for a population of stars with an age of 15 Gyr and for different choices of the $\Delta Y/\Delta Z$ parameter. The mass at the base of the red giant branch (M_{RG}), the mass on the zero-Age HB (M_{HB}) and its corresponding helium core mass(M_H^{HB}), and the mass of ZAHB stars in the RR Lyrae instability strip (M_{RR}) are shown. The figure displays the results of the evolution for different choices for the mass-loss rate during the RGB phase, with Reimers' canonical case (with $\eta = 0.36$) indicated by the triple dot-dashed line. The solid, dashed, and dotted lines refer, respectively, to three progressively lower values for the Z_{crit} parameter (see text). In order of increasing $\Delta Y/\Delta Z$, the values of Z_{crit} in each panel are, respectively, (0.08, 0.06, 0.05), (0.10, 0.08, 0.07), (0.14, 0.10, 0.08), and (0.25, 0.12, 0.09).

helium and therefore larger values of $\Delta Y/\Delta Z$ imply slightly larger values of M_{RR} at high Z. The core mass at the helium flash decreases with increasing both Z and Y, and therefore the larger $\Delta Y/\Delta Z$ the more pronounced the high-Z drop of M_H^{HB}.

We now come to comment on the behavior of M_{HB} and therefore of the HB morphology as a function of Z. From the adopted calibration of η it follows that for $Z \lesssim 0.001$ one has $M_{HB} < M_{RR}$, and mostly blue HBs (most stars on the blue side of the RR Lyrae strip), while for $Z \gtrsim 0.001$ one has $M_{HB} > M_{RR}$, and most stars lie to the red of the RR Lyrae strip. With the standard mass-loss rate (i.e., for $Z_{crit} = \infty$) this trend persists all the way to the highest metallicities. However, by allowing the mass-loss rate to moderately increase with Z, the trend of the HB to become redder with increasing Z can be reversed, and hot HB stars can be produced above some relatively high Z. It is worth stressing that the required increase of $\dot{M}$ over the standard value is really very modest, for example, in the most unfavorable case $\Delta Y/\Delta Z = 0$, hot HB stars are produced for $Z \gtrsim 0.03$ when $Z_{crit} \simeq 0.06$, which implies a 60% increase of $\dot{M}$ at $Z = 0.03$. For the case of high helium enrichment ($\Delta Y/\Delta Z = 3$), $Z_{crit} \simeq 0.12$ is sufficient, which implies an increase of $\dot{M}$ of less than 30%. These numerical experiments illustrate very emphatically the main point: *the possibility of producing high-Z hot HB stars is very critically controlled by the precise trends of mass-loss and helium enrichment with metallicity.* We shall further illustrate this point in § III*c* below. From Figure 1 one sees that for each combination ($\Delta Y/\Delta Z$, Z_{crit}) up to four metallicity ranges can be defined from the interception(s) of M_{HB} with M_{RR} and M_H^{HB}. For $Z \leq 0.001$ one has $M_{HB} < M_{RR}$, and blue HB stars are produced (such as in metal-poor globular clusters); for $0.001 < Z < Z_{RR}$ one has $M_{HB} > M_{RR}$, and red HB stars are produced (such as in metal-rich globulars); for $Z_{RR} < Z < Z_{WD}$ one has $M_{RR} < M_{HB} < M_H^{HB}$ and blue HB stars are produced again; and, finally, for $Z > Z_{WD}$ helium WDs of mass $M_{WD} \simeq M_H^{HB}$ are generated. We first discuss this latter case.

b) Post–RGB Stars

When the assumed mass-loss rate during the RGB is sufficiently large the whole hydrogen-rich envelope is lost before the core can reach the critical mass for helium ignition (see Fusi Pecci and Renzini 1975; Harpaz, Kovetz, and Shaviv 1987). Such models will then fail to experience the core helium flash as well as the HB, AGB, and P–AGB evolutionary phases and will rather become helium WDs after a relatively short post–RGB phase.[3] Thanks to the large luminosity dependence

[3] In these cases, the mass of the helium P–RGB remnant is obtained following the buildup of M_H and the decrease of M until virtually the whole envelope is lost, i.e., until $M \simeq M_H \simeq M_{P-AGB}$.

of the mass-loss rate, most of the mass-loss takes place when the red giants are already close to the tip of the RGB. Therefore, the mass of the post–RGB remnant is always close to the corresponding $M_{\rm H}^{\rm HB}$, as can be appreciated from Figure 1. Evolutionary models for low-mass stars which lose their envelope before having a chance of experiencing the core helium flash have been recently calculated by Iben and Tutukov (1986) in the context of mass transfer scenarios in close binary systems. For the adopted binary parameters, Roche-lobe overflow removes most of the envelope when the core mass of the RGB star has reached 0.3 $M_\odot$. The models then leave the RGB as soon as the envelope mass M_e drops below $\sim 4 \times 10^{-3}$ $M_\odot$, and start contracting at nearly constant luminosity, toward the region of the H-R diagram usually occupied by planetary nebula (PN) nuclei. It takes about 2 million yr to reach temperatures in excess of $\sim$20,000 K, and in the meanwhile the envelope mass has decreased to $\sim 3 \times 10^{-3}$ $M_\odot$. Most of this hydrogen-rich envelope is then burnt before the star approaches the helium WD cooling sequence, and in two successive hydrogen-shell flashes, which cause the star to undergo very extended loops in the H-R diagram. Still, the residual envelope is burnt while the star is at fairly high effective temperatures ($T_e \gtrsim$ 20,000 K), and therefore post–RGB stars would contribute to the UV upturn of old stellar populations

The fuel consumption $F_{\rm P\text{–}RGB}(M_{\rm H}, Z)$ during the hot post–RGB phase is a decreasing function of the core mass and metallicity, which unfortunately remains to be systematically explored. The larger $M_{\rm H}$ the higher the luminosity, which favors larger envelope dimensions. So the larger $M_{\rm H}$ the less massive the envelope has to become before the star departs from the RGB, and then the smaller $F_{\rm P\text{–}RGB}$. For the case investigated by Iben and Tutukov (1986), i.e., for $(M_{\rm H}, Z) =$ (0.3, 0.02), one has $F_{\rm P\text{–}RGB} \simeq X_e \times 3 \times 10^{-3}$ $M_\odot = 2 \times 10^{-3}$ $M_\odot$, where $X_e = 0.7$ is the hydrogen abundance in the envelope. To find another reliable value one has to go back to the old works of Kippenhahn, Thomas and Weigert (1968), whose model with $(M_{\rm H}, Z) = (0.26, 0.044)$ gives $F_{\rm P\text{–}RGB} \simeq 4 \times 10^{-3}$ $M_\odot$, and Giannone, Refsdal, and Weigert (1970) whose models for the same composition give $F_{\rm P\text{–}RGB} \simeq 4\times$, $2\times$, and 0.6×10^{-3} $M_\odot$, respectively for $M_{\rm H} = 0.265$, 0.363, and 0.426 $M_\odot$. From Figure 1 we see that core masses greater than $\sim$0.4 $M_\odot$ are to be expected for hypothetical high-metallicity post–RGB remnants. We correspondingly adopt $\log F_{\rm P\text{–}RGB} \simeq -2.7\text{–}7.5(\log M_{\rm H} + 0.44)$, which is a linear fit to the latter two values above, and we expect a rather modest fuel consumption to be associated to this kind of hot component, i.e., $F_{\rm P\text{–}RGB} \simeq 6 \times 10^{-4}$ $M_\odot$. The corresponding contribution to the total bolometric light of a galaxy will then be

$$\frac{L_{\rm P\text{–}RGB}}{L_T} \simeq 9.75 \times 10^{10} B(t) F_{\rm P\text{–}RGB} f_{\rm P\text{–}RGB} \simeq 1.2 \times 10^{-3} f_{\rm P\text{–}RGB} , \tag{14}$$

where $f_{\rm P\text{–}RGB} = \int_{Z_{\rm WD}}^{Z_{\rm max}} dZ\Phi(Z)$ is the fraction of the total galaxy light which is provided by stars with $Z > Z_{\rm WD}$.

c) Hot HB Stars

Within the metallicity range $Z_{\rm RR} < Z < Z_{\rm WD}$ ZAHB models span effective temperatures which go from those typical of RR Lyrae stars ($\sim$7000 K) up to those of helium main-sequence models ($\sim$30,000–40,000 K, see Paczyński 1971*a*), while their mass decreases from $M_{\rm RR}$ to $M_{\rm H}^{\rm HB}$. As $\log T_e$ is fairly linear with $M_{\rm HB}$ in this mass range, stars with envelope mass lower than about half that of RR Lyrae with the same composition have fairly high effective temperatures ($\log T_e \gtrsim 4.2$). We then assume that hot ZAHB stars are produced for $M_{\rm H}^{\rm HB} \lesssim M_{\rm HB} \lesssim M_{\rm H}^{\rm HB} + 0.5(M_{\rm RR} - M_{\rm H}^{\rm HB})$, which corresponds to metallicities roughly in the range $Z_{\rm HHB} \lesssim Z < Z_{\rm WD}$, with $Z_{\rm HHB} = 0.5(Z_{\rm RR} + Z_{\rm WD})$, as seen in Figure 1. It is worth stressing that the sensitivity of the HB morphology (i.e., ZAHB temperature) to $M_{\rm HB}$ greatly increases with increasing Z. This is clearly shown by Figure 1, where we see how dramatically the mass difference $M_{\rm RR} - M_{\rm H}^{\rm HB}$ decreases with increasing metallicity. For example, in the case $\Delta Y/\Delta Z = 1$ for $\log Z = -1.5$ one has $M_{\rm RG} \simeq 0.92$, $M_{\rm RR} \simeq 0.54$, and $M_{\rm H}^{\rm HB} \simeq 0.45$ $M_\odot$. Therefore an increase in the adopted RGB mass loss from $\sim$0.38 to 0.47 $M_\odot$ (only a 20% increase!) will move the ZAHB location of the models from the middle of the RR Lyrae strip ($T_e \simeq 7000$ K) all the way to the helium main-sequence ($T_e \simeq 40{,}000$ K). An even larger sensitivity is predicted at higher Z. This illustrates the basic difficulty one encounters when theoretically predicting the HB morphology of high-Z stars: minor changes in adopted mass loss produce vastly different results. Moreover, there is very little chance that observations could provide mass-loss rates estimates with the required accuracy, at least in a foreseeable future.

For a regular metallicity distribution a quite uniform population of the ZAHB is obviously to be expected, a property which is not very attractive in the context of the UV upturn of ellipticals, which rather requires a bimodal temperature distribution in order to generate the 2000 Å dip (e.g., Nesci and Perola 1985). However, starting from a rather uniform ZAHB population, the subsequent HB evolution might by itself introduce a bimodality in the temperature distribution of core helium-burning stars and their progeny. This, in fact, would be the case if a fairly modest mass loss ($\sim 10^{-10}$ $M_\odot$ yr^{-1}) operates in these hot HB stars, so as to confine their evolution to high temperatures rather than allowing them to evolve toward the AGB as in the case of slightly more massive HB stars (Renzini 1984). So a bimodal distribution could be produced even starting from a continuous ZAHB population, and a dip in the SED would then result since what actually matters is at which effective temperatures most of the fuel gets burned.

We then assume that the HB evolution of ZAHB models initially hotter than $\sim$25,000 K is entirely confined to higher effective temperatures, while initially cooler models evolve to even cooler temperatures. Indeed, existing hot HB evolutionary models indicate that both the whole core helium-burning stage, and the subsequent shell helium-burning stage are spent at high effective temperatures. These models fail to experience the AGB phase close to the Hayashi track, but rather climb to high luminosities ($\sim$2000 $L_\odot$) maintaining temperatures in excess of $\sim$25,000 K, and eventually join with the usual P–AGB tracks for the same mass (Sweigart, Mengel, and Demarque 1974; Sweigart 1978; Caloi 1989). Hot HB models have initially a very small envelope mass, $M_e = M_{\rm HB} - M_{\rm H}^{\rm HB} \simeq$ few 10^{-2} $M_\odot$, and this envelope is likely to be totally lost by winds (rather than burned) during either the core helium-burning (HB) phase, or at latest during the subsequent shell helium-burning phase, that we may call the AGB–manqué phase. Therefore, the cumulative (HB+AGB–manqué) fuel consumption of the hot HB stars and their progeny is just given by $\sim M_{\rm H}^{\rm HB}$ solar masses of helium, as virtually the whole helium core is converted to carbon and oxygen, i.e., $F_{\rm HHB} \simeq 0.1 \times M_{\rm H}^{\rm HB} \simeq 0.05$ $M_\odot$. Correspondingly, the contribution to the total bolometric light of a galaxy will be

$$\frac{L_{\rm HHB}}{L_T} \simeq 9.75 \times 10^{10} B(t) F_{\rm HHB} f_{\rm HHB} \simeq 0.1 \times f_{\rm HHB} \,, \qquad (15)$$

where $f_{\rm HHB} = \int_{Z_{\rm HHB}}^{Z_{\rm WD}} dZ \Phi(Z)$ is the fraction of the total galaxy light which is provided by stars in the metallicity range $Z_{\rm HHB} \lesssim Z < Z_{\rm WD}$.

One interesting aspect of the hot HB and AGB–manqué candidates is that a fair fraction of them may have already completed the wind ejection of the hydrogen-rich envelope, therefore exposing materials in which hydrogen burning has gone to completion, i.e., basically helium and nitrogen, but very little carbon and oxygen. Such objects (that hereafter we call helium stars) would then exhibit considerably weakened C IV lines at 1550 Å, compared to stars of similar effective temperature but with normal composition. As a consequence, the observed weakness (or absence) of the C IV feature in the spectra of ellipticals would not necessarily provide a strong constraint for the temperatures of the hot star content of these galaxies. (The absence of a strong C IV feature has on occasion been taken as evidence against a major UV contribution from stars B0 and earlier, cf., B3FL; Fanelli, O'Connell, and Thuan 1987; Welch 1982). The proportion of helium star light in the population will basically depend on the wind mass-loss rate during the core and shell helium-burning stages. Figure 1 shows that hot HB stars of high metallicity must have a fairly thin hydrogen-rich envelope, with $M_e \lesssim$ few $10^{-2}\ M_\odot$. Since the HB lifetime is $\sim 10^8$ yr, mass-loss rates of a few $10^{-10}\ M_\odot$ yr^{-1} will be sufficient for the formation of helium stars during the core helium-burning stage, with a prompter transition the larger the mass-loss rates. For lower rates the formation of helium stars would be postponed to the AGB–manqué phase, when $\dot{M}$ presumably increases as a consequence of the increased luminosity of shell helium-burning models (see Caloi 1989). From an observational point of view hot HB stars which have not yet exposed a helium surface would probably be classified as SdO, SdOB, or SdB stars. The empirical mass-loss rates for this class of objects are very uncertain, with values from $\sim 10^{-11}$ up to $\sim 10^{-8}\ M_\odot$ yr^{-1} being reported (see Willis 1988 for a recent review), therefore generously encompassing the astrophysically interesting range ($\sim 10^{-10} - 10^{-9}\ M_\odot$ yr^{-1}).

IV. POST–AGB CANDIDATES

Post–AGB stars, i.e., stars on their way to becoming WDs after having left the AGB, were soon regarded as potentially important UV producers in old stellar populations (see Rose and Tinsley 1974, in particular their Fig. 1), well before *OAO 2* observations confirmed the existence of the UV rising branch of elliptical galaxies (Code and Welch 1979). Indeed, *OAO 2* had previously given just a hint on the UV spectrum of early-type galaxies (Code, Welch, and Page 1972). It has been subsequently realized that the P–AGB light contribution is extremely sensitive to the age (and possibly metallicity) of the population (Renzini 1981*b*; RB) therefore offering an attractive chance of accounting for the large galaxy-to-galaxy variations of the UV rising branch, a property that has made the P–AGB candidates particularly fashionable (e.g., Bohlin *et al.* 1985; Mochkovitch 1986; Kjærgaard 1987; B3FL; Bertola 1988*a*; Barbaro and Olivi 1989; Bertelli, Chiosi, and Bertola 1989; Brocato *et al.* 1990).

For those stars which experience an AGB phase (close to the Hayashi line), the transition to the P–AGB phase is controlled by mass loss processes. Models start departing from the AGB and evolve to high temperatures as soon as the envelope mass decreases below a critical value M_e^D which is function of mass, luminosity, and composition. According to the early calculations of Paczyński (1971*b*) $M_e^D \lesssim$ few times $10^{-3}\ M_\odot$, but the actual behavior of this function remains to be systematically explored, in particular for what concerns the composition dependence.

We can distinguish three cases concerning the evolutionary phase at which mass-loss processes eventually succeed in reducing the envelope mass below M_e^D (Renzini and Fusi Pecci 1988; Renzini 1989*a*): (*a*) during the early AGB (E–AGB) phase before the first thermal pulse (TP) has a chance to take place, (*b*) at the very first TP which can trigger a fast reduction of the hydrogen envelope having survived E–AGB losses, and (*c*) after the completion of one or more TPs. All these possibilities are now discussed in detail.

a) Synthetic HB and AGB Evolution

To investigate the behavior of P–AGB candidates we now need to extend the synthetic description of the evolution through the HB and AGB phases, in the same spirit as for the synthetic RGB evolution described in § III*a*. In doing so we have been forced to introduce some simplifying assumptions which will be adequately signalled, as these are the points where improvements over the present approach may be easier to achieve. However, none of our main conclusions is seriously dependent on the accuracy of the adopted approximations. These simplifications basically concern the E–AGB phase of low-mass stars, for which only partial exploration of the parameter space is available yet.

The mass of the hydrogen-exhausted core at the end of the HB phase (terminal age HB, TAHB) is given by the initial core mass (at the core helium flash) increased by the amount of mass which has been processed by the hydrogen-burning shell during the HB phase:

$$M_{\rm H}^{\rm TAHB} = M_{\rm H}^{\rm HB} + \Delta M_{\rm H}^{\rm HB}(M_{\rm HB}, M_{\rm H}^{\rm HB}, Y, Z) \,, \qquad (16)$$

where the function $\Delta M_{\rm H}^{\rm HB}$ is given by equation (A5).

At the end of the HB phase helium burning in the core ceases, while in the meantime helium burning in the shell quickly develops. An expansion of the layers above the helium shell causes the power released by the hydrogen-burning shell to temporarily decrease during the first part of the ascent along the E–AGB. Later, the strength of the hydrogen burning resumes, and so does the buildup in the core mass $M_{\rm H}$ (Gingold 1976). In the meantime also the helium-burning front propagates, and therefore both the mass of the carbon-oxygen core ($M_{\rm CO}$) and $M_{\rm H}$ keep building up as the luminosity increases during the E–AGB. We approximate the core mass-luminosity relation during this phase with the expression:

$$M_{\rm H}^{\rm EAGB}(L) = M_{\rm H}^{\rm TAHB} + \Lambda(L) \Delta M_{\rm H}^{\rm EAGB} \,, \qquad (17)$$

where $\Delta M_{\rm H}^{\rm EAGB}$ is the amount of mass which is processed by the hydrogen-burning shell during the whole E–AGB phase, as given by equation (A6), and the fitting function $\Lambda(L)$ is given by equation (A7). For the mass of the carbon-oxygen core $M_{\rm CO}(L)$ we use a rather complex analytic approximation given by equations (A8)–(A10).

During the E–AGB phase mass loss resumes at the rate given by equation (11), and therefore the evolutionary rate

$dt(L)$ is required in order to follow the decrease in mass by integrating the equation $dM = \dot{M}\,dt$. For consistency with the adopted core mass-luminosity relations the evolutionary rate is obtained by imposing that the amount of energy released Ldt is proportional to the amount of fuel which is burned. This gives

$$dt = 9.75 \times 10^{10}\left(X_e \frac{dM_H}{dL} + 0.1\,\frac{dM_{CO}}{dL}\right)d\ln L\,, \qquad (18)$$

where X_e is the hydrogen abundance in the envelope. Equation (18) is used also for the subsequent, thermally pulsing regime, where, however, the core mass-luminosity relation is replaced by the appropriate expressions given in the Appendix. The calculations are started by setting $M = M_{HB}$, $M_H = M_H^{TAHB}$, $M_{CO} = 0.5M_H^{TAHB}$, $\log L = 2.1$; the equation $dM = \dot{M}\,dt$ is integrated using equations (11) and (18), until the envelope mass drops below 0.02 $M_\odot$, when we assume that a *superwind* quickly removes most of this residual envelope. In some cases this happens during the E–AGB phase, and when so the subsequent evolution is supposed to follow a P–EAGB sequence to be discussed in the following subsection.

Figure 2 shows the progressive increase in the core mass M_H and decrease of the total mass M through the HB and AGB phases, as functions of metallicity. The initial conditions on the ZAHB are characterized by $(M, M_H) = (M_{HB}, M_H^{HB})$. During the HB phase hydrogen-shell burning leads to an increase of the core mass from M_H^{HB} to M_H^{TAHB}, while the stellar mass remains the same (apart from a small reduction in hot HB stars). During the E–AGB phase the hydrogen shell moves from M_H^{TAHB} to M_H^1, the core mass at the first thermal pulse. Two options now open: either the first thermal pulse is supposed to trigger the superwind envelope ejection and a P–AGB star with $M \simeq M_H^1$ is produced (option A), or the star is allowed to continue through the thermally pulsing AGB phase until its envelope mass is reduced below 0.02 $M_\odot$ (option B). In this latter case the core mass further increases during the TP–AGB phase, up to the value $M_H = M_H^F$, the final core mass.

It is worth noting that the run of the final mass with metallicity is rather different depending on whether option A or B are used. In particular, at the low metallicities encompassed by galactic globular clusters ($\log Z \lesssim -2$) the final mass increases with Z with option A, and decreases with option B. Correspondingly, thanks to the core mass-luminosity relation one would expect the maximum AGB luminosity to increase with Z in the case of option A, and decrease with option B. Actually, among galactic globulars the maximum AGB luminosity is observed to increase with the cluster metallicity (Frogel and Elias 1988), and for this reason it has been argued that option

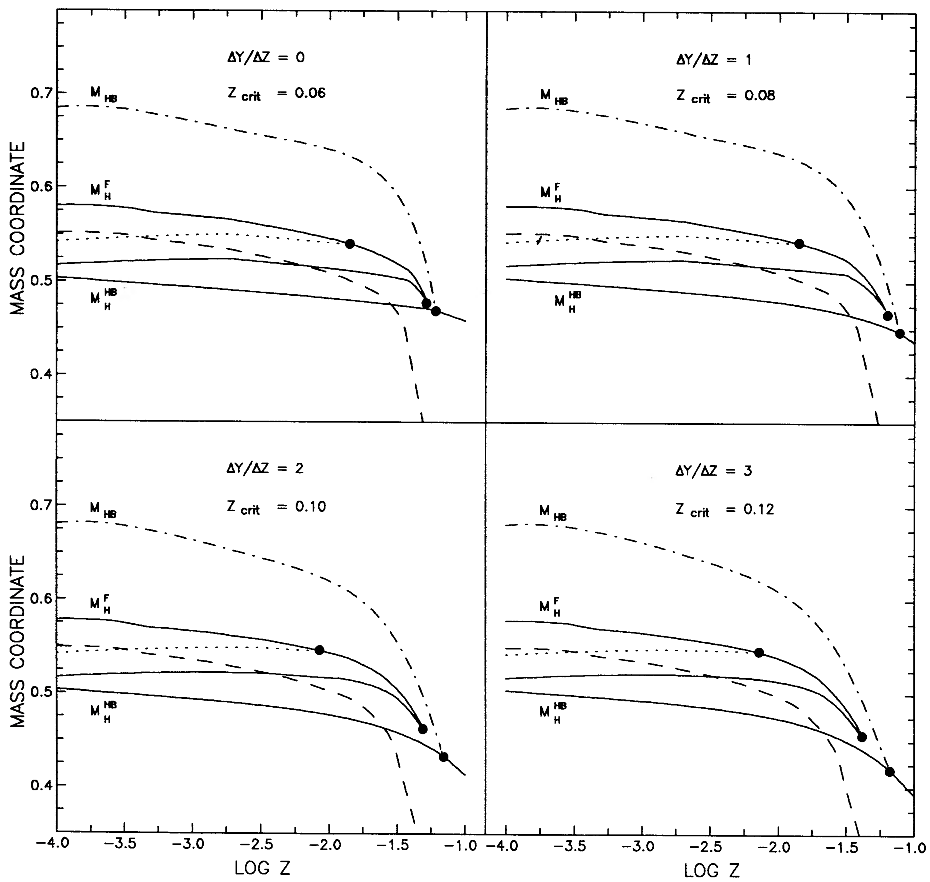

FIG. 2.—Further relevant mass values (in $M_\odot$ units) as functions of metallicity for a population of stars with an age of 15 Gyr. The results refer to several choices for the $\Delta Y/\Delta Z$ and Z_{crit} parameters. Besides M_H^{HB} and M_{HB}, reproduced from Fig. 1, the mass of the hydrogen exhausted core at the end of the HB phase is indicated (M_H^{TAHB}, *unlabeled solid line*), together with the value it has reached at the first thermal pulse on the AGB (M_H^1, *dotted line*), and its final value at the end of the AGB phase (M_H^F). Also shown is the final mass of the carbon-oxygen core when $M_H = M_H^F$ (*dashed line*). In each panel the interceptions identified by the three filled circles mark the transitions between the four different final evolutionary behaviors: P–AGB, P–EAGB, hot HB and P–RGB models are produced for progressively larger values of log Z.

A may be more appropriate to the case of the low-mass stars currently evolving in either globular clusters or elliptical galaxies (Renzini and Fusi Pecci 1988).

Some of the lines in Figure 2 intercept each other, therefore defining relevant values of Z for which a qualitative change in the evolutionary behavior takes place. These interceptions are marked with filled circles in Figure 2. In order of increasing Z, the first interception is between $M_{\rm H}^{1}$ and $M_{\rm H}^{\rm F}$. For lower values of Z stars experience the first thermal pulse while still having an envelope mass in excess of 0.02 $M_{\odot}$. For them the A/B option is therefore open, and either helium-burning (option A) or hydrogen-burning (option B) P–AGB stars are produced. For larger values of Z the envelope mass has already decreased below 0.02 $M_{\odot}$ before the first thermal pulse has a chance to take place, and P–EAGB stars are produced. The second filled circle marks the near interception of $M_{\rm H}^{\rm F}$ with $M_{\rm H}^{\rm TAHB}$, and corresponds to stars which barely manage to reach the AGB. Correspondingly, for slightly larger values of Z, hot HB stars are produced. Finally, the third interception (already shown in Fig. 1) is that between $M_{\rm HB}$ and $M_{\rm H}^{\rm HB}$ which marks the transition to the P–RGB case. In summary, the three interceptions circled in Figure 2 define four ranges of Z in which P–AGB, P–EAGB, hot HB, and P–RGB stars are, respectively, produced. Having already discussed the fuel consumptions of hot HB and P–RGB stars in § III, we now discuss the remaining cases.

b) Post–Early–AGB Stars

Model stars which complete the core helium-burning (HB) phase with a small envelope mass ($M_e \sim$ few 10^{-2} $M_{\odot}$) barely manage to reach the AGB, rather than venturing through the hot AGB-manqué phase. However, as red giant winds resume eroding the envelope the star will leave the AGB, the departure occurring when the envelope mass is reduced below the critical value M_e^D. The departure from the AGB will then be controlled by the envelope mass at the start of the AGB, and by the adopted wind mass-loss rate. In turn, the value of M_e^D depends on the luminosity reached on the EAGB, and by analogy with existing constant-mass models (Gingold 1976; Caloi 1989) we infer that it decreases from $\sim$0.01 $M_{\odot}$ down to roughly a few 0.001 $M_{\odot}$, in the relevant luminosity range ($\sim$200–2000 $L_{\odot}$). The larger value of M_e^D then applies to those models which depart shortly after their arrival at the base of the AGB, while the smaller value applies to those stars which manage to maintain an envelope with $M_e > M_e^D$ long enough for climbing almost the whole EAGB, and leave it shortly before the first helium shell flash. As EAGB and P–EAGB stars are burning helium in a shell, their surface luminosity is not greatly affected by how much envelope mass is left, i.e., the actual value of M_e can have a strong influence on the effective temperature, but not on the luminosity. As these stars leave the EAGB, and proceed to hot temperatures in their P–EAGB evolution, their path in the H-R diagram eventually merges with that of AGB-manqué stars of similar mass. There again, fast winds typical of hot stars are likely to complete the envelope removal, thereby exposing He-N layers. We can easily estimate the mass-loss rates required to complete this job, as $\langle \dot{M} \rangle = M_e^D / t_{\rm P-EAGB}$. Stars which quickly leave the EAGB with $M_e^D \simeq 0.01$ $M_{\odot}$ will spend the whole shell helium-burning stage at high temperatures ($t_{\rm P-EAGB} \simeq 1.5 \times 10^7$ yr), and correspondingly an average mass-loss rate $\lesssim 10^{-9}$ $M_{\odot}$ yr^{-1} is sufficient to remove the H-rich envelope. Those stars which leave the AGB toward the end of the EAGB phase will spend only some 10^6 yr in the P–EAGB phase, but with an initial envelope mass $M_e^D \simeq$ few 10^{-3} $M_{\odot}$. Again, an average mass-loss rate $\sim 10^{-9}$ $M_{\odot}$ yr^{-1} should be sufficient to complete the envelope removal. These bare-core objects will then exhibit strongly enhanced nitrogen and strongly reduced carbon and oxygen in their spectra, and we conventionally refer to them as WN P–EAGB stars, for the possible similarity with the Wolf-Rayet stars of the WN variety. On the other hand, those hot stars which have not already completed the envelope removal will exhibit a normal O- or B-type spectrum. Therefore, the actual WN:O light proportion in the integrated spectrum of a pure P–EAGB population will be related to the efficiency of the fast winds which operate at high temperatures.

High metallicity may favor a larger WN:O light ratio, as M_e^D is likely to decrease with increasing metallicity, while $t_{\rm P-EAGB}$ remains substantially the same. Moreover, the average mass-loss rate during the P–EAGB may increase with metallicity, as expected if line radiative acceleration plays a rôle in driving the fast winds from hot stars, thus favoring the production of WN objects.

As far as the fuel consumption is concerned, for the reasons we have just outlined above, we consider more likely that the hydrogen-rich envelope of hot P–EAGB stars will be lost by winds, rather than burned by the hydrogen shell. This leaves helium as the main fuel for this stage. The amount of helium fuel burned during the P–EAGB will obviously be larger, the sooner the star leaves the AGB, and the corresponding fuel consumption is given by

$$F_{\rm P-EAGB} = 0.1(M_{\rm H} - M_{\rm CO}) = 0.1\,\Delta M_{\rm He}\,, \qquad (19)$$

where $M_{\rm H}$ and $M_{\rm CO}$ are, respectively, the mass interior to the hydrogen shell and the mass of the carbon-oxygen core, both at the moment of departure from the AGB. Correspondingly, $\Delta M_{\rm He}$ is the buffer mass of helium contained in the star, i.e., the mass between the two shells. With this relation we assume that the whole helium buffer mass is burned during the P–EAGB phase, while actually some fraction of it can survive unburned as a helium skin for the non–DA WD progeny. On the other hand, equation (19) may somewhat underestimate the fuel in those cases in which the star leaves the AGB toward the end of the EAGB, when the hydrogen shell has fully reactivated (see Gingold 1976). In such cases part of the mass M_e^D of the hydrogen-rich envelope could be burned, rather than lost in a wind, thus further contributing to $F_{\rm P-EAGB}$.

With $M_{\rm H} \simeq 0.5$ $M_{\odot}$, $M_{\rm CO} = 0.4$ $M_{\odot}$, and $\Delta M_{\rm He} = 0.1$ $M_{\odot}$ the P–EAGB star luminosity would be $\sim$1000 $L_{\odot}$, the P–EAGB phase would last for more than $\sim 10^6$ yr, and thanks to equation (4) such objects would contribute $\sim$2% of the total bolometric light of a population (Renzini and Fusi Pecci 1988), just what is required even in the extreme case of NGC 4649. Detailed model calculations for precisely this choice of the stellar parameters have confirmed this theoretical expectation (see Brocato *et al.* 1990). However, there remains to see how fast these *late* P–EAGB objects are able to complete their transition to the high temperatures required by the observed upturns (slow P–EAGB models would tend to produce rather flat UV spectra, an aspect which might have been overlooked by Brocato *et al.* 1990).

c) Helium-burning P–AGB Stars

When the mass losses suffered during the RGB and EAGB phases are not sufficient to remove the whole hydrogen-rich envelope, then stars manage to enter the thermally pulsing

AGB phase (for an extensive review see Iben and Renzini 1983). There are good physical and astrophysical reasons to suspect that the envelope of low-mass AGB stars may not survive the very first helium shell flash, when the surface luminosity experiences a sudden, major increase (Renzini 1989*a*; Renzini and Fusi Pecci 1988). This may indeed trigger the transition to the *superwind* regime ($\dot{M} \simeq 10^{-5}\ M_{\odot}\ \mathrm{yr}^{-1}$) which rapidly gets rid of the tiny envelope (a few $\sim 10^{-2}\ M_{\odot}$) which has survived EAGB losses. Models which leave the AGB in response to a thermal pulse do so while burning helium in the shell. Helium-burning P–AGB models of $\sim 0.6\ M_{\odot}$ burn perhaps 3 times more fuel than hydrogen-burning P–AGB models of the same mass do (Iben 1984; Iben and Tutukov 1984*b*, in particular their Fig. 1). These kind of objects are therefore producing ~ 3 times more UV photons than their hydrogen-burning analogs, a very attractive property in the present context. Unfortunately, this applies to a 0.6 $M_{\odot}$ P–AGB star, and there is no physical reason why the ratio of helium and hydrogen P–AGB fuel consumption should remain the same at lower M_{H} values. Actually, the available models indicate that this is not the case.

Like for P–EAGB stars, the luminosity of these objects is not sensitive to the mass $M_e \lesssim M_e^D$ of the residual hydrogen-rich envelope, and therefore envelope removal can go to completion, until the carbon-rich intershell material is eventually exposed (Renzini 1979; Iben and Renzini 1983; Iben 1984). A simple population in which all the stars lose their envelope at the first helium shell flash will therefore contain hot P–AGB stars with helium/carbon rich atmospheres, and neither hydrogen or nitrogen, the latter being rapidly burned in the convective shell produced by the flash. Conventionally, we then refer to the spectrum of these objects as WC, and indeed some 10% of PN nuclei in the solar neighborhood do show this kind of Wolf-Rayet spectrum (Heap 1983).

The fuel consumption during the P–AGB phase of these objects is just given by the amount of helium burned during the thermal pulse and the ensuing quiescent helium-burning phase, which corresponds to the mass growth of the carbon-oxygen core, i.e.,

$$F_{\mathrm{P-AGB,He}} = 0.1\,\Delta M_{\mathrm{CO}} \simeq 10^{-3}\ , \tag{20}$$

with a rather modest dependence on M_{H}. This estimate follows from various AGB models of small core mass (Gingold 1974; Iben 1982*b*; Caloi 1989), and contrary to all other P–AGB fuel consumptions, it is not subject to the uncertainties introduced by the mass loss rates during either the AGB or the P–AGB phases (see next subsection). Note that we do not impose to equations (19) and (20) to match each other at the transition from the P–EAGB to the P–AGB regime. This will indeed help identifying the transition in the figures that we shall present later.

d) *Hydrogen-burning P–AGB Stars*

We finally consider the case of hydrogen-burning post–AGB stars, namely, those objects which leave the AGB in between two thermal pulses, while the hydrogen-burning shell is fully active. These stars are certainly the most widely known candidates among those which have been considered in connection with the UV rising branch. There are two reasons for this popularity: (1) for the low-mass P–AGB stars which are pertinent to ellipticals only the Schönberner's (1983) tracks are available, and these refer only to hydrogen-burning P–AGB objects; and (2) in these models the fuel consumption is dramatically sensitive to the mass of the P–AGB stars, an attractive characteristics when dealing with the large galaxy to galaxy variations in the level of the UV rising branch.

A P–AGB star which burns hydrogen is produced every time that AGB mass-loss processes succeed in reducing the envelope mass below M_e^D while the star is in one of its quiescent phases, in between two thermal pulses (Iben 1984 and references therein). We do not precisely know how much mass M_e^R is left in the envelope when the AGB *superwind* ceases, and it has been argued that it may considerably fluctuate from one to another otherwise identical star, a result of the hydrodynamical nature of the ejection process (Renzini 1983). There exists some observational evidence supporting this notion, such as for example the case of the PN in the globular cluster M15, for which M_e^R must have been much less than M_e^D otherwise the star would have taken a much longer time to complete its transition to the observed high temperature, compared to the ~ 1000 yr demanded by its kinematical *age* (Renzini 1989*a*). Anyway, whatever its mass, this residual envelope is progressively depleted by hydrogen burning at its base, and possibly mass loss at its surface. Clearly, mass loss will reduce the available fuel, and therefore the ability of this kind of stars of producing UV photons. Yet, P–AGB mass loss rates are sufficiently uncertain (Cerruti-Sola and Perinotto 1985; Perinotto 1989) to allow every possibility, from no substantial reduction, to severe depletion. In conclusion, the current uncertainties in both the AGB and P–AGB mass-loss processes prevent any precise determination of the fuel consumption in hydrogen-burning P–AGB stars.

In any case, fuel consumptions can be estimated from the track of Paczyński (1971*b*) and Schönberner (1983), and analytical approximations to model values give, respectively,

$$F_{\mathrm{P-AGB,H}} \simeq 4 \times 10^{-5} M_{\mathrm{H}}^{-7.33}\ , \tag{21}$$

$$F_{\mathrm{P-AGB,H}} \simeq 2 \times 10^{-11} M_{\mathrm{H}}^{-30.6}\ , \tag{22}$$

where the first equation has been derived from Paczyński's 0.6 and 0.8 $M_{\odot}$ models (Renzini 1981*b*), and the second from Schönberner's 0.546 and 0.565 $M_{\odot}$ models (see RB, where, however, an incorrect coefficient was given). The striking difference between the two expressions derives entirely from the fuel consumption in the 0.546 $M_{\odot}$ model, which is much larger than one would have expected by extrapolating from larger mass models.

The strong mass dependence of $F_{\mathrm{P-AGB,H}}$ given by equation (22) soon suggested a possible way of explaining the UV-metallicity correlation (see RB), a way further explored by numerous investigators (Kjærgaard 1987; B3FL; Barbaro and Olivi 1989; Bertelli, Chiosi, and Bertola 1989). Indeed, a very modest decrease of the final mass M_{H} with metallicity would have been sufficient to produce the desired increase in P–AGB fuel consumption: the above authors estimate that a final mass $M_{\mathrm{H}} \simeq 0.53\ M_{\odot}$ would also explain the strongest UV upturns. Taking equation (22) at face value, this would give $F_{\mathrm{P-AGB,H}} \simeq 5 \times 10^{-3}\ M_{\odot}$, or $L^{\mathrm{UV}}/L_T \simeq 0.01$, thanks to equation (4), which might be a little small. If one insists for having 2% of the light in the far UV then equations (4) and (22) would demand $M_{\mathrm{H}} \simeq 0.52$. We note, however, that this result relies entirely on the applicability of equation (22) beyond the mass limits for which it has been derived.

To further investigate this latter aspect, in Figure 3 the fuel consumptions from the two sets of models are compared to each other, together with those from other sources. The sharp change in slope of $F_{\text{P-AGB,H}}(M_H)$ exhibited by Schönberner's models should suggest caution when guessing its values for M_H smaller than the minimum for which models have been actually constructed (0.546 $M_\odot$). This function must flatten out just below 0.546 $M_\odot$ (as indeed warned in RB), otherwise it would give unrealistically large fuel consumptions at low M_H values. Indeed, some additional information can now be obtained from the the evolutionary models constructed by Caloi (1989) for masses even lower than those investigated by Schönberner. These models refer to a composition $(Y, Z) = (0.20, 0.001)$, masses from 0.503 to 0.535 $M_\odot$, all with a core mass at the ZAHB $M_H^{HB} = 0.5\ M_\odot$. They were computed primarily to investigate the evolution of hot HB models, but provide adequate information also on the behavior of models which start from the ZAHB with different mass, but eventually converge to the same structure (i.e., M and M_H) while on the AGB. In Figure 3 we report the corresponding P–AGB fuel consumptions, that have been estimated from Caloi's 0.51 and 0.52 $M_\odot$ models by taking the minimum mass of hydrogen in the envelope when models finally depart from the AGB and assuming that 90% of it is burned (i.e., we have adopted $F_{\text{P-AGB,H}} = 0.8 \times 0.9 \times M_e^D$). As anticipated in RB, there is indeed a clear indication that the function $F_{\text{P-AGB,H}}(M_H)$ does actually flatten for $M_H \lesssim 0.546\ M_\odot$, compared to the extrapolation given by equation (22). In our calculations we have therefore adopted for $M_H < 0.546\ M_\odot$:

$$\log F_{\text{P-AGB,H}}(M_H) = -9.27 M_H + 2.32\,, \tag{23}$$

which has been obtained by connecting the fuel consumption of Caloi's 0.51 $M_\odot$ model to that of Schönberner's 0.546 $M_\odot$ model. Admittedly, this is a very rough recipe, which may be taken as a generous estimate.

In the straightforward application of equation (22) one important aspect of the P–AGB evolution might have been dangerously overlooked. This is the direct metallicity dependence of the minimum envelope mass with which model stars can still remain on the AGB (by definition this is M_e^D), and therefore the direct metallicity dependence of the P–AGB fuel consumption, which is a fraction of M_e^D. More formally, in general we have the functional dependence:

$$F_{\text{P-AGB,H}} = g[M_H(Z, Y), Z, Y]\,, \tag{24}$$

but only the Z-dependence via M_H has been actually considered. However, for fixed M_H the g function must decrease with Z, i.e., $(\partial g/\partial Z)_{M_H=\text{const}} < 0$, and therefore a final mass M_H decreasing with Z does not necessarily ensure that the fuel consumption will increase. The negative sign of this derivative follows from an opacity effect: as metallicity increases the envelope opacity will increase, helping maintaining red giant dimensions until a smaller M_e^D is reached. Indeed, as a rule of thumb the more opaque the envelope, the bigger its dimensions. This is clearly shown by Figure 1 in Iben and MacDonald (1986), in which two P–AGB sequences with the same mass but different Z are compared to each other: when Z is increased from 0.001 to 0.02 the fuel consumption drops from $\sim 9 \times 10^{-4}$ to $\sim 4 \times 10^{-4}\ M_\odot$. These values are also shown in Figure 3, together with the fuel consumptions in the P–RGB models of Giannone, Refsdal, and Weigert (1970), which refer to $Z = 2\ Z_\odot$. Clearly, these latter fuel consumptions are far below the extrapolations from the trend exhibited by P–AGB models with $Z \lesssim Z_\odot$.

The full composition-dependence of the P–AGB fuel consumption therefore remains to be explored, but we suspect that at metallicities several times above solar it will be very hard to get a value as large as that required to account for the UV output of the brightest galaxies, such as NGC 4649. Besides this, the possible increase with metallicity of the P–AGB mass-loss rates would reduce the UV output of these objects, by subtracting hydrogen fuel to the burning shell. In conclusion, extensive model explorations of the actual behavior of the function g in equation (24) are required to better assess the potential capability of this variety of P–AGB stars as UV producers, but the prognosis does not look encouraging.

All the hot star candidates discussed above are listed in Table 1, together with their main fuel (hydrogen or helium), the typical fuel consumption, luminosity, lifetime, and spectral type. Also listed are two double-star candidates to be discussed in § VI.

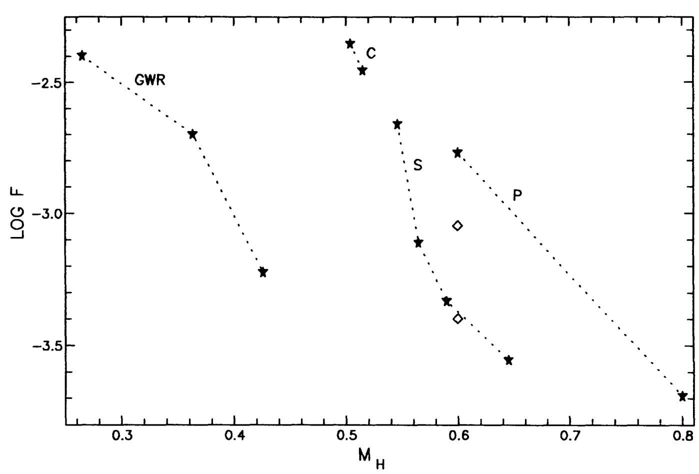

FIG. 3.—The hydrogen fuel consumption as a function of the final core mass M_H, both in $M_\odot$ units. For $M_H \leq 0.5\ M_\odot$ the fuel consumption refers to P–RGB models (Giannone, Refsdal, and Weigert 1970). For larger values of M_H it refers to P–AGB models from various studies, namely, C: Caloi (1989) for $Z = 0.001$; S: Schönberner (1983) for $Z = 0.02$; P: Paczyński (1971*b*) for $Z = 0.02$. The two diamonds at $M_H = 0.6\ M_\odot$ refer to the $Z = 0.001$ (*upper*) and $Z = 0.02$ (*lower*) models of Iben and MacDonald (1986).

V. UNIFIED SCENARIOS FOR SINGLE STAR CANDIDATES

We are now in the position to predict the total UV output of simple stellar populations of any given age and metallicity, for various values of the parameters $\Delta Y/\Delta Z$ and Z_{crit}, and for the two options concerning the envelope ejection at the end of the AGB. We first discuss the metallicity dependence for models at $t = 15$ Gyr; the age (lookback time) dependence is presented next. Finally, the case of stellar populations characterized by a metallicity distribution is briefly addressed in the last subsection.

a) Metallicity Dependence

As already discussed, different P–AGB fuel consumptions apply depending on whether the superwind envelope ejection is triggered by the first thermal pulse on the AGB (option A), or whether the star is allowed to evolve through the thermally pulsing regime before the wind removes most of the envelope (option B). We first describe this latter option. Figure 4 (*left panel*) shows some relevant values of the total mass and core mass through the AGB phase, together with the resulting $F^{UV}(Z)$ for the most favorable case $\Delta Y/\Delta Z = 3$, and for a

TABLE 1

MAIN PROPERTIES OF HOT-STAR CANDIDATES

Candidate	Fuel	F_j ($M_\odot$)	Luminosity ($L_\odot$)	Lifetime (yr)	Spectra
Single Star Candidates					
P-RGB	H	0.0006	~1000	~10^6	O-B
Hot HB	He	0.025	20	10^8	sdOB-WN
AGB-Manqué	He	0.025	10^2–10^3	10^7	O-WN
P-EAGB	He	0.003–0.025	10^2–10^3	10^7–10^6	O-WN
P-AGB (He)	He	$\lesssim$0.003	~1000	~10^6	O-WC
P-AGB (H)	H	$\lesssim$0.005:	~1000	~3×10^5	O-B
Double Star Candidates					
P-RGB	H	~0.003	10–10^3	~10^6–10^7	O-B
AWD[a]	H + He	$\lesssim$0.3	10–6×10^4	~5×10^7	O-B

[a] With a typical duty cycle of ~0.03 their effective lifetime as bright UV sources is reduced by the same factor.

straight Reimers's mass-loss rate, i.e., $Z_{\rm crit} = \infty$ in equation (11). We can notice that (1) there is a modest increase of $F^{\rm UV}(Z)$ with Z, which, however, is far less than the factor ~10 required to account for the $(1550-V)$–Mg_2 correlation, (2) for metallicities above solar ($\log Z = -1.7$) stars lose their envelope before the first thermal pulse, and then experience a P-EAGB evolution, and (3) one fails to reach fuel consumptions of ~0.01 $M_\odot$ which would be required to account for the brightest galaxies. We further note that a final mass less than 0.546 $M_\odot$ applies only to stars which evolve through a P-EAGB phase, and therefore the results are insensitive as to whether equation (22) or equation (23) is used for the P-AGB fuel consumption.

This result is partly at variance with the finding of Bertelli, Chiosi, and Bertola (1989), and an exploration of the input models used in the algorithms for the synthetic evolution reveals a likely origin for the discrepancy. Indeed, our equation (A5) gives an advancement of the hydrogen-burning shell during the HB phase which is somewhat larger than given by the stellar evolutionary models used by Bertelli *et al.* This difference is partly the results of a genuine difference in the models, partly of a different definition of the core mass $M_{\rm H}$. Following Sweigart (1987) we use for $M_{\rm H}$ the mass coordinate of the point where the hydrogen abundance is half its surface value, while Bertelli *et al.* (1986) use the point where $X = 0$. Usually it makes no difference what definition is adopted, unless the hydrogen shell becomes rather thick in mass, as unfortunately is the case in HB models. To explore the potential effect of an overestimate of the core mass on the AGB, we have run a calculation similar to the one displayed in Figure 4 (*left panel*), but having reduced $\Delta M_{\rm H}^{\rm HB}$ by 30% over the value given by equation (A5), all the rest being the same. This experiment should also illustrate the sensitivity of the results to possible small errors in the core masses (of order of ~0.01 $M_\odot$). The result is displayed in the right panel of Figure 4, having used both equations (22) and (23) for the P-AGB fuel con-

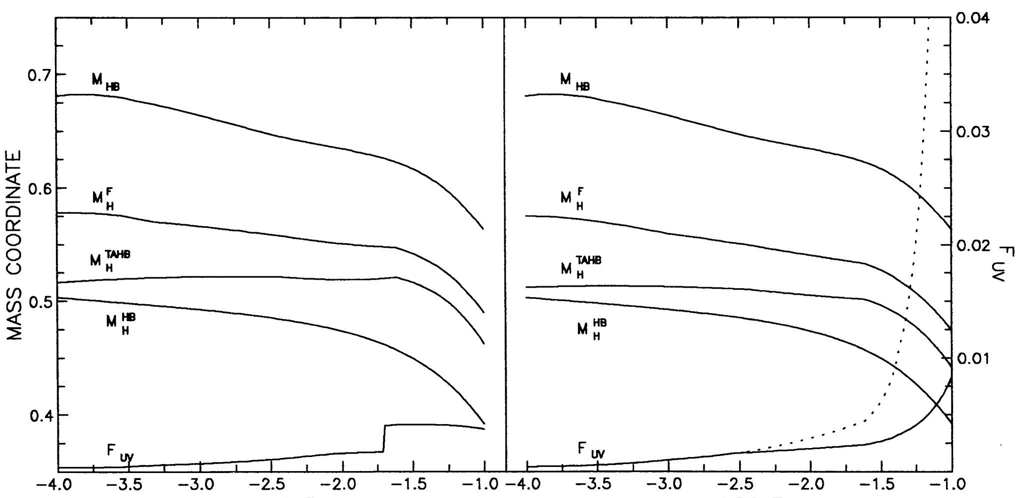

FIG. 4.—Various relevant masses as in Fig. 2 for $\Delta Y/\Delta Z = 3$, $\eta = 0.36$, and $Z_{\rm crit} = \infty$ in eq. (11), and option B concerning the AGB termination (no envelope ejection enforced at the first thermal pulse), together with the resulting fuel consumption $F_{\rm UV}$ (on the right scale). The left panel shows the results following from the synthetic HB and AGB evolution described in the text. The sharp discontinuity in $F_{\rm UV}$ marks the transition from the P-AGB to the P-EAGB regime. The right panel shows the same quantities as obtained following the same procedures apart from a 30% reduction in the amount of envelope material processed during the HB phase. Two values of $F_{\rm UV}$ are displayed, the dotted and solid lines referring to the use of eqs. (22) and (23), respectively. No transition to the P-EAGB regime now takes place.

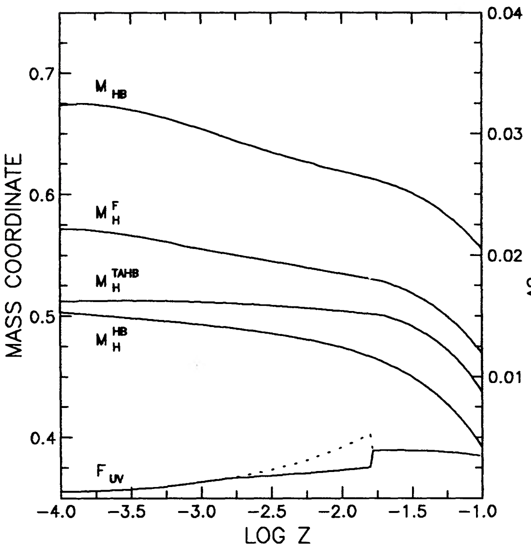

FIG. 5.—The same as the right panel in Fig. 4, apart from an increase of the mass-loss parameter η from 0.36 to 0.38. Note that stars with log $Z \gtrsim -1.7$ now experience a P–EAGB evolution.

sumptions. We clearly recover the result of Bertelli *et al.* (1989), but we do not share their optimism on the possibility for hydrogen-burning P–AGB stars to account for the UV upturns in the brightest galaxies. Indeed, it makes a great difference whether equations (22) or (23) is used for the P–AGB fuel consumption: although in both cases $F^{UV}(Z)$ increases with increasing metallicity, only equation (22) gives values able to account for the strongest upturns, while the $F^{UV}(Z)$ given by equation (23) falls short by a large factor. Yet, for the reasons discussed in § IV*d* we are inclined to regard as more algebraic than physical the solution offered by equation (22). Moreover, we rather believe that even equation (23) is more likely to give an *upper limit*, as it is based in part on Caloi's models for low-metallicity P–AGB stars. Therefore, the whole possibility for P–AGB models to account for the UV upturns and the UV-Mg_2 correlation rests on the very weak foundations represented by an extrapolation (eq. [22] from RB) that from Caloi's models we already know must be invalid at low core masses.

Besides this argument, there are additional reasons to regard with suspicion the classical P–AGB candidates, and this is because the very possibility of producing them in low-mass, high-metallicity stars is extremely fragile. Figure 5 further illustrates the case. It has been obtained with precisely the same assumptions used to draw the right panel in Figure 4, apart from the mass loss parameter η which has been increased from 0.36 to 0.38, just a 5% increase. Now all the stars with Z in excess of $\sim Z_{\odot}$ lose their envelope shortly before the first thermal pulse, and then experience a P–EAGB evolution, burning an amount of helium given by equation (19). Clearly this scenario is not able to reproduce the observed trend of the UV upturns, but emphasizes how small differences in the assumed mass loss at high metallicity can produce dramatically different results, an aspect which is further explored next using equation (11).

Allowing the mass-loss rate to slightly increase with metallicity has a rather strong impact on the scenario depicted above. Figure 6 shows the total fuel consumption in the hot

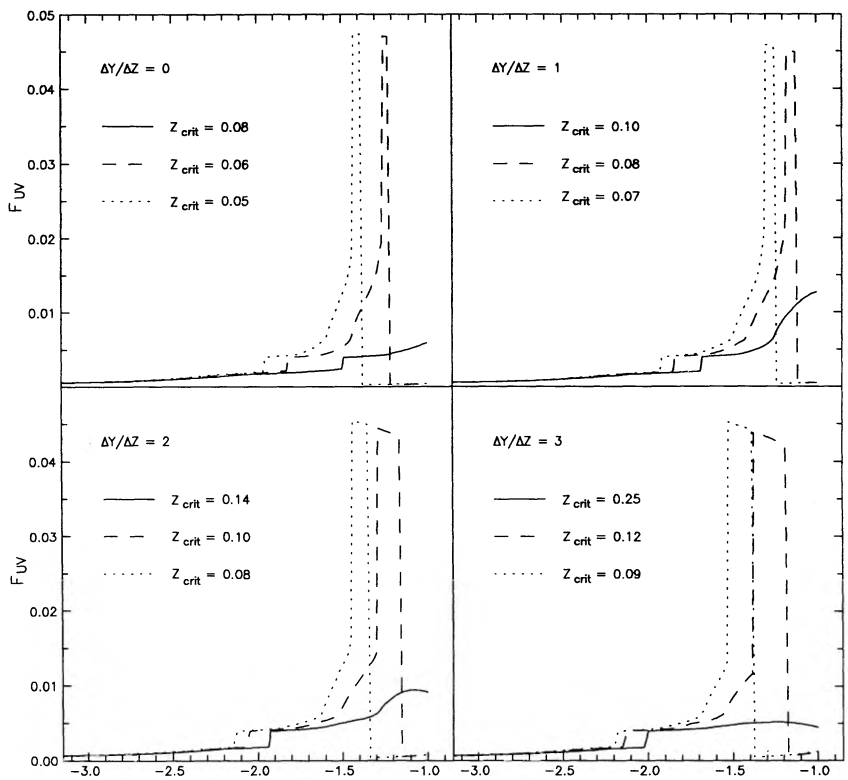

FIG. 6.—The fuel consumption in $M_{\odot}$ units as a function of metallicity Z, for the indicated combinations of the parameters $\Delta Y/\Delta Z$ and Z_{crit}, for t = 15 Gyr, η = 0.36, and option B concerning the AGB termination.

evolutionary phases $F^{UV}(Z)$, for several combinations of the parameters describing mass loss and helium enrichment, and having adopted option B for the AGB termination. The general behavior of $F^{UV}(Z)$ is fairly similar in all the various cases: at low metallicity hydrogen-burning P–AGB stars are produced, the fuel consumption is correspondingly rather small ($\lesssim 0.001\ M_\odot$) and moderately increasing with increasing metallicity. The envelope mass at the first thermal pulse decreases with increasing Z (see Fig. 2), until above a first critical value of Z it turns out that the envelope is lost prior to the first thermal pulse, and the stars experience a P–EAGB evolution. This is marked by the sharp discontinuity in $F^{UV}(Z)$ as equation (19) now replaces equation (22) or (23). For increasing Z the P–EAGB fuel consumption increases, as stars leave the AGB at a progressively earlier stage, thereby completing at high effective temperatures a larger and larger fraction of their shell helium-burning phase. At a second critical value of metallicity, mass loss during the RGB phase has reduced the envelope so much that stars populate the hot HB, later fail to reach the AGB, and rather experience an AGB–manqué phase. Correspondingly, $F_{HHB} \simeq 0.1 M_H^{HB} \simeq 0.05\ M_\odot$ replaces equation (19) for the fuel consumption, thus producing the prominent jump seen in Figure 6. Eventually, at a third critical metallicity ($Z = Z_{WD}$) the RGB mass loss has become so large that the whole envelope is lost prior to the core helium flash, and correspondingly $F^{UV}(Z)$ drops to the small value appropriate to P–RGB stars (see § III*b*).

Various trends can easily be recognized while examining Figure 6. Of course all these *transition metallicities* decrease, as the parameter Z_{crit} is decreased, i.e., when enhancing the mass-loss rate increase with Z. This is indeed equivalent to moving the whole scenario to lower and lower metallicities, by adopting a steeper and steeper Z-dependence of the mass-loss rate. Second, increasing the helium enrichment parameter $\Delta Y/\Delta Z$ facilitates the production of hot stars in two ways. (1) By reducing the initial mass (see eq. [A1]), less mass needs to be lost in order to produce, e.g., P–EAGB or hot HB stars (and a larger Z_{crit} becomes sufficient), and (2) the range of Z's in which hot HB stars are produced becomes wider, therefore encompassing a larger and larger fraction of a galaxy's population. Figure 6 clearly shows the leading rôle played in this scenario by P–EAGB and especially hot HB stars. Indeed, only this kind of hot star appears to burn enough fuel at high temperature so as to provide an adequate supply of UV photons. Another crucial aspect is worth appreciating: we can note from Figure 6 that if one insists on producing P–EAGB stars, almost inevitably hot HB stars are produced as well, at a somewhat larger metallicity. Similarly, when for a certain combination of the parameters hot HB stars are produced at high metallicity, inevitably P–EAGB stars are also produced at a somewhat lower metallicity. We conclude that P–EAGB and hot HB stars tend to come together, with a tendency for the latter ones to dominate the UV output thanks to their larger fuel consumption (see Fig. 6). Finally, we wish to emphasize with particular strength that large fuel consumptions (the hot HB peaks) tend to be confined to a fairly narrow range of Z. Rather than to a generic average metallicity, the UV output of a galaxy will therefore be most sensitive to the actual population of stars in such crucial metallicity range. Again, this emphasizes the crucial rôle of the detailed metallicity distribution, an aspect further explored in § V*c* below.

The above picture is not significantly altered when option A is adopted for the AGB termination, i.e., assuming that low-mass stars depart from the AGB at the very first helium shell flash. In this case the P–AGB fuel consumption is given by equation (20). The result is shown in Figure 7 and is fairly similar to the corresponding cases shown in Figure 6. At low Z now $F^{UV}(Z)$ is practically flat, as the amount of helium burned during the first (and last) thermal pulse and thereafter is much less sensitive to the core mass, compared to the amount of hydrogen fuel consumed by hydrogen-burning P–AGB stars. At larger metallicities P–EAGB and hot HB stars are produced, and the A/B alternative is not activated any more. We conclude that in the frame of mass-loss enhanced models it does not make much difference for the UV output of ellipticals whether hydrogen- or helium-burning P–AGB stars are produced.

b) *Age/Lookback Dependence*

All the results presented in the previous subsection refer to an age of 15 Gyr, and we recall that the mass-loss rate parameter η in equation (11) has been calibrated in order to have evenly populated HBs for $Z = 10^{-3}$ globular clusters of this age. Now, keeping fixed $\eta = 0.36$, we explore what happens when the age is progressively reduced from 15 down to 11 Gyr, so as to elucidate the predictions of the model concerning the evolution of the UV upturn with lookback time, i.e., redshift. Very roughly, 1 Gyr in lookback time corresponds to $\Delta z \simeq 0.1$, the precise value depending on the adopted cosmological model.

As the age is decreased, the initial mass of currently evolving stars increases according to equation (A1), and correspondingly so does M_{HB}, the mass of stars evolving along the AGB, and eventually the mass of the remnant P–AGB stars. This complex behavior is illustrated in Figure 8, which clarifies how the production of P–RGB, hot HB, and P–EAGB stars is progressively shifted to higher and higher metallicities, until eventually discontinued. Figure 9 explicitly shows how the fuel consumptions at the various metallicities is affected when the age is progressively reduced from 15 to 10 Gyr. The contributions of the various types of hot stars can easily be identified thanks to the similarity with Figure 6. One sees that a reduction of just 1 Gyr already makes a sensible difference (see also next subsection), and it is therefore worth emphasizing one distinctive characteristics of the hot HB and P–EAGB candidates (see RB): their contribution to the UV rising branch of elliptical galaxies should rapidly decrease with increasing redshift, with sizable effects being already detectable at fairly low redshifts ($z = 0.1$–0.2). This result is fairly insensitive to the assumed common age of 15 Gyr for galactic globulars and ellipticals, being instead essentially related to the slope of the relation between turnoff mass and age. Had we assumed a younger age, a slightly larger value of η would have been required to reproduce the HB morphology of today globular clusters, but leading to the same trend of $F^{UV}(Z)$ with redshift.

c) *Composite Populations with a Metallicity Dispersion*

We now turn to consider a realistic population characterized by a certain metallicity dispersion (see § II*b*). Merely for illustration purposes, we assume the metallicity distribution $\Phi(Z)$ in a galaxy to be described by a Gaussian with width $\sigma(\log Z) = 0.5$, peaked at $Z = Z_0$, and with a cutoff at $Z = 0.1$. The fuel consumptions $F^{UV}(Z)$ is then convolved with this $\Phi(Z)$ to get the metallicity-averaged fuel consumption $\langle F^{UV}\rangle_Z$ (see § II*d*). For the reference age $t = 15$ Gyr the corresponding results are shown in Figure 10 as functions of the actual

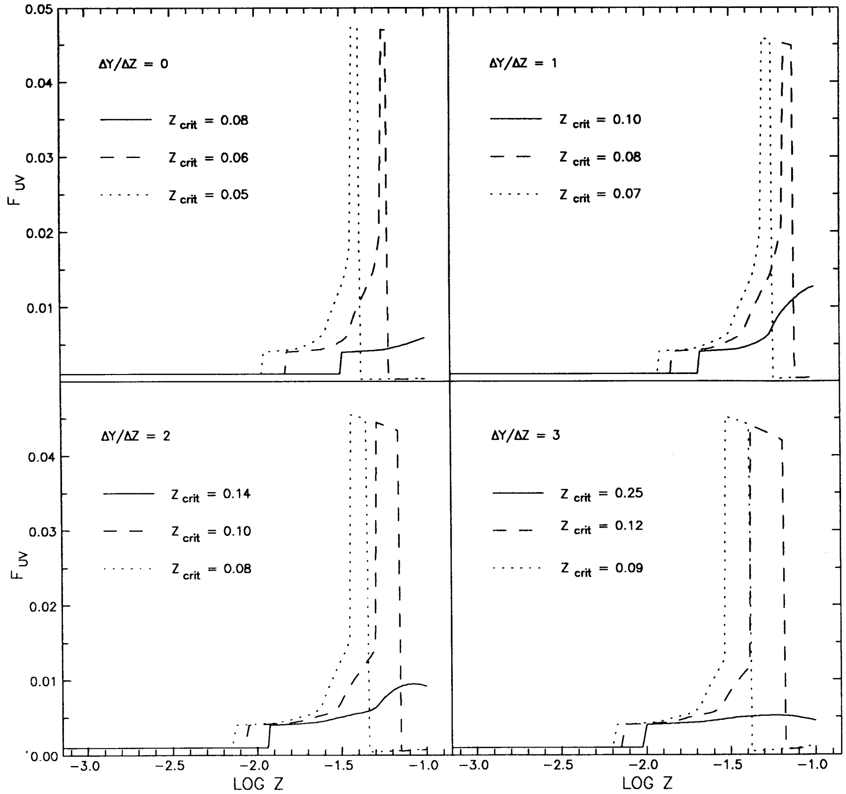

FIG. 7.—The same as Fig. 6 but with option B being now replaced by option A concerning the termination of the AGB, i.e., the final envelope ejection is supposed to be triggered by the very first thermal pulse.

average metallicity (note that the distribution of log Z is asymmetric because of the imposed cutoff, and $\langle Z \rangle$ may tend to saturate at high Z_0). Two main aspects are clearly apparent from Figure 10: The larger the parameter $\Delta Y/\Delta Z$ the larger the slope of the $\langle F^{\mathrm{UV}} \rangle_Z$–$\langle \log Z \rangle$ relation, and therefore the easier to produce the desired trend of $(1550-V)$ versus Mg_2; and for each value of $\Delta Y/\Delta Z$ there is an optimal value of Z_{crit} which gives the largest span in $\langle F^{\mathrm{UV}} \rangle_Z$. This nonmonotonic behavior

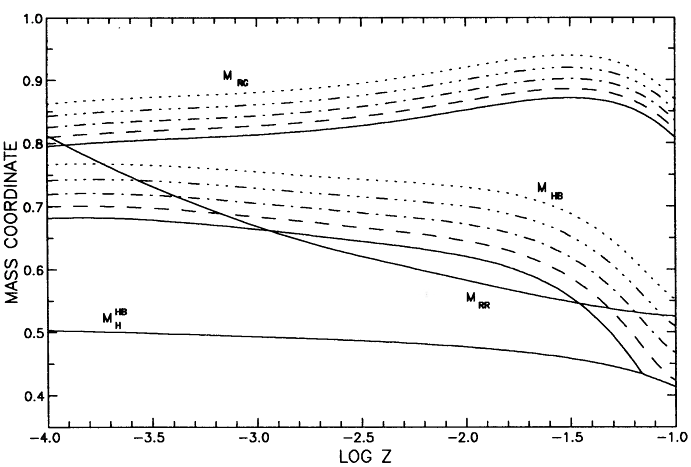

FIG. 8.—Several mass values as in Fig. 1 are displayed vs. log Z for $\eta = 0.36$, $\Delta Y/\Delta Z = 2$, and $Z_{\mathrm{crit}} = 0.10$. While $M_{\mathrm{H}}^{\mathrm{HB}}$ and M_{RR} are virtually insensitive to age, M_{RG} and M_{HB} decrease with increasing age: the solid, dashed, dot-dashed, three dot-dashed, and dotted lines refer, respectively, to t = 15, 14, 13, 12, and 11 Gyr.

is a consequence of the fact that for lower values of Z_{crit} an increasing fraction of the stars experience a P–RGB phase which produces very little ultraviolet radiation, and therefore the average fuel consumption eventually decreases.

The effect of reducing the age of the population is illustrated in Figure 11, analogous to Figure 10, but obtained by convolving the fuel consumptions at the various ages (from Fig. 9) with the adopted metallicity distribution. The model predicts the disappearance of a strong UV-metallicity correlation already at redshifts of a few tenths, and the effect would have been even more pronounced had we truncated $\Phi(Z)$ at a lower cutoff metallicity. We shall briefly return to this point in § VII*d*.

VI. BINARY CANDIDATES

At least in the solar neighborhood a sizable fraction of stars are binaries, and a sizable fraction of them are close enough to allow Roche-lobe overflow(s) during the evolution of the components. It may therefore be rather risky to ignore binaries in population synthesis studies. The evolution of single stars is controlled by two primary parameters, the mass and metallicity (M, Z). The evolution of binary systems is controlled also by two additional parameters, i.e., the initial mass ratio and separation of the two components $(M_2/M_1, A)$, bringing then to four the number of parameters. Correspondingly, as the evolution of a population of single stars is controlled by the distribution function $\psi(M, Z)$, that of a population including some fraction of binary stars is controlled also by the distribution function $\tilde{\psi}(M_1, M_2, A, Z)$. The evolution of single stars presents several complications and uncertainties, and these are obviously greatly amplified in the case of binaries. Moreover, if

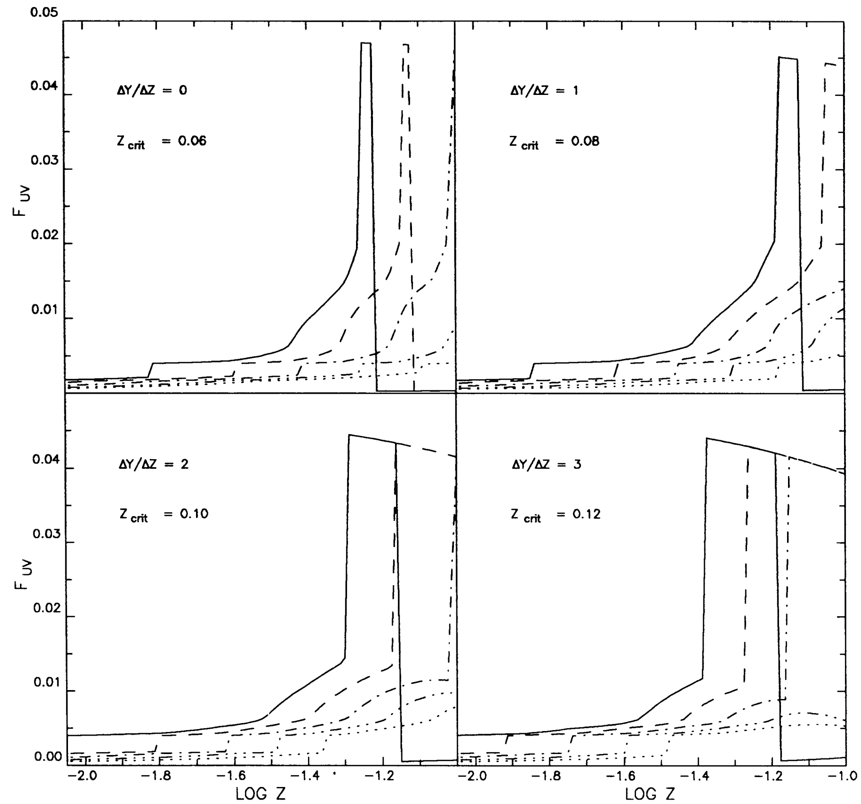

FIG. 9.—The fuel consumptions as in Fig. 6 for selected combinations of the various parameters (as indicated), and for various ages. The various line types refer to the same ages as in Fig. 8.

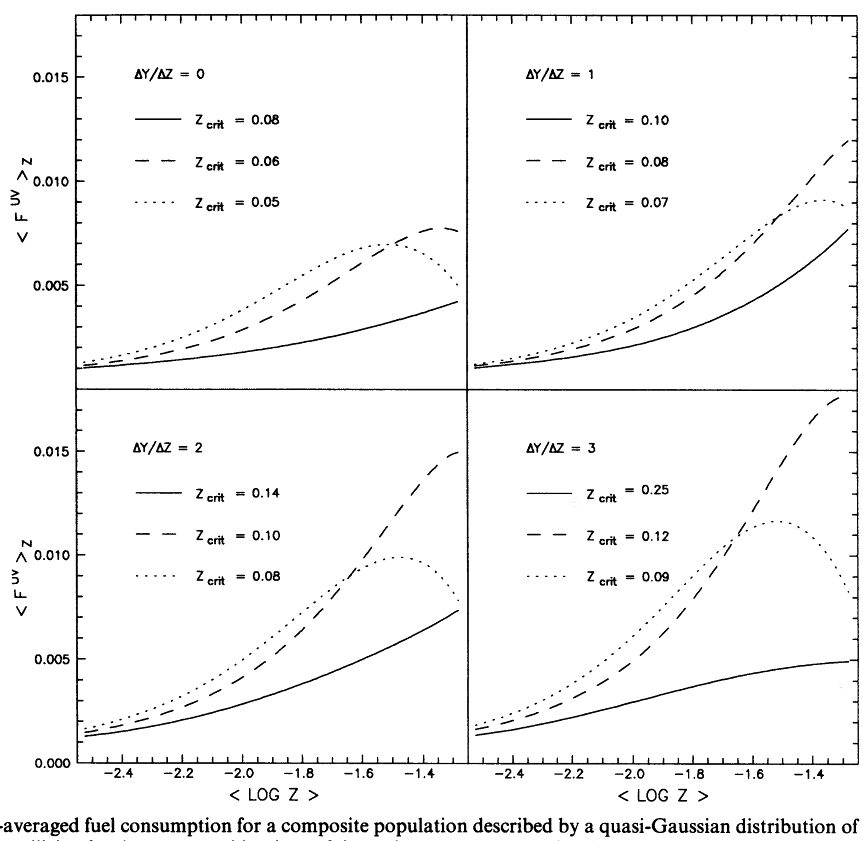

FIG. 10.—The metallicity-averaged fuel consumption for a composite population described by a quasi-Gaussian distribution of metallicities (see text) is displayed as a function of the average metallicity, for the same combinations of the various parameters as in Fig. 6.

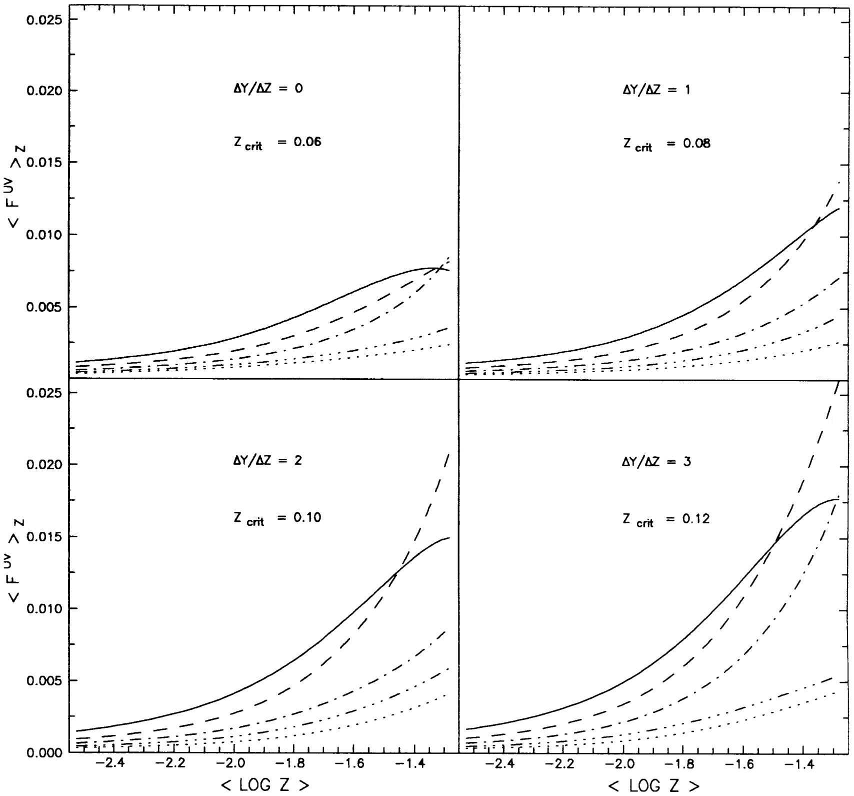

FIG. 11.—The metallicity-averaged fuel consumption as a function of metallicity for different ages and for various combinations of the other parameters. The various line types refer to the same ages as in Fig. 8.

we know little about ψ, we know even less about $\tilde{\psi}$. For example, we still do not know whether the initial mass function (IMF) of single stars is sensitive to metallicity, but we have even less insight as to whether, e.g., the distribution of binary separations is different in different star-forming environments (Zinnecker 1984). It is worth starting from these pessimistic considerations when dealing with populations of binaries, so as to emphasize the very preliminary nature of the following considerations.

Rather than attempting a comprehensive examination of all the possible hot stars that can be produced by binary evolution, we shall here concentrate on a few cases, that we consider more likely to be of potential interest in the context of the UV upturns. Correspondingly, we shall restrict ourselves to the case of low-mass binaries, in which at least the secondary component has a mass lower than that of currently evolving single stars, i.e., $M_2 \leq M_{RG}$. In principle, equations (1) and (5) can be generalized to include arbitrary populations of binaries. This may be an elegant, yet academic exercise because of the limitations introduced by our poor knowledge of $\tilde{\psi}$. Therefore, we will treat interacting binaries as a trace population, whose present death rate is proportional to that of single stars, i.e., to $B(t)L_T$, with $t = 15$ Gyr. The estimate of the proportionality coefficient is then left to empirical arguments. In the following we adopt the classification into case A, B, and C Roche-lobe contacts as those which take place when the Roche-lobe filling star is, respectively, on the main sequence, in the shell hydrogen-burning stage, or in the double-shell (AGB) stage (Kippenhahn and Weigert 1967; Lauteborn 1970).

Roche-lobe overflows can dramatically affect the fuel consumptions in the various stages of evolution, even suppressing some of them and allowing others which are specific of binaries (Renzini 1981*b*). Besides this, from an energetic point of view, it makes a great difference if, e.g., rather than having one 2 $M_\odot$ star one has a wide pair of two $M_\odot$ stars. In the former case a $\sim$0.65 $M_\odot$ WD is left, in the latter two $\sim$0.55 $M_\odot$ WDs are left. The amount of consumed fuel is about twice in the latter case, but the corresponding energy release is diluted and delayed over a time scale about a factor of 10 longer ($\sim 10^{10}$ yr rather than $\sim 10^9$ yr). So it can make a rather great difference for the spectral evolution of a galaxy to have many wide pairs, rather than the same number of single stars with the same mass of the binary. If the binaries interact, then much hydrogen-rich material can be lost during common-envelope stages (Livio 1989 and references therein), thus subtracting fuel to the burning shells. Therefore, the distribution of the mass ratios and separations plays a very important rôle in determining the spectral evolution of the binary component of a galaxy. Clearly, one crucial parameter will thus be the binary frequency, that by properly tuning the normalization coefficients we can choose to be the integral of $\tilde{\psi}$ as the integral of ψ will correspondingly be the fraction of single stars.

We are primarily interested in searching for possible ways of generating the UV-metallicity correlation, and therefore it is appropriate to ask how binaries could contribute in establishing this correlation. At the present stage of our knowldege of binary populations in different environments it would be completely ad hoc to appeal for any particular trend of $\tilde{\psi}$ with Z which would favor the formation of UV-powerful binaries. There is however one binary-metallicity connection which follows naturally from stellar evolution theory, even if $\tilde{\psi}$ is independent of Z. This is the well known fact that the size of stars increases with metallicity, and therefore the chance for a binary component to fill its Roche-lobe is larger, the larger the metallicity. For example, the derivatives of the radius at the RGB tip with respect to Z and Y are given by (e.g., Sweigart, Greggio, and Renzini 1989):

$$\left(\frac{\partial \log R}{\partial \log Z}\right)_Y \simeq 0.18\ , \quad \left(\frac{\partial \log R}{\partial Y}\right)_Z \simeq -0.45\ . \qquad (25)$$

It is then legitimate to expect the frequency of interacting binaries to increase with increasing metallicity. Assuming for the distribution function of binary separations a law of the type $dN \simeq 0.2 d \log A$ (Iben and Tutukov 1984*a*), we estimate that the number of case B interacting binaries will increase by perhaps as much as 20% per dex of Z.

a) Post–RGB Components

In an old stellar population P–RGB stars are produced by a type B Roche-lobe contact of either the primary or the secondary component. For a wide range of binary parameters, most of the envelope of the lobe-filling object is lost during the ensuing common-envelope stage of the system, until the star leaves the RGB to become a helium WD. Thereafter, as in the case of single P–RGB stars (see § III*b*), most of the residual hydrogen-rich envelope is burned during the bright part of this P–RGB phase. As shown in Figure 3 the fuel consumptions of binary P–RGB stars is rather modest, comparable to that of single P–AGB stars, which however are much more abundant. This should ensure a negligible contribution of binary P–RGB stars to the UV upturns.

In any event, in order to assess the contribution of binary P–RGB stars one needs an estimate of the frequency of case B binaries. In a random sample of 54 DA white dwarfs Bragaglia *et al.* (1988, 1990) have found three close binaries in which at least one of the components had to experience a P–RGB phase, and four other objects may also be close helium WD pairs. This may indicate that at least $\sim 6\%$ of currently forming WDs become such through a case B binary evolution, i.e., their death rate in a galaxy could be $\sim 0.06 B(t) L_T$, and their contribution to the total light:

$$\left(\frac{L_{\rm P-RGB}}{L_T}\right)_{\rm binary} \simeq 0.06 \times 9.75 \times 10^{10} B(t) \langle F_{\rm P-RGB} \rangle \lesssim 0.0004\ , \qquad (26)$$

having assumed an upper limit of $\sim 0.003\ M_\odot$ for the fuel consumption.[4] It seems that this kind of binaries can at most contribute of the order of 10%–20% of the observed UV in the faintest galaxies, i.e., a small but not entirely negligible contribution.

b) Accreting White Dwarfs

A potentially very important source of UV radiation may be provided by WDs which accrete *and* burn hydrogen fuel from a nondegenerate companion (Renzini 1981*b*; Nesci and Perola 1985; RB). The fuel accumulating on top of the WD can burn either in a series of quasi-static flashes, or in more violent thermonuclear runaways depending on the accretion rate and on the mass of the accreting white dwarfs (AWD) (e.g., Nomoto 1982; Iben 1982*a*). In the latter case nova outburst are produced, but novae waste at least 90% of the fuel they are given, ejecting it largely unburned (e.g., Prialnik 1982). We defer to § VII*c* a discussion on the possible role of novae, that we will develop on mostly empirical grounds. On the other hand, if the mass transfer rate is high enough to avoid the outbursts the quasi-static burning of potentially large amounts of fuel is ensured. Theoretical evolutionary sequences for this kind of AWDs have been constructed by Iben (1982*a*), and show that most of the accreted fuel is burned while the star is at very high effective temperatures ($T_e \gtrsim 25{,}000$ K), and therefore these systems can be very powerful UV emitters. More difficult is to have reliable indications about their frequency and their contribution to the UV light of old stellar populations.

It is worth emphasizing here that although these systems may have an accretion disk (and therefore some accretion luminosity), what really matters from an energetic point of view is the nuclear burning of the accreted fuel. In fact, the gravitational energy released by dropping 1 g of hydrogen on top of a WD is at most a few percent of the energy that can be obtained by burning it. We will correspondingly neglect any accretion luminosity.

Binary systems in which a WD component accretes and burns hydrogen fuel have been considered as possible precursors of supernovae of type I (Whelan and Iben 1973), as a SN explosion can be produced once the mass of the growing WD exceeds the Chandrasekhar limit ($\sim 1.4\ M_\odot$). In this scenario, the energy output of AWDs which hypothetically would be SNI precursors is given by (Greggio and Renzini 1983):

$$L_{\rm SNIP} \simeq \langle M_{\rm H}^{\rm acc} \rangle X_e E_{\rm H} R_{\rm SNI}\ , \qquad (27)$$

where $\langle M_{\rm H}^{\rm acc} \rangle$ is the average mass which is accreted and burned on top of each WD before the SNI explosion, X_e is the hydrogen abundance of the accreted material, $E_{\rm H}(= 6.6 \times 10^{18}$ ergs) is the energy released by the conversion of 1 g of hydrogen into carbon and oxygen, and $R_{\rm SNI}$ is the SNI rate. The SN rate is usually expressed in SN units (1 SNU = 1 SN every 100 yr per $10^{10}\ L_\odot$ of *blue* luminosity of the parent galaxy), and using equation (10) for the blue-to-bolometric conversion equation (27) eventually becomes

$$\frac{L_{\rm SNIP}}{L_T} \simeq 0.046 \langle M_{\rm H}^{\rm acc} \rangle R_{\rm SNI}\ , \qquad (28)$$

where now the rate is expressed in SNUs, and the accreted mass is in $M_\odot$ units. Adopting 0.2 SNU for the rate in ellipticals (Tammann 1982) and $\langle M_{\rm H}^{\rm acc} \rangle = 0.2\ M_\odot$, equation (28) indicates that some 0.2% of the total bolometric light of an elliptical may come from the progenitors of type I SNs, if these are binary systems in which one WD component accretes and burns fresh fuel from a companion. This is about a factor of 10 less than in most powerful UV ellipticals, i.e., a small, yet not completely negligible contribution. However, for every system which manages to produce a SNI event, many others may exist which fail to conclude their career with such a major display, as, e.g., the fuel reservoir is exhausted before the Chandrasekhar limit is reached.

To explore this possibility we need to look closer to the evolution of AWDs. In order to avoid nova outbursts, and quasi-statically burn the accreted hydrogen the accretion rate must exceed some $10^{-9}\ M_\odot\ {\rm yr}^{-1}$ (e.g., Nomoto 1982; Iben 1982*a*), and indeed low-mass red giant companions with a core mass in excess of $\sim 0.2\ M_\odot$ can sustain such transfer rates

[4] We only caution that it may be somewhat inaccurate to use for such low-mass objects ($M \lesssim M_\odot$) the same P–RGB fuel consumptions which have been derived for stars initially considerably more massive, such as in the case of the models of Giannone, Refsdal, and Weigert (1970) and Iben and Tutukov (1986).

(Taam 1983). However, the binary system must fulfill additional constraints in order to ensure a stable mass transfer, avoiding the formation of a common envelope which would just eject unburned all the available fuel. This condition is

$$M_2 \leq a_{\rm DS} M_{\rm WD} , \qquad (29)$$

where the coefficient $a_{\rm DS}$ has been variably estimated from ~0.6–0.65 (Iben and Tutukov 1984*a*; Pastetter and Ritter 1989) up to ~0.8 (Webbink, Rappaport, and Savonije 1983). We adopt here $a_{\rm DS} = 0.7$, and $M_2 = M_{\rm RG}$ from equation (A1) for the initial mass of the Roche-lobe filling secondary. Therefore, for any given age of the population, the minimum mass of WDs which are able to accrete from a red giant of initial mass $M_{\rm RG}$ is given by

$$M_{\rm WD}^{\rm min} = M_{\rm RG}/a_{\rm DS} \simeq 1.43 \times M_{\rm RG} . \qquad (30)$$

For example, in a ~15 Gyr old population $M_{\rm RG} \simeq 0.8\ M_\odot$ and therefore equation (30) indicates $M_{\rm WD}^{\rm min} \simeq 1.14\ M_\odot$, i.e., only rather massive WD binary components are able to accrete. It is not at all clear whether such massive WDs are actually produced in the evolution of single stars (see Iben 1985); they may be produced, however, in binary systems in which the more massive component fills its Roche-lobe during the E–AGB phase, prior to the reduction in the core size operated by the so-called second dredge-up (Becker and Iben 1979). Having determined $M_{\rm WD}^{\rm min}$, there remains to estimate in which range of the initial mass of the primary $[M_1^{\rm min}(Y, Z, t) - M_1^{\rm max}(Y, Z, t)]$ WDs with mass exceeding $M_{\rm WD}^{\rm min}$ are produced. The lower limit can easily be obtained by solving the equation $\alpha + \beta M_1^{\rm min}(Y, Z, t) = M_{\rm WD}^{\rm min}$, with α and β given by equation (4) and (5) in Becker and Iben (1979). For the upper limit we can adopt $M_1^{\rm max}(Y, Z, t) = M_{\rm UP}$, the upper mass for the production of degenerate carbon-oxygen cores, as given by equation (7) in Becker and Iben (1979). The result is shown in Figure 12 for various choices of the helium enrichment parameter $\Delta Y/\Delta Z$. It appears that the mass range of useful primaries both shrinks and moves to higher masses with increasing Z, an unattractive property in the context of the UV-metallicity correlation. Nor the larger chance of producing case C contacts at high metallicity seems

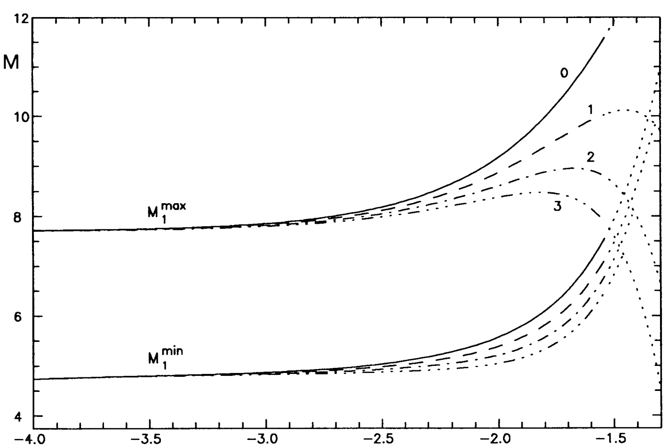

FIG. 12.—The minimum and maximum initial mass (in $M_\odot$ units) of the primary binary components which can evolve into white dwarfs able to stably accrete and burn hydrogen fuel from a red giant companion at t = 15 Gyr, i.e., a binary companion fulfilling eq. (29). The various line types are labeled by the value of the helium enrichment parameter $\Delta Y/\Delta Z$.

able of balancing this trend, unless the extrapolations beyond $Z = 0.03$ shown in Figure 12 are seriously in error.

In any event, these AWDs burn the accreted hydrogen and helium fuels in a series of flashes, characterized by recurrence periods, on/off duty cycles, and luminosity amplitudes which depend on both $M_{\rm WD}$ and $\dot{M}_{\rm WD}$ (Iben 1982*a*). For the case of interest in the present context ($M_{\rm WD} \simeq M_\odot$, $\dot{M}_{\rm WD} \simeq 10^{-8}\ M_\odot$ yr^{-1}; cf. Webbink, Rappaport, Savonije 1983), one typically has one flash every ~500 yr, with an ON phase of ~15 yr (duty cycle = 3%). Once started, the accretion process can be halted in three possible ways:

1. The WD reaches the Chandrasekhar limit and undergoes a SNI explosion. The corresponding AWD fuel consumption is $F_{\rm AWD} = X_e(1.4 - M_{\rm WD}^{\rm min}) \simeq 0.2\ M_\odot$.
2. The donor exhausts its envelope, and becomes a helium WD.
3. The donor experiences its own core helium flash, and then retreats inside its Roche-lobe to quietly spend its core helium-burning stage. If $M_e(L)$ is the envelope mass when the contact is first established, then in the latter two cases $F_{\rm AWD} = X_e \times f \times M_e(L)$, with $f \approx 1$ in case 2, and $f < 1$ in case 3. We then assume:

$$\langle F_{\rm AWD} \rangle \simeq 0.1 . \qquad (31)$$

This is not the whole story, however, as in case 2 we have to allow for the P–RGB fuel consumption of the donor, and a fraction of case 3 systems will experience a hot HB + AGB–manqué evolutionary phases, which are characterized by a fuel consumption of ~0.5 $M_\odot$. All in all, we can conclude that equation (31) may underestimate the total fuel consumption of the hot phases of these binaries by perhaps as much as 50%.

Now, in order to relate the fuel burned by each AWD to the UV output of elliptical galaxies we need an estimate of the actual frequency in nature of systems with these characteristics. This has been for years a matter of inconclusive discussions, as several selection effects may conjure against their discovery. Iben (1982*a*) lists a few galactic objects which may represent real counterparts for these theoretically produced AWDs, but a conclusive word can most likely come from HST observations of the bulge of M31. The rise time of these objects to their ON phase is only a few months, and UV imagery repeated a few months apart should easily identify them, if present, thanks to their large luminosity variations. We shall return to AWDs in M31 in § VII*c*.

Before concluding this section, it is worth mentioning yet two other possibilities which might have great importance in connection with UV upturns and type I supernovae. Until now we have conservatively assumed that when condition (29) is violated a common envelope is formed, which leads to the virtually complete ejection of any hydrogen fuel from the system. By no means this is a proved assumption. Other possibilities exist, such as the formation of polar jets about the super-Eddington AWD (F. Meyer 1989, private communication), which would result in a rapid decrease in the mass of the donor red giant, without much angular momentum losses. In this way, once the mass of the donor decreases below the critical value given by equation (29) a dynamically stable, conservative mass transfer could be established. A second possibility is that although condition (29) could be initially violated, severe mass loss prior to the Roche-lobe contact could reduce enough the mass of the secondary star for a stable mass transfer to be later established, once contact is eventually realized. This extraordinary mass loss could be the result of an exceptionally high *chromospheric* activity induced by the forced co-rotation of the secondary, once it swallows to red giant dimensions (A. V. Tutukov 1989, private communication). Admittedly both possibilities are at the moment rather speculative, but in either case the volume in the binary parameter space (M_1, M_2, A) leading to stable AWDs would enormously inflate over that previously explored in this section, as would the contribution of AWDs to the UV rising branch of elliptical galaxies! Again, the search of UV bright objects in the bulge of M31 may help answering some still open questions about the evolution of binary systems. The main properties of binary candidates are reported in Table 1, together with those of single star candidates.

VII. DISCUSSION

In this section we discuss the observational strategies which can eventually lead to the unambiguous identification of the nature of the hot stars inhabiting elliptical galaxies. Some kinds of objects are certainly there (e.g., P–AGB stars, P–RGB binary members, AWDs, etc.), and in these cases we only need to assess the size of their contribution to the UV rising branch. But in other, more hypothetical cases (such as hot HB and P–EAGB stars) it is their mere existence which has to be proven first. In the previous sections we have often emphasized the critical role played by red giant winds in possibly producing some of these stellar types. Having recognized its importance, a first question naturally comes to mind: is it possible to empirically determine the direct metallicity dependence of the mass-loss rate? I.e., in the frame of the adopted (arbitrary) parameterization of $\dot{M}(Z)$: is an empirical determination of $Z_{\rm crit}$ practically feasible? We believe that the answer to these questions is sharply negative. To be of any interest in the present context, mass loss rate determinations (of super metal-rich RGB stars) should reach at least a 10%–20% accuracy, while even in the best cases current estimates are not any better than a factor of a few. Moreover, most mass-loss rate estimates refer to bright AGB stars (with rates of $\sim 10^{-7}$ $M_\odot$ yr^{-1} or greater) while no direct determination even exists for the RGB stars which are those of interest in the present context (see Dupree 1986). Therefore, the only alternative is to search for individual hot stars in various environments, and then carefully project their properties to the case of elliptical galaxies. Such environments include globular clusters, M32, the bulge of the Milky Way, and that of M31, and are discussed in the next subsections. The remaining part of this section is then dedicated to a discussion of the expected evolution with lookback time of the UV contribution of various hot candidates, and to a brief comment on the possible connection of the UV upturn with other properties of ellipticals, with particular reference to their X-ray luminosity.

a) Globular Clusters

Galactic globular clusters for which there exists UV information cover a range of [Fe/H] which is of very limited interest in the context of elliptical galaxies. Still, these objects represent the best-studied stellar populations, and it is therefore important to check the evolutionary models in at least those cases in which this is feasible in the greatest possible detail. We shall not address here the case of binary UV producers, because globular clusters may have a quite different population of binaries compared to ellipticals, due to the dynamical processes which can efficiently create and/or destroy binaries in the dense core of the clusters. In practice, we discuss here only the case of P–AGB stars. For $t_{\rm P-AGB}$ we adopt the *fading time*, defined as the time it takes to a P–AGB star to fade from its plateau luminosity and $T_e \simeq 30{,}000$ K to a luminosity a factor of 10 lower (Renzini 1983; Iben and Renzini 1983). It is well known that the fading time is extremely sensitive to the mass of P–AGB stars, and with this proviso in mind we can estimate how many P–AGB stars one expects to observe. If one adopts Schönberner's (1983) 0.546 $M_\odot$ track as representative of the P–AGB evolution in globulars, then with $t_{\rm P-AGB} = 3 \times 10^5$ yr, equation (5) indicates that ~ 60 bright P–AGB stars should be recovered by fully analyzing the 20 brightest globulars, globally totaling $\sim 10^7$ $L_\odot$. On the same basis, one can estimate these clusters to contain ~ 100 P–AGB stars brighter than ~ 100 $L_\odot$ (Renzini 1985). Actually, only eight confirmed hot P–AGB stars are known within the whole galactic globular cluster family (de Boer 1985), a discrepancy which most likely stems from strong observational selection. Indeed, these are all fairly *cool* objects ($T_e \lesssim 35{,}000$ K, while P–AGB stars extend up to $\sim 100{,}000$ K or more) found in optical surveys, which are likely to have missed bolometrically bright but optically faint stars (de Boer 1985). Unfortunately, very few globulars have so far been imaged in the UV (for one exception see Bohlin *et al.* 1983), a limitation that the Ultraviolet Imaging Telescope (UIT; see Stecher *et al.* 1978) on board of the Astro-1 mission can help eliminate, provided that a sufficient number of rich clusters is observed. Moreover, the spectroscopic follow-up of the cluster hot P–AGB stars will provide a decisive way for distinguishing among the three possible modes of envelope ejection sketched in § IV. Indeed, stars with WN–like, WC–like, and O-type spectra are expected to be produced, respectively, in the case of P–EAGB, helium-burning P–AGB, and hydrogen-burning P–AGB stars (see Table 1).

The relative contribution $L_{\rm P-AGB}/L_T$ in globular clusters has been estimated by de Boer (1985) and Renzini and Fusi Pecci (1988); for pioneering similar studies see Minkowski and Osterbrock (1959) and Hills (1971). De Boer estimates this contribution to be ~ 0.005, a value coming from the ratio of the bolometric luminosity of the eight studied P–AGB stars to the total luminosity of the seven parent clusters (see his Table 3). This can give only a rough estimate of the contribution, on the one hand because of the noted probable incompleteness of the sample, on the other hand because for L_T one should have taken the total luminosity of all the surveyed clusters, rather than that of only those seven clusters in which P–AGB stars have been found. So, both the numerator and the denominator in the ratio are likely to have been underestimated by perhaps a large factor. Renzini and Fusi Pecci concentrate on the cluster M3, for which there exists extensive stellar photometry. There is one confirmed P–AGB star (vZ1128) with $L = 1260$ $L_\odot$ within a sample of cluster stars totaling $\sim 80{,}000$ $L_\odot$, and formally the ratio of these numbers gives 0.016. There are, however, in the sample two other hot stars (with $V \simeq 18$) which might also be in the P–AGB phase. If they are bolometrically as bright as the confirmed P–AGB star, then the three objects would altogether contribute $3780/80{,}000 \simeq 4.7\%$ of the total luminosity of the sample, which seems too much. On the basis of their $U-V$ color and V magnitude Hills (1971) estimated ~ 500 $L_\odot$ for the combined luminosity of these two objects, which would bring the total P–AGB contribution to $1760/80{,}000 \simeq 2\%$. Clearly, these attempts are totally dominated by small number statistics and may be more useful if replicated on a larger scale. Note that this 2% estimate is likely to be an upper limit, as vZ1128 is the only bright P–AGB star in a sample totaling roughly one-third of the cluster light, yet it contributes ∼25% of the whole cluster light at 1550 Å (de Boer 1985), while the blue HB stars must also give a sizable contribution.

In conclusion, the study of the P–AGB evolution in galactic globulars is a very promising field of investigation (see Renzini 1985 for an extensive discussion), but at the moment the meager data available do not provide useful constraints on the nature of hot stars in ellipticals. It certainly shows, however, that a P–AGB contribution of the order of ∼1% is not at all inconceivable. After all, this is not far from what is required to explain the UV rising branch in ellipticals. Future ultraviolet observations of globular clusters in M31 may also prove useful. Unfortunately, below 2000 Å current *IUE* observations have too poor S/N for providing any stringent information (see Cowley and Burstein 1988).

b) The Galactic Bulge

We now turn to the Galactic nuclear bulge. Here UV studies are hampered by the high extinction, but nevertheless a major effort should be attempted as the bulge is the only region of the Milky Way in which stars with Z approaching that of the metal-rich component of elliptical galaxies have been found (Whitford 1978, 1985; Frogel and Whitford 1987; Rich 1988; Frogel 1988). Extensive stellar photometry of the Galactic bulge has been secured by Terndrup (1988). He notes that the average metallicity rapidly drops off with distance from the Galactic center, and therefore we shall restrict to the innermost studied field of $3' \times 5'$, which lies in Baade's window (BW). The integrated properties of the field are (Terndrup 1988): $\mu_V = 18.7$ (dereddened average surface brightness), $(B-V)_0 \simeq 0.7$, $E(B-V) \simeq 0.45$, $A_V \simeq 1.44$, true distance modulus DM = 14.2. The average metallicity of the stars in the BW field is estimated to be in the range from [Fe/H] = 0.0 to 0.3, depending on the adopted method.

From these data we obtain $L_V^{\rm BW} \simeq 5.7 \times 10^4\ L_{\odot,V}$ for the total visual luminosity framed by Terndrup's $3' \times 5'$ field. Given the fairly blue color (actually closer to that of M3 than to those of giant ellipticals) we adopt $\rm BC^{BW} = 0.6$, or $L_T = 1.55 L_V = 8.8 \times 10^4\ L_\odot$ for the total framed luminosity, fairly similar to the sample of M3 discussed in the previous subsection. Adopting $t_{\rm HB} = 1.25 \times 10^8$ yr for the HB lifetime and then entering with these values into equation (5) one predicts ∼244 HB stars in the frame. Terndrup actually detected 251 HB stars (after statistical foreground decontamination), which is somewhat reassuring. Most of these HB stars are very red *clump* objects, and Terndrup estimates that just ∼30 might belong to a blue HB population.

In order to see whether the BW field contains UV bright stars we have ideally placed vZ1128 (the bright P–AGB star in M3) and the whole HB of NGC 6752 into the BW field, by applying the appropriate distance and reddening compensations. The choice of NGC 6752 is motivated by the very extended blue HB of this cluster (Cannon 1981). The result is shown in Figure 13. It is clear that an extended blue HB such as that of NGC 6752 cannot be exhibited but by a trace fraction of the population of the BW field. Yet, its blue end would be below the frame limit at $V \simeq 20$, and therefore the presence of a bimodal HB (with a clump of very hot objects close to the helium main sequence) cannot formally be excluded by the available data. This would require reaching some 2 mag deeper in this very crowded field, which is certainly feasible with the

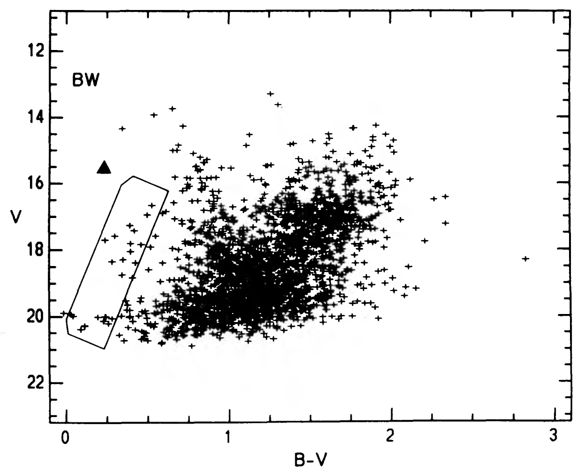

FIG. 13.—The color-magnitude diagram of stars in the Baade Window from Terndrup (1988), with superposed the corresponding approximate position of the HB of the globular cluster NGC 6752 (*inclined box*) and of the hot P–AGB star vZ1128 in the cluster M3 (*filled triangle*).

Hubble Space Telescope (HST) or even from the ground under excellent seeing conditions.

As far as P–AGB are concerned, one object lies close to the expected position for the P–AGB star vZ1128, and might be an interesting candidate for a spectroscopic follow-up. Objects as faint as the other two P–AGB stars in M3 would fall at $V \simeq 20$, i.e., practically at the frame limit. We conclude that a substantially larger fraction of the BW field will have to be explored in order to secure a statistically significant sample of P–AGB stars, so as to possibly check whether they have any special property which could be ascribed to the high metal content of stars in the Galactic bulge.

An alternative way of investigating the P–AGB evolution of bulge stars is to look at planetary nebulae. Two samples of PNs close to the Galactic center have been recently investigated by Zijlstra and Pottasch (1989) and Pottasch and Acker (1989). In the first study it is claimed that these PN nuclei would be systematically much fainter (L = few 100 $L_\odot$) than those in other parts of the Galaxy and with similar effective temperatures which typically have luminosities of a few 1000 $L_\odot$. Such low-luminosity objects would be interesting in the context of the UV production of metal-rich populations, as they could be interpreted as the result of a P–EAGB evolution, therefore implying a lower luminosity but a larger fuel consumption compared to ordinary P–AGB stars. However, Zijlstra and Pottasch result is a consequence of having assumed these nebulae to be optical thick, an assumption which is unlikely to be generally correct. For example, in the case of the PN in the globular cluster M15 one would also obtain an unrealistically low luminosity for its central star. We rather find more plausible that the low-mass stars (in either globular clusters or the Galactic bulge) produce low-mass PNs, which soon become optically thin. In the second study, relative to another sample of bulge PN nuclei, Pottasch and Acker find luminosities which are just those expected from standard P–AGB models, in particular when allowance is made for the uncertainty introduced by the large errors in the estimated effective temperatures. For these latter PNs the assumption about optical thickness may indeed be more justified, although the argument concerning the nebular mass versus radius relation may be undermined by the arbitrary assumption of a constant filling factor. We conclude that the study of PNs in the Galactic bulge is still in a primitive stage of development and does not yet provide valuable indications about the hot star content of metal-rich populations. However, UV and optical spectroscopy of the central stars with HST would certainly provide very useful clues, in particular by checking whether there is any systematic difference in the WN:WC:O spectral type distribution, compared to samples at lower metallicities.

In concluding this discussion we emphasize that the hypothetical super metal-rich stars responsible for the UV upturn in ellipticals with largest Mg_2 index may just be represented by a trace population in the Galactic bulge. Very extensive surveys of hot stars in the bulge are then required to ensure an adequate sampling of this high-metallicity tail.

c) The Bulge of M31 and M32

We first emphasize that M32 and the bulge of M31 do follow, and actually help to define, the Mg_2-$(1550-V)$ correlation (Faber 1983; B3FL). Since none of these objects is an active site of star formation (Bohlin *et al.* 1985), this fact already indicates that the UV output of old stellar populations does indeed increase with increasing metallicity. Yet, rather than blaming metallicity, Bohlin *et al.* attribute the UV difference to an alleged younger age of M32. We do not concur with this interpretation, basically because while the difference in Mg_2 index is a direct observational fact, the younger age of M32 remains a conjecture based on assumptions which have now proved to be invalid (see § II*a*). Certainly, this trend continues well beyond the metallicity of the M31 bulge, and extrapolations are always risky. But we believe that establishing the origin of the differences between these two nearby objects will greatly help in orienting our understanding of elliptical galaxies at large.

The bulge of M31 offers the only other chance of studying individual stars in an old, super metal-rich environment. CCD photometry of bulge fields has just started from the ground (Mould 1986*a*, *b*; Mould and Kristian 1986) and does not reach as faint as it would be required to detect the hot stellar component of the old population. However, this technique will certainly give the most interesting results when HST observations will become available. We note that the rocket UV imagery of Bohlin *et al.* (1985) has revealed no object brighter than a B0V star (or $\sim 36{,}000\ L_\odot$), which leaves ample room for any old/hot component.

Given the limitations of the current observational data, we discuss here only two particular categories of interesting objects, for which valuable information is currently available, namely, planetary nebulae and novae. Based on the survey of Ford and Jacoby (1978), Ford (1978) estimates that ~ 775 PNs within 3 mag of the brightest nebula are present in the bulge of M31, and he derives a stellar death rate of ~ 0.05 stars per year, having assumed a PN lifetime of 14,700 yr, admittedly the weakest point in the whole derivation. For the luminosity of the bulge we adopt $M_B = -19.85$ (Cowley, Crampton, and McClure 1982) and from equations (8) and (10) we estimate $L_B = 1.14 \times 10^{10}\ L_\odot$ and $L_T = 2.7 \times 10^{10}\ L_\odot$. The death rate of the whole bulge of M31 is therefore $B(t)L_T = 2.2 \times 10^{-11} \times 2.7 \times 10^{10} = 0.6$ stars per year, i.e., one order of magnitude more than estimated by Ford. We believe that most of the discrepancy comes indeed from the adopted PN lifetime, which may apply to disk PNs in the Milky Way (typical progenitor mass $\sim 1.5\ M_\odot$), but is likely to give an overestimate when applied to PNs in an old population (typical progenitor mass $\sim 0.85\ M_\odot$). Indeed, two effects probably conspire to shorten the *average* nebular lifetime in low-mass stars. On the one hand, these stars presumably produce low-mass PNs, whose surface brightness will therefore more promptly drop below detection thresholds (Renzini 1983); on the other hand, lower mass progenitors leave lower mass P–AGB remnants which may take longer to complete the transition to high temperatures, thereby more frequently producing *lazy* P–AGB objects, i.e., with zero PN lifetime (Renzini 1981*c*). If the 0.546 $M_\odot$ P–AGB track of Schönberner (1983) is appropriate for stars in the M31 bulge, then these objects take $\sim 500{,}000$ yr to fade 3 mag in bolometric. This compares to ~ 1500 yr for the average PN lifetime that we have just inferred, and therefore we conclude that in old stellar populations PNs trace only a small fraction of bright P–AGB stars.

The comparison with M32 offers additional evidence supporting this conclusion. Indeed, for M32 Ford and Jenner (1975) estimate a content of ~ 34 PNs, within the same limiting magnitude giving the 775 PNs in M31. Since M32 is 53 times less luminous (in B) than the bulge of M31, one can infer that in M32 the production rate of PNs per unit blue luminosity is about twice than in M31. In spite of this, M32 is much fainter in the UV than the bulge of M31. This argument has been used against the P–AGB origin of the UV in these galaxies (Bica and Alloin 1988), while it only proves that the average PN lifetime is different in the two galaxies, which is most likely the result of the different average metallicity between them.

We now turn to novae, which may indeed provide valuable indications about the binary star content of galactic spheroids. Ford (1978) estimates an average of 33 nova outbursts per year in the bulge of M31, and more recently Capaccioli *et al.* (1989) obtain 25 ± 4. Thus the production rate of nova outbursts per unit bolometric luminosity is $B_{NO} \simeq 25/(2.7 \times 10^{10}) \simeq 10^{-9}$ novae per year per $L_\odot$. Note that 85% of all M31 novae occur in the bulge, although the bulge itself represents only $\sim 20\%$ of the whole blue luminosity of the galaxy (see Cowley *et al.* 1982). Therefore, the nova productivity in the bulge is ~ 20 times larger than that of the disk! This is in itself a very interesting aspect, as it shows that an old metal-rich population may at least in some respect be considerably more *active* than a continuously star-forming system. In this connection, we note that spiral galaxies are slightly more prolific SNI producers than ellipticals (Tammann 1982; Cappellaro and Turatto 1988). If an analogy between the bulge of spirals and ellipticals is allowed, then the different population behavior of classical novae and SNIs argues against novae concluding their careers as SNIs, or even against novae and SNI precursor lifetimes being controlled by the same clock. For example, it is believed that magnetic wind braking can play a role in shrinking the binary orbit of nova progenitors (e.g., Verbunt and Zwaan 1981). If so, then different processes must be at work in SNI precursor systems.

A substantial fraction of the energy released in one nova outburst is radiated in the UV (e.g., Stickland *et al.* 1981), so in the present context it makes sense to estimate what fraction of the bulge light is (on a time average) provided by novae. For this purpose we use a generalization of equation (1):

$$\frac{L_{NO}}{L_T} \simeq 9.75 \times 10^{10} B_{NO} \langle F_{NO} \rangle \,, \qquad (32)$$

where $\langle F_{NO} \rangle$ is the average amount of nuclear fuel which is burned in one outburst, in $M_\odot$ units. From, e.g., Prialnik *et al.* (1982) we adopt $\langle F_{NO} \rangle \simeq 8 \times 10^{-7} M_\odot$, and we finally obtain $L_{NO}/L_T \simeq 8 \times 10^{-5}$, i.e., a fairly negligible contribution even to the UV itself. Should we conclude that binaries in general are unimportant contributors to the UV of old populations? We believe that this would be a premature conclusion, as classical novae may just represent the tip of the iceberg of the AWD population in spheroidal systems. In fact, UV photons could be produced by AWDs which do not exhibit the dramatic optical display of novae, and for this reason may have attracted less interest from both observers and theoreticians. As already mentioned, deep UV imagery of the bulge of M31 could detect such objects and distinguish them from other kinds of hot stars (P–AGB, AGB–manqué, etc.) because of their possible variability on time scales of months to years, such as in the case of hydrogen-flashing WD accretors (see § VI*b*). If the SNI rate of ellipticals applies also to the bulge of M31, then it should produce $\sim 2 \times 10^{-13} \times 1.14 \times 10^{10} \simeq 2 \times 10^{-3}$ SN yr^{-1}. Moreover, if the precursors of SNIs are AWDs of the type discussed in § VI*b*, then with a typical lifetime of 5×10^7 yr the expected number of precursor AWDs should be $\sim 10^5$. With a duty cycle of 0.03 this would imply the presence of ~ 3000 objects in the ON phase at any time, and ~ 200 flashes per year if the duration of each ON phase is ~ 15 yr. This is roughly one order of magnitude more than the observed nova frequency, and we conclude that either flashing AWDs do not produce much optical display to resemble classical novae, or (perhaps more likely) SNIs are not the product of hydrogen-burning AWDs. In any event, UV imagery of the bulge of M31 will also provide a crucial test for the possibility (envisaged at the end of § VI) for a much more ubiquitous realization of stable AWDs than demanded by the more restrictive interpretation of equation (29).

d) Spectral Evolution in the UV

It has been frequently pointed out that a potential way of discriminating among the various candidates is to look at the evolution of the UV upturn with redshift (e.g., RB; Bertelli, Chiosi, and Bertola 1989). Indeed, the contribution of each type of candidate tends to evolve in a different way with redshift. In particular, both the P–AGB and hot HB contributions tend to decrease with decreasing age (increasing z), but the SED of the former tends to get harder, while that of the latter tends to become softer (see RB for details). On the contrary, the contribution by AWDs may increase with increasing z. Unfortunately, the real situation may be far more complex than in the case of the *simple stellar populations* discussed by RB. This is so for two main reasons: real galaxies will contain all kinds of candidates in some proportions (rather than just one), and the presence of a metallicity spread further complicates the case. Yet, our results in § V indicate that the *UV upturn can be the most rapidly evolving feature in the spectrum of elliptical galaxies*, and therefore it offers a unique opportunity of intercomparing the stellar evolution clock to the cosmological expansion clock (the lookback time), for redshifts as small as $z = 0.1$–0.2.

To check for evolutionary effects one should build up a galaxy data base for a few redshift bins, and the sample size for each bin should be comparable to that of the local *IUE* sample investigated by B3FL. Certainly this is not a simple task. The local sample basically refers to galaxies with velocities up to ~ 2000 km s^{-1}. Reaching a redshift $z = 0.2$ ($v \simeq 60{,}000$ km s^{-1}) would correspond to galaxies some 30 times more distant or some 1000 times fainter. Yet, the aperture of HST is less than 6 times that of *IUE*. This means that, if aperture effects alone are taken into account, HST should not be able to get low-dispersion UV spectra of ellipticals much beyond $v \simeq 6 \times 2000 = 12{,}000$ km s^{-1}, which corresponds to a redshift (lookback time) too low for evolutionary effects to be detectable. Indeed, it is well known that HST is not much more efficient than *IUE* when UV spectroscopy of extended sources is concerned (e.g., Macchetto and Henry 1987). Broad-band UV imagery of moderate redshift galaxies looks more promising, but our impression is that we may have to wait for a 10 m class telescope in orbit for an unambiguous detection of evolutionary effects.

e) UV–X-Ray Connection

The X-ray emission of elliptical galaxies has sometimes been interpreted in terms of cooling-flow models (see Fabbiano 1989 for a recent review). One problem with such models is the large amount of matter which should disappear in the central regions of these galaxies, and star formation has often been invoked. This kind of process would automatically provide UV photons if massive stars are formed in the flows, and therefore the possible connection between the X-ray and UV properties of ellipticals has attracted the interest of various investigators (Bertola 1988*b*; B3FL; O'Connell and McNamara 1989). The tendency has been to exclude a tight connection, basically because there are cases (e.g., M31) in which there is a fairly strong UV upturn, but no signs of either cooling flows or massive star formation. However, more recently it has become clear that powerful X-ray emission in an elliptical galaxy does not guarantee the presence of a cooling flow (D'Ercole *et al.* 1989; Ciotti *et al.* 1990). It is actually more likely that subsonic outflows prevail among X-ray ellipticals, while the fainter galaxies still support supersonic winds. Only some among the brightest X-ray ellipticals may indeed host a large-scale inflow of material, thereby maintaining a favorable condition for hypothetical ongoing star formation. Therefore, inflow rates derived for individual galaxies under the presumption that steady cooling-flow solutions apply may be nonsense, as most galaxies can actually support an outflow. This new possibility should be considered when correlating the X-ray and UV properties of ellipticals. On the other hand, D'Ercole *et al.* models also show that a large optical luminosity does favor the establishment of galactic inflows. Hence, the possibility exists that only the optically brightest ellipticals host an inflow, form massive stars in their nuclei, and therefore show a stronger UV upturn. The general L_B–Mg_2 correlation will then help to also maintain the $Mg_2 - (1550 - V)$ correlation. If so, the hot and old star component would be basically responsible for this correlation, and *only* in a few of the optically, X-ray, and UV brightest galaxies an extra contribution from young massive stars would be present (e.g., in NGC 4649).[5] This possibility will readily become testable with HST, as individual massive stars will be directly observable in nearby ellipticals (e.g., B3FL; O'Connell and McNamara 1989). Therefore, this scenario would predict that HST should detect massive stars only in the brightest early-type galaxies, while old stars would totally account for the UV upturn in optically and X-ray fainter galaxies.

[5] This does not exclude the possibility of episodical star formation in dwarf ellipticals due to cold gas accretion, as may be the case for NGC 205.

VIII. SUMMARY

Rather than attempting the construction of synthetic spectra, in this paper we have used pure energetic arguments to investigate in detail under which conditions old, low-mass stars could be responsible for the ultraviolet rising branch of elliptical galaxies. The main tool that we have used is the so-called fuel consumption theorem that we have applied to every conceivable kind of low-mass stars of sufficiently high effective temperature. These include post–red giants, hot horizontal branch stars and their AGB–manqué progeny, post–early-AGB stars, and post–AGB stars of the helium- and hydrogen-burning varieties. Among binary star candidates we have considered post–red giants produced by a Roche-lobe overflow, and accreting white dwarfs able to burn the accreted fuels. We argue that the presently available observational data are not sufficient to unambiguously decide which of these candidates provide the dominant contribution, but we make an effort to identify for each candidate the distinctive characteristics which will help a positive identification in the future. Our main conclusions are the following:

1. The possibility for metal-rich, low-mass stars to evolve through sufficiently hot stages, provide enough UV photons, and produce the observed UV-metallicity correlation (see B3FL) is primarily controlled by two poorly known trends with increasing metallicity: that of the helium enrichment, and that of the mass-loss rate during the red giant phases.

2. The classical hydrogen burning post–AGB stars (often regarded as viable candidates, e.g., RB) at a closer scrutiny do not appear able to burn enough fuel to account for the UV most powerful galaxies. This follows from previous extrapolations of the fuel consumption having neglected a direct metallicity dependence, and also from more recent model calculations (Caloi 1989) showing that such extrapolations may dramatically overestimate the fuel consumption and therefore the efficiency of these stars as UV producers.

3. More promising appear to be the other hot star candidates, namely, hot HB stars, AGB–manqué, and post–EAGB stars. However, the numerical experiments show that their production is critically sensitive to the adopted values of the helium enrichment and mass loss parameters.

4. These kinds of hot stars are likely to lose entirely their hydrogen-rich envelope and expose helium-nitrogen layers. The identification of helium-star features in high S/N spectra of ellipticals would then lend strong support to this scenario.

5. It is shown that a very important role is played by the actual metallicity distribution within individual galaxies. Even more important than the average metallicity might in fact be the actual population of the high-metallicity tail.

6. Models in which the bulk of the UV flux is produced by hot HB and P–EAGB stars show a very strong evolution with lookback time. If these models are appropriate for real galaxies, then the UV rising branch should be the most rapidly evolving feature, with strong effects being already detectable at redshifts as small as $z \simeq 0.1$–0.2.

7. Binary stars will certainly contribute to the UV flux of ellipticals, but arguments exist suggesting that such contribution is unlikely to be dominant. There is, however, some theoretical possibility for binaries to experience a stable mass transfer for a wider volume in the binary parameter space than allowed in the traditional scenario. If so, accreting white dwarfs could become very interesting candidates. Their connection with the supernovae of type I is also emphasized.

8. The observability of individual UV bright stars is presently restricted to the local group galaxies. Yet, the UV imaging of Galactic globular clusters, of the bulges of the Milky Way and M31, and of M32 can provide extremely valuable information to help our understanding of the hot star content of elliptical galaxies. Still, such galaxies may contain even more metal-rich stars than the bulges of local spirals. If so, some residual extrapolation will remain unavoidable.

9. It seems that the detection of evolutionary effects in low-redshift objects ($z \simeq 0.1$–0.2) will be out of reach for the Hubble Space Telescope (HST) and may require a 10 m class telescope in orbit. Such effects may become most easily detectable in broad-band ultraviolet imaging mode.

10. Concerning the possible contribution from ongoing forming massive stars and the UV-X-ray connection, we argue that HST should in case detect massive stars only in the brightest ellipticals, while only old stars would be present in optically and X-ray fainter galaxies.

We would like to thank Francesco Bertola, David Burstein, Lucio Buson, Carla Cacciari, Sperello di Serego Alighieri, Mario Livio, Friedrich Meyer, Hans Ritter, Gustav Tammann, and Saša Tutukov, for very useful conversations, Jay Frogel, John Lattanzio, Wendy Freedman, Sergio Ortolani, Michael Rich, Don Terndrup, and Ed Velentijn for having kindly provided their preprints. We would also like to thank Alberto Buzzoni for having computed for us the coefficients in equation (10), and especially Vittoria Caloi for having searched in the computer output the fuel consumption of her low-mass P–AGB models. We are particularly grateful to Jay Frogel and Michael Rich for a critical reading of the manuscript which resulted in a significant improvement in presentation, and to the European Southern Observatory for its kind hospitality in the period during which much of this paper has been written. Finally, we wish to acknowledge for partial support the Italian Ministry of Research and the Piano Spaziale Nazionale (through the grants PSN 85076, 85079, 87035, and 87037).

APPENDIX

We give here all the analytical approximations that we have used for the construction of synthetic HB and AGB evolutionary algorithms.

I. ALONG THE RGB

The mass $M_{\rm RG}(Y, Z, t)$ in equation (13) is obtained via best-fit interpolation on the evolutionary sequences by Mengel *et al.* (1979):

$$\log M_{\rm RG} \simeq c_0 + c_1 \log t_9 + 0.0326 \log^2 t_9 , \tag{A1}$$

with

$$c_0 = 0.7866 + 0.00842 \log^3 Z + 0.0868 \log^2 Z + 0.3 \log Z - 0.783Y , \qquad c_1 = -0.3242 + 0.0061 \log Z + 0.059Y .$$

where t_9 is the age in Gyr. For $-4 \leq \log Z \leq -1$, $0.1 \leq Y \leq 0.4$, and $0.7 \leq M_{RG} \leq 3.5$ this expression has an average accuracy of 1%, and the maximum deviation from tabular values does not exceed 3%.

The functions g and γ appearing in equation (13) are given by

$$g(Y, Z, \alpha, \gamma) = 5.37 \times 10^{-4}\alpha^{-0.4}\left(\frac{0.84}{\gamma} + \frac{\beta - 0.84}{\gamma + \beta}\right)10^{-0.16(Y-0.3)}(10^3 Z)^{-0.04}\left(\frac{X}{0.7}\right)^{1.36} ,$$

$$\gamma = 0.754 + 0.02(4 + \log Z)^{1.79} ,$$

with

$$\beta = 0.91 + 0.295(4 + \log Z) - 0.09(4 + \log Z)^2 + 0.67Y .$$

The luminosity at the onset of the helium flash at the tip of the RGB is given by

$$\log L_{\rm fl} = 3.32 + 0.09(4 + \log Z) + 0.45(X - 0.7) - 0.16(M_{\rm fl} - 0.8) , \tag{A2}$$

where $X = 1 - Y - Z$ is the hydrogen mass fraction.

The core mass-luminosity relation for RGB models has been obtained by best fit over the RGB sequences of Sweigart and Gross (1978):

$$\log M_{\rm H} = -0.7657 - 0.18Y - 0.045M + (0.12552 + 0.018M - 0.007 \log Z) \log L , \tag{A3}$$

which has been derived using models spanning the ranges $-4 \leq \log Z \leq -2$, $0.1 \leq Y \leq 0.4$, and $0.7 \leq M_i \leq 2.2$. In absence of the explicitly computed quantities, we have used the above relations also for larger Z and Y values, but none of our conclusions is seriously affected by the actual accuracy of these extrapolations.

II. THE HB PHASE

For the mass of RR Lyrae stars $M_{\rm RR}$ we have adopted the mass of ZAHB models at $\log T_e = 3.85$, from Sweigart (1987):

$$M_{\rm RR}(Y, Z) = M_{\rm RR}^{Y=0.2} + 10(Y - 0.2)[M_{\rm RR}^{Y=0.3} - M_{\rm RR}^{Y=0.2}] , \tag{A4}$$

with

$$\begin{aligned} M_{\rm RR}^{Y=0.2} &= 0.02 \log^2 Z + 0.02 \log Z + 0.54 && (\log Z \leq -2.5) , \\ &= 0.415 - 0.08 \log Z && (\log Z > -2.5) , \\ M_{\rm RR}^{Y=0.3} &= 0.05 \log^2 Z + 0.17 \log Z + 0.74 && (\log Z \leq -2.5) , \\ &= 0.428 - 0.08 \log Z && (\log Z > -2.5) . \end{aligned}$$

For the amount of mass which is processed by the hydrogen-burning shell during the HB phase we adopt:

$$\Delta M_{\rm H}^{\rm HB} = \min\,[(\Delta M_{\rm H}^{\rm HB})_1 ; (\Delta M_{\rm H}^{\rm HB})_2] , \tag{A5}$$

where

$$(\Delta M_{\rm H}^{\rm HB})_1 = 0.296(Y - 0.25) + [0.0583 + 0.254(Y - 0.25)]M_{\rm HB} - 0.001 ,$$

$$(\Delta M_{\rm H}^{\rm HB})_2 = [0.218 + 0.844(Y - 0.25) + 0.11(3 + \log Z)][M_{\rm H}^{\rm HB} - M_{\rm crit} - 0.02(3 + \log Z)] ,$$

$$M_{\rm crit} = \max\,[0.5; 0.525 - 0.5(Y - 0.25)] .$$

These expressions have been obtained from the HB evolutionary sequences of Sweigart (1987) and Sweigart and Gross (1976), having incremented by 5% the tabular values of $\Delta M_{\rm H}^{\rm HB}$ so as to take into account the final HB evolution which was not included in the tabulations. This assumes that the so-called *breathing pulses* sometime encountered in the calculations (Sweigart and Demarque 1972) do not actually take place, as indeed is indicated by the AGB to HB lifetime ratios derived for galactic globular clusters (Renzini 1987; Renzini and Fusi Pecci 1988).

III. ALONG THE E–AGB

The advancement of the hydrogen shell during the whole E–AGB phase is given by

$$\Delta M_{\rm H}^{\rm EAGB} = \min\,[0.4(M_{\rm HB} - M_{\rm H}^{\rm TAHB}); \quad 0.01 - 0.0485(M_{\rm HB} - 1)] , \tag{A6}$$

a relation that we have obtained from various E–AGB models (Gingold 1974; Lattanzio 1986; Caloi 1989). A possible direct dependence on composition has been ignored because of insufficient model data. The function $\Lambda(L)$ in equation (17) has been constructed in such a way as to reproduce the E–AGB luminosity function of the models of Gingold (1974):

$$\Lambda(L) = a \log^2 L + b \log L + c , \tag{A7}$$

where $a = -1.6$, $b = 8.32$, and $c = -10.42$ for $\log L \leq 2.6$, and $a = 1.98$, $b = -10.31$ and $c = 13.81$ for $\log L > 2.6$. This formulation assumes that $\sim 40\%$ of the total amount of hydrogen-rich envelope which is processed during the whole E–AGB (ΔM_H^{EAGB}) is burned at luminosities in the range $2.1 \leq \log L \leq 2.6$, while the remaining 60% is burned in the range from $\log L = 2.6$ and 3.15, where the first thermal pulse is assumed to take place, irrespective of the mass, core mass, and composition. This is certainly a rough approximation (see Renzini and Fusi Pecci 1988 for a slightly different approach), but it is fairly reasonable for the present applications. After all, the luminosity at the first flash is found to within $\Delta \log L = \pm 0.05$ of this adopted value in most available model calculations for low-mass stars (see Fig. 3 in Boothroyd and Sackmann 1988). For $\log L = 3.15$ then equation (17) gives the core mass M_H at the first helium-shell flash, hereafter M_H^1.

For the mass of the C-O core during the E–AGB phase we adopt

$$M_{CO}(L) = [1 - \phi(M, L)]M_H(L) , \quad (A8)$$

where we use

$$\phi(M, L) = \max\left[1.47 - 0.483 \log L + \frac{0.0613(M - 0.75)}{0.118 + M^{5.5}} ; \qquad 0.453 - 0.124 \log L\right] \quad (A9)$$

for $\log L \leq 3$, and

$$\phi(M, L) = \max\,[0.24 - 0.053 \log L; 0.01] \quad (A10)$$

for $\log L > 3$. These expressions have been obtained by fitting existing model tabulations (Gingold 1976; Castellani, Chieffi, and Pulone 1989) and can certainly be improved as new calculations will hopefully become available, in particular for high metallicities. None of our main conclusion is seriously affected by the actual accuracy of the approximation of function ϕ in equation (A8). During the E–AGB the helium shell advances in mass at a rate which is somewhat larger than the corresponding rate of the hydrogen shell, and the mass of intershell helium-rich zone ($\Delta M_{He} = M_H - M_{CO}$) decreases. Note that equation (A8) implies $\Delta M_{He} = \phi M_H$, i.e., ϕ is the helium buffer mass in units of the total core mass.

IV. ALONG THE THERMALLY PULSING AGB

For the core mass-luminosity relation during the thermally pulsing regime we distinguish between an early phase when the thermal pulses have not yet reached their asymptotic strength, and a subsequent full-amplitude phase. Using various AGB models (Gingold 1974; Boothroyd and Sackmann 1988) for these two phases we respectively adopt:

$$M_H(L) = M_H^1 + 10^{-5}(L - 1400) , \quad (A12)$$

$$M_H(L) = 0.488 - 0.01(3 + \log Z) + L/47, 700 , \quad (A13)$$

and we switch from the first to the second value by taking the larger of them.

REFERENCES

Arimoto, N., and Yoshii, Y. 1987, *Astr. Ap.*, **173**, 23.
Barbaro, G., and Olivi, F. 1989, *Ap. J.*, **337**, 125.
Barbuy, B., Bica, E., and Ortolani, S. 1989, *The Messenger*, **58**, 42.
Becker, S. A., and Iben, I., Jr. 1979, *Ap. J.*, **232**, 831.
Bertelli, G., Bressan, A., and Chiosi, C. 1984, *Astr. Ap.*, **130**, 279.
Bertelli, G., Bressan, A., Chiosi, C., and Angerer, K. 1986, *Mem. Soc. Astr. Italiana*, **57**, 427.
Bertelli, G., Chiosi, C., and Bertola, F. 1989 *Ap. J.*, **339**, 889.
Bertola, F. 1988*a*, in *A Decade of UV Astronomy with the IUE Satellite*, ESA SP-281, Vol. **2**, p. 63.
———. 1988*b*, in *Cooling Flows in Clusters and Galaxies*, ed. A. C. Fabian (Dordrecht: Kluwer), p. 127.
Bertola, F., Capaccioli, M., Holm, A. V., and Oke, J. B. 1980, *Ap. J. (Letters)*, **237**, L65.
Bertola, F., Capaccioli, M., and Oke, J. B. 1982, *Ap. J.*, **254**, 494.
Bertola, F., Gregg, M. D., Gunn, J. E., and Oemler, A., Jr. 1986, *Ap. J.*, **303**, 624.
Bica, E., and Alloin, D. 1987, *Astr. Ap.*, **186**, 49.
———. 1988, *Astr. Ap.*, **192**, 98.
Bica, E., Alloin, D., and Schmidt, A. A. 1990, *Astr. Ap.*, **228**, 23.
Bohlin, R. C., Cornett, R. H., Hill, J. K., Smith, A. M., Stecher, T. P., and Sweigart, A. V. 1983, *Ap. J. (Letters)*, **267**, L89.
Bohlin, R. C., Cornett, R. H., Hill, J. K., Hill, R. S., O'Connell, R. W., and Stecher, T. P., 1985, *Ap. J. (Letters)*, **298**, L40.
Boothroyd, A. I., and Sackmann, I.-J. 1988, *Ap. J.*, **328**, 653.
Boulade, J., Rose, J. A., and Vigroux, L. 1988, *A. J.*, **96**, 1319.
Bragaglia, A., Greggio, L., Renzini, A., and D'Odorico, S. 1988, *The Messenger*, **52**, 35.
———. 1990, *Ap. J. (Letters)*, submitted.
Brocato, E., Matteucci, F., Mazzitelli, I., and Tornambè, A. 1990, *Ap. J.*, **349**, 458.
Bruzual, G. A. 1983, *Ap. J.*, **273**, 105.
Burstein, D., Bertola, F., Buson, L. M., Faber, S. M., and Lauer, T. R. 1988*a*, *Ap. J.*, **328**, 440 (B3FL).
Burstein, D., Davis, R. L., Dressler, A., Faber, S. M., Lynden-Bell, D., Terlevich, R., and Wegner, G. 1988*b*, in *Towards Understanding Galaxies at Large Redshift*, ed. R. G. Kron and A. Renzini (Dordrecht: Kluwer), p. 17.
Burstein, D., Faber, S. M., Gaskell, C. M., and Krumm, N. 1984, *Ap. J.*, 287, 586.
Caloi, V. 1989, *Astr. Ap.*, **221**, 27.
Cannon, R. D. 1981, in *Astrophysical parameters for Globular Clusters*, ed. A. G. D. Philip and D. S. Hayes (Schenectady: L. Davis), p. 501.
Capaccioli, M., Della Valle, M., D'Onofrio, M., and Rosino, L. 1989, *A.J.*, **97**, 1622.
Cappellaro, E., and Turatto, M. 1988, *Astr. Ap.*, **190**, 10.
Castellani, V., Chieffi, A., and Pulone, L. 1989, *Ap. J.*, **344**, 239.
Cerruti-Sola, M., and Perinotto, M. 1985, *Ap. J.*, **291**, 237.
Chiosi, C., and Maeder, A. 1986, *Ann. Rev. Astr. Ap.*, **24**, 329.
Ciotti, L., D'Ercole, A., Pellegrini, S., and Renzini, A. 1990, *Ap. J.*, submitted.
Code, A. D., and Welch, G. A. 1979, *Ap. J.*, **228**, 95.
Code, A. D., Welch, G. A., and Page, T. L. 1972, in *The Scientific Results from the Orbiting Astronomical Observatory* (OAO-2), ed. A. D. Code, NASA SP-310, p. 559.
Cowley, A. P., and Burstein, D. 1988, *A.J.*, **95**, 1071.
Cowley, A. P., Crampton, D., and McClure, R. D. 1982, *Ap. J.*, **263**, 1.
Davies, R. L., and Sadler, E. M. 1987, in *Structure and Dynamics of Elliptical Galaxies*, ed. P. T. De Zeeuw (Dordrecht: Reidel), p. 441.
Davidge, T., and Jones, J. 1989, *CFHT Inf. Bull.*, **20**, 16.
de Boer, K. S. 1985, *Astr. Ap.*, **142**, 321.
D'Ercole, A., Renzini, A., Ciotti, L., and Pellegrini, S. 1989, *Ap. J. (Letters)*, **341**, L9.
Dupree, A. K. 1986, *Ann. Rev. Astr. Ap.*, **24**, 377.
Efstathiou, G., and Gorgas, J. 1985, *M.N.R.A.S.*, **215**, 37P.
Fabbiano, G. 1989, *Ann. Rev. Astr. Ap.*, **27**, 87.
Faber, S. M. 1983, *Highlights Astr.*, **6**, 165.
Fanelli, M. N., O'Connell, R. W., and Thuan, T. X. 1987, *Ap. J.*, **321**, 768.
Ford, H. C. 1978, *Ap. J.*, **219**, 595.
Ford, H. C., and Jacoby, G. H. 1978, *Ap. J.*, **219**, 437.
Ford, H. C., and Jenner, G. H. 1975, *Ap. J.*, **202**, 365.
Freedman, W. L. 1989, *A. J.*, **98**, 1285.
Frogel, J. A. 1988, *Ann. Rev. Astr. Ap.*, **26**, 51.
Frogel, J. A., and Elias, J. H. 1988, *Ap. J.*, **324**, 823.
Frogel, J. A., and Whitford, A. E. 1987, *Ap. J.*, **320**, 199.

Fusi Pecci, F., and Renzini, A. 1975, *Astr. Ap.*, **39**, 413.
———. 1976, *Astr. Ap.*, **46**, 447.
Giannone, P., Refsdal, S., and Weigert, W. 1970, *Astr. Ap.*, **4**, 428.
Gingold, R. A. 1974, *Ap. J.*, **193**, 177.
———. 1976, *Ap. J.*, **204**, 116.
Greggio, L., and Renzini, A. 1983, *Astr. Ap.*, **118**, 217.
Guiderdoni, B., and Rocca-Volmerange, B. 1987, *Astr. Ap.*, **186**, 1.
Gunn, J. E., Stryker, L. L., and Tinsley, B. M. 1981, *Ap. J.*, **249**, 48.
Harpaz, A., Kovetz, A., and Shaviv, G. 1987, *Ap. J.*, **323**, 154.
Heap, S. R. 1982, in *Wolf-Rayet Stars: Observations, Physics, Evolution*, ed. C. W. H. de Loore and A. J. Willis (Dordrecht: Reidel), p. 423.
Hills, J. G. 1971, *Astr. Ap.*, **12**, 1.
Humphreys, R. M., and McElroy, D. B. 1984, *Ap. J.*, **284**, 565.
Iben, I., Jr. 1982*a*, *Ap. J.*, **254**, 244.
———. 1982*b*, *Ap. J.*, **260**, 821.
———. 1984, *Ap. J.*, **277**, 333.
———. 1985, *Quart. J.R.A.S.*, **26**, 1.
Iben, I., Jr., and MacDonald, J. 1986, *Ap. J.*, **301**, 164.
Iben, I., Jr., and Renzini, A. 1983, *Ann. Rev. Astr. Ap.*, **21**, 271.
Iben, I., Jr., and Tutukov, A. V. 1984*a*, *Ap. J. Suppl.*, **54**, 335.
———. 1984*b*, *Ap. J.*, **282**, 615.
———. 1986, *Ap. J.*, **311**, 742.
Kjærgaard, P. 1987, *Astr. Ap.*, **176**, 210.
Kippenhahn, R., Thomas, H.-C., and Weigert, A. 1968, *Zs. Ap.*, **69**, 265.
Kippenhahn, R., and Weigert, A. 1967, *Zs. Ap.*, **65**, 251.
Kunth, D. 1983, in *Primordial Helium*, ed. P. A. Shaver, D. Kunth, and K. Kjär (Garching: ESO), p. 305.
Lattanzio, J. 1986, *Ap. J.*, **311**, 708.
Lauteborn, D. 1970, *Astr. Ap.*, **7**, 150.
Livio, M. 1989, *Space Sci. Rev.*, **50**, 299.
Longo, G., Capaccioli, M., Bender, R., and Busarello, G. 1989, *Astr. Ap.*, **225**, L17.
Macchetto, F. D., and Henry, R. C. 1987, in *Exploring the Universe with the IUE Satellite*, ed. Y. Kondo (Dordrecht: Reidel), p. 753.
Maeder, A. 1983, in *Primordial Helium*, ed. P. A. Shaver, D. Kunth, and K. Kjär (Garching: ESO), p. 89.
Mengel, J. G., Sweigart, A. V., Demarque, P., and Gross, P. G. 1979, *Ap. J. Suppl.*, **40**, 733.
Minkowski, R., and Osterbrock, D. 1959, *Ap. J.*, **129**, 583.
Mochkovitch, R. 1986, *Astr. Ap.*, **157**, 311.
Mould, J. R. 1978, *Ap. J.*, **220**, 434.
———. 1986*a*, in *Stellar Populations*, ed. C. A. Norman, A. Renzini, and M. Tosi (Cambridge: Cambridge University Press), p. 9.
———. 1986*b*, in *Spectral Evolution of Galaxies*, ed. C. Chiosi, and A. Renzini (Dordrecht: Reidel), p. 133.
Mould, J. R., and Kristian, J. 1986, *Ap. J.*, **305**, 591.
Nesci, R., and Perola, G. C. 1985, *Astr. Ap.*, **145**, 236.
Nomoto, K. 1982, *Ap. J.*, **253**, 798.
O'Connell, R. W. 1986, in *Stellar Populations*, ed. C. A. Norman, A. Renzini, and M. Tosi (Cambridge: Cambridge University Press), p. 167.
———. 1988, in *Starburst and Galaxy Evolution*, ed. T. X. Thuan, T. Montmerle, and J. Tran Thanh Van (Gif sur Yvette: Editions Frontières), p. 367.
O'Connell, R. W., and McNamara, B. R. 1989, *A. J.*, **98**, 180.
O'Connell, R. W., Thuan, T. X., and Puschell, J. J. 1986, *Ap. J.* (*Letters*), **303**, L37.
Oke, J. B., Bertola, F., and Capaccioli, M. 1981, *Ap. J.*, **243**, 453.
Paczyński, B. 1971*a*, *Acta Astr.*, **21**, 1.
———. 1971*b*, *Acta Astr.*, **21**, 417.
Pagel, B. E. J. 1989, in *Evolutionary Phenomena in Galaxies*, ed. J. E. Beckman and B. E. J. Pagel (Cambridge: Cambridge University Press), p. 368.
Pastetter, L., and Ritter, H. 1989, *Astr. Ap.*, **214**, 186.
Peimbert, M. 1983, in *Primordial Helium*, ed. P. A. Shaver, D. Kunth, and K. Kjär (Garching: ESO), p. 267.
Peletier, R. F., Valentijn, E. A., and Jameson, R. F. 1990, *Astr. Ap.*, **233**, 62.
Perinotto, M. 1989, in *Planetary Nebulae*, ed. S. Torres-Peimbert (Dordrecht: Kluwer), p. 291.
Perola, C., and Tarenghi, M. 1980, *Ap. J.*, **240**, 447.
Pickles, A. 1987, in *Structure and Dynamics of Elliptical Galaxies*, ed. T. de Zeeuw (Dordrecht: Reidel), p. 203.
Pottasch, S. R., and Acker, A. 1989, *Astr. Ap.*, **221**, 123.
Prialnik, D., Livio, M., Shaviv, G., and Kovetz, A. 1982, *Ap. J.*, **257**, 312.
Reimers, D. 1975, *Mém. Soc. Roy. Sci. Liège*, 6th Ser. **8**, 369.
Renzini, A. 1977, in *Advanced Stages in Stellar Evolution*, ed. P. Bouvier and A. Maeder (Geneva: Geneva Observatory), p. 149.
———. 1979, in *Stars and Star Systems*, ed. B. Westerlund (Dordrecht: Reidel), p. 155.
———. 1981*a*, in *Mass Loss and Stellar Evolution*, ed. C. Chiosi and R. Stalio (Dordrecht: Reidel), p. 319.
———. 1981*b*, *Ann. Phys. Fr.*, **6**, 87.
———. 1981*c*, in *Physical Processes in Red Giants*, ed. I. Iben, Jr., and A. Renzini (Dordrecht: Reidel), p. 431.
———. 1983, in *Planetary Nebulae*, ed. D. R. Flower (Dordrecht: Reidel), p. 267.
———. 1984, in *Observational Tests of the Stellar Evolution Theory*, ed. A. Maeder and A. Renzini (Dordrecht: Reidel), p. 21.
———. 1985, in *Horizontal Branch and UV-Bright Stars*, ed. A. G. D. Philip (Schenectady: L. Davis), p. 19.
———. 1986, in *Stellar Populations*, ed. C. A. Norman, A. Renzini, and M. Tosi (Cambridge: Cambridge University Press), p. 213.
———. 1987, in *Stellar Evolution and Dynamics in the Outer Halo of the Galaxy*, ed. M. Azzopardi and F. Matteucci (Garching: ESO), p. 289.
———. 1989*a*, in *Planetary Nebulae*, ed. S. Torres-Peimbert (Dordrecht: Kluwer), p. 391.
———. 1989*b*, in *Evolutionary Phenomena in Galaxies*, ed. J. E. Beckman and B. E. J. Pagel (Cambridge: Cambridge University Press), p. 422.
Renzini, A., and Buzzoni, A. 1986, in *Spectral Evolution of Galaxies*, ed. C. Chiosi, A. Renzini (Dordrecht: Reidel), p. 135 (RB).
Renzini, A., and Fusi Pecci, F. 1988, *Ann. Rev. Astr. Ap.*, **26**, 199.
Rich, R. M. 1988, *A. J.*, **95**, 828.
Rich, R. M., Mould J., Picard, A., Frogel, J. A., and Davis, R. 1989, *Ap. J.* (*Letters*), **341**, L51.
Rocca-Volmerange, B. 1989, *M.N.R.A.S.*, **236**, 47.
Rose, W. K., and Timsley, B. M. 1974, *Ap. J.*, **190**, 243.
Schönberner, D. 1983, *Ap. J.*, **272**, 708.
Sharov, A. S., and Lyuti, V. M. 1983, *Soviet Astr.*, **27**, 1.
Stecher, T. P. 1978, NASA UIT-680-78-01.
Steigman, G., Gallagher, J. S., and Schramm, D. N. 1989, *Comments Ap.*, **14**, 97.
Stickland, D. J., Penn, C. J., Seaton, M. J., Snijders, M. A. J., Storey, P. J. 1981, *M.N.R.A.S.*, **197**, 107.
Sweigart, A. V. 1978, in *The HR Diagram*, ed. A. G. D. Philip and D. S. Hayes (Dordrecht: Reidel), p. 333.
———. 1987, *Ap. J. Suppl.*, **65**, 95.
Sweigart, A. V., and Demarque, P. 1972, *Astr. Ap.*, **20**, 445.
Sweigart, A. V., Greggio, L., and Renzini, A. 1989, *Ap. J. Suppl.*, **69**, 911.
Sweigart, A. V., Gross, P. G. 1978, *Ap. J. Suppl.*, **36**, 405.
Sweigart, A. V., Mengel, J. G., and Demarque, P. 1974, *Astr. Ap.*, **30**, 13.
Taam, R. E. 1983, *Ap. J.*, **270**, 694.
Tammann, G. 1982, in *Supernovae: A Survey of Current Research*, ed. M. Rees and R. Stoneham (Dordrecht: Reidel), p. 371.
Terndrup, D. M. 1988, *A. J.*, **96**, 884.
Tinsley, B. M. 1980, *Fund. Cosmic Phys.*, **5**, 287.
Verbunt, F., and Zwaan, C. 1981, *Astr. Ap.*, **100**, L7.
Webbink, R. F., Rappaport, S., and Savonije, G. J. 1983, *Ap. J.*, **270**, 678.
Welch, G. A. 1982, *Ap. J.*, **259**, 77.
Whelan, J., and Iben, I., Jr. 1973, *Ap. J.*, **186**, 1007.
Whitford, A. E. 1978, *Ap. J.*, **226**, 777.
———. 1985, *Pub. A.S.P.*, **97**, 205.
Whitford, A. E., and Rich, R. M. 1983, *Ap. J.*, **274**, 723.
Willis, A. J. 1988, in *A Decade of UV Astronomy with the IUE Satellite*, ESA SP-281, Vol. **2**, p. 53.
Zijlstra, A. A., and Pottasch, S. R. 1989, *Astr. Ap.*, **216**, 245.
Zinnecker, H. 1984, *Ap. Space Sci.*, **99**, 41.

LAURA GREGGIO and ALVIO RENZINI: Dipartimento di Astronomia, Università di Bologna, CP 596, I-40100 Bologna, Italy