THE ASTROPHYSICAL JOURNAL, 363:255–264, 1990 November 1

LARGE MAGELLANIC CLOUD HELIUM-RICH PECULIAR BLUE SUPERGIANTS AND SN 1987A

Y. TUCHMAN AND J. CRAIG WHEELER[1]
Department of Astronomy, University of Texas at Austin
Received 1989 June 8; accepted 1990 May 2

ABSTRACT

The theoretical distribution of massive stars in the H-R diagram is compared to the revised data of Fitzpatrick and Garmany for the LMC. Preferred models of $\sim 20\ M_{\odot}$ undergo a thermal contraction at $T_{\rm eff} \sim$ 35,000 K at the end of core hydrogen burning but reestablish thermal equilibrium to the red of the main sequence at $T_{\rm eff} \sim$ 20,000 K after ignition of a hydrogen-burning shell. They then evolve on a nuclear time scale to $T_{\rm eff} \sim$ 6000 K where they lose thermal equilibrium and jump to the Hayashi track. The theoretical and observed distributions agree with two significant exceptions: the blue thermal contraction "gap" is overpopulated compared to the theory, and there is a "ledge" crossing the center of the H-R diagram. We explore the hypothesis that some of the observed stars in the blue "gap" are secondaries that have accreted helium-rich matter from deep within the hydrogen envelope of a red supergiant primary. Some preliminary observational justification is given. Other features of the observed H-R diagram and alternative modes of evolution are also discussed along with implications for Type Ib supernovae and SN 1987A.

Subject headings: stars: evolution — stars: individual (SN 1987A) — stars: interiors — stars: late-type — stars: supernovae

I. INTRODUCTION

One of the major ways to confirm our understanding of the physical processes which determine the evolution of massive stars beyond the main sequence is by comparing their theoretical evolutionary rates with the corresponding observed number density distribution in the Hertzsprung-Russell diagram (HRD).

For a reliable comparison, one needs a nearly complete survey of the massive stars for a certain galaxy, including an accurate measurement of luminosities and effective temperatures. The relatively well-known distances and the low reddening of stars in the Magellanic Clouds make these galaxies ideal candidates. The observational data even for the Magellanic Clouds are still inadequate for a detailed comparison; however, some prominent general features of theoretical massive star evolutionary rates across the HRD can and should be verified by comparison with appropriate observational data. Such a comparison for the Large Magellanic Cloud (LMC) is described in this article (§§ II and III).

A principal outcome of this analysis is the identification of a specific region in the the HRD, located just to the red of the main sequence, which should be unpopulated according to the theoretical models which otherwise fit the data, but which is found to contain a relatively large number of stars. We investigate the hypothesis that all these stars are or were members of binary systems which went through a mass accretion process, creating thereby an outer helium-enriched layer. Some preliminary theoretical and observational justifications and implications of this suggestion are discussed (§ IV). Section V summarizes the current understanding of the systematics of the HRD of the LMC.

[1] AURA Visiting Professor, National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy, Inc. (AURA), under cooperative agreement with the National Science Foundation.

II. BLUE TO RED—THEORETICAL ANALYSIS

In a previous paper (Tuchman and Wheeler 1989*a*, hereafter Paper I) it has been shown that the post–main-sequence evolution of massive stars from blue to red effective temperatures can be traced using only equilibrium models of the hydrogen-rich stellar envelopes. Models which agree with the coarse features of the HRD during this blue to red phase burn hydrogen in a shell surrounding the compact helium-burning core (Brunish and Truran 1982*a*, *b*). The hydrogen shell provides a definite inner boundary condition for the envelope: the temperature at the bottom of the envelope should be equal to the typical hydrogen-burning temperature ($\sim 2 \times 10^7$ K). This condition determines, for any given values of the total mass and luminosity, a unique locus of thermodynamic equilibrium solutions in the [helium core mass ($M_{\rm He}$) – effective temperature ($T_{\rm eff}$)]-plane (Fig. 1), where the helium core mass is determined as the mass interior to the envelope, i.e., $M_{\rm He} = M_r(T = 2 \times 10^7\ {\rm K})$.

The features of these equilibrium lines have been described in detail in Paper I. Each of these equilibrium solution lines in the ($M_{\rm He} - T_{\rm eff}$)-plane contains three distinct branches. The upper "blue" branch starts at effective temperatures above 35,000 K and extends down to about 10,000 K where the line curves backward to form the middle "reddish" branch. At about 3000 K the equilibrium solution line bends again into the red Hayashi branch. Points located to the right and above the line represent envelope models in which the rate of energy transport through the envelope is *smaller* than the rate of energy that enters the inner boundary (= the rate of energy generated in the core). These models will therefore *accumulate* energy and will expand while *decreasing* their effective temperatures, until reaching the equilibrium line. A similar analysis shows that envelope models located to the left of the line will lose energy while *increasing* their effective temperatures. The middle "reddish" branch is therefore *thermally unstable*.

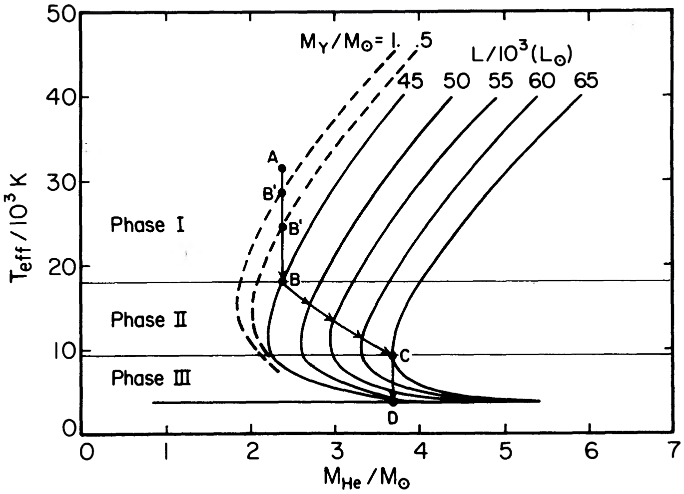

FIG. 1.—The relation between the effective temperature and the helium core mass for a 15 $M_\odot$ star is presented for selected luminosities along its evolution. ABCD is an example for a typical evolutionary track going from blue to red. The three distinct phases determined by this evolution (see text) are also marked. The dashed curves show the shift of the equilibrium solution line due to a helium-enriched ($Y = 0.5$) outer layer of 0.5 and 1 $M_\odot$.

In order to follow the post–main-sequence blue to red evolution for a star of a certain total mass, one should create equilibrium loci covering the relevant luminosity range (Fig. 1 is an example that has been created for 15 $M_\odot$). Since the post–main-sequence evolution of massive stars takes place at a relatively constant luminosity, the necessary luminosity range is usually quite narrow. This technique is not applicable to the main-sequence evolution prior to hydrogen shell ignition and we make no attempt to analyze that phase, but shall discuss the work of others in that regard in § V.

Once we have these data on the thermal equilibrium solutions, the evolution across the HRD can be determined in the following way.

At the initiation of hydrogen burning in a shell, the star still retains its typical post–main-sequence blue temperature ($\sim$40,000 K, point A in Fig. 1). At this point, the luminosity carried through the envelope (L_e) has not yet readjusted to the newly increased energy production rate, L_c, generated by the helium core and by the hydrogen-burning shell. Since $L_c > L_e$ the star is located, on the ($M_{\rm He} - T_{\rm eff}$)-plane, above the thermodynamic equilibrium line for $L = L_c$ (=45,000 $L_\odot$ in the example of Fig. 1). The envelope effective temperature will therefore decrease until reaching the equilibrium line, at B, where a complete thermal balance is achieved.

This readjustment process (A → B in Fig. 1) proceeds on a typical thermal time scale ($\sim 10^4$ yr) and thus at a relatively constant helium core mass (see eq. [3] below). The subsequent evolution of the star after attaining its equilibrium position is dictated by the internal evolutionary changes. The hydrogen-burning shell advances outward, thereby increasing the helium core mass and causing a corresponding increase of the total luminosity. In order to compare theoretical models with observations, we need to derive appropriate expressions for the rate of evolution of models in effective temperature and luminosity corresponding to the locus B-C in Figure 1. Physically, this locus arises in the following way. As the core mass grows, the luminosity increases in response to the higher gravity and pressure at the hydrogen-burning shell in a manner which is independent of $T_{\rm eff}$. This dictates the specific $L(M_{\rm He})$ locus B-C in Figure 1 which connects the curves of constant L. The response of $T_{\rm eff}$ to the growth in $M_{\rm He}$ and hence in L is given by the envelope solutions $T_{\rm eff}(L, M_{\rm He})$ represented by the family of curves in Figure 1. Once $M_{\rm He}$ is specified, L is uniquely determined through the luminosity-core mass relation and $T_{\rm eff}$ is then uniquely specified by the envelope solution.

This physical logic can be expressed analytically by writing $T_{\rm eff} = T_{\rm eff}[L(M_{\rm He}), M_{\rm He}]$ which expresses the fact that $M_{\rm He}$ is the appropriate independent variable which, along with the $L(M_{\rm He})$ relation, dictates $T_{\rm eff}$. The rate of change of the effective temperature can thus be written as

$$\frac{dT_{\rm eff}}{dt} = \frac{dM_{\rm He}}{dt}\frac{dT_{\rm eff}(L, M_{\rm He})}{dM_{\rm He}}$$

$$= \frac{dM_{\rm He}}{dt}\left[\left(\frac{\partial T_{\rm eff}}{\partial L}\right)_{M_{\rm He}}\frac{dL}{dM_{\rm He}} + \left(\frac{\partial T_{\rm eff}}{\partial M_{\rm He}}\right)_L\right]. \quad (1)$$

Equation (1) explicitly manifests the independent luminosity-core mass relation and the fact that the evolution of $T_{\rm eff}$ is determined by the $T_{\rm eff}(L, M_{\rm He})$ relation implicit in Figure 1. The partial derivatives in equation (1) could be evaluated directly from Figure 1, but to clarify the relationships among the variables determined by evolution and those determined by the envelope solutions in Figure 1, we invoke a standard identity among partial differentials of three variables to get $(\partial T_{\rm eff}/\partial M_{\rm He})_L = -(\partial T_{\rm eff}/\partial L)_{M_{\rm He}}(\partial L/\partial M_{\rm He})_{T_{\rm eff}}$. Note that the partial differentials apply to the self-contained envelope solutions $T_{\rm eff}(L, M_{\rm He})$ which are independent of the evolutionary constraint of the $L(M_{\rm He})$ relation. The result is that equation (1) can also be written as

$$\frac{dT_{\rm eff}}{dt} = \frac{dM_{\rm He}}{dt}\left(\frac{\partial T_{\rm eff}}{\partial L}\right)_{M_{\rm He}}\left[\left(\frac{dL}{dM_{\rm He}}\right) - \left(\frac{\partial L}{\partial M_{\rm He}}\right)_{T_{\rm eff}}\right]. \quad (2)$$

Equation (2) explicitly differentiates the derivative $dL/dM_{\rm He}$ which arises from the evolutionary luminosity-core mass relation from the derivative $(\partial L/\partial M_{\rm He})_{T_{\rm eff}}$ which arises from the envelope solutions. The former is that implicit in the locus B–C in Figure 1; the latter is represented by a horizontal line in Figure 1. Although these derivatives are of comparable magnitude, they are distinctly different with the evolutionary gradient being somewhat greater. It is just this difference that drives $T_{\rm eff}$ to lower values in Figure 1.

We can now evaluate equation (2). The increase of the helium core mass is controlled by the hydrogen burning rate, thus

$$\frac{dM_{\rm He}}{dt} = \frac{L_{\rm H}}{Q_{\rm H}} = 9.7 \times 10^{-12}\, L_{\rm H}\, M_\odot\ {\rm yr}^{-1}\ , \quad (3)$$

where $L_{\rm H}$ is the luminosity, in solar units, produced by the hydrogen-burning shell and $Q_{\rm H}$ is the hydrogen-burning Q-value (=6×10^{18} ergs g^{-1}). Since the equilibrium lines of constant luminosity in the ($T_{\rm eff} - M_{\rm He}$) plane turn out to be nearly parallel to each other and almost equally spaced for a constant luminosity shift, the partial derivatives in equation (2) can easily be derived.

For ($M \sim$ 15–20 $M_\odot$) one gets the following relations:

$$\left(\frac{\partial T_{\rm eff}}{\partial L}\right)_{M_{\rm He}} \simeq -1.35 \times (3 - T_{\rm eff}/10^4)\ K/L_\odot\ , \quad (4)$$

where T_{eff} is in kelvins and the expression in parentheses on the left-hand side is due to the increase in the slope of the equilibrium lines while approaching the first turning point (see Fig. 1). The second (partial derivative) term in brackets in equation (2) is also derived from Figure 1 to be

$$\left(\frac{\partial L}{\partial M_{\rm He}}\right)_{T_{\rm eff}} = 1.1 \times 10^4 \; L_\odot/M_\odot \; . \tag{5}$$

The first term in brackets in equation (2) must be evaluated independently from evolutionary calculations. There is a helium core mass–luminosity relation which is found, according to previously published evolutionary calculations, to be approximately linear (Barkat and Wheeler 1989). Such a relation resembles the well-known luminosity–core mass relation for asymptotic giant branch stars (Paczyński 1971). One gets

$$\frac{dL}{dM_{\rm He}} \sim 1.8 \times 10^4 \; L_\odot/M_\odot \; . \tag{6}$$

Since, according to equations (5) and (6),

$$\frac{dL}{dM_{\rm He}} \geq \left(\frac{\partial L}{\partial M_{\rm He}}\right)_{T_{\rm eff}} , \tag{7}$$

the effective temperature of the star will *decrease* moderately with time (see eq. [2]) along a path similar to the one presented in Figure 1 (B → C), in accord with appropriate evolutionary calculations. The rate of the effective temperature decrease, during this stage, will therefore be given by (inserting eqs. [3]–[6] into eq. [2])

$$\frac{d \log T_{\rm eff}}{dt} \simeq -2 \times 10^{-8} \frac{(L/L_\odot)}{T_{\rm eff}} \times (3 - T_{\rm eff}/10^4) \; {\rm yr}^{-1} \; , \tag{8}$$

where we have assumed $L_{\rm H} \sim L/2$. Although this relation is not very accurate quantitatively, it has the correct order of magnitude and it includes the correct dependence upon the stellar luminosity and effective temperature.

The following main conclusions should therefore be noted.

1. Since the post–main-sequence evolution of massive stars occurs at a relatively constant luminosity, the effective temperature dependence in equation (8) means that the star is accelerating its rate of advance while evolving across this portion of the HRD.

2. Assuming that relation (8) holds approximately for all massive stars, one may use the luminosity dependence to conclude that the more massive the star (the larger its luminosity), the faster it will evolve across the HRD.

The duration of this phase (given by eq. [11] below) is about two orders of magnitude larger than the previous thermal readjustment phase (A to B in Fig. 1). This evolutionary phase lasts about 2 million years for a star of 15 $M_\odot$ and half a million years for a star of 40 $M_\odot$.

This evolutionary phase continues until the upper turning point (C in Fig. 1) is reached. Since the equilibrium solutions along the middle branch are thermally unstable the star is obliged to "jump" downward, again on a thermal time scale, to the red Hayashi branch (point D in Fig. 1; see Paper I).

In order to compare the results of the above theoretical analysis with observations, it is convenient to define a normalized "duration function"—$D(L, T_{\rm eff})$ given by

$$D(L, T_{\rm eff}) = \frac{1}{t_{br}} \frac{dt}{d \log T_{\rm eff}} , \tag{9}$$

where

$$t_{br}(L) = \int_{T_{\rm eff(blue)}}^{T_{\rm eff(red)}} \left(\frac{dt}{d \log T_{\rm eff}}\right) d\,(\log T_{\rm eff}) \; , \tag{10}$$

is the total time spent by a star in crossing the HRD from blue to red.

Using equation (8) for the effective temperature range of the second phase (B → C) assuming $T_{\rm eff}({\rm blue}) \cong 35{,}000$ K and $T_{\rm eff}({\rm red}) \cong 3000$ K, and since the contribution to t_{br} from the first and the last thermal readjustment phases is negligible, one gets

$$t_{br}(L) \simeq \frac{1.5 \times 10^{11}}{L/L_\odot} \; {\rm yr} \; , \tag{11}$$

and thus

$$D(L, T_{\rm eff}) \simeq \begin{cases} \sim 0.1, & \text{Phase I (A} \to \text{B)} \\ 3.32 \times 10^{-4} \; T_{\rm eff}/(3 - T_{\rm eff}/10^4), & \text{Phase II (B} \to \text{C)} \\ \sim 0.1 & \text{Phase III (C} \to \text{D)} \end{cases} \tag{12}$$

where the effective temperature ranges of these phases (shown in Fig. 1) are given approximately by

$$\text{Phase I: } (20{,}000 \text{ K} \leq T_{\rm eff} \leq 35{,}000 \text{ K});$$

$$\text{Phase II: } (10{,}000 \text{ K} \leq T_{\rm eff} \leq 20{,}000 \text{ K});$$

$$\text{Phase III: } (3000 \text{ K} \leq T_{\rm eff} \leq 10{,}000 \text{ K}) \; .$$

III. COMPARISON WITH OBSERVATIONS

For comparison we have chosen the observational survey of the Large Magellanic Cloud as published by Garmany and Fitzpatrick (1989; the revised diagram presented by Fitzpatrick and Garmany 1990 which became available after this paper was first submitted will be discussed in § V). This survey, in which effective temperatures were assigned from *UBV* colors, shows some remarkable features of which there were only hints in previous works (Humphreys and Davidson 1979; Humphreys and McEloy 1984). One feature is the appearance of a "ledge" which cuts across the HRD from the upper left to the lower right. Another feature, possibly the more interesting one, is the main issue of the discussion below. From the observed HRD presented by Garmany and Fitzpatrick (1989) we have derived the normalized stellar density across the HRD defined by

$$N(L, T_{\rm eff}) = \frac{1}{N_{br}} \frac{dn(L, T_{\rm eff})}{d \log T_{\rm eff}} , \tag{13}$$

where $n(L, T_{\rm eff})$ is the local number density of stars in the HRD and N_{br} is given by

$$N_{br}(L) = \int_{T_{\rm eff(blue)}}^{T_{\rm eff(red)}} \left(\frac{dn}{d \log T_{\rm eff}}\right) d\,(\log T_{\rm eff}) \; . \tag{14}$$

In Figure 2 we present the normalized distribution function $N(L, T_{\rm eff})$ as compiled from the observational data for stars located in the HRD between $M_{\rm bol} = -7$ and $M_{\rm bol} = -8$. Solid dots are from Garmany and Fitzpatrick (1984); open circles the revised data from Fitzpatrick and Garmany (1990). This

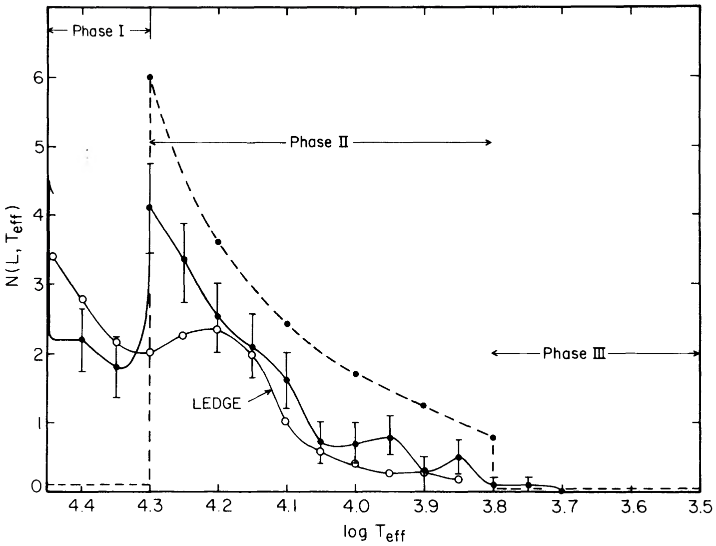

FIG. 2.—The observed normalized number density of stars across the HRD according to Garmany and Fitzpatrick (1989; *solid dots*) and the revised data of Fitzpatrick and Garmany (1990; *open circles*) are compared with the theoretical normalized duration function (eq. [12]). The three evolutionary phases (see text) are marked as well. The error bars represent $N^{1/2}$ counting statistics and are similar for both data sets but are shown only for one for clarity.

range corresponds to stars with total mass between 15 and 20 $M_{\odot}$. The observed blue boundary to Phase I is especially distinct because of a tendency to assign any star of early spectral type to "OB" or "OB0" and plot it in the same place, whereas more careful differentiation would move it to the blue or red. Theoretically, the blue boundary to Phase I represents the termination of hydrogen core burning on the main sequence prior to thermal contraction and the ignition of hydrogen in a shell. In the models of Brunish and Truran (1982*a*, *b*) with the standard Schwarzschild convection and moderate mass loss, the corresponding $T_{\rm eff}$ is insensitive to the stellar mass and metallicity and does fall at about spectral type B0 or log $T_{\rm eff} \sim 4.45$. Our theoretical analysis starts only beyond this blue boundary. The expected "duration function" (eq. [12]) for stars in this mass range is also shown in Figure 2 (*dashed line*). Models with substantial convective core overshoot can have a considerably cooler boundary to the main sequence. These models will be discussed in § V.

From Figure 2 one can draw the following main conclusions.

1. The three distinct phases that have been identified by the theoretical analysis seem to be present in the preliminary observational data as well. Phases I and II are not easily distinguished in the revised data.

2. The theoretical prediction for the dependence of evolutionary rates upon the effective temperature during the second phase (eq. [12]) is found to be compatible with observations.

3. The number of stars observed in the effective temperature range that corresponds to the "gap" of Phase I (35,000 K $\geq T_{\rm eff} \geq$ 20,000 K) is *much greater* than the theoretically expected value (by a factor of 20!). The lack of a Phase I gap is even more pronounced in the revised data. This last point is the major discrepancy between the theory and the observations; thus it deserves an explanation. A possible explanation will be described in the next section.

The "ledge" described by Garmany and Fitzpatrick (1989) and Fitzpatrick and Garmany (1990) is represented by the small kink at log $T_{\rm eff} = 4.1$ in Figure 2. At lower luminosities it is more prominent, appearing as a sharp discontinuity in the corresponding density function. One factor contributing to this ledgelike behavior may be the change in the slope of the equilibrium solution lines as the upper turning point is approached (Fig. 1). We have introduced the expression in parentheses (eq. [4]) as representing this effect of the change of slope. The value of the adopted analytic expression changes linearly between a value of 1, at the beginning of Phase II, to about 2 at the end of Phase II. This linear dependence was chosen for numerical convenience. In reality, however (see Fig. 1), the slope remains relatively constant until it grows sharply close to the turning point. This behavior should obviously cause an abrupt change (a "ledge") in the number density distribution across the HRD. Other possible contributions to this ledge will be discussed in § V.

IV. THE NATURE OF THE STARS FILLING THE "GAP"

One trivial way to "solve" the discrepancy of the number of stars observed in the Phase I "gap" is by assuming the theory to be incorrect despite the fact that such a gap in some effective temperature range is a generic feature of all evolutionary calculations. The satisfactory agreement between the theoretical analysis and the observations concerning the general trends of the evolutionary rates across the HRD demonstrated above makes this possibility even less likely. We therefore propose the following contribution towards filling the gap.

Many stars observed in the effective temperature region that corresponds to the Phase I gap are not common LMC stars.

To clarify this statement one has to provide answers to the following three questions.

a. What is the peculiarity of these stars?

b. In what way does this peculiarity cause stars to evolve through the effective temperature range of Phase I with a relatively slow (evolutionary rather than thermal) rate?

c. Is the probability for stars having this peculiarity high enough to be compatible with the observational data?

We suggest that a peculiarity of these stars that might account for at least a partial filling in of the gap is the existence of an outer mass region, of the order of 1 $M_{\odot}$, which is *helium-enriched.*

In Figure 1 (*dashed lines*) we present the effect of such a helium-enriched outer region upon the upper blue branch of the equilibrium solution curves in the ($M_{\rm He} - T_{\rm eff}$)-plane. The dashed lines in Figure 1 show the effect of adding an outer layer enriched to $Y = 0.5$ keeping the mass constant ($=15\ M_{\odot}$). As can be seen, the increase in mass of this helium-enriched layer causes a parallel shift of the equilibrium line to *higher* effective temperatures, thereby *reducing* the effective temperature range of Phase I (AB′ in Fig. 1). This region (AB′) completely disappears when the helium-rich zone (with $Y = 0.5$) exceeds about 1 $M_{\odot}$. The same general trend is found for a constant amount of enriched mass and an increased helium abundance. Thus, a star with an outer helium-enriched layer will start its moderately slow evolution along Phase II at a *higher* effective temperature, which will be determined by the exact amounts of mass and helium enrichment in this outer layer. These stars will therefore have a relatively high probability (according to eq. [12]) to be observed along the effective temperature range of Phase I which was found to be "forbidden" for common LMC stars.

We further suggest that the excess blue supergiants in the Phase I gap may have gained their outer enriched helium layer through a previous mass accretion process. The gap may be obscured by relatively large uncertainties in $T_{\rm eff}$ (see § V). A small fraction ($<3\%$) of the stars truly in the gap may be normal single stars evolving on a thermal time scale. We propose that all the other true gap stars are, or at least were, members of binary systems with more massive compansions. These companions have probably filled their Roche lobe while evolving through the red giant stage, thereby initiating extensive mass transfer and mass loss from the system. In order to determine the extent of this mass accretion process we have checked the changes in the static surface radius of a typical massive star (a 20 $M_\odot$ red supergiant) caused by the reduction of its total mass. As demonstrated in Figure 3, as long as the inner helium-rich layers (formed by the retreat of convection during the hydrogen core burning phase) are not exposed at the surface, the surface radius of the star *increases* considerably with the reduction of its total mass. This result means that the mass accretion process will not cease before some amount of helium-rich material is transferred. This process automatically leaves both the red supergiant primary and the main-sequence secondary with helium-enriched outer layers. The precise amount of helium-rich matter which will finally be accreted is determined by the specific initial conditions of the binary system; however, according to calculations presented in Figure 3 this helium-enriched accreted mass is at least a few tenths of a solar mass and not much more than 1 $M_\odot$. For the case demonstrated in Figure 3, the Roche lobe radius at the initiation of the mass transfer process is about 970 $R_\odot$. The radius of the donating star will drop below this value only after losing about 1 $M_\odot$ of helium-enriched material and total loss of envelope mass of about 13 $M_\odot$.

We may therefore summarize this issue by the following crucial conclusion.

Every mass transfer process that occurs between massive stars causes the accreting member to become peculiar, according to our previous definition, by leaving the secondary with an

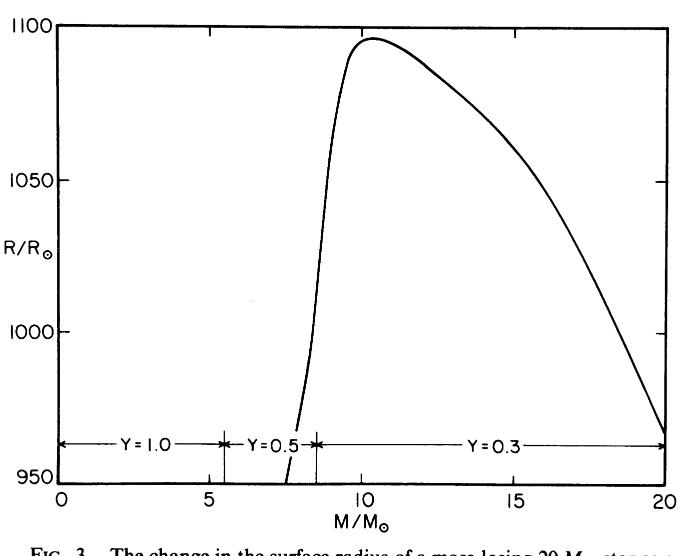

FIG. 3.—The change in the surface radius of a mass-losing 20 $M_\odot$ star as a function of its remaining mass is presented. The helium content as a function of mass is shown along the X-axis.

appreciable amount of helium-rich material at its surface. The probability of observing these peculiar massive blue stars in their Phase II compared to normal (redder) Phase II stars thus becomes comparable to the relative occurrence of stars in this mass range that belong to binary systems.

Using equation (12) for evaluating the rate of advance of such helium-enriched stars across the Phase I effective temperature region for normal stars (neglecting the helium enrichment on the rate of evolution in thermal equilibrium) and the distribution of Figure 2, one finds that the required relative frequency for these helium-enriched stars must be about 1 in 4 in order to be compatible with the observational data of Garmany and Fitzpatrick (1989). This value is not unreasonable for the fraction of stars in close binaries. These peculiar blue giants may thus be cousins to the lower mass barium stars (McClure 1983) that are believed to result from a similar process of mass accretion. On the other hand, the revised HRD of Fitzpatrick and Garmany (1990) shows even higher densities where the gap should be. This emphasizes the problem of the absence of the predicted gap, but begins to strain the interpretation purely in terms of duplicity. We return to this point below.

The obvious direct way to test our proposal for these peculiar giants is by checking their composition in comparison with common LMC stars scattered along the effective temperature region that corresponds to Phases I and II. If the density of normal stars evolving on a thermal time scale in Phase I is about 1/50 that of the density at the onset of Phase II and the observed density is about 1/2, then about 90% of the stars in Phase I should be helium-rich. The fraction of stars at a given $T_{\rm eff}$ in Phase II that are helium-rich should just be the fraction that undergo helium enrichment, that is, approximately the binary fraction. Stars to the blue of Phase I should be predominantly normal main-sequence stars, although there could be a small proportion of helium-enriched shell-burning stars.

Kudritzki *et al.* (1989) have used non–LTE atmosphere models to determine helium abundances in the photospheres of five blue LMC stars. *All* the stars that have been checked were found to be highly helium-enriched ($Y \geq 0.5$). Due to the specific spectral type selection criteria, however, *all* these stars have been selected from the Phase I region where the density is anomalously high compared to basic (unenriched) theoretical models. The results of Kudritzki *et al.* (1989) demand some explanation. They are *compatible* with our suggested scenario, but they are not sufficient to prove it.

The suggested mass transfer phase probably takes place when the accreting star is still in its main-sequence stage while its companion is already a red supergiant. Since we observe this accreting star as a peculiar blue giant only beyond the main-sequence stage, its companion is most likely to have completed its evolution. Such a star is expected to explode as a supernova or to collapse, leaving behind a neutron star or a black hole. Since any explosion is expected to occur after mass has been stripped from the primary, there is a fair probability that less than half the mass of the system will be ejected, meaning the collapsed companion is likely to be retained in binary orbit. These peculiar blue giants should therefore be examined closely for evidence of duplicity. They could be (perhaps weak) X-ray sources. Likewise, high-mass binary X-ray sources could and should have helium-rich companions. If the original primary explodes as a hydrogen-stripped core, the result could resemble a Type Ib supernova. The peculiar blue giants could thus be another clue that SNIb events occur in binary systems, as argued by Wheeler and Levreault (1985) and Wheeler *et al.* (1987).

Alternatively, Figure 3 shows that the Roche lobe overflow is apt to cease when the primary has shed mass down to the helium-enriched layers left by the retreating convective core during main-sequence burning. If further wind-type mass loss does not expose the core, then the primary could have a helium-enriched hydrogen envelope at the time of explosion. Figure 3 suggests that a 20 $M_\odot$ star would be left with an envelope of 4–5 $M_\odot$. Barkat and Wheeler (1989; their Fig. 7) have shown that the observed conditions of the progenitor of SN 1987A, Sk $-69°202$, require an envelope of ~ 2–4 $M_\odot$ for $Y \sim 0.3$–0.4 if the envelope is uniformly enriched. Envelopes of this mass are at the lower end of those favored by studies of the light curve (e.g., Woosley 1988). The suggestion of helium and barium enrichment in SN 1987A (Arnett *et al.* 1989, and references therein) leads one to speculate that the progenitor may have been one of these stars left in a helium-enriched state after undergoing Roche lobe overflow.

This possibility is tempting since some wind mass loss in addition to transfer would be compatible with the observed low-velocity circumstellar shells (Fransson *et al.* 1987), and it overcomes the difficulties connected with the return of the star from the red to its final blue location (Tuchman and Wheeler 1989*b*) by adding the physical mechanism of mass loss to a companion. One might also consider that Sk $-69°202$ were the accreting secondary in a binary system and hence one of the peculiar helium-enriched blue giants that filled the post–main-sequence "gap." This suggestion would not easily account for the circumstellar matter because such a star is likely never to have evolved to the Hayashi track.

V. DISCUSSION

The analyses of Garmany and Fitzpatrick (1989) and Fitzpatrick and Garmany (1990) and in the current paper suggest that questions of the evolution of Sk $-69°202$ from blue to red and return (Tuchman and Wheeler 1989*a*, *b*) require continued examination. As argued in Paper I, helium-enrichment during the passage from blue to red, as suggested here for a significant fraction of the massive stars in the LMC, would tend to prevent the star from reaching the Hayashi track, but might lead it to end its evolution near the location on the HRD where Sk $-69°202$ was observed. If the progenitor of SN 1987A were peculiar, then one cannot constrain its evolution by comparison to "average" evolution in the LMC. A key question is what comprises average evolution in the LMC.

Figure 4 shows the revised HRD from Fitzpatrick and Garmany (1990) with several relevant loci determined by the analysis of the systematics of the blue to red evolution as presented in Paper I. The key features to be explained are the paucity of stars that begins around log $T_{\rm eff} \sim 3.8$, the unexpectedly high concentration of stars where a gap is predicted from log $T_{\rm eff} \sim 4.3$–4.45 and the "ledge" which runs across the diagram from upper left to lower right shown by the thin solid line in Figure 4. We will discuss each of these features in turn.

a) The Red "Hertzsprung" Gap

Brunish and Truran (1982*b*) and Tuchman and Wheeler (1989*a*) identified the paucity of stars beginning at about log $T_{\rm eff} \sim 3.8$ as that corresponding to the "Hertzsprung gap" in clusters of moderate turn-off mass. As shown in Figure 4, models with $Y = 0.3$, $Z = 0.005$, and no mass loss place the locus of the breakdown of thermal equilibrium too far to the blue and with the wrong slope to account for this feature. Models with moderate mass loss (adopted from Brunish and Truran 1982*b*) put the locus of the breakdown of the hydrogen shell-burning thermal equilibrium very close to the observed boundary of the gap. Models with mass loss but $Y = 0.4$ again put the locus much too far to the blue.

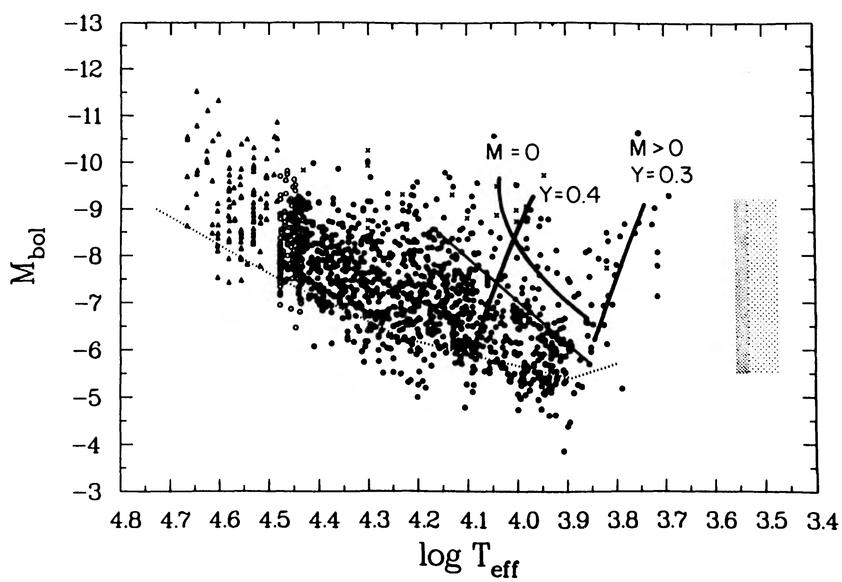

FIG. 4.—The Hertzsprung-Russell diagram of Fitzpatrick and Garmany (1990) is presented along with several observational and theoretical loci. The dashed line is an estimate of the completeness limit and the thin solid line represents the boundary of the "ledge." Other lines represent the blue edge of the Hertzsprung gap where models are predicted to jump to the Hayashi track for the standard case with moderate mass loss ($\dot{M} > 0$) and normal helium abundance in the envelope ($Y = 0.3$), with moderate mass loss and enhanced helium abundance ($Y = 0.4$), and with normal helium abundance and no mass loss, as indicated. The gray strips show the location of the Hayashi track.

A very different interpretation of the red gap has been suggested by people investigating convective core overshoot in the core hydrogen-burning phase. For instance, Meylan and Maeder (1982), Bertelli, Bressan, and Chiosi (1984), and Doom (1985) argue that the main sequence iself extends to log $T_{\text{eff}} \sim$ 3.9–4.25 as a result of core overshoot or other phenomena. In this interpretation, the red gap plays the role of the blue Phase I gap in the current discussion. Several factors contribute to this interpretation. One is that previous HRDs of the Galaxy, LMC, and SMC were relatively sparse, suggesting an approximately uniform distribution from O to A stars (compare the diagrams from Humphreys and Davidsen 1979 with Fig. 4). Another factor is the focus on the ratio of main-sequence to post–main-sequence stars. The argument presented in these papers is that there are too few main-sequence stars compared to B and A supergiants. This argument assumes the completeness of the sample of main-sequence O stars, whereas there are problems in this regard as discussed below. A third factor is that the overshoot models make the transition from the main sequence to the Hayashi track on the thermal time scale of $\sim 10^4$ yr, so that they predict virtually no B and A supergiants unless they extend the main-sequence phase to those spectral types. This property is intrinsic to the overshoot models because they generically predict larger core masses for a given total mass or, equivalently, a smaller total mass for a given helium core mass. One can see from Figure 5 of Paper I that if a helium core of a given mass resides in a larger mass star, the envelope can find an intermediate temperature thermal equilibrium solution, whereas the same mass core in a lower mass star will yield no thermal equilibrium solutions between the main sequence and the Hayashi track.

If the concentration of stars at log $T_{\text{eff}} \sim 4.45$ in Figure 4 has any reality, then it is difficult to avoid the conclusion that it represents the boundary of the main sequence. This conclusion is reinforced by the models of Brunish and Truran (1982*a*, *b*) that put the main sequence there and account for the distribution of the B and A supergiants as a slowly evolving post–main-sequence phase. There thus seems no need to invoke a main sequence that extends to A stars in this mass range. Indeed, models with sufficient overshoot to accomplish this may be too extreme to account for other observational features (Maeder and Meynet 1987, 1988, 1989).

An observational caveat should be noted here. C. D. Garmany (private communication, 1989) cautions that the yellow portion of the HRD may simply not be searched thoroughly because of the problem of contamination by abundant Galactic yellow dwarfs. R. M. Humphreys (private communication, 1989) comments that she has never had a problem with contamininating foreground stars and believes the red gap is real. Clearly, this is an issue that must be resolved with some certainty lest theorists overinterpret the data. Paper I showed that the expected change in locus of the boundary of this gap with metallicity seemed to correspond to observations of the Galaxy, LMC, and SMC. For now, the agreement with the observations thus seems satisfactory, and physically well motivated, but other interpretations must be borne in mind.

b) The Missing Blue Post–Main-Sequence Gap

There is no sign in Figure 4 of the post–main-sequence contraction gap that is predicted by all evolutionary models, and by the analysis presented here. Before discussing this feature, a comment is appropriate concerning the concentration of stars at log $T_{\text{eff}} \sim 4.45$ and the paucity of stars to the blue of that. As mentioned earlier, the concentration arises because of a tendency for observers using low-dispersion classification spectra to identify a star as simply "OB" or "OB0." Such stars then get plotted with the T_{eff} of a B0 star, i.e. log $T_{\text{eff}} \sim 4.45$. This increase is thus partly artificial, although it is also affected by the physical increase in stellar density expected as one goes blueward of the terminal-age main sequence (TAMS) and begins to pick up main-sequence stars. The low density at log $T_{\text{eff}} > 4.5$ is due to a deliberate decision by Fitzpatrick and Garmany (1990) to classify as O-type only those stars with slit spectral classification. Photometry alone becomes degenerate for O stars, also leading to the concentration at $\sim$B0. There must be an increase in stellar density as one moves into temperatures corresponding to the main sequence, and yet this is not seen. This is surely a major challenge to observers to find and catalog the missing main-sequence population of massive stars in the LMC which tend to be hidden in the cores of H II regions and associations. There are main-sequence stars in associations that are beginning to be recorded (Massey *et al.* 1989), but which are not yet in the study of Fitzpatrick and Garmany.

Bearing these uncertainties in mind, the blue distribution of stars in Figure 4 is still very interesting. Fitzpatrick and Garmany (1990) summarize a number of evolutionary calculations, and, although they have different assumptions and different behavior at cooler temperatures, they have one feature of the post–main-sequence evolution in common. They all have a post–main-sequence thermal contraction gap such that no models or combination of models predicts that there should be any concentration of stars in the range of effective temperature log $T_{\text{eff}} = 4.3$–4.4. For individual models the predicted gap can shift around somewhat and is generally wider. As shown in Figure 2, this gap seemed to be discernible in the preliminary LMC HRD of Garmany and Fitzpatrick (1989), but the temperature range to the red of the main sequence which should be evacuated due to the rapid evolution of the stars was, in fact, well-populated. Figures 2 and 4 show that the revised data give virtually no sign of the post–main-sequence gap at all. We note that a very distinct, partially filled gap was also a feature of the HRD of the SMC presented by Garmany and Fitzpatrick (1989). It is very important to know whether continued analysis will preserve or remove this apparent post–main-sequence gap in the SMC.

The large increase in the stellar density at log $T_{\text{eff}} \sim 4.45$, the blue boundary of the Phase I gap, while confused by the tendency to assign artificially this effective temperature, is, nevertheless, the locus of the TAMS for reasonable models. Models with moderate overshoot (0.25 times the pressure scale height) and solar metallicity by Maeder and Meynet (1988), as illustrated in Fitzpatrick and Garmany (1990), have a TAMS at log $T_{\text{eff}} = 4.4$. This is still insufficiently red to explain the population of the predicted gap to the blue of log $T_{\text{eff}} = 4.3$, and this TAMS would be further to the blue if a metallicity appropriate to the LMC, $Z < 0.01$, were invoked. As remarked earlier, these overshoot models predict virtually no B and A supergiants during the first pass from the blue to the red. It is conceivable, in principle, to invoke somewhat more overshoot than done by Maeder and Meynet and move the red edge of the main sequence sufficiently further to the red to cover the gap predicted with the "standard" models of Brunish and

Truran (1982*a*, *b*). There are several problems with this possibility. It would give no means to explain the concentration at log $T_{\rm eff} \sim 4.45$, and hence this feature would have to be entirely artificial. If the red edge of the main sequence were moved to log $T_{\rm eff} \sim 4.3$ with overshoot, another gap would just open up to the red of that, extending all the way to the Hayashi track. Unless this gap could be filled with subsequent blue loops, the problem would not be avoided, merely displaced. Finally, such a hypothesis would give no account as to why the stars analyzed by Kudritzki *et al.* (1989) in the range log $T_{\rm eff} =$ 4.3–4.45 were helium-rich.

The red edge of the Phase I gap expected theoretically, the boundary between Phases I and II where thermal equilibrium is reestablished in the hydrogen shell-burning phase, corresponds to the observed increase in stellar densities at log $T_{\rm eff} \sim 4.3$ in the preliminary HRD of Garmany and Fitzpatrick (1989). The observed rise was so steep that some crudeness or artificiality in spectral type assignment was again suspected. In the revised data (Fig. 2), this rise has disappeared, even, perhaps, to be replaced by a local minimum.

As argued in the previous section, the predicted thermal gap in Phase I could be partially filled in by stars which have accreted helium-enriched matter from a companion star. That this cannot be the whole explanation for the filling of the gap is strongly suggested by the presentation of the final HRD of Fitzpatrick and Garmany (1990) as given in Figures 2 and 4. Between the preliminary analysis of Garmany and Fitzpatrick (1989) and the final version, more stars from the "OB" bin were assigned effective temperatures by means of *UBV* photometry rather than objective prism spectra. Many of these stars moved to cooler effective temperatures (E. L. Fitzpatrick 1989, private communication) and there was apparently some redistribution of the stars near log $T_{\rm eff} = 4.3$. The result is that there is now absolutely no sign of the post–main-sequence "gap." The conflict with all evolutionary models is thus more severe. The binary hypothesis can contribute to filling the gap, but it does not provide a natural way of precisely obliterating it. Considering the different rates of evolution of the normal stars at log $T_{\rm eff} \sim 4.2$ and the putative helium-enriched stars filling the gap at log $T_{\rm eff} \sim 4.4$ with comparable density in Figure 2 (see eq. [12]), one would now need about one star in two to be suitably helium-enriched, and this begins to seem extreme. The mystery of the missing "gap" thus is made more severe by the distribution shown in Figure 4.

If anything, there are even more stars where the gap should be at log $T_{\rm eff} > 4.3$ compared to the distribution at log $T_{\rm eff} <$ 4.3 where normal stars should reestablish thermal equilibrium. Helium-enrichment of the outer layers would tend to fill the gap as illustrated in Figure 1. Binary mass transfer is one plausible way to provide such helium-enrichment, but there may be others.

Another possibility that must be considered is that the gap is real and present, but is filled in because the assigned effective temperatures are uncertain and thus the boundaries of the gap are smeared and the gap itself obscured. Uncertainties of 2000–4000 K or of order 10–20% in $T_{\rm eff}$ could severely erode the definition of the gap. Another factor to be considered is duplicity which is suspected to lead to apparently broader, redder main sequences for Galactic open clusters by adding the light from a redder, undetected companion. Neither of these explanations for the partial filling of the gap accounts for the helium-enrichment observed by Kudritzki *et al.* (1989), but they may play some role.

c) The Ledge

The other important feature of the HRD of Fitzpatrick and Garmany (1990) is the prominent "ledge." Fitzpatrick and Garmany estimate that the stellar density jumps by a factor of about 6 across the ledge. The change in density between log $T_{\rm eff} = 4.2$ and log $T_{\rm eff} = 4.05$ in Figure 2 is indeed a factor of 5.7. The ledge feature in Figure 2 appears somewhat less sharp than it actually is because the effective integration from $M_{\rm bol} = -7$ to -8 covers a range in the location of the ledge from log $T_{\rm eff}$ of 4.0–4.1. The local maximum in the revised data at log $T_{\rm eff} \sim 4.2$ may be a statistical fluctuation. If real, a contribution to it may again come from the integration from $M_{\rm bol} = -7$ to -8 which includes the greater number of lower luminosity stars which are further from the ledge and dominate this hump.

Note two juxtapositions that may be coincidental or significant. One is that Sk $-69°202$ fell near the edge of the ledge at $M_{\rm bol} \sim -8$ and log $T_{\rm eff} \sim 4.2$. The second is that the locus of the boundary of the theoretical Hertzsprung gap in models with no mass loss in Figure 4 falls rather near, and roughly parallel to, the locus of the boundary of the ledge. The origin of this ledge, and its possible relation to Sk $-69°202$ which sat astride it, emerges as one of the central problems of the evolution of massive stars in the LMC.

Fitzpatrick and Garmany point out that Chiosi and Summa (1970) computed models with the Ledoux criteron that depart from the Hayashi track, or near it, and move back to the blue to spend an appreciable time in core helium burning in the blue before moving back to the Hayashi track near the end of their lives. Fitzpatrick and Garmany suggest that such models might give a natural explanation for the ledge. The idea is that the models should evolve to the red, then return to the blue for most of core helium burning. The models of Chiosi and Summa then lose thermal equilibrium once again and return to the Hayashi track. This onset of the rapid return to the red would then represent the ledge.

Fitzpatrick and Garmany (1990) examine the models of Brunish and Truran (1982*b*) and argue that models invoking the Schwarzschild criterion undergo a thermal jump to the Hayashi track at a nearly constant effective temperature, independent of luminosity. This seems to be the case for the Brunish-Truran models with $Z = 0.01$, but not in general for their models with either higher or lower Z, as illustrated by Paper I and Fig. 4, where the boundary is a function of luminosity, mass loss, and Y for $Z = 0.005$. Fitzpatrick and Garmany argue that models with the Ledoux criterion have blue loops with a blueward extent that depends on mass, but the blueward extent may not be relevant to the ledge, since the ledge is hypothesized to correspond to the breakdown in thermal evolution as the stars evolve back to the Hayashi track. Fitzpatrick and Garmany imply that the Ledoux models do seem to put the redward thermal contraction gap in the correct place, but this is not clear since many of the models they cite also seem to give the boundary of the gap at log $T_{\rm eff} \sim 3.95$ which is too red for the location of the ledge for a star of 20 $M_\odot$, $M_{\rm bol} \sim -8$. Furthermore, no model with such blue loops places any stars sufficiently blue to fill in the post–main-sequence gap, so such models are surely no panacea.

The analysis of the blue to red and red to blue evolution in the LMC presented by Tuchman and Wheeler (1989*a*, *b*) omitted the possibility of a blue loop during core helium burning that can, in principle, place a significant number of stars in the middle of the HRD for a significant time. We note that our analysis of thermal equilibrium envelopes should be useful to better understand the existence and properties of blue loops during core helium burning. In this case the pertinent diagrams are those like Figure 1 because the hydrogen shell is still active during the blue loops with core helium burning. The tentative hypothesis would be that the helium cores exceed the maximum permissible value to have thermal equilibrium solutions on the Hayashi track. This maximum core mass is again presumably a function of the parameters of the model and especially the distribution of helium in the hydrogen-rich envelope as affected by assumptions concerning convective mixing. Gaps and ledges aside, any theory of the distribution of stars in the HRD of the LMC based on blue loops during core helium burning must be able to explain the distribution of stars from log $T_{\rm eff} \sim 4.3$ to log $T_{\rm eff} \sim 3.8$ in Figure 2 at least as well as the hydrogen-shell burning interpretation presented in Figure 2.

Returning to Figure 4, we emphasize that the boundary of the Hertzsprung gap derived for models with $Z = 0.005$, $Y = 0.3$, and no mass loss falls very close to the ledge of Fitzpatrick and Garmany. Our models with mass loss fall very close to the apparent gap at log $T_{\rm eff} \sim 3.8$. This raises the speculation as to whether there are two populations of massive stars in the LMC.

We have suggested here that there is a second population of post–main-sequence stars which have helium-enriched layers on the outside, specifically the result of binary mass transfer. We note that a small enhancement of the helium abundance in the models with no mass loss would plausibly move the boundary of the thermal contraction gap in Figure 4 to very nearly coincide with the observed locus of the ledge. Computations are underway to determine where the thermal contraction gap is predicted to be for stars with sufficient helium-enhancement in the outer layers to close the main-sequence gap. The factor of 6 decrease in the observed density across the ledge in Figure 4 is less than would be expected from equation (12) at log $T_{\rm eff} \sim 4.1$ by about a factor of 3 if the ledge were to be explained solely as a boundary where thermal equilibrium is lost. This suggests that the stars to the red of the ledge are still in thermal equilibrium and evolving on a nuclear time scale, but it does not rule out that they are evolving on a thermal time scale.

Two other potential explanations for the
worth mentioning. We note that some evolutionary calculations (Arnett 1987; Hillebrandt *et al.* 1987) never reach the red, but halt near the final resting place of Sk $-69°202$. In principle, such evolution might account for the ledge, if there were some other population or phenomenon to account for the stars to the red of the ledge. In practice, the ledge slopes toward higher luminosity at higher $T_{\rm eff}$, whereas the tracks of Hillebrandt *et al.* form a nearly vertical locus for stars of 15 and 20 $M_\odot$ for $Z = 0.005$, and for $Z = 0.001$ they give a locus with the opposite slope to that observed. Nevertheless, this hypothesis has the feature that it would give a natural explanation for the location of Sk $-69°202$ near the ledge. It would not, however, account for the evidence from the circumstellar emission that the star had recently been a red supergiant (Fransson *et al.* 1989). This interpretation seems natural and quantitatively justified. It is, nevertheless, the only direct evidence that Sk $-69°202$ had been a red supergiant. Since this conclusion is so crucial in the current context, and there is no independent confirmation, it would thus be very interesting to consider whether there are other ways to interpret these observations than resulting from a red supergiant wind.

Another possibility is that models with overshoot are successful despite the arguments given earlier and the ledge represents the boundary of the true main sequence. In principle, overshoot models with mass loss can give a locus to the main sequence that is roughly parallel to the observed ledge, but they would have to be very extreme to put the main sequence as red as the ledge, especially at lower luminosities, $M_{\rm bol} \sim -6$ (Maeder and Meynet 1987). This explanation could not account for the location of Sk $-69°202$ near the ledge.

We are thus left with a number of questions. What is the cause of the obliteration of the predicted post–main-sequence gap in Figure 4? What is the distribution of surface helium abundance as a function of luminosity to the red and blue of the predicted edge of the thermal contraction gap for normal stars at log $T_{\rm eff} \sim 4.3$? What is the effect of mass transfer on the main sequence and post–main-sequence evolution of a mass accreting star? Do such stars linger near the main sequence longer than their luminosity and effective temperature would indicate? If so, where? Where is the predicted locus of thermal contraction to the Hayashi track of the putative helium-enriched stars that we propose to fill partially the post–main-sequence gap? Can helium core-burning blue loops account for the basic distribution of stars in the HRD of the LMC? For the ledge? Are the stars to the red of the ledge evolving on a nuclear or thermal time scale? Have these stars lost more mass than the stars to the blue of the ledge? Are the stars to the blue of the ledge more helium-enriched than those to the red? Is the gap beginning at log $T_{\rm eff} \sim 3.8$ real or an artifact of small number statistics? If the ledge is related to the breakdown of thermal equilibrium of helium-enriched stars or those in helium core-burning blue loops, etc., as they evolve to the red and then jump to the Hayashi track, then why did Sk $-69°202$ sit near the ledge, since it was presumably engaged in a blueward loop when it exploded? Could the ledge be the boundary of a population of stars that never do evolve to the Hayashi track? If so, is it significant that Sk $-69°202$ was among them or just a coincidence? The answers to these observational and theoretical questions should give very useful insight into the evolution of Sk $-69°202$, stars in the LMC, and massive star evolution in general.

We are indebted to Katy Garmany and Ed Fitzpatrick for permission to share the fruits of their labors prior to publication and to both of them and to Roberta Humphreys for stimulating discussions of the distributions of stars in the Magellanic Clouds. J. C. W. is especially grateful to AURA and to Bob Williams and the staff of CTIO for hospitality while this paper was completed. This work is supported in part by NSF grant 8717166, by a grant from the University Research Institute of the University of Texas at Austin, and by the computational facilities of the University of Texas System Center for High Performance Computing.

REFERENCES

Arnett, W. D. 1987, *Ap. J.*, **319**, 136.
Arnett, W. D., Bahcall, J. N., Kirshner, R. P., and Woosley, S. E. 1989, *Ann. Rev. Astr. Ap.*, **27**, 629.
Barkat, Z., and Wheeler, J. C. 1989, *Ap. J.*, **341**, 925.
Bertelli, G., Bressan, A. G., and Chiosi, C. 1984, *Astr. Ap.*, **130**, 279.
Brunish, W. M., and Truran, J. W. 1982*a*, *Ap. J.*, **256**. 247.
———. 1982*b*, *Ap. J. Suppl.*, **49**, 447.
Chiosi, C., and Summa, C. 1970, *Ap. Space Sci.*, **8**, 478.

Doom, C. 1985, *Astr. Ap.*, **142**, 143.
Fitzpatrick, E. L., and Garmany, C. D. 1990, *Ap. J.*, **363**, 119.
Fransson, C., Cassatella, A., Gilmozzi, R., Kirshner, R. P., Panagia, N., Sonneborn, G., and Wamsteker, W. 1989, *Ap. J.*, **336**, 429.
Garmany, C. D., and Fitzpatrick, E. L. 1989, *IAU Colloquium 113, Physics of Luminous Blue Variables*, ed. K. Davidsen, A. F. J. Moffatt, and H. J. G. L. M. Lamers (Dordrecht: Kluwer), p. 83.
Hillebrandt, W., Höflich, P., Truran, J. W., and Weiss, A. 1987, *Nature*, **327**, 597.
Humphreys, R. M., and Davidson, K. 1979, *Ap. J.*, **232**, 409.
Humphereys, R. M., and McEloy, D. B. 1984, *Ap. J.*, **284**, 565.
Kudritzki, R. P., Gabler, A., Gabler, R., Groth, H. G., Pauldrach, A. W. A., and Puls, J. 1989, in *IAU Colloquium 113, Physics of Luminous Blue Variables*, ed. K. Davidson, A. F. J. Moffatt, and H. J. G. L. M. Lamers (Dordrecht: Kluwer), p. 67.
Maeder, A., and Meynet, G. 1987, *Astr. Ap.*, **182**, 243.
———. 1988, *Astr. Ap. Suppl.*, **76**, 411.
———. 1989, *Astr. Ap.*, **210**, 155.
Massey, P., Garmany, C. D., Silkey, M., and Degoia-Eastwood, K. 1989, *A.J.*, **97**, 107.
McClure, R. D. 1983, *Ap. J.*, **268**, 264.
Meylan, G., and Maeder, A. 1982, *Astr. Ap.*, **108**, 148.
Paczyński, B. 1971, *Acta Astr.*, **21**, 417.
Tuchman, Y., and Wheeler, J. C. 1989*a*, *Ap. J.*, **344**, 835 (Paper I).
———. 1989*b*, *Ap. J.*, **346**, 417 (Paper II).
Wheeler, J. C., Harkness, R. P., Barker, E. S., Cochran, A. L., and Wills, D. 1987, *Ap. J.* (*Letters*), **313**, L69.
Wheeler, J. C., and Levreault, R. 1985, *Ap. J.* (*Letters*), **294**, L17.
Woosley, S. E. 1988, *Ap. J.*, **330**, 218.

Y. Tuchman: Department of Physics, Hebrew University of Jerusalem, Jerusalem, Israel

J. Craig Wheeler: Department of Astronomy, University of Texas, Austin, TX 78712