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RAPIDLY ROTATING PULSARS AND THE EQUATION OF STATE

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ABSTRACT

It has recently been claimed that a submillisecond pulsar has been observed in the remnant of SN 1987A. *If the pulsations are due to rotation of a neutron star at this frequency*, the equation of state (EOS) must be severely limited. In general, soft equations of state allow rapid enough rotational frequencies, but the softness is limited by the observed neutron star mass of 1.44 $M_{\odot}$ in the binary pulsar PSR 1913+16. In order to simultaneously satisfy these two constraints, we show that in the vicinity of ordinary nuclear density, the pressure must vary relatively slowly with density, but at higher densities, the EOS must become relatively stiff and approach the causality limit. Specifically, in the absence of phase transitions above nuclear density and for cases in which the nuclear symmetry energy is an increasing function of density, the compression modulus of symmetric nuclear matter at the saturation density 0.16 fm^{-3} must be less than about 160 MeV. If either of these situations occurs, higher values for the compression modulus may be possible, but in any case the EOS must still stiffen to the causal limit at high density. Examples of phase transitions that might permit rapid enough rotation are those due to pion or kaon condensates or to parity-doublet matter occurring around a few times the nuclear matter density. On the other hand, phase transitions to ordinary or strange quark matter well above nuclear densities soften the EOS to well below the causal limit and make rapid rotation much more difficult. Self-bound stars composed entirely of perturbative strange quark matter are not quite able to satisfy the requirements of rapid enough rotation and sufficient mass even in the optimum case where strange quarks are assumed to be massless. Other schematic self-bound stars can satisfy these constraints, but physical motivations for such equations of state are presently lacking.

Subject headings: equation of state — pulsars — stars: individual (SN 1987A) — stars: neutron

I. INTRODUCTION

The recent supernova in the Large Magellanic Cloud, SN 1987A, has allowed for the first time observational data on the birth of a neutron star. The neutrinos detected by Kamioka (Hirata *et al.* 1987) and IMB (Bionta *et al.* 1987) basically confirmed the standard theoretical model (Burrows and Lattimer 1986) for neutron star formation. The binding energy of the neutron star, 99% of which is converted to neutrino emission, can thus be estimated. However, because of statistics, it is uncertain by about 50% (Lattimer and Yahil 1989). Nevertheless, the rest mass of the neutron star formed may be estimated from the neutrino observations to lie in the range 1.3–1.8 $M_{\odot}$ (Burrows 1988; Lattimer and Yahil 1989). Moreover, an independent estimate of the neutron star's mass from the observed nucleosynthesis has been made by Thielemann, Hashimoto, and Nomoto (1989), who find a rest mass of $1.60 \pm 0.05\ M_{\odot}$. If the rotational energy of the neutron star is ignored, these estimates imply that a gravitational mass of approximately 1.4 $M_{\odot}$ has been formed in SN 1987A. Despite this information, the equation of state (EOS) for matter above nuclear density cannot be severely constrained by these mass estimates alone (Prakash, Ainsworth, and Lattimer 1988).

The more recent observation of a submillisecond pulsar (Kristian *et al.* 1989) in the remnant of SN 1987A with period $P = 0.508$ ms is very exciting because the possibility exists of pinning down the EOS at supra-nuclear density, *if the pulsations are due to rotation.* The maximum rotation rate of a star must be less than or equal to the Keplerian rate, at which the equatorial surface velocity equals the orbital velocity of a particle at the equator. Friedman, Ipser, and Parker (1989) have shown that only a few of the many published equations of state they tested can simultaneously satisfy the constraints of Keplerian rotation with periods less than or equal to 0.5 ms and nonrotating masses greater than or equal to the more massive (gravitational mass, 1.44 $M_{\odot}$) of the neutron stars in the binary pulsar PSR 1913+16 (Weisberg and Taylor 1984, 1989). However, more recent calculations of the EOS, which contain improvements both from technical and physical standpoints, are available today. In addition, some of the equations of state tested by Friedman, Ipser, and Parker do not include beta equilibrium, which we show to be an important omission. Finally, some violate causality at densities found in neutron stars, although we will show that this does not greatly affect the rotational properties of these equations of state.

It is the purpose of this paper to identify those properties of the EOS that will give rise to rapid enough rotation while at the same time sustain sufficient mass. Our viewpoint, that both rotation and mass constraints must apply to the EOS, represents the simplest hypothesis. Alternative scenarios, in which the object in SN 1987A is not a normal neutron star that is rapidly rotating, are possible, but they involve a greater number of assumptions, and, in addition, they may be inconsistent with other observations (e.g., the mass estimates referred to earlier). To achieve our objective, we will explore equations of state calculated using nucleonic degrees of freedom as well as those in which phase transitions to other types of matter, e.g., pions, kaons, quarks, etc., are considered. Where possible, our calculations will include effects due to beta equilibrium. The presence of leptons softens the EOS of neutron star matter, and, in general, imposing beta equilibrium decreases *both* the maximum mass and the maximum Keplerian rotation rate. We will also require that the EOS satisfies the causality condition, i.e., sound speed is less than or equal to the speed of

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light. As shown by Bludman and Ruderman (1970), this condition is rigorously satisfied in bulk matter in its ground state. In many cases, some of the parameters entering the calculations of the EOS are constrained by experiments on laboratory nuclei. We will therefore include a brief discussion about the plausibility for the validity of each EOS considered where such constraints do apply.

We will examine especially the "successful" equations of state employed by Friedman, Ipser, and Parker (1989), because one could argue that the *functional form*, i.e., the energy-density relation, of the successful equations of state might eventually be reproduced by more modern calculations. Indeed, a number of authors (Sato and Suzuki 1989; Friedman, Ipser, and Parker 1989; Haensel and Zdunik 1989; Shapiro, Teukolsky, and Wasserman 1989) have argued that satisfying both the above constraints implies that the EOS of matter above nuclear density (2.7×10^{14} g cm^{-3} or 0.16 fm^{-3}) is relatively soft. However, we will show that this softening of the EOS is effective only if it occurs in the vicinity of nuclear densities. Furthermore, the EOS is required to become extremely stiff, i.e., nearly causal, at higher densities (beyond about 6 times nuclear density).

We have established general limits on potentially *observable* parameters of the EOS, *if* both the binary pulsar mass and the 0.5 ms rotation period are valid for neutron stars. In particular, we demonstrate that *in the absence of extreme softening of the equation of state just above nuclear densities*, such as that accompanying pion or kaon condensation (but *not* a phase transition to quark matter), the compressibility parameter of symmetric nuclear matter must be less than about 160 MeV if the EOS at higher densities maintains causality. This limit is somewhat model dependent, being sensitive to the form of the nuclear symmetry energy and to the possible existence of phase transitions above nuclear density. In addition, we show that stars containing quark cores as a result of transitions to quark matter, either normal or strange, rotate less rapidly than stars without such cores. Furthermore, stars made up entirely of perturbative quark matter with strangeness to baryon ratio of the order of unity are unable, barely, to satisfy the criteria of rapid rotation and sufficient mass, even if the strange quarks are assumed to be massless.

This paper is organized as follows. In § II, we discuss general limits to the rotation of neutron stars and give a prescription for establishing the rotational properties for a given equation of state. This prescription is based upon fully general relativistic, axisymmetric structural calculations, but it may be effectively stated in terms of the properties of the maximum mass, nonrotating star. Section III summarizes current theories of the equation of state of baryonic matter and the rotational properties of a representative sample. Section IV establishes, via semianalytic arguments, the required properties of an equation of state needed to satisfy *both* the mass and rotation constraints. The possibilities of phase transitions above nuclear density and self-bound equations of state are explored in § V. Our general relativistic structure calculations are summarized in § VI.

II. ROTATIONAL PROPERTIES OF NEUTRON STARS

An absolute upper limit to the rotation rate of a neutron star can be found when the equatorial surface velocity equals the Keplerian velocity, the orbital velocity of a particle at the equator. In reality, the rotation may be more severely limited by a gravitational instability to nonaxisymmetric perturbations. Although gravitational radiation makes all rotating *perfect* fluids unstable, the instability is damped out by viscous effects, except in sufficiently rapidly rotating objects (Lindblom and Detweiler 1977). Until recently, the best estimates were based on gravitational-radiation instability calculations for Newtonian polytropes (Imamura, Durisen, and Friedman 1985; Managan 1985). These calculations express the onset of instability in terms of the ratio t of the rotational kinetic energy to the gravitational potential energy. The critical t for the onset of instability decreases with increasing softness of the EOS and also decreases with increasing mode number. On the other hand, viscous effects become more stabilizing as the mode number increases. Friedman, Ipser, and Parker (1986) concluded that, due to this competition, the instability point is around $t \simeq 0.08$ for uniformly rotating stars. (Differential rotation is not expected due to the high viscosity of neutron star matter.) For a given EOS, this still translates into a maximum rotation rate that is at least 90% of the Keplerian rate, since the dependence of rotation rate on t is rather weak in this regime.

Recently, Ipser and Lindblom (1989) have carried out a general relativistic instability analysis. They considered only the stabilizing effects of the shear viscosity, which has a temperature dependence of T^{-2}, and they conclude that maximum rotation rate is greater than 90% of the Keplerian rate only for neutron star temperatures less than 1 MeV. At higher temperatures, they suggest the instability may be triggered earlier. When, however, effects due to the bulk viscosity are included, this caveat may disappear. Sawyer (1989) has estimated that bulk viscosity varies as T^8 and that it exceeds the shear viscosity above 1 MeV. Hotter neutron stars would then be stabilized by the bulk viscosity. In addition, Sawyer (1989) emphasizes that if there exist physical mechanisms for increasing the neutrino fluxes in young neutron star matter, such as quark matter or meson condensates, the viscosity may, in fact, be much larger, perhaps by a factor of 10. This would mean that the maximum rotation rate would be very nearly the Keplerian rate. To summarize, the Keplerian rate is a reasonable estimate of the maximum rotation rate of a neutron star, but one should be aware that a slightly more conservative limit may yet exist.

In general, the Keplerian rotation rate, at which mass is "shed" from the equator, is larger for more compact neutron stars with higher central densities. Thus, the maximum rotation rate is an increasing function of the "softness" of the EOS. If the observed pulsation rate of 0.508 ms is due to rotation, a severe limit to the EOS may be established. On the other hand, the masses (1.44 and 1.38 $M_\odot$) of the components of the binary pulsar PSR 1913+16, which have been determined to great accuracy (Weisberg and Taylor 1984, 1989), set a minimum limit to the maximum mass for a neutron star. The maximum mass is a decreasing function of the "softness" of the EOS and therefore establishes a second constraint. These two constraints will restrict the EOS to lie in a narrow region.

The calculations of the maximum rotation rate and the maximum mass permitted by a given EOS has to be done with full general relativity, since the relativity parameter GM/R is maximized for these situations. Heretofore the only published calculations of the Keplerian rotation rate for general equations of state are those of Friedman, Ipser, and Parker (1986, 1989), who considered the case of uniform rotation. It is worthwhile to point out the existence of an approximate empirical formula for the Keplerian angular frequency Ω_K in terms of

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properties of the maximum mass *nonrotating* star for a given EOS, discovered by Haensel and Zdunik (1989):

$$\Omega_K = 7.7 \times 10^3 \left(\frac{M_{max}}{M_\odot}\right)^{1/2} \left(\frac{R_{max}}{10 \text{ km}}\right)^{-3/2} \text{s}^{-1} . \quad (1)$$

Here the subscripts "max" denote the maximum mass nonrotating star of a given equation of state, and M_{max} is the gravitational mass. We have determined that this approximation has an error of less than about 4% for all the equations of state discussed by Friedman, Ipser, and Parker (1986, 1989) and for the equations of state described below, whose Keplerian rates were calculated by us using a code (Masak, Lattimer, and Yahil 1989) similar to the one of Friedman, Ipser, and Parker. We summarize these results in § VI below and in Figure 5. If we restrict our attention to those equations of state that support the largest rotation rates, say greater than 10^4 s^{-1}, then this formula has an even smaller error. We therefore believe that it has a very broad application for any causal EOS. We note, however, that a completely incompressible fluid also obeys equation (1), but with the coefficient 7.7 replaced by 9.6 (Butterworth and Ipser 1976). Since the calculation of the Keplerian rate for a given EOS is extremely time-consuming, this approximation is particularly useful.

Some motivation for this simple relationship comes from the so-called Roche model (Shapiro and Teukolsky 1983) in Newtonian gravity. This model assumes that the distribution of the bulk of the mass is unchanged by rotation, which, for highly centrally condensed stars, the ones with the largest Keplerian rates, is a good approximation. In this model the Keplerian rate for a star of mass M and *nonrotating* radius R is

$$\Omega_K = \left(\frac{2}{3}\right)^{3/2} \left(\frac{GM}{R^3}\right)^{1/2} = 6.3 \times 10^3 \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{10 \text{ km}}\right)^{-3/2} \text{s}^{-1} . \quad (2)$$

The small difference between the numerical factors of equations (1) and (2) can be attributed to the facts that, in general relativity (GR), the maximally rotating configuration has 10%–20% larger (Friedman, Ipser, and Parker 1986) mass than M_{max}, and gravity is stronger. Thus, we see that M_{max} and R_{max} represent two parameters for the assumed EOS, and that the Keplerian rate is well described by the combination of the two parameters suggested by Newtonian mechanics.

Shapiro, Teukolsky, and Wasserman (1989) claim that a relation identical to equation (2) can be established in general relativity, but they restricted their attention to sequences of models with constant *rest mass*. In fact, rotation stabilizes the star so that the most massive rotating star has more mass than the maximum mass nonrotating star. The amount of this mass increase cannot be easily established without performing the full GR structure with rotation. Therefore, equation (2) cannot establish the maximum Keplerian rate, Ω_K, for a given equation of state. It is, however, a good approximation for the Keplerian rate of a star with a *fixed number of baryons M/m*, where m is the baryon mass, whose nonrotating radius is R. The fact that the rest mass is about 10% larger than the gravitational mass fortuitously pushed equation (2) even closer to equation (1).

It is important not to confuse R_{max} and M_{max} in equation (1) with the actual radius and gravitational mass of the star rotating with Ω_K. At the surface of a rotating star, the spacetime geometry cannot be simply written in terms of this radius and mass, but the exterior Kerr geometry is a close approximation. The orbital frequency at the equator in this geometry is

$$\Omega_0 = \frac{G^{1/2}M}{(MR^3)^{1/2} + G^{1/2}J/c^2} , \quad (3)$$

where J, R, and M are the angular momentum, the actual radius, and the mass of the rotating star. This formula is accurate to about 1% for all equations of state that we have checked. However, J, R, and M are only obtainable if the full calculation for the structure of the rotating star has been done. The beauty of equation (1) is that only properties of the *nonrotating* star enter.

The challenge for the EOS is therefore to compress at least 1.44 $M_\odot$ within a radius limited by $\Omega_K = 2\pi/P \geq 1.237 \times 10^4$ s^{-1}: Employing equation (1), this implies that

$$R_{max} \leq 8.23 \left(\frac{M_{max}}{1.44 \, M_\odot}\right)^{1/3} \text{km} . \quad (4)$$

We will see that this constrains the EOS to be relatively soft around nuclear densities but quite stiff at higher densities. The stiffness at high densities is necessary to ensure that the binary pulsar constraint of $M_{max} \geq 1.44 \, M_\odot$ is satisfied. The softness at nuclear density is required to raise the central density, if the limit $\Omega_K \geq 1.237 \times 10^4$ s^{-1} is imposed.

III. ROTATIONAL PROPERTIES WITH NUCLEONIC EQUATIONS OF STATE

Matter in neutron stars is in beta equilibrium, that is, the energy is optimized with respect to the proton/baryon ratio. In the density range below nuclear density ($n_s \simeq 0.16$ fm^{-3}), the EOS is dominated by both leptons and nuclei, the description of which is very well understood (Baym, Pethick, and Sutherland 1971; Negele and Vautherin 1974). For the low-density ($n < 0.001$ fm^{-3}) EOS we use the results of Baym, Pethick, and Sutherland, and for the mid-density regime ($0.001 < n < 0.08$ fm^{-3}) we use the results of Negele and Vautherin. (A change of about −0.2 km occurs in R_{max} if the EOS of Baym, Pethick, and Sutherland is used throughout.)

Above $\frac{2}{3}$ nuclear density, the nuclei merge to form a sea of baryons. It is often useful to express the ground-state energy of nucleons using the energy of symmetric nuclear matter, $E(n, x = \frac{1}{2})$, as a reference since properties of symmetric matter near n_s are, in principle, obtainable from experiment. Expanding in terms of the proton fraction x, a reasonable approximation for the EOS is:

$$\epsilon = n[E(n, x = \tfrac{1}{2}) + S(n)(1 - 2x)^2 + \cdots + E_l] . \quad (5)$$

Here ϵ is the mass-energy density and $S(n)$ is the density-dependent symmetry energy. We have found that higher order terms in the parameter $1 - 2x$ than that shown in equation (5) are very small in practically all equations of state. $E_l = E_e + E_\mu$ represents the lepton contributions, where E_e and E_μ are the energies of the free electron and muon gases, respectively. In beta equilibrium, the optimal proton fraction is determined by the relation $\partial(\epsilon/n)/\partial x = 0$. *Near* n_s, the energy of symmetric nuclear matter can be expanded as

$$E\left(n, x = \frac{1}{2}\right) = m - 16 + \frac{K}{18}\left(1 - \frac{n}{n_s}\right)^2 + \cdots , \quad (6)$$

where m is the baryon mass (939.5 MeV); we use units of MeV and fm throughout. The binding energy of normal nuclear matter is −16 MeV, and the compression modulus of symmetric nuclear matter is $K = 9(\partial P/\partial n)_{x=1/2}$, where P is the

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pressure. The nucleon kinetic energies are implicitly included in equations (5) and (6).

From nuclear mass systematics, it is known that $S(n_s) \simeq 30$ MeV, but the functional form of $S(n)$ remains a mystery. Roughly one-half the symmetry energy arises from the nucleon kinetic energies, and their contribution to the symmetry energy varies as $n^{2/3}$. However, the density dependence of the potential contribution to the symmetry energy is unknown. Unfortunately, it is not possible to employ nuclear mass systematics to reliably determine the compression modulus. Some estimates, derived from experimental determinations of the giant monopole resonance, for the compression modulus of symmetric matter K exist and suggest that it lies in the range 200–250 MeV (Blaizot 1980). Theoretical analyses of heavy ion experiments below 2 GeV per nucleon laboratory energy are currently being used (Bertsch and Das Gupta 1988; Gale *et al.* 1989) to extract the behavior of the nuclear mean field for densities up to $4n_s$. Accounting for the momentum dependence crucial in such analyses, a mean field that gives rise to an EOS with $K = 200$–250 MeV is found to be consistent with the heavy ion data. However, both these experimental determinations must be regarded with caution at present (Brown 1988).

In equation (6), K is the compression modulus of *symmetric* nuclear matter. It is related to a similar modulus for *neutron-only* matter, $K_n = 9\, dP/dn|_{x=0}$ by (Prakash and Bedell 1985)

$$K_n = K + 9[n^2 S''(n) + 2nS'(n)] , \qquad (7)$$

using equation (5). Primes denote derivatives with respect to density. Thus, at nuclear density, K_n is generally much larger than K and may be dominated by the symmetry term. Since neutron star matter is in beta equilibrium, and the proton fraction at nuclear density is small, approximately 0.03–0.05, the compressibility of neutron star matter is closer to that of neutron matter than to that of symmetric matter. However, it is the compression modulus for symmetric matter that is experimentally accessible. No direct measurements of the compressibility of pure neutron matter exist. Thus, the density dependence of the symmetry energy is a potentially important variable for the structure of neutron stars (Prakash, Ainsworth, and Lattimer 1988).

The many attempts to determine the EOS for dense nucleon matter can be conveniently grouped into three categories: (1) nonrelativistic potential models, (2) field theoretical models, and (3) the relativistic Dirac-Brueckner approach. In our work we will use only the most recent calculations from each group, since these represent both technical and physical improvements over the older models.

a) Nonrelativistic Potential Models

In this approach, one starts from a Hamiltonian with a two-nucleon potential that fits nucleon-nucleon (NN) scattering data and the properties of deuteron. The quantum many-body problem is traditionally handled either by a selective summation of diagrams in perturbation theory (the Brueckner-Bethe-Goldstone approach) or using a variational method with correlation operators (Pandhaharipande 1971*a*, *b*). Nuclear matter with two-body forces alone consistently saturated near $2n_s$ rather than n_s (Day and Wiringa 1985). This result holds for all phase shift–equivalent potentials and for the different methods (Brueckner-Bethe-Goldstone or variational) used to calculate the energy.

To achieve saturation at the empirical density, Friedman and Pandharipande (1981) added a density-dependent three-nucleon interaction, which effectively incorporates the suppression of nonnucleonic degrees of freedom, e.g., Δ isobar resonances, in the construction of a two-body potential. The work of Wiringa, Fiks, and Fabrocine (1988) is a recent update of such variational calculations with explicit forms for three-nucleon interactions. This work also includes several technical improvements in the calculations of energy expectation values and a more thorough search for the best variational wave functions. The work of Friedman and Pandharipande (1981, hereafter FP) and Wiringa, Fiks, and Fabrocine (1988, hereafter WFF) represent definite improvements over earlier calculations (e.g., Pandharipande 1971*a*, *b*) because they are able to come closer to the empirical properties of nuclear matter. However, an important shortcoming of many potential models, including these calculations, is that the speed of sound exceeds the speed of light at densities relevant for maximum mass neutron stars. For example, the EOS of Friedman and Pandharipande becomes acausal at about $6n_s$, and some of Wiringa, Fiks, and Fabrocine do so in the range 6–$9n_s$.

In Table 1 we show properties of maximum mass neutron stars for the equations of state described above. These calculations are for neutron star matter with beta equilibrium including muons. The Keplerian rate as calculated from equation (1) is also displayed. In some cases, full general relativistic structural calculations were performed by Friedman, Ipser, and Parker (1989) or by us (Masak, Lattimer, and Yahil 1989). In every one of these cases, equation (1) proved to be valid to better than 4%.

It is seen that two of the entries in this table have, simultaneously, $\Omega_K > \Omega_{1987A}$ and $M_{max} > M_{1913+16} = 1.44\ M_\odot$. However, given the accuracy with which the EOS can be determined from the published tables and figures, and the accuracy of the determination of the rotational properties, these suc-

TABLE 1

MAXIMUM MASS NEUTRON STARS FROM POTENTIAL MODELS

EOS Reference	n_s (fm^{-3})	K (MeV)	$M_{max}/M_\odot$	R_{max} (km)	n_c/0.16 fm^{-3}	Ω_K ($10^4\ s^{-1}$)
$FP_{V14+TNI}$	0.160	240	1.95	9.07	8.79	1.25
$WFF_{AV14+UVII}$	0.194	209	2.13	9.34	7.96	1.24(1.24)
$WFF_{UV14+UVII}$	0.175	202	2.19	9.76	7.35	1.18
$WFF_{UV14+TNI}$	0.163	269	1.84	9.45	8.71	1.14

NOTES.—n_s and K are the nuclear matter saturation density and compression modulus; M_{max}, R_{max}, and n_c are the mass, radius, and the central density of the nonrotating configuration. Ω_K is the Keplerian frequency according to eq. (1). The number in parentheses is the maximum rotational frequency from a general relativistic calculation assuming uniform rotation.

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cesses are only marginal. If nonaxisymmetric instabilities set in within few percent of Ω_K, then these equations of state must be viewed as failures. As an example, Friedman, Ipser, and Parker (1989) found $\Omega_K = 1.23 \times 10^4\ s^{-1} < \Omega_{SN\,1987A}$ for the FP EOS, although their result is within 1% of our own.

The FP EOS has an additional problem. The UV14+TNI interaction used in the FP EOS leads to a symmetry energy that decreases beyond about 3 times the saturation density and approaches zero at 5 times n_s. Thus, the softening of the EOS from leptons occurs only near nuclear matter density and not also at higher densities, as is the case for most other equations of state. It is important to note that for the same interaction, Wiringa, Fiks, and Fabrocine (1988) have recalulated the EOS with improved techniques, and, as seen in Table 1, Ω_K decreases by more than 10%, to values below $\Omega_{SN\,1987A}$.

As our analysis below will make clear, model AV14+UVII succeeds because it is relatively soft close to nuclear densities and relatively stiff at high densities. In fact, it exceeds the causality limit above 1 fm^{-3}, which is about $\frac{3}{4}$ of the central density of the maximum mass stars. Although the maximum mass and radii did not appreciably change when we forced causality (as in eq. [9] below) at high densities with this EOS, the effective compressibility at normal saturation densities (0.16 fm^{-3}) is quite small. This EOS saturates at a density 20% larger than this and has a compression modulus of 209 MeV there. Since one expects the modulus to increase rapidly with density, perhaps as n^2–n^3, the compression modulus evaluated at the *fiducial* density of 0.16 fm^{-3} is only about 100–150 MeV. In addition, this equation of state has a further bit of softening in the range 0.2–0.3 fm^{-3}, which is attributed to neutral pion condensation. As we discuss in § V, this is an important ingredient in obtaining rapid rotation.

The nonrelativistic potential models that are included in the often-quoted compendium of Arnett and Bowers (1977) include those of Pandharipande (1971*a*, hereafter A, pure neutrons) and Pandharipande (1971*b*, hereafter B, hyperons in beta equilibrium). These calculations were superceded by the calculations of Friedman and Pandharipande (1981) and were improved once again by the work of Wiringa, Fiks, and Fabrocine (1988). Another such model is the EOS of Arponen (1972, hereafter F). Above 2×10^{15} g cm^{-3}, it is matched onto EOS B. Friedman, Ipser, and Parker (1989) showed that this set of equations of state (A, B, F) *does* lead to neutron stars that rotate at rates nearly equal to or greater than Ω_{1987A}, so one could argue that their functional forms are of interest in deciding what kinds of equations of state have this property. Results for these equations of state are: F (Ω_K = 12,400 s^{-1}), A (Ω_K = 12,800 s^{-1}) and B (Ω_K = 15,700 s^{-1}). However, EOS B has a maximum mass of 1.414 $M_\odot$ and therefore is ruled out by the binary pulsar PSR 1913+16. Equation of state A is for neutrons only. When beta equilibrium with muons is included, its Ω_K falls below $\Omega_{SN\,1987A}$. Beyond 3.9×10^{15} g cm^{-3}, equations of state F, A, and B become superluminal. This density is below the central densities of the maximum mass neutron stars for B and F.

Although Pandharipande (1971*b*) does not give either $S(n)$ or K for equations of state A and B, their values may be inferred from his published results: we find $K \simeq 60$ MeV and $S(n_s) \simeq$ 35 MeV, but the density dependence of S is such that the compressibility of neutron-rich matter is not much different from K. The F equation of state (Arponen 1972) has $K \simeq 200$ MeV and $S(n_s) \simeq 30$ MeV. Clearly, the constraints described above are satisfied: the effective compression modulus of symmetric matter is relatively low, and these equations of state are all soft near nuclear matter density but become very stiff at higher densities.

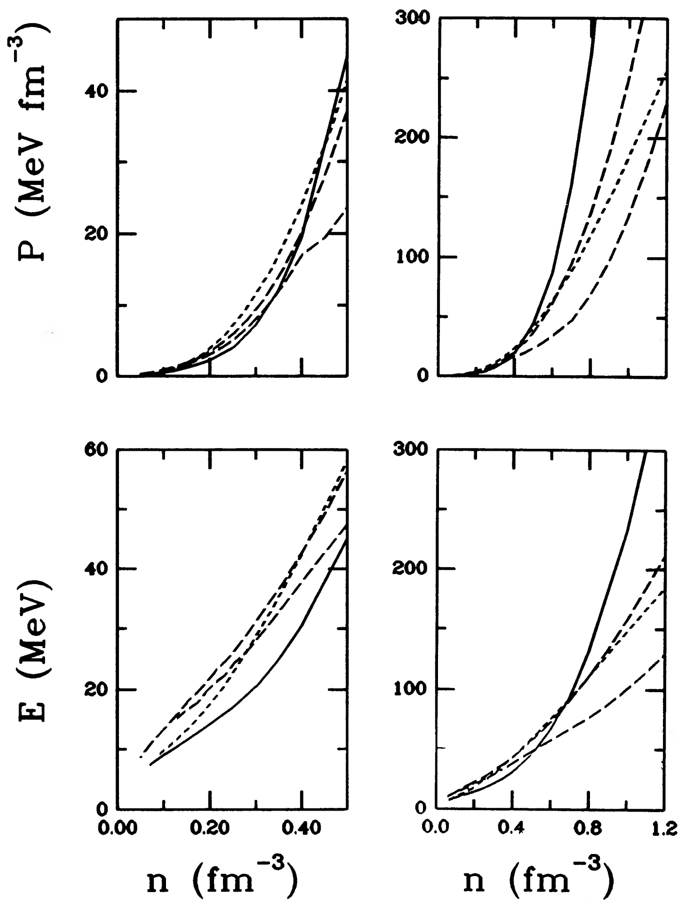

FIG. 1.—Comparison of energy and pressure as a function of baryon density for potential model equations of state enabling rotation nearly equal to or greater than that of the pulsar in SN 1987A. Solid lines show results of Wiringa, Fiks and Fabrocine (1988, model AV14+UVII) for neutron star matter in beta equilibrium. Long dashed lines are from Pandharipande (1971*a*, *b*) and are referred to as models A (pure neutron matter, upper dashed curve) and B (matter in beta equilibrium with hyperons, lower dashed curve). Short dashed lines are from Arponen (1972, model F) for beta equilibrium matter. For clarity, the low- and high-density parts are shown separately.

In Figure 1 we compare the energy and pressure of models A, B, F, and WFF (AV14+UVII), all of which enable rotation nearly equal to or greater than that of the pulsar in SN 1987A. At low density ($0.2 < n < 0.4$ fm^{-3}), the EOS of Wiringa, Fiks, and Fabrocine is the softest, while for high densities it is the stiffest. Wiringa, Fiks, and Fabrocine attribute the softening at low densities to neutral pion condensation (we will return to charged pion condensation later). This additional softening allows for rapid enough rotation, while the stiffening at high densities (in this case to the causal limit) provides for a maximum mass significantly above 1.44 $M_\odot$. Note that, in comparison, models A, B, and F all have maximum masses very close to 1.44 $M_\odot$. This result implies that the condition $\Omega_K > \Omega_{SN\,1987A}$ may be met by stars with rest masses of order 2 $M_\odot$ in contrast to the upper limit of 1.7 $M_\odot$ quoted by Friedman, Ipser, and Parker (1989).

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b) Field-Theoretical Models

Here one starts from local, renormalizable Lagrangian densities with baryon and meson degrees of freedom (Serot and Walecka 1986). The models rigorously satisfy causality but sacrifice the connection to NN scattering data, since the coupling constants and masses are chosen to fit only the nuclear matter saturation properties. The traditional approach is to start with a mean-field approximation and then to include effects of vacuum fluctuations and correlations. At the mean-field level, calculations with and without scalar self-interactions have been reported (Serot 1979; Boguta 1981; Glendenning 1986). Inclusion of nucleon and meson vacuum fluctuation terms at the one-loop level generally improves fits to nuclear matter saturation properties (Chin 1977; Jackson, Rho, and Krotschek 1985; Prakash and Ainsworth 1987; Glendenning 1989*a*).

Recently full two-loop corrections for nuclear matter in the Walecka model including contributions from vector meson exchanges have been calculated (Furnstahl, Perry, and Serot 1989). The size and nature of the corrections indicate that the loop expansion is not convergent at two-loop order in either the strong or weak sense. The result is that good fits to saturation properties are no longer possible with acceptable values of coupling constants and masses, pointing to a need for alternative approximation schemes. From a phenomenological point of view, results of mean-field theory and one-loop corrections to it are nevertheless useful in that they have been successful in describing many nuclear systems and phenomena. For this reason, our calculations of star structure will include results up to only one-loop corrections. All such models have potential contributions to the symmetry energies that are roughly proportional to density.

The properties of nonrotating maximum mass neutron stars resulting from these equations of state are shown in Table 2. The EOS of Serot (1979) is calculated in the mean-field approximation to the Walecka model including σ-ω and ρ mesons and that of Chin (1977) with one-loop corrections to the σ-ω model, using coupling constants and masses quoted by these authors. Our calculations, however, include the effects of beta equilibrium, which these authors did not originally consider. Both these equations of state are rather stiff close to and above nuclear densities and therefore can support sufficiently massive stars. However, the corresponding radii are also large, with the result that they cannot sustain rotation with rates greater than 10^4 s^{-1}. It is possible to improve somewhat the saturation properties of the Chin EOS (Prakash and Ainsworth 1987) by a suitable choice of coupling constants. This model is referred to in Table 2 as Chin(PA). The lowering of the saturation density without a corresponding decrease in the compression modulus has effectively led to an overall stiffening of the EOS, with the result that both the maximum mass and the radius increase. This lowers Ω_K for this case.

The EOS of Prakash and Ainsworth (1987, hereafter PA) was calculated using the linear sigma model with one-loop corrections. The recent work of Glendenning (1989*a*, hereafter Gle) also includes one-loop corrections to a model with scalar self-interactions (up to quartic order) whose magnitudes are adjusted to reproduce empirical saturation properties. This work includes the effect of the hyperons in beta equilibrium in addition to electrons and muons, which further softens the EOS above 3 times nuclear density, and as far as rapid rotation is concerned, is counterproductive. It is clear that none of these models can rotate rapidly enough. This is because all field theoretic models, even those with reasonable values for the compressibility parameter, approach causality at high densities very slowly, in constrast to nonrelativistic potential models.

c) The Relativistic Dirac-Brueckner Approach

This approach (Brockman and Machleidt 1984; ter Haar and Malfliet 1986; Horowitz and Serot 1987) is based on the idea that the self-energy of a nucleon in medium is made up from a large attractive scalar potential and a repulsive vector potential of comparable magnitude. The magnitude of these large self-energy terms indicates that relativistic effects are important even at low energies. The nucleon spinors that are solutions of the Dirac equation are characterized by an effective mass equal to the sum of the free mass and the scalar potential. In a self-consistent calculation, these spinors are used to evaluate the matrix element of the NN potential, e.g., the one-boson-exchange potential, in the nuclear medium. Following the lines of Brueckner theory, the Bethe-Goldstone equation is solved to account for the effects of NN correlations. The nucleon self-energy is evaluated in the Brueckner Hartree Fock approximation. There are two sets of calculations (Horowitz and Serot 1987, hereafter HS; Muether, Prakash, and Ainsworth 1987, hereafter MPA) which have been carried out to high enough densities for neutron star structures to be calculated; results are shown in Table 3. Once again, the potential contributions to the symmetry energy are roughly linear in density.

These equations of state also fail to give rapid enough rotation. However, they do approach the causality limit somewhat more rapidly than the field theoretic models, which results in higher Keplerian rotation rates.

To summarize this section, we find that the only functional forms for the energy-density relation that approach the rate of

TABLE 2

MAXIMUM MASS NEUTRON STARS FROM FIELD THEORETICAL APPROACHES

EOS Reference	n_s (fm^{-3})	K (MeV)	$M_{max}/M_\odot$	R_{max} (km)	n_c/0.16 fm^{-3}	Ω_K (10^4 s^{-1})
Serot	0.193	540	2.54	12.28	4.92	0.90
Chin	0.193	471	2.10	10.87	6.54	0.96
Chin(PA)	0.160	462	2.30	12.10	5.36	0.88
PA	0.160	225	1.45	9.89	7.94	0.94
Gle	0.153	300	1.79	11.18	6.88	0.87(0.84)

NOTES.—n_s and K are the nuclear matter saturation density and compression modulus; M_{max}, R_{max} and n_c are the mass, radius, and the central density of the nonrotating configuration. Ω_K is the Keplerian frequency according to eq. (1). The number in parentheses is the maximum rotational frequency from a general relativistic calculation assuming uniform rotation.

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TABLE 3

MAXIMUM MASS NEUTRON STARS FROM DIRAC-BRUECKNER APPROACHES

EOS Reference	n_s (fm^{-3})	K (MeV)	$M_{max}/M_\odot$	R_{max} (km)	$n_c/0.16\ fm^{-3}$	Ω_K ($10^4\ s^{-1}$)
HS	0.193	275	1.93	10.23	7.57	1.04(1.04)
MPA	0.157	200	2.44	11.25	5.68	1.01

NOTES.—n_s and K are the nuclear matter saturation density and compression modulus; M_{max}, R_{max}, and n_c are the mass, radius, and the central density of the nonrotating configuration. Ω_K is the Keplerian frequency according to eq. (1). The number in parentheses is the maximum rotational frequency from a general relativistic calculation assuming uniform rotation.

$\Omega_{SN\,1987A}$ are those of the nonrelativistic potential model variety, which achieve maximum stiffness ($\partial P/\partial\epsilon = 1$) by about 6 times nuclear density. In addition, in successful models the effective compressibility at saturation density is relatively low (in the range 100–150 MeV), and the presence of additional softening above nuclear density (in the range of 2–$3n_s$) is helpful.

IV. RECIPE FOR RAPID ROTATION

From the previous discussion, both the compressibility at the saturation density and the stiffness at high density are important parameters for neutron star structure. In this section we explore the dependence of the mass and radius of the maximum mass stars to these features of the equation of state. In most models of the EOS, the sound speed [$(\partial P/\partial\epsilon)^{1/2}$] approaches a limiting value by 5–8 times nuclear density (this value is sometimes greater than c!). If equation (5) were valid up to these densities, the sound speed would equal the speed of light when $\partial P/\partial\epsilon = 1$, or when

$$\frac{n}{n_s} \simeq 7.44\left(\frac{100\ \text{MeV}}{K}\right)^{1/2} . \quad (8)$$

This estimate is insensitive to the function chosen for $S(n)$ and whether or not the matter is in beta equilibrium. Equation (8) shows that the extrapolation of equations of state to moderate to high densities must be done carefully if the causality limit is to be obeyed.

Practically nothing is known experimentally about the EOS above a few times nuclear matter density. It is generally thought, however, that the causality limit must hold. This suggests that a suitable parameterization for the high-density region, above some transition density n_t, could be written using the relation $s = \partial P/\partial\epsilon =$ constant in this region. Denoting by P_t and ϵ_t the respective values of P and ϵ at n_t, we have:

$$\left.\begin{aligned} P &= \frac{1}{s+1}\left[P_t - s\epsilon_t + s(\epsilon_t + P_t)\left(\frac{n}{n_t}\right)^{s+1}\right] \quad (9a)\\ \epsilon &= \epsilon_t + (P - P_t)/s \quad (9b)\end{aligned}\right\} n > n_t .$$

Below n_t, we will employ the recent parameterization of the EOS proposed by Prakash, Ainsworth, and Lattimer (1988, hereafter PAL). It is constrained to reproduce known nuclear properties (binding energy, saturation density, effective mass, single particle potential, symmetry energy) and to be causal at all densities. This EOS is able to mimic, with suitable choices of its parameters (compressibility, finite range interaction lengths, and the functional form of the symmetry energy), a wide variety of potential, mean-field, and hybrid models of the equation of state. As far as the maximum mass stars are concerned, its single most important parameter is K. When coupled with the high-density EOS, equation (9), the additional parameters n_t and s are introduced and are important.

With these equations of state, we have calculated the masses and radii of maximum mass neutron stars in beta equilibrium, including muons. The case $s = 1$ may be compared to the calculations performed by Rhoades and Ruffini (1974), who coupled an $s = 1$ EOS onto the low-density EOS of Baym, Pethick, and Sutherland (1971). For the case $s = 1$, for $n_t/n_s \leq 6$, and for relatively low values for K (i.e., $K < 150$ MeV), we find the following approximate relations for the maximum mass stars:

$$\frac{M_{max}}{M_\odot} \simeq 4.1\left(\frac{n_s}{n_t}\right)^{1/2} ; \quad R_{max} \simeq 18.5\left(\frac{n_s}{n_t}\right)^{1/2}\ \text{km} . \quad (10)$$

The first of these, in fact, is almost exactly the result found in Rhoades and Ruffini (1974). We have found that these relations are remarkably insensitive to all reasonable choices for the other EOS parameters such as the symmetry energy. For the condition $n_t/n_s < 6$ and $K \ll 150$ MeV, we may approximate $n_t/n_s \simeq \epsilon_t/\epsilon_s$ and $P_t \simeq 0$. The equation of state has then the form $P = \epsilon - \epsilon_t$. Haensel and Czerny (1989) have shown that such an EOS gives the relations

$$\frac{M_{max}}{M_\odot} = 4.1\left(\frac{n_s}{n_t}\right)^{1/2} ; \quad R_{max} = 17.4\left(\frac{n_s}{n_t}\right)^{1/2}\ \text{km} . \quad (11)$$

The difference between these relations and equation (10) is due to our inclusion of a low-density "tail" on the EOS, which does not affect the total mass, but changes the radius. Substituting equation (10) into equation (1), we find

$$\Omega_K = 6.2\left(\frac{n_t}{n_s}\right)^{1/2} \times 10^3\ s^{-1} ; \quad (12)$$

that is, in order to achieve a rotational frequency as fast as Ω_{1987A} would require $n_t/n_s \geq 4.0$. Furthermore, the nonrotating maximum mass would have to exceed 1.98 $M_\odot$, according to equation (10). It is worth pointing out that in the limit $P_t = 0$, equation (11) implies that the coefficient in equation (12) becomes 6.9×10^3. With the additional constraint that $M_{max} > 1.44\ M_\odot$, or $n_t < 1.2\ fm^{-3}$ from equation (11), we find a maximum Ω_K of $1.98 \times 10^4\ s^{-1}$, which is the ultimate rotational speed limit for any causal equation of state that has vanishing pressure and energy density at low baryon densities.

However, for values of $K \geq 150$ MeV, the relationships of equation (10) break down. Specifically, the radii of maximum mass neutron stars deviate substantially from this formula, and for large enough K, they even increase with increasing n_t. Figure 2 displays the Keplerian frequencies, as deduced from

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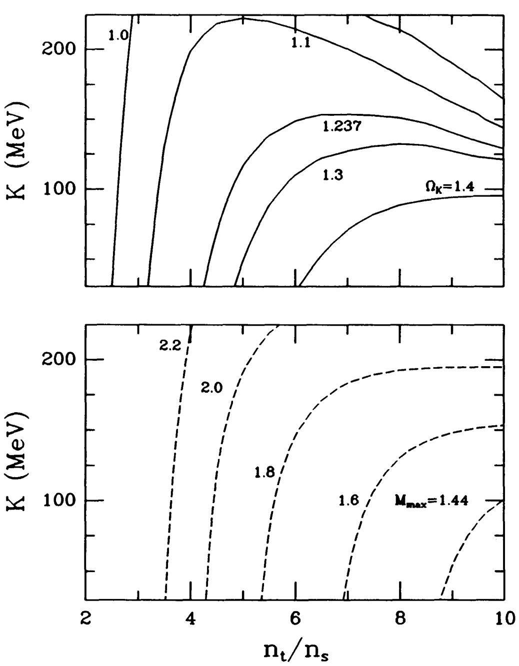

FIG. 2.—Solid lines show contours of Keplerian frequencies according to eq. (1) as a function of the compression modulus of nuclear matter and the transition density beyond which the speed of sound is set to the speed of light. Dashed lines are contours of fixed maximum mass.

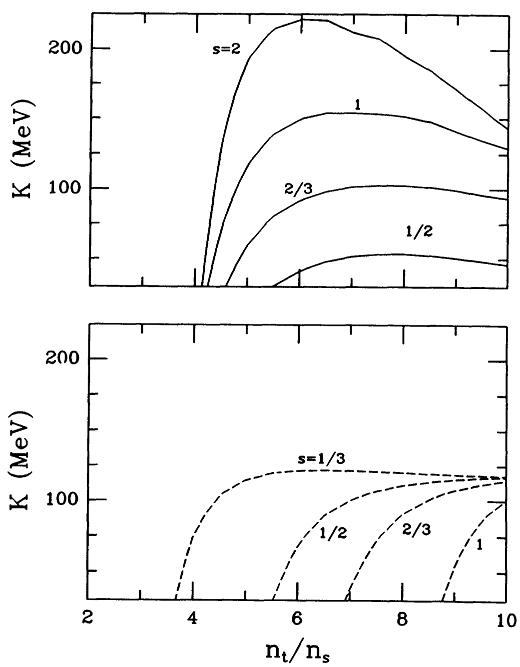

FIG. 3.—Solid lines show contours of $\Omega_K = \Omega_{SN\,1987A}$, and dashed lines are contours of $M_{max} = 1.44\ M_\odot$ as a function of the compression modulus of nuclear matter and the transition density. The different curves are for different values of the square of the sound speed, $s = dP/d\epsilon$.

equation (1), as a function of the parameters K and n_t, for the case $s = 1$ and assuming the potential contributions to the symmetry energy vary linearly with density. It is clear from Figure 2 that a maximum value of the compressibility is permitted, if $\Omega_K \geq \Omega_{SN\,1987A}$ is taken as a limit; this value is approximately 155 MeV. This value is relatively insensitive to the choices $S(n_s)$ and its density dependence so long as the total symmetry energy increases with density: for example, in the case with $S(n_s) = 25$ MeV and density-independent potential contributions, the limit increases slightly to 165 MeV. Only in cases for which the total symmetry (kinetic plus potential) energy vanishes beyond a few times nuclear density can this limit be substantially increased. An example is the force UV14+TNI, which is the underlying potential in the FP and WFF(UV14+TNI) equations of state. As previously noted in Table 1, both these equations of state result in relatively rapid rotation even though their compression moduli are considerably larger than 160 MeV.

In Figure 2, the dashed lines are contours of fixed M_{max}. The dashed line in the far right represents the contour $M_{max} = 1.44\ M_\odot$: the region below and to the right of this curve is not an allowed region because of the binary pulsar constraint. It is of interest to note that the model with the largest permissible value of K has a mass of about 1.8 $M_\odot$, in rough agreement with the estimate of about 2 $M_\odot$ inferred earlier. Also note that the contour lines become vertical in the low n_t, low K region, for which equation (10) applies.

The sensitivity of these results to s was also examined. In Figure 3 contours of $\Omega_K = \Omega_{SN\,1987A}$ and $M_{max} = 1.44\ M_\odot$ are shown for s values of $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, and 2. Note that the constraint on the compressibility parameter becomes progressively more severe as s is lowered. Also, the region to be excluded because of the mass constraint becomes larger as s is lowered. In particular, *note that there are no acceptable solutions for* $s \leq \frac{1}{2}$; the entire region with $\Omega_K \geq \Omega_{SN\,1987A}$ is then excluded. This is an important result: the case $s = \frac{1}{3}$ is of great interest because this is the limiting EOS of a quark gas having the property of asymptotic freedom, as in the MIT bag quark model. Thus, such equations of state will have extreme difficulty if the pulsations observed in SN 1987A represent rotation (this point is addressed in more detail in the next section). We have also included in Figure 2 a contour for the unphysical case $s = 2$, because this represents the behavior of the equations of state A, B, F, and FP at extremely high densities. The upper limit for K is increased in this case to about 220 MeV. This value is, interestingly enough, very close to the compression moduli of EOS F and EOS FP, which are among the few "successful" equations of state.

We stress that these results do not depend in detail on the effective mass or finite range interaction lengths of the PAL EOS, as long as the nuclear symmetry energy was taken to be a monotonically increasing function of density. We have also made computations using the simpler, parabolic EOS equation (5) below n_t, and we have obtained nearly identical results. Thus, our conclusions appear to be quite general. The Friedman, Ipser, and Parker (1986, 1989) computations of Ω_K can be understood in terms of the effective parameters K and s of the respective underlying equations of state. It must be concluded that only equations of state that are (a) soft (low compressibility parameter) near nuclear density and (b) very stiff, i.e., causal or supercausal, above 5–8 times nuclear density can simultaneously have $\Omega_K \geq \Omega_{SN\,1987A}$ and $M_{max} \geq 1.44\ M_\odot$. In the next section we explore some possible alterations of the standard EOS that may ameliorate the constraint on the compressibility.

V. PHASE TRANSITIONS AND NEUTRON STAR STRUCTURE

We have observed that in order to obtain the rapid rotation implied by the observation of a 0.5 ms pulsar in SN 1987A, it is

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necessary for the EOS to remain soft in the vicinity of nuclear density and to stiffen at higher densities to near the causal limit. Phase transitions can soften the equation of state. Many different physical mechanisms have received attention that can lead to phase transitions in neutron star matter above nuclear density: (*a*) pion condensation, (*b*) kaon condensation, (*c*) parity doubling of the nucleon, and (*d*) quark matter, either the two- or three-flavored variety.

a) *Pion Condensation*

Considerable work has been carried out, both theoretically and experimentally, to find pion condensates (Migdal 1978; Rho and Wilkinson 1979; Gyulassy and Greiner 1977). Early applications of pion condensation to neutron star structure were considered by Hartle, Sawyer, and Scalpino (1975), Weise and Brown (1975), and Maxwell and Weise (1975). In the latter works, π^- condensation was discussed using chiral-invariant πN Lagrangians (i.e., the σ model) which lead to a considerable softening of the EOS due to the attractive pion-nucleon *p*-wave interaction. In such models, the critical density at which the transition occurs is given by the relation

$$n_t \simeq \frac{0.32}{g_A^*(g_A^{*2} - 1)^{1/2}}\ \mathrm{fm}^{-3}\ . \tag{13}$$

The effective *p*-wave coupling strength is $g_A^{*2} \simeq (1 - g')(1 + S)g_A^2$, where $g_A = 1.36$ is the free axial-vector coupling constant. The term g' summarizes local field corrections due to short-range correlations between nucleons from ρ- and ω-meson exchanges, and S accounts for interactions with isobars. Originally, values of $g' = 0.4$–0.5 and $S = 0.8$–0.9 were considered to be appropriate, leading to values of $g_A^* = 1.3$–1.5 for which the critical density $n_t = 1.9$–1.2 times the nuclear saturation density $n_s = 0.16\ \mathrm{fm}^{-3}$.

Using the neutron-only matter EOS A of Pandharipande (1971*a*), substantial softening due to pion condensation was found for g_A^* of 1.3 and 1.5, with the result that the radii of maximum mass stars were reduced to about 7.5 and 6.7 km, respectively, from 8.6 km for the case without pion condensation. The corresponding maximum masses were 1.51 and 1.46 $M_\odot$ to be compared with 1.66 $M_\odot$ for the case without phase transition. Using equation (1), Keplerian frequencies larger than $1.237 \times 10^4\ \mathrm{s}^{-1}$ are found with these masses and radii, in agreement with the full calculations of Friedman, Ipser, and Parker (1989) (their model π). However, effects due to beta equilibrium were not included in these earlier calculations. Imposing beta equilibrium, we find the maximum mass to be lower than 1.44 $M_\odot$, so that for this EOS, the two constraints are not simultaneously satisfied. It may, however, be possible to satisfy both these constraints using more modern nuclear equations of state.

The above results must, however, be viewed with caution in light of the findings of recent experiments that have determined the crucial ingredient g' of these models. Pion condensation leads to an effective softening of the EOS only if the attractive *p*-wave interaction predominates over the repulsive *s*-wave interaction. This requires $g_A^{*2} - 1 > 0$. The condensation mechanism considered here deals chiefly with spin-isospin sound, involving mainly valence nucleons. The Fermi liquid parameter g' refers to the nucleon particle-hole spin-isospin branch and is now experimentally measured (Carey *et al.* 1984; Moss 1985) to be ≥ 0.9. The calculations of Brown, Osnes, and Rho (1985) also suggest similar values. *The upshot is that the above mechanism invoked for π^- condensation is ruled out from experiments. Thus the energy-density relationships given by these models are only suggestive of effects due to pion condensation, provided alternative mechanisms can reconstitute pion condensation.* Recent investigations (Brown 1989) do find a softening of pionic modes due to the admixture of isobar-hole components, which may indicate the onset of pion condensation, but only at several times the nuclear density.

b) *Kaon Condensation*

The possibility of kaon condensation due to chiral symmetry breaking for densities above 3–4 times the nuclear density has recently been proposed (Kaplan and Nelson 1986; Brown *et al*, 1988). Kaon condensation is expected to be only weakly affected by nuclear dynamics, unlike the case for pion condensation. The condensation is brought about chiefly due to attractive *s*-wave kaon-baryon interactions. Possible attractive *p*-wave interactions in certain channels are thought to help in driving the condensation, since there appears to be no strong repulsion in kaon-baryon interactions that counterbalances the attraction gained due to symmetry breaking.

The threshold for K^- condensation is estimated to be (Brown, Kubodera, and Rho 1987)

$$n_t = \frac{f_K^2(m_K^2 - \mu^2)}{\Sigma^{KN}} \simeq 2.7 n_s\ , \tag{14}$$

where f_K is the kaon decay constant, m_K is the kaon mass, and μ is the chemical potential associated with charge conservation. The above numerical estimate was obtained using a value of 570 MeV for the symmetry breaking term, Σ^{KN}. Its value is rather uncertain; lower values would push the threshold to higher density.

As of now only schematic models exist for estimating the amount of energy gained by kaon condensation (Brown *et al.* 1988). Our calculations using these estimates show that the softening provided by kaon condensation has much the same effect on maximum mass configurations of neutron stars as that from π^- condensation considered earlier, provided the condensation occurs around 3–5 times the nuclear density. For higher threshold densities (recall that Σ^{KN} is poorly known), the softening due to condensation increases the radius of the star with an attendant decrease in the maximum mass from values obtained without condensation. This results in values of Ω_K substantially smaller than $\Omega_{\mathrm{SN\,1987A}}$. A more realistic calculation of the energy-density relationship for neutron star matter in beta equilibrium is therefore a promising direction for further study.

c) *Chiral Parity Doubling Transitions*

Recent lattice calculations (DeTar and Kogut 1987) at finite temperature have suggested that there exist parity-doublet baryons with finite mass above the critical temperature for chiral transition. Hatsuda and Prakash (1989) have shown that such a phenomena occurs also at finite baryon densities leading to a first order chiral phase transition. They start from the generalized σ model of DeTar and Kunihiro (1987) in which two kinds of nucleons with opposite parity $N_+(939)$ and $N_-(1535)$ are introduced without contradicting chiral symmetry. To make the calculation realistic, they also take into account ρ and ω mesons in addition to π and σ mesons. They find a first-order chiral transition for both nuclear and neutron matter around 3–5 times the nuclear density within the mean field approximation. Once the phase transition occurs, the

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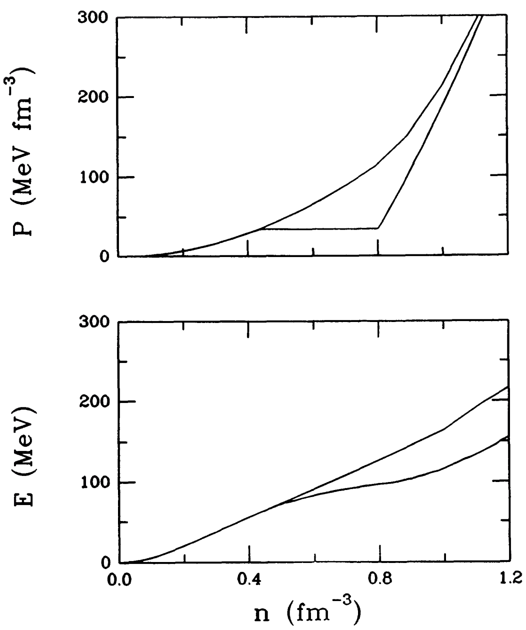

FIG. 4.—Energy and pressure as a function of baryon density for the parity-doublet model of Hatsuda and Prakash (1989). The lower (upper) curve in both panels corresponds to the case with (without) a first-order phase transition.

system prefers a chirally symmetric phase, and symmetry restoration occurs. As a consequence, the chirally symmetric matter composed of nucleons with a mass of $\sim$300 MeV leads to a soft equation of state.

Figure 4 shows the energy and pressure as a function of density, in this model, for neutron star matter in beta equilibrium. Values of the parameters used for these calculations are only slightly different from the published work (Hatsuda and Prakash 1989). The nuclear matter saturation density and compression modulus were chosen to be 0.16 fm^{-3} and 150 MeV, respectively. The minimum and maximum densities across the phase transition are $2.75n_s$ and $5n_s$, respectively. Although there is some ambiguity about the contribution of the vacuum fluctuations to the effective potential, this model leads to an energy-density relation that leads to stars with sufficient mass and small enough radii so that Keplerian rotation frequencies larger than SN 1987A are possible. This is due to the fact that the phase transition to parity doublet matter softens the EOS near nuclear densities but retains the causal behavior of the underlying equation of state at high densities. Results from this model are given in Table 4.

TABLE 4

MAXIMUM MASS NEUTRON STARS FOR THE PARITY-DOUBLING MODEL

$M_{max}/M_\odot$	R_{max} (km)	n_c/0.16 fm^{-3}	Ω_K (10^4 s^{-1})
1.58..............	8.63	11.41	1.21
1.47..............	7.74	13.10	1.37

NOTE.—The top (bottom) row corresponds to matter in beta equilibrium without (with) a first-order phase transition. M_{max}, R_{max}, and n_c are the mass, radius, and the central density of the nonrotating configuration. Ω_K is the Keplerian frequency according to eq. (1).

d) Quark Matter

We turn now to the possibility of neutron stars containing quark cores and stable collapsed stars containing quark-only matter. We discuss these two categories separately in what follows.

i) *Quark Cores*

Phase transitions to quark matter composed of *u* and *d* quarks as well as *u*, *d*, and *s* (strange) quarks have received a great deal of attention in the last decade. At present the energy-density relationship of the quark phase is calculable only using perturbative QCD. Starting with massless *u* and *d* quarks, the interactions between quarks are generally included via a constant positive energy per unit volume in the vacuum, *B* (bag constant), and by gluon exchange corrections to second order in the QCD coupling, g_c. For constant (density independent) values of g_c, the quark matter EOS then reads

$$P = (\epsilon - 4B)/3 . \tag{15}$$

The bag constant *B* is constrained so that the stable configuration of normal nuclear matter is the hadronic phase.

The transition density n_t above which the quark phase has a lower energy than the hadronic phase depends sensitively on *B* and also on whether the hadronic EOS is soft or stiff at the relevant densities. For stiff hadronic equations of state, n_t is usually very much larger than n_s and often even larger than the central densities n_c of neutron stars constructed from such a two-phase EOS. This has led many authors (Chapline and Nauenberg 1976; Baym and Chin 1976; Bowers, Gleeson, and Pedigo 1977; Serot and Uechi 1987) to conclude that stable stars cannot contain any quark cores.

It is, however, possible to arrange n_t to be a few times the nuclear density by an appropriate choice of *B* and the use of a soft hadronic EOS, and still have stars with $n_c > n_t$. In contrast to pion and kaon condensation considered earlier, a phase transition to quark matter has the effect that the EOS continues to be soft, with sound speed $\surd(1/3)$, beyond the transition region. Table 5 shows a few examples of maximum mass stars with quark cores for which the quark matter equations of state were calculated using $B \simeq 65$ MeV fm^{-3} and $g_c = 0$. In general, such stars have smaller masses and larger radii than those of neutron star models based on conventional hadronic-matter equations of state. Consequently, the Keplerian frequencies of stars containing quark cores are less than that of the hadronic EOS alone. These conclusions are not affected by choosing $g_c \neq 0$ in the calculation of the quark matter EOS.

If macroscopic quark matter does exist, then equilibrium with respect to weak interactions implies that at sufficiently high densities, quark matter must contain massive *s* (strange) quarks in addition to the massless *u* and *d* quarks. Above the threshold density for the appearance of strange quarks, strange quark (*u*, *d*, and *s*) matter, with strangeness per baryon of the order of unity, is energetically preferred to nonstrange quark (*u* and *d*) matter. The three-flavor quark matter EOS, again using perturbative QCD, is given by (Baym and Chin 1976; Freedman and McLerran 1978)

$$P = \tfrac{1}{3}(\epsilon - 4B) - \tfrac{1}{3}\epsilon_c(\epsilon - B) , \tag{16}$$

where the function ϵ_c represents small corrections due to the mass of the *s* quark. The phenomenological parameters *B*, g_c and m_s thus delineate the quark EOS. In this context, two classes of models have been considered in the literature. The

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TABLE 5

MAXIMUM MASS NEUTRON STARS WITH QUARK (u) AND (d) CORES

EOS Reference	$n_1/0.16\ \mathrm{fm}^{-3}$	$n_2/0.16\ \mathrm{fm}^{-3}$	$M_{max}/M_\odot$	R_{max} (km)	$n_c/0.16\ \mathrm{fm}^{-3}$	Ω_K ($10^4\ \mathrm{s}^{-1}$)
WFF(AV14+UVII)	5.74	7.91	1.97	10.07	7.51	1.07(1.06)
WFF(UV14+UVII)	4.88	6.07	1.92	10.71	6.63	0.96
WFF(UV14+TNI)	8.1	10.23	1.83	9.60	9.38	1.11

NOTES.—n_1 and n_2 are the minimum and maximum densities across the phase transition. M_{max}, R_{max}, and n_c are the mass, radius, and the central density of the nonrotating configuration. Ω_K is the Keplerian frequency according to eq. (1). The number in parentheses is the maximum rotational frequency from a general relativistic calculation assuming uniform rotation.

first class corresponds to the case of slightly unbound strange quark matter, in which matter at nuclear density is in the hadronic phase and a phase transition occurs above nuclear density. The second possibility is the existence of self-bound strange quark matter, to which we will return later.

The addition of mass to one of the flavors (the s quarks) causes very little deviation of the quark matter EOS from the massless case. The qualitative aspects of the EOS for neutron star matter are similar to the case of two-flavor matter as are the mass-radius relationships for the star. Fechner and Joss (1978) and Haensel, Zdunik, and Schaeffer (1986) have calculated properties of the maximum mass stars using nuclear equations of state from the Arnett and Bowers (1977) compendium. Their results and the use of equation (1) show that the resulting Keplerian frequencies are well below that required by the pulsar in SN 1987A. We have verified that use of more modern nuclear equations state such as that of Wiringa, Fiks, and Fabrocine does not affect this conclusion.

ii) *Self-Bound Strange Quark Stars*

The possibility that strange quark matter may be an *absolute* ground state of matter, i.e., $E_Q < 939$ MeV or more stringently, $E_Q < M(^{56}\mathrm{Fe})c^2/56 = 930.4$ MeV, has received considerable attention recently (Witten 1984; Fahri and Jaffe 1984; Haensel, Zdunik, and Schaeffer 1986; Alcock, Fabri, and Olinto 1986; Alcock and Olinto 1988). Employing the EOS in equation (16) given by perturbative QCD and for suitable choices of B, g_c, and m_s, self-bound strange star models have been calculated by Haensel, Zdunik, and Schaeffer (1986) and by Alcock, Fahri, and Olinto (1986). Rotational properties of strange quark stars have been recently discussed by Haensel and Zdunik (1989) and by Friedman and Olinto (1989). For completeness, we summarize results of their work here. Since models with $m_s = 0$ represent the best possible case for the most rapid rotation, we discuss their mass-radius relationships first.

The configurations with the maximum mass scale with the bag constant B as (Witten 1984; Haensel, Zdunik, and Schaeffer 1986)

$$\frac{M_{max}}{M_\odot} = 2.033\left(\frac{B_0}{B}\right)^{1/2} ; \quad R_{max} = 11.09\left(\frac{B_0}{B}\right)^{1/2}\ \mathrm{km}\ , \quad (17)$$

where $B_0 = 56\ \mathrm{MeV\ fm}^{-3}$ is a fiducial value for the bag constant. The value of B, however, is bounded from above, if self-bound strange matter is to be energetically preferred over the normal nucleonic matter. If the energy corresponding to ^{56}Fe crystal (930.4 MeV) is used as the ground state of cold matter, then $B < 1.634B_0$. A slightly larger upper bound, $B < 1.695\ B_0$, is obtained using 939 MeV as the ground-state energy. Use of the maximum allowed bag constant ensures the smallest radius and hence gives rise to the maximum Keplerian frequency. Using equation (1) to estimate the Keplerian frequency, we find that $\Omega_K = 0.94(B/B_0)^{1/2} \simeq 1.20 \times 10^4\ \mathrm{s}^{-1} < \Omega_{\mathrm{SN\,1987A}}$. We also performed a general relativistic calculation of the maximum rotation rate for this case, with the result that $\Omega = 1.20 \times 10^4\ \mathrm{s}^{-1}$, in good agreement with the approximation provided by equation (1).

For the case when $m_s \neq 0$, the numerical coefficients multiplying $(B_0/B)^{1/2}$ decrease from their values when $m_s = 0$ in both the mass and the radius. In addition, from energetic considerations the maximum allowed B decreases with increasing m_s. As a consequence, the Keplerian frequencies are systematically lower than for the $m_s = 0$ case. It must be mentioned that fits (Gasser and Leutwyler 1982) to data such as the mass of the kaon require m_s to be ~ 175 MeV. It may be concluded that the perturbative quark matter EOS cannot simultaneously satisfy $M > 1.44\ M_\odot$ and $\Omega_K > 1.237 \times 10^4\ \mathrm{s}^{-1}$.

The quark matter equations of state equations (15) and (16) are results of perturbative QCD. The use of perturbation theory in the strong coupling limit has been criticized by Bethe, Brown, and Cooperstein (1987), who advocate that quark matter with strangeness per baryon of order unity is well approximated by a close packed gas of Λ-particles at nuclear densities. Choosing the QCD coupling g_c to decrease with the Fermi momentum k_F of matter, they find that a phase transition occurs only at densities well beyond the central densities to be found in neutron stars with maximum masses in excess of $1.44\ M_\odot$. In view of these cautionary remarks regarding the use of perturbation theory, further investigations of the strong coupling regime are useful.

iii) *Self-Bound Stars with Schematic Equations of State*

The relation

$$P = s(\epsilon - \epsilon_0)\ , \quad (18)$$

where ϵ_0 is a constant energy density, represents an EOS with a constant speed of sound $(s)^{1/2}$ for all densities. The bag model result equation (15), which applies for massless quarks of both two and three flavors, corresponds to $s = \frac{1}{3}$ and $\epsilon_0 = 4B$. The linear dependence of pressure on the energy density in equation (18) has the interesting consequence that nonrotating configurations with maximum mass scale with ϵ_0 according to (Witten 1984)

$$M_{max}(\epsilon_0') = \left(\frac{\epsilon_0}{\epsilon_0'}\right)^{1/2} M_{max}(\epsilon_0)\ ; \quad R(\epsilon_0') = \left(\frac{\epsilon_0}{\epsilon_0'}\right)^{1/2} R(\epsilon_0)\ . \quad (19)$$

Haensel and Zdunik (1989) have recently studied the rotational properties of a star for the case of $s = 1$ which corresponds to an EOS that is at the causal limit for all densities. We have calculated the properties of the maximum mass configurations for a few other typical values of s. In Table 6 we

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TABLE 6

MAXIMUM MASS SELF-BOUND STARS FOR THE EOS $P = s(\epsilon - \epsilon_0)$

s	$\left(\frac{M_{max}}{M_\odot}\right)$	R_{max} (km)	$\Omega_K(10^4\ s^{-1})$
1/3	2.033	11.09	0.940
2/3	2.880	12.98	0.884
4/5	3.095	13.48	0.866
1	3.347	13.97	0.853

NOTES.—Results are for $\epsilon_0 = 4 \times 56$ MeV fm^{-3}; values for other values of ϵ_0 are obtained using the scaling law, eq. (19). The term s is the square of the speed of sound of matter; M_{max} and R_{max} are the mass and radius of the non-rotating configuration. Ω_K is the Keplerian frequency according to eq. (1).

summarize these results for a fiducial value of $\epsilon_0 = 4B = 4 \times 56$ MeV fm^{-3}. Results for other values of ϵ_0 are obtained using equation (19). From the results in Table 6, we find the approximate additional scaling relations

$$M_{max}(s') \simeq \left(\frac{s'}{s}\right)^{1/2} M_{max}(s) \quad \text{and} \quad R(s') \simeq \left(\frac{s'}{s}\right)^{1/4} R(s) , \quad (20)$$

which imply

$$\Omega_K(s') \simeq \left(\frac{s}{s'}\right)^{1/8} \Omega_K(s) . \quad (21)$$

The success or failure of these models depends crucially on the value of ϵ_0. In the bag model, as mentioned earlier, ϵ_0 is bounded from above: $(\epsilon_0)_{max} \leq 1.634/(4B_0)$. Lacking a detailed knowledge of the underlying physics, similar upper bounds are not calculable for the $s > \frac{1}{3}$ models at present. For the $s = 1$ EOS, Haensel and Czerny (1989) advocate the value 1.24×10^{15} g cm^{-3} (4×173.9 MeV fm^{-3}) for ϵ_0, for which $\Omega_K = 1.5 \times 10^4\ s^{-1} > \Omega_{SN\,1987A}$ and $M_{max} = 1.9\ M_\odot > 1.44\ M_\odot$ are obtained. Such a high value of ϵ_0 would imply that the density at which pressure vanishes is at several times the nuclear density. Somewhat lower densities can, however, be obtained by choosing a smaller value for s.

Subject to the constraint that $M_{max} \geq 1.44\ M_\odot$ and abandoning the notion that ϵ_0 has any physical bounds, the largest possible value of Ω_K may be obtained for a given s. These values are $\Omega_{K,max}/10^4\ s^{-1} = 1.33$, 1.77, 1.85, and 1.98, for $s = \frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{5}$, and 1, respectively, all of which are larger than $\Omega_{SN\,1987A}$. In the bag model ($s = \frac{1}{3}$), strange quark matter would have the baryon density $n_b = (4B/3\pi^{2/3}\hbar c)^{3/4}$ at the surface where the pressure vanishes. In order to obtain the above largest possible Ω_K, the bag pressure must be $B = 111.6$ MeV fm^{-3}, for which $n_b \cong 2.85$ times the nuclear saturation density. However, for this case, the energy per particle $\epsilon/n_b \cong 977.8$ MeV, which is substantially greater than the nucleon mass of 939 MeV, in contrast to the scenario proposed by Witten in which the energy per particle of strange quark matter would be less than the nucleon mass. Such a star, if it exists, would therefore be metastable. Similar considerations might apply for other self-bound equations of state as well, so that the maximum possible values of Ω_K may be more limited than those given above.

We must also bear in mind that in contrast to the bag model, models with $s > \frac{1}{3}$ suffer from the drawback that the constituents and the laws of interactions are unspecified. Therefore the success of the $s > \frac{1}{3}$ models can at best be viewed as an interesting possibility for which there is as yet no connection to identifiable physics. It should also be remembered that unless these self-bound equations of state apply at all densities, the addition of a low-density baryonic equation of state will substantially increase the radii of these stars. Thus equation (18) must apply *at all densities* in order for both mass and rotation constraints to be satisfied.

Summarizing this section, we observe that if the EOS around 3–5 times the nuclear density is softened by a phase transition, such as pion or kaon condensation or the occurrence of parity doublet matter, *and* a nearly causal behavior is attained at high densities, then a maximum mass greater than 1.44 $M_\odot$ and rapid enough rotation to match that of the purported pulsar in SN 1987A is possible. A phase transition to perturbative quark matter at high densities softens the EOS, and stars with quark cores can only rotate less rapidly than those without quark cores. If stars are made up entirely of perturbative quark matter with strangeness to baryon ratio of the order of unity, compact stars with enough mass can almost, but not quite, spin as fast as the pulsar reported in SN 1987A. The dual constraints of mass and rotation are met with schematic equations of state that are characterized by the square of speed of sound ≥ 0.30; however, these models lack a physical basis at present.

VI. GENERAL RELATIVISTIC CALCULATIONS

In this section, we summarize results from general relativistic calculations assuming uniform rotation using several equations of state considered in the earlier sections. These calculations were done following the approach developed by Butterworth and Ipser (1976). Table 7 and Figure 5 contain our results. In all cases considered, the calculated maximum angular velocity is very close to the approximation provided by equation (1). Figure 5 shows a comparison of the calculated maximum angular velocity (*open circles*) with the approximation (*solid line*) for the maximum Keplerian frequency in equation (1). As mentioned earlier, a similarly good fit is also obtained for the equations of state examined by Friedman, Ipser, and Parker (1986, 1989). These additional points are shown as solid dots in Figure 5. The largest deviation is of the order of ~4%. This goodness of fit highlights the utility of equation (1).

It is of interest to note from Table 7 that the largest percentage increase (~30%) of the gravitational mass over the maximum mass of the corresponding spherical model occurs for the star made entirely of perturbative strange quark matter. This increase is substantially larger than for the other cases considered here and may be understood as due to the sharp surface of the strange quark star. For the same reason, the strange quark star has the largest ratio of rotational energy to

TABLE 7

MAXIMUM MASS AND ROTATION FOR SELECT EQUATIONS OF STATE

EOS Reference	Ω ($10^4\ s^{-1}$)	M ($M_\odot$)	M_b ($M_\odot$)	R_E (km)	t	e	β
$WFF_{AV14+UVII}$	1.24	2.54	2.75	11.3	0.11	0.60	0.37
Gle	0.84	1.98	2.21	13.7	0.09	0.61	0.51
HS	1.04	2.06	2.38	13.3	0.11	0.66	0.40
$WFF^{QC}_{AV14+UVII}$	1.06	2.28	2.57	11.7	0.14	0.58	0.38
$P = (\epsilon - 4B)/3$	1.20	2.02	2.40	11.2	0.18	0.72	0.34

NOTES.—Properties listed are Ω, the angular velocity; M, the gravitational mass; M_b, the baryon mass; R_E, equatorial radius; t, the ratio of rotational energy to gravitational energy; e, eccentricity, and β, injection energy.

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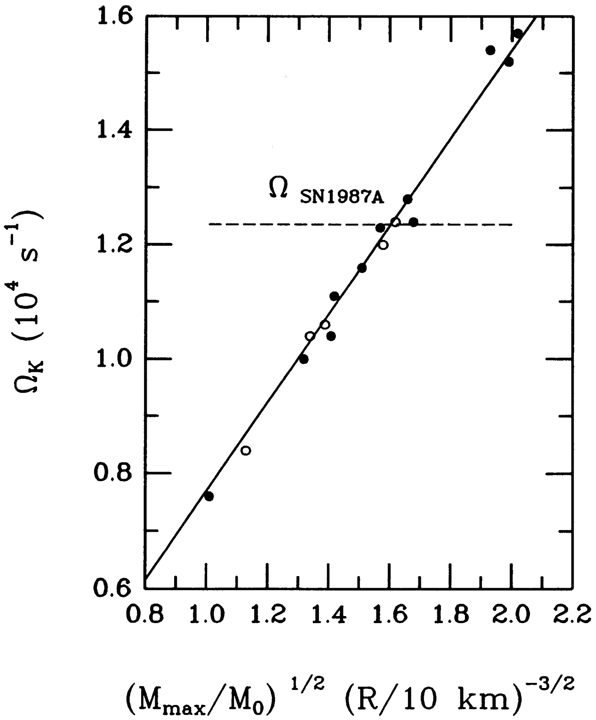

FIG. 5.—Comparison of angular velocities (*points*) from general relativistic calculations assuming uniform rotation and the approximation (*solid line*) for the maximum Keplerian frequencies from eq. (1). Open circles are results of this work for select equations of state shown in Table 7. Solid dots are results for the equations of state examined by Friedman, Ipser, and Parker (1989). The angular velocity of the purported pulsar in SN 1987A is shown by the horizontal dashed line.

gravitational energy, t. The value of t for the Keplerian frequency is so large, in fact, that triaxial deformation may have already set in, limiting the rotation rate to lower values. In Newtonian calculations, this instability is triggered when $t \geq 0.08$. Fully general relativistic calculations of this instability have not yet been performed.

VII. CONCLUSIONS

The recent report of a half-millisecond pulsar from supernova SN 1987A highlights the promise of pinning down the equation of state of dense matter if the pulsations are due to rotation of a neutron star at this frequency. Given the significance of this opportunity, the need for more detailed observations confirming the period of rotation cannot be overemphasized.

Our objective in this paper was to explore the energy-density relationships that could satisfy the rotation constraint imposed by the purported pulsar in SN 1987A together with the mass constraint imposed by the neutron star in the binary pulsar PSR 1913+16. To this end, a detailed study of rotational properties was made using recent equations of state with nucleonic degrees of freedom and also those in which phase transitions to other types of matter, such as pions, kaons, parity doublets, and quarks were considered. The subnuclear (e.g., $0.001 < n < 0.08$ fm^{-3}) EOS has very little effect on the maximum mass of a star, but we found differences of up to 0.2 km in the radius between subnuclear equations of state. Such differences may be sufficient to make some borderline cases "successful," and they represent part of the 1%–2% uncertainty in calculated values of Ω_K.

We found that the requirements of rapid rotation and sufficient mass were fulfilled by only those nucleonic equations of state that are soft around 1–4 times the nuclear density but attain a stiffness approaching the causal limit by about 6 times nuclear density. We have found that the inclusion of beta equilibrium is very important and eliminates some otherwise "successful" equations of state. We determine that, in absence of phase transitions, these requirements imply the existence of an upper bound on the compression modulus of nuclear matter, $K < 160$ MeV. This limit applies to equations of state in which the nuclear symmetry energy does not vanish beyond a few times nuclear density. Even including phase transitions, our calculations suggest that the upper limit to K might not be much greater.

With the types of phase transitions considered in this work, both mass and rotation constraints are met only with equations of state that regained a nearly causal stiffness at higher densities. The phase transition to perturbative quark matter at high density leads to matter that is less stiff and therefore results in a decrease of both the maximum mass and the maximum rotational frequency of a star with a quark core. Self-bound stars whose masses are at least 1.44 $M_\odot$ and made up entirely of perturbative quark matter with strangeness to baryon ratio of the order unity can only approach, but not exceed, the rotation rate of the SN 1987A pulsar. The optimum case is when the strange quarks are assumed massless. Other schematic, self-bound equations of state may be possible, but only if they apply at all densities, including those below nuclear density. This may not be physically reasonable in all cases.

The microscopic physics underlying phase transitions in dense matter is at best suggestive. This is highlighted by experiences with pion condensation for which experiments have ruled out the original condensation mechanism. Theoretical work to reconstitute pion condensation must therefore seek alternative mechanisms. Similar cautionary remarks apply to other types of phase transitions as well, especially in view of the simplistic manner in which they are treated at present. Notwithstanding these caveats, our findings here should impel investigations of these matters in greater detail.

Given the possible existence of nonaxisymmetric instabilities in young, rapidly rotating neutron stars, Friedman, Ipser, and Parker (1989) conclude the only equation of state they tested that simultaneously satisfied $M_{max} > 1.44$ $M_\odot$ and rapid enough rotation was the Pandharipande (1971*b*) EOS with pion condensation. We, however, have ruled out even this possibility, by imposing the condition of beta equilibrium. Also, experiments now indicate that the mechanism of pion condensation used in this EOS was invalid. In fact, all the published equations of state without phase transitions we tested had rotational frequencies less than $\Omega_{SN\,1987A}$, to within the errors of the calculation. Even when phase transitions were included, we did not find a case in which Ω_K was more than about 10% greater than $\Omega_{SN\,1987A}$, although our search was not exhaustive. Again, considering that nonaxisymmetric perturbations are likely, we can rule out all equations of state except those that become causal beyond about 6 times nuclear density and either have compression moduli below 160 MeV or an extensive first-order phase transition around a few times nuclear density.

At this time, it is not possible to rule out these possibilities through nuclear experiments and/or theory. Therefore, the

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interpretation that a rapidly rotating neutron star is the source of the pulsations observed by Kristian *et al.* (1989) cannot be excluded. Alternative suggestions to explain the pulsations are that they are due to radial vibrations (Wang *et al.* 1989) or that the compact star in SN 1987A is not a normal neutron star and hence not subject to the binary pulsar mass constraint (Glendenning 1989*b*). Each of these models is not without serious attendant problems. Should further observations verify the rotating neutron star interpretation, the implications for nuclear physics, and nuclear astrophysics, will be very exciting.

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Note added in proof.—It now appears that the report of Kristian *et al.* (1989) of the 0.5 ms pulsar in SN 1987A is erroneous (C. Pennypacker *et al.* 1990, *Nature*, submitted). Although this report provided a strong motivation to set limits on the maximum rotation rates of neutron stars, the theoretical analysis in our paper is valid even without reference to the existence of the 0.5 ms pulsar. For nonexotic equations of state, with compression moduli greater than or equal to 200 MeV, the limiting rotational frequency appears to be about 1×10^4 s^{-1}. The establishment of a limiting frequency through future observations should provide a powerful constraint on the high-density equation of state.

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