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ON THE SANDAGE PERIOD SHIFT EFFECT AMONG FIELD RR LYRAE STARS

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ABSTRACT

The synthetic horizontal branch (HB) calculations of Lee, Demarque, and Zinn are extended to all HB types over the metallicity range of the Galactic halo. It is shown that the period shifts of the RR Lyrae stars in globular clusters depend sensitively on their HB type as well as metallicity. These models suggest that the disagreement between the slopes, $\Delta \log P(T_{\rm eff})/\Delta$[Fe/H], obtained by Lee, Demarque, and Zinn from their theoretical simulations and the one obtained by Sandage from his recent analysis of Lub's sample of field RR Lyrae stars can be explained if Lub's sample contains many highly evolved stars in the metallicity range of $-2.0 \leq$ [Fe/H] ≤ -1.6. The analysis of Lee, Demarque, and Zinn did not include these stars. When such an effect is taken into account, the correlation between the period shift and [Fe/H] among the field RR Lyrae stars matches the theoretical model calculations to within the errors, as is the case for the RR Lyraes in globular clusters.

It is shown, however, that the mean RR Lyrae luminosity–[Fe/H] relationship is less affected by this effect. The present models for 33 clusters suggest that the best estimate for the mean RR Lyrae luminosity is $\langle M_V^{\rm RR} \rangle = 0.19$[Fe/H] $+ 0.97$ (for $Y_{\rm HB} \approx 0.22$), in excellent agreement with the recent Baade-Wesselink measurements of field RR Lyrae stars. Consequently, a significant correlation is found between age and [Fe/H] among the globular clusters that spans $\sim$4 Gyr (for constant [O/Fe]) over the metallicity range of the Galactic halo. However, the slope of this correlation between age and metallicity will be reduced if [O/Fe] varies steeply with [Fe/H] as recently suggested by Abia and Rebolo.

Subject headings: clusters: globular — stars: abundances — stars: horizontal branch — stars: pulsation — stars: RR Lyrae

I. INTRODUCTION

From the new calculations of synthetic horizontal branch (HB) models and more detailed analyses of the period shift observations of RR Lyrae stars in globular clusters, Lee, Demarque, and Zinn (1990*a*, hereafter LDZ; see also 1987, 1988) have obtained several important results. First, evolution away from the zero-age horizontal branch (ZAHB) leads to longer periods at a given $T_{\rm eff}$ in the Oosterhoff group II clusters than in the group I clusters by increasing the luminosity and lowering the mass of the stars in the instability strip, compared with the values predicted by ZAHB models. Second, Sandage's (1982*a*) assumption that light curve shape (rise time) and amplitude are unique functions of $T_{\rm eff}$ is at odds with the observations of the RR Lyrae stars in the globular cluster ω Cen and of a sample of field variables, which show that both quantities depend on [Fe/H] as well as $T_{\rm eff}$. And third, when these effects are taken into account, the observed relationship between period shift (determined from rise time data; Sandage 1982*a*) and [Fe/H] matches the theoretical model calculations to within the errors.

However, Sandage (1990*b*) has recently presented a new analysis of the period shifts between clusters which involves the measurements of the position of the ZAHB in the log P–log $T_{\rm eff}$ diagram. In this analysis, he made no use of amplitude and rise time data, but the period shifts were determined directly from temperature data, making moot the criticisms of LDZ and Caputo (1988) concerning the former approach. From the ZAHB positions (which he defines as the lower envelope of the scatter of the variables in the log P–log $T_{\rm eff}$ diagram), Sandage (1990*b*) obtained $\Delta \log P(T_{\rm eff})/\Delta$[Fe/H] $\approx -0.12(\pm 0.03)$, which is much steeper than the slope given by the synthetic HB models ($\Delta \log P(T_{\rm eff})/\Delta$[Fe/H] ≈ -0.047; see LDZ and below).

As is well-known (see Caputo 1988; LDZ; Sandage 1990*b*), the reality of this correlation depends on the correctness of the reddening corrections. For example, if we use Zinn's (1980) reddening values, instead of the values adopted by Sandage, the correlation becomes $\Delta \log P(T_{\rm eff})/\Delta$[Fe/H] $\approx -0.061(\pm 0.03)$, excluding the uncertain data for NGC 6712 (see Sandage 1990*a*). This value is consistent with the synthetic HB calculations to within the fitting error (see LDZ and Sandage 1990*b* for extensive discussions concerning the sensitivity of the correlation to errors in the adopted reddenings). Furthermore, LDZ have already pointed out the following uncertainties involved in Sandage's (1990*b*) analysis: (1) LDZ models suggest that in the Oosterhoff group II clusters, including M15, M92, and ω Cen (see Lee 1990), which were analyzed by Sandage, very few (if any) of the variables are near their ZAHB locations. If this is correct, Sandage's (1990*b*) analysis is likely to attribute to the ZAHB larger period shifts for these clusters relative to M3 than they actually have. (2) Even in clusters that are expected to have variables near the ZAHB, their positions in the log P–log $T_{\rm eff}$ diagram are much less well-determined than the mean location of the variables, because the lower boundary of a distribution is influenced more by the observational errors in the photometry and by statistical fluctuations stemming from the finite samples of variables. It is important to note that LDZ have judged the success of their models on how well they can reproduce the *mean* positions of the cluster variables in the log P^N log $T_{\rm eff}$ diagram.

Sandage (1990*b*) has justified the correctness of his reddening values and other uncertainties involved in his analysis of the cluster data by showing that a similar period shift–metallicity relation [$\Delta \log P(T_{\rm eff})/\Delta$[Fe/H] ≈ -0.08] is obtained for Lub's (1977, 1987) sample of field RR Lyrae stars (where reddening values are well-determined). He considered this as evidence against the models presented by LDZ and to be strong support for a steep variation in the HB luminosity with [Fe/H] that yields the same age (for constant [O/Fe]) for all globular clusters regardless of [Fe/H]. (Of course, Sandage has implicitly assumed that the RR Lyrae stars in the field are similar to the ones in the globular clusters.) Consequently, the current debate has now moved to Lub's (1977, 1987) sample of field RR Lyrae stars.

The purpose of this paper is to investigate the origin of the disagreement between the slopes obtained by LDZ from the synthetic HB models and by Sandage (1990*b*) from the analysis of Lub's (1977) data. In the following section (§ II), we review some important aspects of the observed relationship between the period shift and [Fe/H] for Lub's (1977) sample of field RR Lyrae stars. In § III, the synthetic HB calculations are extended to all HB types over the metallicity range of the galactic halo to investigate the effects of HB type on period shifts. It is shown that the disagreement can be resolved if Lub's (1977) sample contains many highly evolved stars in the metallicity range of $-2.0 \leq$ [Fe/H] ≤ -1.6, something not considered in detail by LDZ. Finally, implications of this result on the luminosity of the HB and the ages of globular clusters are briefly discussed in § IV. The related problem of the Oosterhoff separation of globular clusters into two RR Lyrae period groups is a sufficiently involved question that we postpone the full discussion of it to a forthcoming paper (see footnote 1 of this paper to anticipate the results to be discussed there).

II. PERIOD SHIFT AS A FUNCTION OF [Fe/H] FOR FIELD RR LYRAE STARS

The investigations by Sandage (1982*b*), Kemper (1982), and Lub (1987) have established that period shifts exist among field RR Lyrae stars of different [Fe/H], much like the ones observed between globular clusters. The observations by Lub (1977) in the Walraven photometric system comprise a sufficiently large sample whose values of reddening, mean values of $T_{\rm eff}$ and metallicities can be measured in a uniform way from different combinations of the Walraven colors. His reddening values for the field variables are very well-determined; hence the period shifts (and the equivalent quantity $L/M^{0.81}$; see Lub 1987) can be determined from the temperature and period data directly. Therefore, one can study the field star correlation of the period shifts with [Fe/H] in more detail than was possible before Lub's (1977) data were available.

The field star correlation of the period shifts at constant $T_{\rm eff}$[$\Delta \log P(T_{\rm eff})$] with [Fe/H] is shown in Figure 1*a*, from the data listed in Table 3 of Sandage (1990*b*). The data for field stars scatter more than the cluster data in the $\Delta \log P(T_{\rm eff})$–[Fe/H] diagram because the post–ZAHB luminosity evolution cause scatter in period shifts and also because only single determinations exist of period and [Fe/H] for individual field stars, whereas those for the clusters represent averages over many stars. The values of $\Delta \log P(T_{\rm eff})$ in Figure 1*a* are defined as deviations in period from the lower envelope of the M3 fiducial period temperature line (see Figure 5 of Sandage 1990*b*). Sandage obtained the values of [Fe/H] for field variables from Lub's (1977) photometric blanketing index $\Delta[B-L]$ and abundance calibration (i.e., [Fe/H] $=$

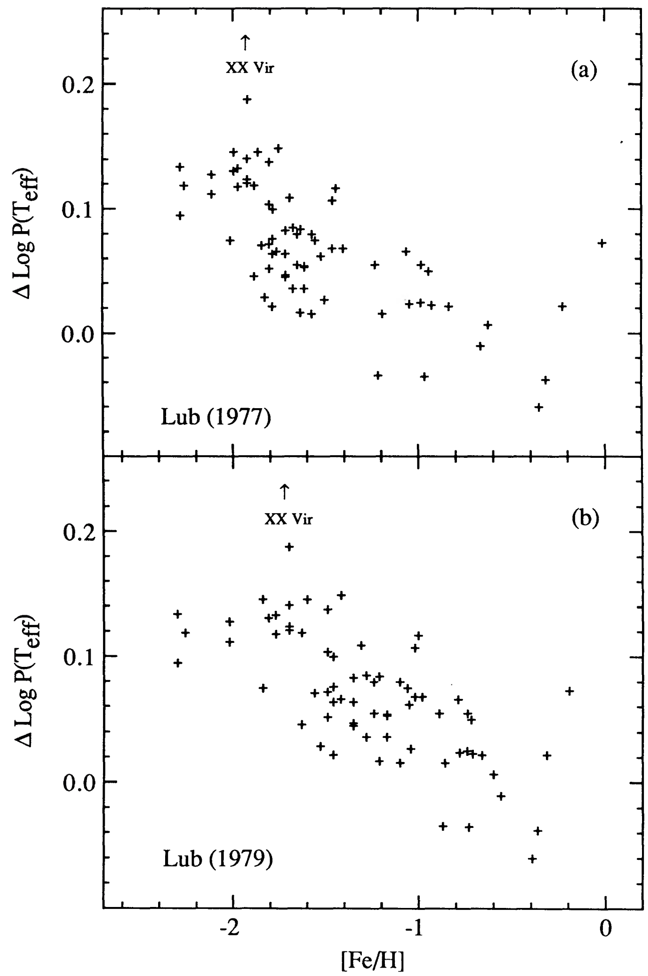

FIG. 1.—Correlation of the period shifts (at constant temperature) with metallicity from Lub's (1977) sample of field RR Lyrae stars. (*a*) Values of [Fe/H] from Sandage's (1990*b*) analysis (i.e., Lub's (1977) old calibration of $\Delta[B-L]$ index). (*b*) Values of [Fe/H] determined from Lub's (1979) new calibration of $\Delta[B-L]$ index. Note that the relationship becomes nonlinear at [Fe/H] ≈ -1.6.

$-2.15 + 21\Delta[B-L]$; see eq. [6] of Sandage 1990*b*), which was based upon the metallicity scale of Butler (1975). For comparison with the cluster data, Figure 1*a* is shifted in [Fe/H] by 0.15 for the known difference between the metallicity scales of Butler (1975) and of Zinn and West (1984) in order for all the data to be on a consistent metallicity scale. The least-squares fit for the correlation in Figure 1*a*, taking [Fe/H] as the independent variable, is

$$\Delta \log P(T_{\rm eff}) = -0.068(\pm 0.009)[\mathrm{Fe/H}] - 0.037(\pm 0.015)\,, \tag{1}$$

with a correlation coefficient of $r = 0.67$. One deviant star, XX Vir, has been excluded from the least-squares fit because it should be classified as a BL Herculis type variable on the basis of its long period (~ 1.3 days). The slope obtained here [$\Delta \log P(T_{\rm eff})/\Delta$[Fe/H] ≈ -0.07] is slightly smaller than the one obtained by Sandage (1990*b*) [$\Delta \log P(T_{\rm eff})/\Delta$[Fe/H] ≈ -0.08], who has excluded XX Vir as well as four other deviant RR Lyrae stars (HH Pup, AN Ser, AT Ser, DX Del), but still steeper than the slope given by LDZ from their synthetic HB models for nine clusters [$\Delta \log P(T_{\rm eff})/\Delta$[Fe/H] $= -0.043$; assuming $Y_{\rm HB} \approx 0.22$].

Recently, both H. Smith (1990, private communication) and A. Sandage (1990, private communication) have kindly pointed out that Lub made a more extensive comparison of his $\Delta[B-L]$ index with ΔS in his later paper (see Table III of Lub 1979). According to a plot of the relationship between ΔS and $\Delta[B-L]$, kindly provided by Smith (1990), two straight lines, rather than one, give a better representation:

$$\Delta S = -74.1\Delta[B-L] + 6.9 \quad \text{for } \Delta[B-L] \geq 0.035 ; \quad (2)$$

$$\Delta S = -220\Delta[B-L] + 12.0 \quad \text{for } \Delta[B-L] < 0.035 . \quad (3)$$

Then, using Butler's (1975) relation between ΔS and [Fe/H] (i.e., $[\mathrm{Fe/H}] = -0.16\Delta S - 0.23$), we now obtain a new [Fe/H] calibration of $\Delta[B-L]$:

$$[\mathrm{Fe/H}]_{\mathrm{Butler}} = 11.9\Delta[B-L] - 1.33 \quad \text{for } \Delta[B-L] \geq 0.035 ; \quad (4)$$

$$[\mathrm{Fe/H}]_{\mathrm{Butler}} = 35.2\Delta[B-L] - 2.15 \quad \text{for } \Delta[B-L] < 0.035 . \quad (5)$$

The effect of this new [Fe/H] calibration on the relationship between [Fe/H] and $\Delta \log P(T_{\mathrm{eff}})$ is illustrated in Figure 1*b*, which shows less clumping in metallicity. As in Figure 1*a*, Figure 1*b* is also shifted in [Fe/H] by 0.15 for the difference between the metallicity scales of Butler and of Zinn and West. The least-squares fit for the correlation in Figure 1*b*, taking [Fe/H] as the independent variable and excluding the XX Vir, is

$$\Delta \log P(T_{\mathrm{eff}}) = -0.073(\pm 0.009)[\mathrm{Fe/H}] - 0.025(\pm 0.013) , \quad (6)$$

which is only slightly different from equation (1). Consequently, despite the revision in [Fe/H] calibration, the field star correlation remains steeper than the slope given by LDZ from their nine model clusters.

Although the correlation is definite, the fine structure in the diagram suggests a potential problem. Inspection of Figure 1*b* indicates that Lub's (1977) sample contains many stars in the metallicity range of $-2.0 \leq [\mathrm{Fe/H}] \leq -1.6$, whereas the analysis of synthetic HB models by LDZ did not include the clusters in this range, except M3 (see Fig. 10 of LDZ). As shown in Figure 2, the majority of clusters in this metallicity range have very blue HBs (M2 or M13–like), particularly those at the galactocentric distances < 10 kpc (data from Lee 1989; Lee, Demarque, and Zinn 1990*b*). The quantity $(B-R)/(B+V+R)$ is analogous to Mironov's (1973) index $B/(B+R)$ which has been widely used to characterize HB morphology (B, V, and R are the numbers of blue HB, RR Lyraes, and red HB stars, respectively). $(B-R)/(B+V+R)$ is more sensitive than $B/(B+R)$ because the number of RR Lyrae variables is included (see Lee 1989). With only two exceptions, M3 and NGC 4147, the clusters in this metallicity range that contain RR Lyrae variables are classified Oosterhoff group II on the basis of their values of $\langle P_{ab} \rangle$ (see Castellani and Quarta 1987). These clusters have not received as much attention as the more metal-poor group II clusters (e.g., M15, M53, NGC 5466), because in general they contain fewer variables.

LDZ have suggested that most of the RR Lyrae variables in metal-poor group II clusters are highly evolved stars from the blue HB. Even though the variables in the range $-2.0 \leq [\mathrm{Fe/H}] \leq -1.6$ are more metal-rich than those in more metal-poor group II clusters, the majority of clusters in this abundance range tend to be blue HB types more so than the clusters that are more metal-poor (see Fig. 2).[1] One might expect, similarly, that some of the field variables with metallicities in the range $-2.0 \leq [\mathrm{Fe/H}] \leq -1.6$ are very highly evolved stars from the blue HB, perhaps to an even greater extent than the variables in more metal-poor group II clusters.[2] Consequently, it is likely that those variables are shifted in period relative to

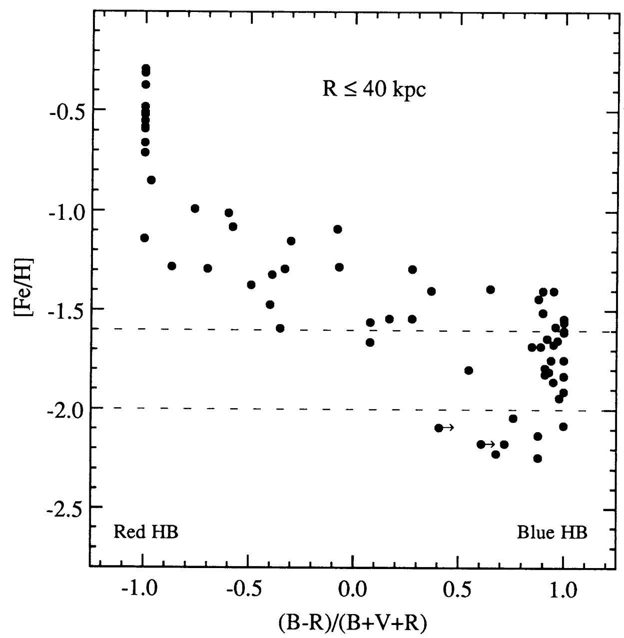

FIG. 2.—HB type, $(B-R)/(B+V+R)$, is plotted against [Fe/H] for all globular clusters with reasonably well-studied CM diagrams ($R \leq 40$ kpc). Note that the majority of clusters (e.g., M2, M13, M22) in the metallicity range $-2.0 \leq [\mathrm{Fe/H}] \leq -1.6$ have very blue HBs $[(B-R)/(B+V+R) > +0.85]$.

[1] This nonmonotonic behaviour of the HB morphology was first mentioned by Renzini (1983) and less explicitly by Castellani (1983). Figure 2 of this paper demonstrates this effect more clearly than before with the new HB morphology classification scheme, together with the improved observational data of globular cluster CM diagrams and metal abundances. Renzini (1983), Castellani (1983), and more recently Sandage (1990*b*) have argued that the Oosterhoff dichotomy (gap) of the ensemble average type *ab* RR Lyrae period ($\langle P_{ab} \rangle$) is caused by this nonmonotonic behavior of the HB morphology with progressive variation of [Fe/H]. Anticipating the models to be discussed in a forthcoming paper on the Oosterhoff dichotomy itself, we note here that this alone cannot be the whole explanation because there are clusters on each side of the empty period range that have almost the same values of [Fe/H] and HB type, yet have very different values of $\langle P_{ab} \rangle$ (e.g., M2 and NGC 6626; see Fig. 1 of Sandage 1990*b*). This abrupt change in $\langle P_{ab} \rangle$ at $[\mathrm{Fe/H}] \approx -1.6$ could be explained by Fig. 3 [see, e.g., $Z = 0.0004$ and 0.0007 loci; $(B-R)/(B+V+R) > +0.8$] of this paper, where one can see a large change in the period shift (and hence $\langle P_{ab} \rangle$, as will be shown in a forthcoming paper) as the HB type gets bluer $[(B-R)/(B+V+R) \geq +0.8]$ with a small decrease in [Fe/H] (see Fig. 2). The separation into the two Oosterhoff period groups may, then, be due to a combination of (1) the abrupt increase in $\langle P_{ab} \rangle$ as all type *ab* RR Lyraes become highly evolved stars from the blue side of the fundamental blue edge as the morphology of the HB gets bluer with decreasing [Fe/H], and (2) the nonmonotonic behavior of the HB morphology with decreasing [Fe/H] (perhaps partly due to the steep variation of [O/Fe] with [Fe/H] as recently suggested by Abia and Rebolo 1989), which together with the first effect makes the correlation between $\langle P_{ab} \rangle$ and [Fe/H] look like a step function. The intermediate values of $\langle P_{ab} \rangle$ between the two groups can be produced in principle, but only by clusters within a very narrow range of HB morphology and [Fe/H]. It is understandable, then, that the ~50 clusters in the Galaxy containing sufficient numbers of RR Lyrae variables fall into one or the other Oosterhoff group (see also § III*c* of LDZ).

[2] In his recent discussion of the Oosterhoff effect, Sandage (1990*b*) has also pointed out the blue HB clusters M2, M22, ω Cen, and NGC 4833 as having longer periods due to HB evolution. It is also worth noting that M13 was the cluster to be recognized as having this characteristic (see Sandage 1970).

M3 variables of the same T_{eff} by approximately the same amounts as the variables in more metal-poor group II clusters even though they are more metal-rich. One prediction is, therefore, that the relationships between [Fe/H] and period shift and the related quantity $L/M^{0.81}$, which was plotted by Sandage (1990*b*), will become nonlinear with decreasing [Fe/H] at approximat ly [Fe/H] ≈ -1.6. This effect is clearly shown in Figure 1*b* for period shifts and in Figure 6 of Sandage (1990*b*) for the quantity $L/M^{0.81}$. To investigate the effect of HB type on period shift in more detail, the synthetic HB calculations are extended to all HB types over the metallicity range of the Galactic halo, and compared with the observations in following section.

III. COMPARISON WITH MODELS

It was shown previously (LDZ) that the morphology of the synthetic HB depends almost entirely on two parameters, the mean HB mass ($\langle M_{HB} \rangle$) and the mass-dispersion parameter. It was also found there that the standard deviation of the mass dispersion [$\sigma(M)$] is about $\sim 0.03\ M_\odot$ for most clusters (e.g., M3, M4, M5, M92, NGC 288, NGC 362, NGC 6171, NGC 6723, and 47 Tuc), whereas values of approximately a factor of 2 smaller [$\sigma(M) = 0.01$–$0.02\ M_\odot$] are required for some metal-poor clusters (e.g., M53, NGC 5466). It follows, then, for given composition and $\sigma(M)$, one can construct a sequence of synthetic HB models with $\langle M_{HB} \rangle$ as a free parameter. The details of the model constructions and the HB evolutionary tracks are described by LDZ and by Lee and Demarque (1990), respectively.

The synthetic HB models yield the luminosity (L), mass (M), and effective temperature (T_{eff}) of each star within the instability strip; hence it is easy to calculate the fundamental periods (P_f) of the model RR Lyrae stars from the theoretical pulsation equation (van Albada and Baker 1971; Iben 1971; Cox 1987). One can then obtain the period shifts at a given T_{eff} between the clusters and M3 from the theoretical log P–log T_{eff} diagrams. Figure 3 presents the results of such calculations for period shift ($\Delta \log P'_{383} = \log P'_{cluster} - \log P'_{M3}$ at $\log T_{eff} = 3.83$) as functions of HB type and metallicity under two different assumptions regarding $\sigma(M)$, and can thus be viewed as an extension of Figure 5 of LDZ. These calculations are based on the HB evolutionary tracks of Lee and Demarque (1990) for

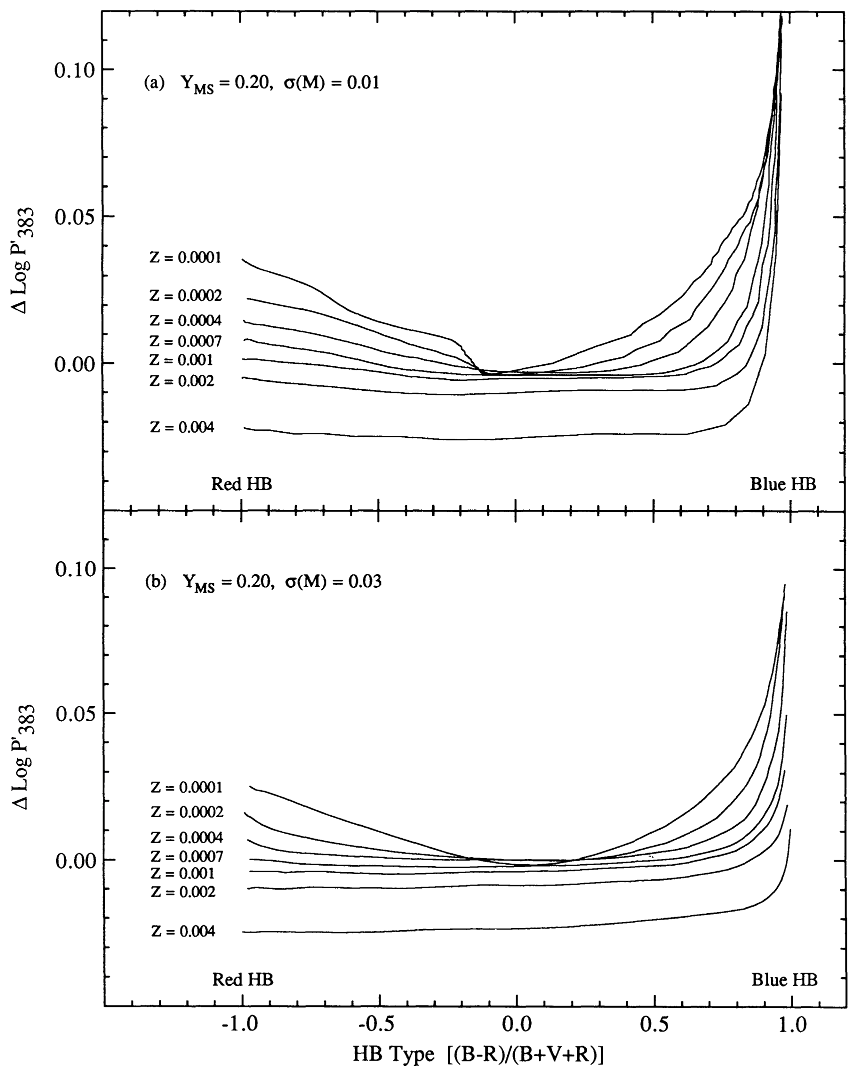

FIG. 3.—The synthetic HB calculations for period shift at a given T_{eff}($\Delta \log P'_{383} = \log P'_{cluster}$–$\log P'_{M3}$at $\log T_{eff} = 3.83$) as functions of HB type and metallicity under two different values for $\sigma(M)$.

$Y_{MS} = 0.20$ ($Y_{HB} \approx 0.22$). LDZ have shown that this choice of Y gives the best agreement with observation for the period shifts (see their paper on the sensitivity of the derived period shifts on helium abundance). Following Roberts and Sandage (1955), the intrinsic scatter in log P–log T_{eff} diagram due to the spread in luminosity among the RR Lyrae variables has been removed by calculating the "reduced period" (P') from the equation $\log P' = \log P + 0.336(M_{bol} - \langle M_{bol} \rangle)$, where M_{bol} is the absolute bolometric magnitude and $\langle M_{bol} \rangle$ is the mean magnitude of all variables in the model cluster. The width of the instability strip was taken to be 0.065 in log T_{eff}, and the period shifts for the synthetic HBs were found at log T_{eff} = 3.83 (near the center of the instability strip; see Fig. 5 of LDZ). It is important to note, however, that the measured period shifts are nearly independent of the choices of log T_{eff} and the width of the strip (see LDZ). The period shift calculations presented in Figure 3 are the mean values obtained from 50 random simulations, each with a sample size of 1000 HB stars—so these calculations are not affected by small number statistics. Figure 3 clearly demonstrates that the period shifts of the model RR Lyrae stars in globular clusters sensitively depend on their HB type as well as metallicity.

The synthetic HB models in Figure 4 explain why the period shifts measured at a given T_{eff} strongly depend on the HB type. The model in the lower panel suggests that the variables in the extremely blue HB clusters of intermediate metallicity [i.e., $Z \approx 0.0004$, and $(B - R)/(B + V + R) \approx +0.95$] are very

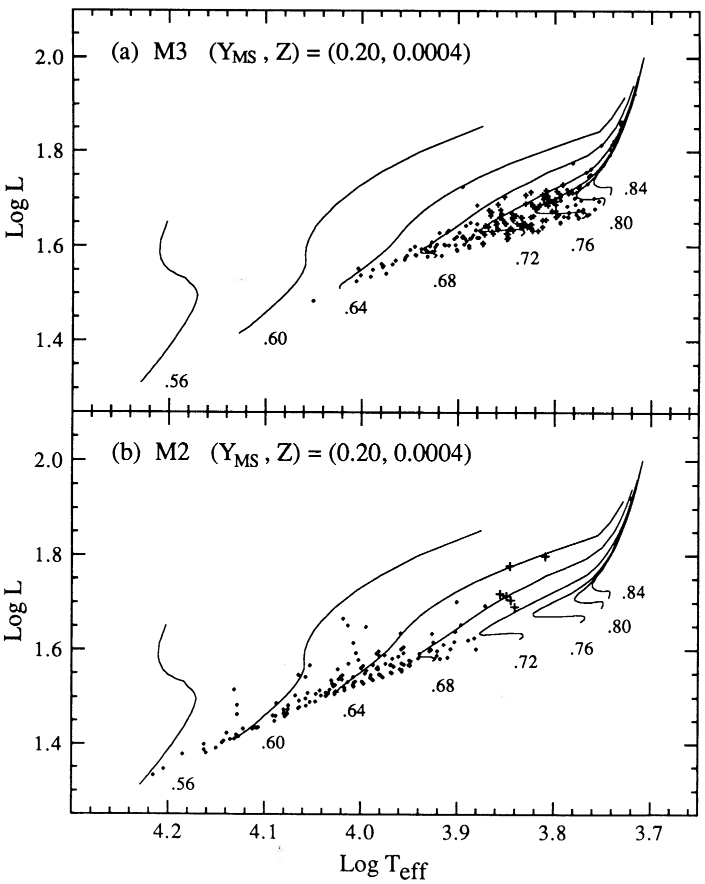

FIG. 4.—Superposition of HB evolutionary tracks on synthetic HB models for M3 and M2. RR Lyrae variables are represented by plus signs (+) and each track is labeled by its total mass in solar units.

highly evolved stars from the blue HB, even more so than the variables in more metal-poor group II clusters [$Z \approx 0.0001$, and $(B - R)/(B + V + R) \approx +0.80$; see Fig. 2 of LDZ]. The variables in these clusters (e.g., M2, M22, and M13) are less massive and more luminous than the ones in clusters that contain both red and blue HB stars [e.g., M3; $(B - R)/(B + V + R) \approx 0.0$]. Hence, they are shifted in period relative to M3 variables of the same T_{eff} by substantial amounts (since, as shown in the upper panel, the M3 variables tend to be close to the ZAHB). Consequently, the variables in these clusters having $-2.0 \leq$ [Fe/H] ≤ -1.6, which have the bluest HBs, are shifted in period relative to M3 variables of the same T_{eff} by approximately the same (or even larger) amounts as the variables in the more metal-poor group II clusters.

The upturn of the curves in Figure 3 at the red HB end (for $Z \leq 0.0007$) is due to the sensitivity of the slope of the ZAHB locus to metallicity within the instability strip, as well as the effect of evolution away from the ZAHB (see Fig. 10 of Sweigart 1987; Lee and Demarque 1990; Sandage 1990*a*). When the HB type is very red [$(B - R)/(B + V + R) < -0.8$], all model RR Lyrae stars are redder than log T_{eff} = 3.83. Consequently, for those models, the values of $\Delta \log P'_{383}$ are estimated by the extrapolation in the log P–log T_{eff} diagram and therefore are subject to some uncertainty. It is also worth noting that the period shift–metallicity correlation measured at $(B - R)/(B + V + R) = 0.0$ (i.e., in the middle), where the evolutionary effect is smallest, is virtually identical (i.e., $\Delta \log P'_{383}/\Delta$[Fe/H] ≈ -0.004; $0.0001 < Z < 0.002$) to the one prediced by ZAHB models [$\Delta \log P(T_{eff})/\Delta$[Fe/H] ≈ -0.003; Sweigart, Renzini, and Tornambè 1987].

From Figure 3, the values of $\Delta \log P'_{383}$ have been determined for all clusters plotted in Figure 2, except those that have only blue or red HB stars. They are listed in Table 1 with the data for HB type and [Fe/H] from Lee (1989) and Lee, Demarque, and Zinn (1990*b*). To determine the values of $\Delta \log P'_{383}$, it was assumed that [Fe/H] $= -1.66$ corresponds to $Z = 0.0004$. The values of $\sigma(M)$ that were adopted to determine the values of $\Delta \log P'_{383}$ are also given in column (5) of Table 1. They were estimated by visual comparisons of the observed HB CM diagrams (see Lee 1989 for references) with the series of theoretical simulations having different values of $\sigma(M)$. The uncertainties involved in these comparisons are estimated to be $\sim \pm 0.005\ M_{\odot}$, but in some cases they may be larger due to the poor quality of the available CM data; consequently, their values of $\sigma(M)$ are marked with colons in Table 1. For clusters with very blue HB type [$(B - R)/(B + V + R) > +0.8$], small uncertainties in the HB type (see Lee 1989) would yield large uncertainties in the derived values of $\Delta \log P'_{383}$ and $\langle M_{bol}^{RR} \rangle$ (see § IV); hence their values of $\Delta \log P'_{383}$ and $\langle M_{bol}^{RR} \rangle$ are also marked with colons in Table 1.

The effect of very highly evolved variables having abundances in the range $-2.0 \leq$ [Fe/H] ≤ -1.6 on the relationship between [Fe/H] and $\Delta \log P'_{383}$ is illustrated in Figure 5. Plotted in the upper panel of Figure 5 is the correlation for all clusters listed in Table 1, including the very blue HB clusters [$(B - R)/(B + V + R) > +0.85$] in the range $-2.0 \leq$ [Fe/H] ≤ -1.6. The least-squares fit for the correlation, taking [Fe/H] as the independent variable, is

$$\Delta \log P'_{383} = -0.071(\pm 0.009)[\mathrm{Fe/H}] - 0.085(\pm 0.015)\ , \quad (7)$$

with a correlation coefficient of $r = 0.73$. It should be noted

TABLE 1

Results of Synthetic HB Calculations for 48 Clusters (Y_{MS} = 0.20)

NGC	Name	$(B-R)/(B+V+R)$	[Fe/H]	$\sigma(M)$	$\Delta \log P'_{383}$	$\langle M_{bol}^{RR} \rangle$
288		0.95	−1.40	0.030	0.022:	0.643:
362		−0.87	−1.28	0.030	−0.005	0.711
1261		−0.70	−1.29	0.030	−0.003	0.708
1851		−0.33	−1.29	0.035	−0.004	0.713
1904	M79	0.89	−1.68	0.015:	0.054:	0.544:
2298		0.93	−1.81	0.015:	0.067:	0.499:
2808		−0.49	−1.37	0.035	−0.003	0.693
3201		0.08	−1.56	0.030	−0.002	0.656
4147		0.55	−1.80	0.020	0.005	0.630
4590	M68	>0.41	−2.09	0.010:	>0.014	0.550
4833		0.95	−1.86	0.030	0.063:	0.523:
5024	M53	0.76	−2.04	0.015	0.044	0.520
5053		>0.61	−2.17	0.010:	>0.028	0.525
5272	M3	0.08	−1.66	0.030	0.000	0.647
5286		0.91	−1.79	0.015:	0.064:	0.518:
5466		0.68	−2.22	0.010	0.034	0.518
5897		0.85	−1.68	0.015	0.044:	0.576:
5904	M5	0.37	−1.40	0.030	−0.002	0.695
5986		0.95	−1.67	0.020	0.076:	0.511:
6093	M80	0.92	−1.64	0.025	0.044:	0.575:
6121	M4	−0.07	−1.28	0.030	−0.004	0.720
6171	M107	−0.76	−0.99	0.030	−0.010	0.776
6205	M13	0.97	−1.65	0.030	0.075:	0.516:
6218	M12	0.92	−1.61	0.030	0.030:	0.607:
6235		0.90	−1.40	0.020·	0.027:	0.640:
6254	M10	0.94	−1.60	0.035	0.040:	0.580:
6266	M62	0.28	−1.29	0.035	−0.003	0.722
6341	M92	0.88	−2.24	0.030	0.055	0.473
6362		−0.58	−1.08	0.030	−0.008	0.758
6366		−0.97	−0.85	0.030	−0.016	0.809
6397		0.93	−1.91	0.015	0.091:	0.485:
6402	M14	0.65	−1.39	0.020:	0.003	0.693
6626	M28	0.88	−1.44	0.025	0.015	0.660
6638		−0.30	−1.15	0.030:	−0.007	0.745
6656	M22	0.94	−1.75	0.025	0.058:	0.540:
6712		−0.60	−1.01	0.035	−0.010	0.776
6723		−0.08	−1.09	0.035	−0.009	0.760
6779	M56	0.98	−1.94	0.030:	0.103:	0.428:
6809	M55	0.91	−1.82	0.030	0.045:	0.564:
6864	M75	−0.39	−1.32	0.030	−0.004	0.715
6934		0.28	−1.54	0.030:	−0.001	0.672
6981	M72	0.17	−1.54	0.030	−0.001	0.672
7006		−0.35	−1.59	0.030	0.002	0.652
7078	M15	0.72	−2.17	0.015:[a]	0.040	0.514
7089	M2	0.96	−1.58	0.030	0.050:	0.586:
7099	M30	0.88	−2.13	0.020	0.055	0.500
7492		0.90	−1.51	0.015	0.040:	0.594:
—	Pal 5	−0.40	−1.47	0.030:	−0.001	0.684

[a] From model M15A of LDZ.

that the slope is very close to the value obtained in equation (6), and that the overall relationship between [Fe/H] and period shift looks just like that in Figure 1*b* (for [Fe/H] $\lesssim -0.85$). NGC 4590 and NGC 5053 have been excluded from Figure 5, because their HB types are uncertain (see Lee 1989), and consequently, estimated values of $\Delta \log P'$ are only lower limits of true values for these clusters. The inclusion of these clusters would change the slope by only $\sim$0.005. Plotted in the lower panel of Figure 5 is the correlation obtained by excluding the very blue HB clusters $[(B-R)/(B+V+R) > +0.85]$ in the range $-2.0 \leq$ [Fe/H] ≤ -1.6. The least-squares correlation is

$$\Delta \log P'_{383} = -0.047(\pm 0.005)[\mathrm{Fe/H}] - 0.062(\pm 0.008) , \quad (8)$$

with a correlation coefficient of $r = 0.86$. It might be noted that this solution has virtually the same slope as was found by LDZ based on much smaller sample of model clusters without the very blue HB clusters (M2 or M13–like). It is clear from Figure 5 that a larger slope is obtained when these highly evolved variables in the range $-2.0 \leq$ [Fe/H] ≤ -1.6 are included—one that is very close to the value obtained by Sandage (1990*b*) from Lub's (1977) data. *This suggests that the disagreement between the theoretical slope obtained by LDZ and the one obtained by Sandage (1990b) from his analysis of Lub's (1977) data may be due to the fact that the analysis of LDZ did not include very blue HB clusters in the range $-2.0 \leq$ [Fe/H] ≤ -1.6, whereas Lub's (1977) sample contains many variables in this range, that are predicted to be very highly evolved stars from*

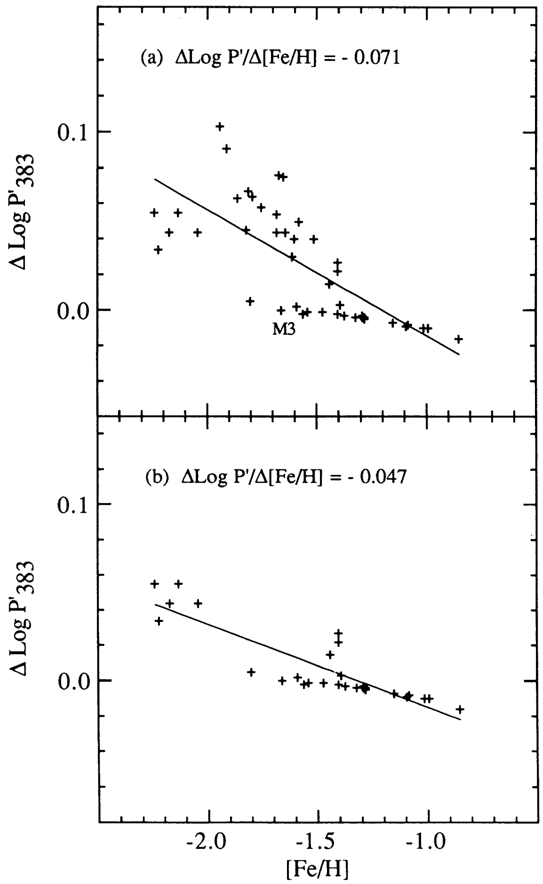

FIG. 5.—The effect of highly evolved variables on the relationship between [Fe/H] and $\Delta \log P'_{383}$. The slope obtained including the very blue HB clusters $[(B-R)/(B+V+R) > +0.85]$ in the metallicity range $-2.0 \leq$ [Fe/H] ≤ -1.6 is close to the value obtained by Sandage (1990*b*) from Lub's (1977) data (*a*), while the slope obtained by excluding them is much smaller and consistent with the result by LDZ (*b*).

the blue HB. Therefore, it seems likely that Sandage's (1990*b*) result obtained from Lub's (1977) sample of field stars is not necessarily evidence that the models presented in LDZ are incorrect.[3]

[3] Allan Sandage has pointed out the failure of eq. (8) to satisfy the observed Oosterhoff correlation (i.e., $\Delta \log \langle P_{ab} \rangle / \Delta$[Fe/H] ≈ -0.090; see eq. [1] of Sandage 1990*b*). However, it is important to note that eq. (8) in this paper is the correlation between the period shift of the clusters relative to M3 (measured from the log *P*–log $T_{\rm eff}$ diagrams at log $T_{\rm eff} = 3.83$) and [Fe/H], and is *not* the prediction of the Oosterhoff correlation between the ensemble average of type *ab* RR Lyraes and [Fe/H], though they are related. It is also worth noting that Sandage's (1990*b*) eq. (1) is the mean of the two least-squares solutions made by exchanging the dependent and the independent variables (the so-called impartial slope; see footnote 1 of his paper). If we take [Fe/H] as the independent variable, to be consistent with the procedure adopted in this paper, the slope is reduced to $\Delta \log \langle P_{ab} \rangle / \Delta$[Fe/H] ≈ -0.074. LDZ have already discussed (see § III*c* of LDZ for details) that their models would produce a satisfactory match to the observed value of $\Delta \log \langle P_{ab} \rangle$ (i.e., $\log \langle P_{ab} \rangle_{\rm cluster} - \log \langle P_{ab} \rangle_{\rm M3}$) if there is a small amount of hysteresis or, equivalently, a small difference in transition $T_{\rm eff}$ (i.e., $\Delta T_{\rm eff} \approx 100$ K) between the Oosterhoff groups. If, as suggested by LDZ calculations, the difference in transition $T_{\rm eff}$ is only ~ 100 K $[\Delta(B-V) \approx 0.017]$, rather than ~ 300 K as suggested by van Albada and Baker (1973) and Stellingwerf (1975), it will be difficult to prove or disprove this observationally by comparing the color boundaries of the type *ab* RR Lyrae stars in two groups. Sandage (1990*b*) has also pointed out the lack of agreement of the LDZ mass prediction with the "observed" variation of double-mode RR Lyrae masses with [Fe/H]. I refer the reader to LDZ (see § III*d*), Cox (1987), and Kovacs (1985) for explicit discussions concerning the uncertainties of double-mode RR Lyrae masses derived from the Petersen diagram (see also Andreasen 1988; Iglesias *et al.* 1990).

IV. EFFECT OF HORIZONTAL BRANCH EVOLUTION ON THE MEAN RR LYRAE LUMINOSITY–[Fe/H] RELATIONSHIP

Having discussed the effect of highly evolved variables on the relationship between the period shifts and metallicity, it is of considerable interest to see whether it also affects the relationship between the mean RR Lyrae luminosity ($\langle M_{\rm bol}{}^{\rm RR} \rangle$) and [Fe/H], which, of course, is the central issue of the debate because it is one way of determining whether the age of the Galactic cluster system depends on [Fe/H] or is independent of it.

To estimate the mean luminosity of the RR Lyrae variables and how it varies with HB type and metallicity, the values of $\langle M_{\rm bol}{}^{\rm RR} \rangle$ predicted by the synthetic HB calculations ($Y_{\rm MS} = 0.20$) have been plotted in Figure 6 as functions of HB type and metallicity under two assumptions regarding $\sigma(M)$ (assuming $M_{\rm bol} = +4.79$ for the Sun). This illustrates that the *ensemble* average of the RR Lyrae luminosities in a given cluster is less sensitive to the HB type than the period shift at a given $T_{\rm eff}$, unless the HB type is extremely blue.[4] This is because (1) the period shifts are due to mass as well as luminosity effects, and (2) the distribution of individual variables along the HB in the blue HB clusters tends to be shifted more toward the blue, where the luminosity is smaller than its value at the middle of the instability strip (see Figs. 2 and 14 of LDZ).

The effect of very highly evolved variables in the range $-2.0 \leq$ [Fe/H] ≤ -1.6 on the $\langle M_{\rm bol}{}^{\rm RR} \rangle$–[Fe/H] relationship is illustrated in Figure 7, where the values of $\langle M_{\rm bol}{}^{\rm RR} \rangle$ have been determined from Figure 6 for all clusters listed in Table 1, assuming that [Fe/H] $= -1.66$ corresponds to $Z = 0.0004$ (the values of $\langle M_{\rm bol}{}^{\rm RR} \rangle$ are also listed in the final column of Table 1). Plotted in the upper panel of Figure 6 is the correlation for all 48 clusters listed in Table 1, while the lower panel is the correlation excluding the very blue HB clusters in the ranges $(B-R)/(B+V+R) > +0.85$ and $-2.0 \leq$ [Fe/H] ≤ -1.6. The least-squares correlation in Figure 7*a* is

$$\langle M_{\rm bol}{}^{\rm RR} \rangle = 0.257(\pm 0.017)[\mathrm{Fe/H}] + 1.03(\pm 0.028)\,, \quad (9)$$

while the correlation in Figure 7*b*, excluding the very blue HB clusters, is

$$\langle M_{\rm bol}{}^{\rm RR} \rangle = 0.224(\pm 0.008)[\mathrm{Fe/H}] + 1.00(\pm 0.013)\,, \quad (10)$$

which has virtually the same solution as was found in LDZ based on a much smaller sample of model clusters (see eq. [4] of LDZ). This suggests that the inclusion of highly evolved variables does not alter the relationship between the mean RR Lyrae luminosity and [Fe/H] by substantial amounts.

It is likely, however, that the best estimate for the relationship between the HB luminosity and [Fe/H] is that which excludes the very blue HB clusters, because the observed level of the HB is usually (including the estimation by Buonanno *et al.* 1989) not estimated from the few variables in these systems, but from the blue HB itself. Using the bolometric corrections

[4] The model calculations for $\sigma(M) = 0.01$ in Figs. 6 and 3 deserve special comment because they provide important implications for the RR Lyrae stars in ω Cen. While the detailed composite model for ω Cen that includes the [Fe/H] distribution observed in this cluster will be constructed in a separate paper (Lee, in preparation), we note here that the evolutionary models in Figs. 3*a* and 6*a* could account for the observed lack of $\langle M_v \rangle$–[Fe/H] and period shift–[Fe/H] relations in ω Cen (Sandage 1982*a*; Gratton, Tornambè, and Ortolani 1986)—provided that $(B-R)/(B+V+R) > +0.9$ as suggested by the number counts of its HB stars by Gratton, Tornambè, and Ortolani (1986). This is a consequence of the variation of the intrinsic HB width with metallicity and the effect of redward evolution away from the blue ZAHB (cf. Gratton, Tornambè, and Ortolani 1986; Sandage 1990*a*).

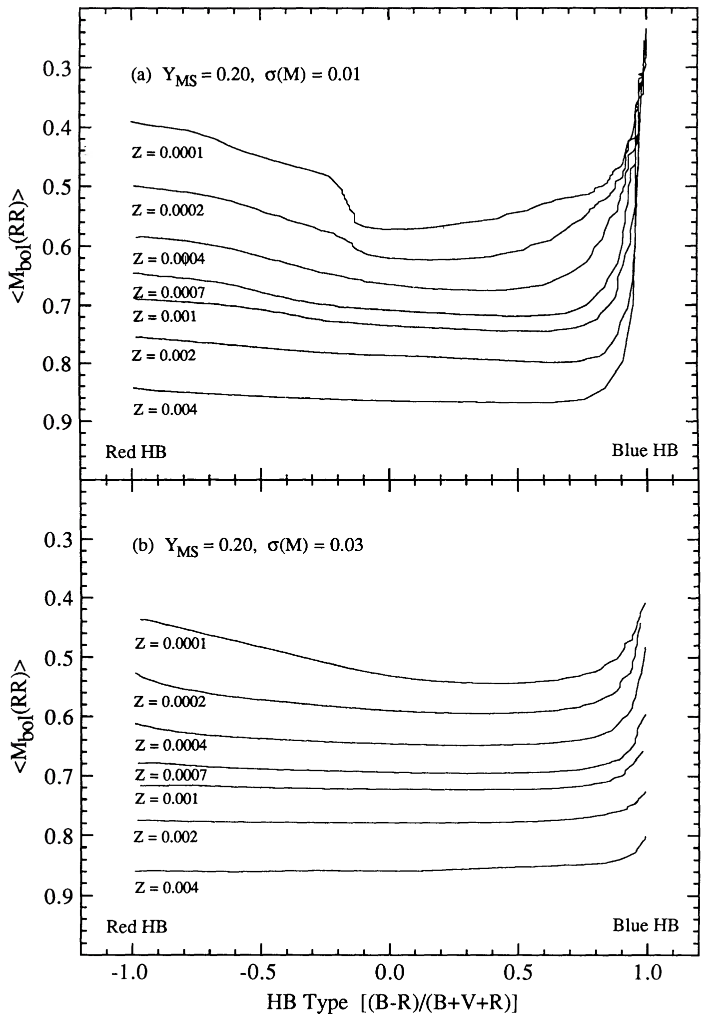

FIG. 6.—Synthetic HB calculations for the ensemble average of the RR Lyrae luminosities in a model cluster as functions of HB type and metallicity under two different values for $\sigma(M)$.

that have been used in the construction of the Revised Yale Isochrones (i.e., $M_V = 4.86$ [Hardorp 1980] and B.C. $= -0.07$ for the Sun to be consistent with the value of $M_{\rm bol}^{\odot} = +4.79$ used in the $\langle M_{\rm bol}^{\rm RR} \rangle$ calculations; see Green 1988), equation (10) gives the theoretical relationship ($Y_{\rm MS} = 0.20$; $Y_{\rm HB} \approx 0.22$) for the absolute visual magnitude as

$$\langle M_V^{\rm RR} \rangle = 0.19[\mathrm{Fe/H}] + 0.97 , \qquad (11)$$

with a high correlation coefficient of $r = 0.98$. As shown by LDZ, both the slope and the zero point of this relation would be decreased if $Y_{\rm MS} = 0.23$ ($Y_{\rm HB} \approx 0.25$), but the increase in $Y_{\rm MS}$ has a bigger effect on the zero point than on the slope (i.e., $\langle M_V^{\rm RR} \rangle = 0.17[\mathrm{Fe/H}] + 0.79$). It should be noted that the bolometric correction scale used by LDZ (i.e., $M_V =$ 4.90, B.C. $= -0.11$, and $M_{\rm bol} = 4.79$ for the Sun) for their equations (5) and (7) is not similarly consistent, though the difference is only ~ 0.04 mag (Green and Demarque, private communication). It is encouraging to see that the recent Baade-Wesselink measurements of field RR Lyrae stars (e.g., Cacciari *et al.* 1989; Liu and Janes 1990) give virtually the same relationship as equation (11). (Since the above relationships between the HB luminosity and [Fe/H] are almost identical to the ones reported by LDZ, I refer the reader to § IV of LDZ for an explicit comparison of these model predictions with observational calibrations.)

As suggested by LDZ, when combined with the result of Buonanno *et al.* (1989) that, in the mean, there is no variation in the difference in V mag between the HB and the main-sequence turnoff, the above relationship yields a significant age-metallicity relation among the Galactic globular clusters that spans ~ 4 Gyr (assuming constant [O/Fe]) over the metallicity range of the Galactic halo (see also Sandage and Cacciari 1990; Sarajedini and King 1989). However, this result does not take into account the strong variation of [O/Fe] with [Fe/H] as recently suggested by Abia and Rebolo (1989). If such a revision applies to globular clusters, the above age spread would be reduced to ~ 1 Gyr (as inferred from VandenBerg 1985). It is important to note here that if [O/Fe] varies steeply with [Fe/H] as suggested by Abia and Rebolo (1989), then Sandage and Cacciari's (1990) much steeper relationship between the HB luminosity and [Fe/H] (i.e., $\Delta M_V/\Delta[\mathrm{Fe/H}] \approx$ 0.39; see their eq. [8]) than that given by equation (11) would yield an anticorrelation between the cluster age and metallicity, in the sense that the metal-poor clusters are younger

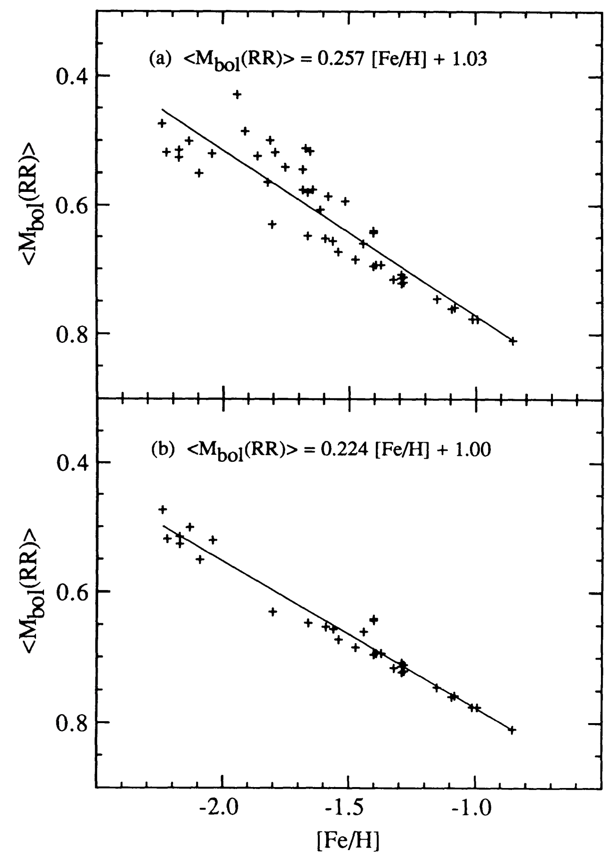

FIG. 7.—The mean RR Lyrae luminosity–[Fe/H] relationship predicted by the synthetic HB models ($Y_{\rm MS} = 0.20$; $Y_{\rm HB} \approx 0.22$), including the very blue HB clusters $[(B-R)/(B+V+R)] > +0.85]$ in the range $-2.0 \leq [\mathrm{Fe/H}] \leq -1.6$ (*a*), and excluding them (*b*). Unlike the period shift–[Fe/H] relationship (see Fig. 5), $\langle M_{\rm bol}^{\rm RR} \rangle$–[Fe/H] relationship is less affected by the very blue HB clusters in the range $-2.0 \leq [\mathrm{Fe/H}] \leq -1.6$, where most variables are believed to be highly evolved stars.

than the metal-rich ones by a few billion years.[5] This would, of course, be against intuition. This comparison underscores the importance of determining [O/Fe] for globular cluster stars and its variation with [Fe/H].

This work has developed from chapter 3 of my doctoral dissertation at Yale University. I am greatly indebted to Pierre Demarque and Robert Zinn for many helpful discussions and suggestions during the course of this research. I am similarly indebted to Allan Sandage (the referee), and Don VandenBerg for so carefully reading an early draft of this paper and for making many valuable suggestions, which led to several improvements in the manuscript. It is a pleasure to thank Horace Smith and Allan Sandage for pointing out Lub's new [Fe/H] calibration of $\Delta[B-L]$ index, now outlined in § II. It is also a pleasure to thank Elizabeth Green, David Hartwick, and Allen Sweigart for some helpful discussions. This research was partially supported by NASA grant NAGW-778 (to P. Demarque), and by the Natural Sciences and Engineering Research Council of Canada (through an operating grant to D. VandenBerg).

[5] It is important to note that the relationship between [O/Fe] and [Fe/H] suggested by the observations of Abia and Rebolo (1989) is much steeper than the one adopted by Sandage and Cacciari (1990) in § VII of their paper [namely, a nearly flat [O/Fe] variation with [Fe/H] for [Fe/H] < −1.0 (e.g., Sneden 1985)]. (See Fig. 5 of Abia and Rebolo 1989.)

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