THE ASTROPHYSICAL JOURNAL, 354:483–491, 1990 May 10

CLUSTERED SUPERNOVAE VERSUS THE GASEOUS DISK AND HALO

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Received 1989 June 5; accepted 1989 November 10

ABSTRACT

Recent developments, both observational and theoretical, require a reevaluation of the effects of clustered supernovae on the two-dimensional porosity parameter Q_{2D} and the rates of mass injection into the halo $\dot{M}$ of both cold and hot gas. Theoretically, the effects of the high-$|z|$, low-density extension of the neutral gas layer have been calculated. Observationally, the distribution of Hα luminosities of extragalactic H II regions has been determined, which allows us to estimate the birthrate of star clusters having N supernovae as a function of N. For the Galaxy, we obtain a Galaxy-wide average $Q_{2D} \approx 0.30$, which corresponds to an area filling factor of 0.23, probably close to the true situation.

There are two types of bubble. Most clusters produce breakthrough bubbles, which do no more than break through the dense gas disk. But clusters with large N produce enough energy to make blowout bubbles, which blow gas up into the halo. We calculate area filling factors and mass injection rates into the halo for different types of galaxy. We relate our calculations to two observables, the area covered by H I "holes" and the area covered by giant H II regions. We also discuss the effects of clusters that are too small to produce breakthrough bubbles, and we reiterate the difficulty of producing the very largest supershells by clustered supernovae.

Subject headings: galaxies: The Galaxy — galaxies: structure — interstellar: matter — nebulae: supernovae remnants — stars: supernovae

I. INTRODUCTION

In a spiral galaxy the gas and new stars are concentrated into a relatively thin disk. The stars are formed in clusters. The massive stars become supernovae, and these explosions are correlated in space and time. These correlated supernovae produce one large bubble instead of many small ones. If the bubble is large enough, it becomes larger than the thickness of the disk and "breaks through."

Mac Low and McCray (1988, hereafter MM) and Mac Low, McCray, and Norman (1989, hereafter MMN) have made detailed calculations of this process and made a very important point: just because a bubble breaks through the "classical" dense gas disk does not mean that it "blows out" into the halo. This is because of the extensive, low-density $|z|$ extensions of disk gas, the neutral "Lockman" (1984) component and the ionized "Reynolds" (1989) component. Communication with the halo requires that the shell break through these components and open out into the halo, which requires much more energy.

Here we define *breakthrough* bubbles as those that break out of the dense, relatively low scale-height part of the disk; and *blowout* bubbles as those that actually break through all disk gas and communicate with the halo. There are two ways for blowout to occur. One is to have enough energy to blow out of the extended components. Alternatively, the extended components are not perfectly uniform, so that in regions where they are thinner than usual, blowout may occur almost as easily as breakthrough.

Breakthrough bubbles can be observed in the 21 cm line as large H I shells and supershells in our own Galaxy and as H I "holes" in external galaxies. But only blowout bubbles will inject mass into the halo. The mass is injected in two forms, cold clouds and hot gas.

Calculations of breakthrough and blowout rates been done using crude theory (Heiles 1987, Paper I) and refined theory (Norman and Ikeuchi 1989; review by Tenorio-Tagle and Bodenheimer 1988). Previous calculations have neglected the extended components, and thereby overestimated the mass injection rate into the halo. The extent to which the components are pervasive and smoothly distributed determines the degree to which they affect the shell dynamics. Here we concentrate on the Lockman component, because numerical simulations have been oriented toward this case and because the Reynolds component is highly clumped with a volume filling factor of about 0.11 at $|z| = 0$ (Kulkarni and Heiles 1988). However, the filling factor of the Reynolds component increases at large $|z|$, and this component may greatly reduce the injection rates of mass into the halo below those calculated explicitly for the Lockman component, as mentioned briefly below.

Below in § III we discuss the interstellar medium (ISM) parameters for the Galaxy. Uncertainties in these parameters, plus, of course, incompleteness of the theory, generates some uncertainty in the results. Additional uncertainty is caused by our ignorance of the formation rate of clusters large enough to produce breakthrough or blowout. Earlier calculations have assumed that all clusters are identical and derived the number of supernova per cluster N empirically, for example, by dividing the overall supernova rate by the overall rate of cluster formation. Estimates of N differ by nearly an order of magnitude.

Not only is the value of N highly uncertain, but even more seriously the very concept that all clusters have identical values of N is surely wrong. The fact that clusters have different values of N is extremely important, because it means that some clusters are too small to produce breakthough, while the largest ones can produce not only breakthrough but also blowout. Recently, we have been blessed with a remarkable paper that allows us to calculate the fraction of clusters of each type. Kennicutt, Edgar, and Hodge (1989; KEH) have derived the frequency and spatial distribution of Hα luminosities L(obs) of bright H II regions in external galaxies, as functions of galactic type. These bright H II regions are produced by precisely those clusters that produce superbubbles. We can use KEH to obtain the formation rates per unit area on the disk of clusters as a function of N. We discuss these details below in § II.

In § IV we apply these results to breakthrough bubbles. Consideration of KEH's paper highlights another important fact: a certain fraction of the disk area is occupied by the H II regions themselves. Twenty-one cm line observers will see these H II regions as "holes" in the H I disks. Such holes are usually interpreted as resulting from supernovae. We estimate the fraction of holes produced by H II regions in § V; it turns out to be small, but nevertheless the number of giant H II regions may be a useful observational diagnostic.

Blowout bubbles are another matter, because they require more energy. In § VI we use KEH's work to estimate the fraction of bubbles that become blowout bubbles, and then estimate the rate of mass injection into the halo from such bubbles.

Supernovae in clusters that are too small to produce breakthrough bubbles affect the ISM in roughly the same way as individual uncorrelated supernovae. These small clusters are discussed in § VII. The largest supershells cannot easily be made by correlated supernovae, because once blowout occurs most additional energy is vented into the halo. Section VIII discusses the probable origin of the largest supershells. Finally, we summarize and discuss our results in § IX.

II. FORMATION RATES OF CLUSTERED SUPERNOVAE

a) The Relation between L (obs) *and N*

In order to use KEH's observations of L(obs) to calculate the influence of supernovae on the interstellar medium, we require one crucial piece of data: the relation between L(obs) and the number of supernovae N. This is not a simple matter and requires a detailed discussion.

We follow conventional wisdom and assume that all stars more massive than 8 $M_\odot$ become supernovae. Since the initial mass function (IMF) favors less massive stars, most of the supernovae come from stars with masses just above this value. However, most of the ionizing photons that produce the H II regions come from the very massive stars. Thus, both the slope and upper mass cutoff of the assumed IMF affect the desired relation.

Fortunately, careful calculations have been done by Lequeux *et al.* (1981), Melnick, Terlevich, and Eggleton (1985), and McKee (1989). All used theoretical evolutionary tracks for stellar masses ranging up to about 100 $M_\odot$, included the effects of mass loss, and considered various slopes of the IMF. They all agree within less than $\sim \pm 40\%$, and here we adopt McKee's (1989) result both because it uses more modern stellar evolution calculations and because it lies between the other two values. For the IMF slope, we adopt the value derived for high-mass stars in clusters by Scalo (1986), -1.5, which is somewhat steeper than Salpeter's (1955) classic value of -1.35.

The most direct way to express the result is in terms of the number of Lyman continuum photons emitted per supernova, where both quantities are summed over the whole cluster: the result is 4.8×10^{62}.

McKee (1989) has evaluated the reliability of this relation by comparing the overall Galactic Type II supernova rate it predicts with the rate estimated from other considerations. (In this paper, "Type II" means all supernovae whose progenitors are young, massive stars, and "supernovae" means "Type II" supernovae.) Güsten and Mezger (1982) used measurements of Galactic thermal radio emission to derive the total Lyman continuum photon production rate in the Galaxy to be $L(\mathrm{uv}) = 2.0 \times 10^{53}$ photons s^{-1} (after multiplying their number by 0.85^2 to account for the currently adopted value of the Sun's Galactocentric radius, 8.5 kpc). The majority of this emission is from "extended low-density" diffuse gas, not individual H II regions. This L(uv) corresponds to a Type II SN rate of one per 76 yr (0.013 yr^{-1}), which is close to independent estimates for the rate expected in our Galaxy. Tammann (1982) estimates 0.023 yr^{-1}, while van den Bergh (1988) and Ratnatunga and van den Bergh (1989) estimate 0.019 yr^{-1} for both Type Ib and II supernovae. Pulsar birthrate statistics in the Galaxy by Lyne, Manchester, and Taylor (1985) suggest a rate between 0.008 and 0.033 yr^{-1}, while Narayan (1987) suggests 0.013 yr^{-1} (after adjustment to the Sun's Galactocentric radius of 8.5 kpc). Thus the relation should be quite reliable, although perhaps it predicts a supernova rate that is a bit too low.

Now consider the applicability of this relation to KEH's observations. L(uv) is directly related to L(obs) because it takes about two Lyman continuum photons to produce an Hα photon; thus we should obtain one supernova for every 7.2×10^{50} ergs of Hα radiation. KEH present L(obs), measured photometrically over the entire galaxy in earlier work by Kennicutt and Kent (1983), for a number of galaxies. These integrated measurements of L(obs) include not only discrete H II regions but also the weaker, extended emission, and thus should be good measures of the total luminosity of ionizing photons in the gaaxies, although of course they do not account for absorption by dust in the external galaxy.

The Sb galaxies observed by Kennicutt and Kent (1983) have an average total $L(\mathrm{obs}) \approx 4 \times 10^{40}$ ergs s^{-1}. This corresponds to a supernova rate of one per 571 yr, or 0.0018 s^{-1}. This is about 7 times lower than the Galactic rate. As our Galaxy is also an Sb, this is a major discrepancy. We do not know the origin of this discrepancy. The more straightforward possibilities include errors in the measurements and internal absorption by dust.

In any case, we will severely underestimate the overall Type II supernova rate in these galaxies unless we *adjust* the relation between L(obs) and N empirically to bring the predicted and expected supernova rates into agreement. We use the adjusted relation, which gives a higher supernova rate, in our calculations, and we predict clustered supernovae to have too large an effect on galaxies, relative to observations. In contrast, if we were to use the unadjusted relation, we would predict too small an effect.

For a cluster, L(obs) is equal to the total Hα energy emitted divided by the lifetime of the cluster's H II region $\tau_{\mathrm{H\,II}}$. With $\tau_{\mathrm{H\,II},7}$ being $\tau_{\mathrm{H\,II}}$ in units of 10^7 yr and $L(\mathrm{obs})_{38}$ the Hα luminosity in units of 10^{38} ergs s^{-1}, this gives for the *unadjusted* relation

$$N = 43.8\ L(\mathrm{obs})_{38}\, \tau_{\mathrm{H\,II},7}\ . \tag{1a}$$

We adopt an *adjusted* relation by applying a factor of 5, which provides an Sb supernova rate of one per 110 yr (0.009 yr^{-1}):

$$N = 222\ L(\mathrm{obs})_{38}\, \tau_{\mathrm{H\,II},7}\ . \tag{1b}$$

This is on the low side of the values derived from supernova and pulsar statistics and represents a compromise between those values and the unadjusted relation.

We must now specify an appropriate value for $\tau_{\rm H\,II}$. Lequeux *et al.* (1981) and Melnick *et al.* found that the Lyman continuum luminosity of the clusters persists for about 4 Myr. However, this is not the proper value of $\tau_{\rm H\,II}$ for our application because not all stars in a cluster are formed simultaneously. There exists ample observational evidence for star formation occurring over a substantial time interval. In our Galaxy, individual clusters such as Orion and Scorpius/Ophiuchus have undergone sequential star formation over intervals of some 15 Myr (Blaauw 1964). Thus the ionizing photons are emitted over a total time interval of some 20 Myr, and we adopt $\tau_{\rm H\,II,7} = 2$ in equation (1b).

b) *Summary of Results from KEH*

KEH's Figure 6 gives the surface density of the observed H II regions on the disks. However, the surface density is not independent of radius; it increases toward the center, approximately exponentially, as does the overall starlight from a galaxy (Kennicutt 1989), into the central portions where in many galaxies there is no interstellar matter. With this radial dependence, the peak surface density is higher than the average, by a factor of perhaps 2 or so.

The stars in the clusters emit ionizing radiation for time $\tau_{\rm H\,II}$. The H II regions remain visible for this length of time, and the cluster birthrate per unit surface area is equal to the observed H II region density divided by $\tau_{\rm H\,II}$. As discussed in § II*a*, we take $\tau_{\rm H\,II} = 20$ Myr. Note, specifically, that we assume that $\tau_{\rm H\,II}$ is independent of the cluster luminosity.

KEH give the distribution in L(obs) of extragalactic H II regions in the form of a power law. In our calculations, we shall require weighted averages over the ensemble of clusters of the form $\langle L({\rm obs})^y \rangle$, and it is convenient to develop simple algebraic approximation for these quantities. Let σ_L be the formation rate per kpc^2 on the galactic disk of clusters having L(obs) between L and $L + dL$, and take $\sigma_L = \kappa L^a$. Our definition of a is similar in spirit to that of KEH, and is in fact identical if the cluster lifetime is independent of L(obs); as stated above, we assume that this is the case. Since $a \approx -2$, with this power-law approximation we have, for the ensemble of clusters having $L_{\rm min} < L({\rm obs}) < L_{\rm max}$:

$$\sum = \int_{L_{\rm min}}^{L_{\rm max}} \sigma_L \, dL = -\kappa \frac{L_{\rm min}^{(a+1)}}{(a+1)} \left[1 - \left(\frac{L_{\rm max}}{L_{\rm min}} \right)^{(a+1)} \right]$$
$$\approx -\kappa \frac{L_{\rm min}^{(a+1)}}{(a+1)} \tag{2}$$

for Σ, the formation rate per kpc^2 on the disk of clusters having L(obs) in the range $L_{\rm min}$ to $L_{\rm max}$; and

$$\langle L({\rm obs})^y \rangle = \frac{\int_{L_{\rm min}}^{L_{\rm max}} L^y \sigma_L \, dL}{\Sigma} L_{\rm min}^y \left(\frac{a+1}{a+1+y} \right)$$
$$\times \left[\frac{1 - (L_{\rm max}/L_{\rm min})^{a+1+y}}{1 - (L_{\rm max}/L_{\rm min})^{a+1}} \right] \approx L_{\rm min}^y \frac{(a+1)}{(a+1+y)} \tag{3}$$

for the weighted average $\langle L({\rm obs})^y \rangle$ of those clusters.

For our application, $L_{\rm min}$ is the luminosity of the smallest cluster of interest, for example, the smallest whose supernovae will produce a breakthrough bubble; we find below that $L_{\rm min,38} \approx 0.12$, where L_{38} has units of 10^{38} ergs s^{-1}. $L_{\rm max}$ is the upper L(obs) cutoff in KEH's observed power-law distributions, which depends on galactic type. For Sb galaxies, $L({\rm obs})_{\rm max,38} \approx 20$.

KEH quote a's of about -2.3, -2.0, and -1.7 for three representative Sb, Sc, and Irr galaxies, respectively. However, KEH's curves are not strict power laws, and in the region of interest $[L({\rm obs})_{38} \sim 1]$, an eyeball fit to their Figure 8 for Sb galaxies yields $a \approx -2.14$. We adopt

$$\mathcal{N}[>L({\rm obs})] = 1.85 \; L({\rm obs})_{38}^{-1.14} \; , \tag{4}$$

where $\mathcal{N}[>L({\rm obs})]$ is the number of clusters per $10^{10}\,L_\odot$ of galactic luminosity having the observed Hα luminosity $>L({\rm obs})_{38}$. From KEH's Figure 6, the surface density of H II regions in Sb galaxies having $L({\rm obs})_{38} > 0.5$ is about 0.15 kpc^{-2}. Combining all this, and neglecting factors involving $(L_{\rm max}/L_{\rm min})$ in equations (2) and (3), we obtain $\kappa = 7.8 \times 10^{-3} \tau_{\rm H\,II,7}^{-1}$ and

$$\langle L({\rm obs})_{38}^y \rangle \Sigma \approx \frac{7.8 \times 10^{-3}}{1.14 - y} \tau_{\rm H\,II,7}^{-1} \, L({\rm obs})_{\rm min,38}^{-(1.14-y)} \; {\rm kpc^{-2}\; Myr^{-1}} \; . \tag{5}$$

Note that $L({\rm obs})_{\rm max,38} \approx 20$ for the Sb galaxies, which from equation (1b) corresponds to a cluster that contains about $4440\tau_{\rm H\,II,7} = 8880$ supernovae. The steepness of the L(obs) distribution guarantees that the clusters with fewer supernovae, which are much more numerous, dominate the interaction with the interstellar medium. In fact, most clusters are not even large enough to produce breakout (§ IV*b*). However, the existence of some very large clusters guarantees that blowout does occasionally occur.

We must stress a caveat with the whole approach of this subsection. We have assumed that the IMF is the same from cluster to cluster, so that variations in L(obs) can be related directly to variations in N. This is certainly not true in an exact sense. The alternative extreme would be to assume that N is the same for all clusters and that variations in L(obs) result from wide swings in the IMF for large masses. This also is certainly not true. KEH have discussed these matters in some detail and we will not elaborate here. We have also assumed that $\tau_{\rm H\,II}$ is the same for all clusters, which also cannot be exactly true. Our approach here is valid so long as, in an ensemble average of clusters, L(obs) is a good measure of N. Similarly, in mentioning differences in L(obs) among different types of galaxy we have implicitly assumed that these factors do not vary systematically from one galaxy type to another. As discussed by KEH, this may not be the case, but they argue that at least some of the increased values of L(obs) in late-type galaxies really does arise from larger clusters instead of only variations in the IMF.

III. ISM PARAMETERS

In Paper I we used numerical parameters for the ISM that seemed appropriate. Numerical values adopted for the important ISM parameters were as follows: the "intercloud" gas density $n_0 = 0.24$ cm^{-3}; the "scale height" $h_{100} = 1.85$ (see below for definition), where the subscript indicates units of 100 pc; the pressure $P_{04} = 0.40$, where the subscript indicates units of $nT = 10^4$ cm^{-3} K; and the rms velocity $v_{\rm rms} = 9.9$ km s^{-1}.

Two new developments force us to reconsider n_0 and h. First, Paper I used "intercloud" values, i.e., values appropriate to the warm neutral medium (WNM), and excluded the cloud component (CNM). We did this because clouds fill a small fraction of space, while the propagation of shocks should be determined by the medium having the largest filling factor. A shock should overrun a cloud, leaving it intact (albeit grossly perturbed).

However, the supernovae explosions are preceded by ionizing photons, which produce the large H II regions observed by KEH. In the case of the clusters considered here, these H II regions are very large. Clouds within an H II region should be photoevaporated quickly, leading to a homogeneous medium within a radius $R_{\rm H\,II}$, the Strömgren radius calculated using the mean density that would exist if the ISM were homogenized (McKee, van Buren, and Lazareff 1984). A supershell expands to a radius larger than $R_{\rm H\,II}$, so the ambient ISM is not homogenized throughout the whole volume. Nevertheless, we proceed assuming that the entire ISM into which the shock propagates has been homogenized.

Second, the $|z|$ structure of the ISM is not the classical, simple thin disk. Instead, there are two extended components, the Lockman and Reynolds components. Here we consider only the Lockman component, because the Reynolds component is clumpy at $|z| = 0$. We follow Lockman, Hobbs, and Shull (1986; LHS) and approximate the $|z|$ distribution as

$$n(|z|) = n_c \exp(-|z|^2/z_c{}^2) + n_{\rm L} \exp(-|z|/z_{\rm L}) . \quad (6)$$

This equation lumps the "classical" CNM and WNM, which have different scale heights, together into the first term; the new "Lockman" component is represented by the second term. LHS find that the total column density above $|z| = 0$ is about 3×10^{20} cm^{-2}, that the column densities in the two terms are about equal, that $z_c \approx 190$ pc, and that $z_{\rm L} \approx 500$ pc. Thus we adopt $n_c = 0.316$ cm^{-2} and $n_{\rm L} = 0.107$ cm^{-2}. The quantity n_0, which we use in various equations below, is the total density at $|z| = 0$, equal to 0.422 cm^{-2}. This is 1.76 times larger than the value used in Paper I.

Below we use an artificial scale height h. In our approximate theory we assume that $n(|z|) = $ const. for $|z| < h$ and $n(|z|) = 0$ for $|z| > h$. We relate h to z_c or $z_{\rm L}$ by requiring that the column densities to $|z| = \infty$ be correct. Thus, if we are discussing the classical component, we take $h_{100} = [(\pi)^{1/2}/2]z_c = h_{c,100} \approx 1.7$; if we are discussing the Lockman component, we take $h_{100} = z_{{\rm L},100} = 5.0$.

IV. BREAKTHROUGH: Q_{2D} FROM SUPERNOVAE

a) *Rederivation of* Q_{2D}

The two-dimensional porosity parameter, roughly equal to the fraction of the disk area occupied by breakthrough bubbles, is denoted by Q_{2D}. Q_{2D} is equal to the integral of the bubble area over time, divided by the time interval for bubble formation, or

$$Q_{2D} = \sum \left[\int_{\rm expansion} \pi R(t)^2 \, dt + \int_{\rm contraction} \pi R(t)^2 \, dt \right] . \quad (7)$$

It was pointed out to me by B. C. Koo (1989) that Paper I considered only the contraction portion, and then incorrectly because it assumed that the bubble remains its largest size R_f during the entire time that it is being repenetrated. These two errors partially cancel. We note that our expression for Q_{3D} in Paper I differs somewhat from that given by McKee and Ostriker (1977) and was derived properly using the three-dimensional analog of equation (7).

We assume that the bubble dynamics are those outlined in Paper I, namely we assume that the energetic winds and sequential explosive impulses of the N supernovae in the cluster act as a "superwind" and produce bubble dynamics equal to that of a continuous stellar wind in the manner described by Weaver *et al.* (1977). The bubble expands to radius R_f, which is the radius at which the bubble expansion velocity slows to $v_{\rm rms}$. At this point, we assume that the bubble immediately begins contracting with velocity $v_{\rm rms}$.

For the contraction phase, the second integral in equation (7) is equal to $(\pi R_f^3)/(3v_{\rm rms})$, just $\frac{1}{3}$ the value taken for the full value of Q_{2D} in Paper I. The expansion phase consists of two parts. The first has $R < h$ and is characterized by the standard stellar-wind behavior, $R \propto t^{3/5}$. The second has $R > h$, and because the hot gas pressure escapes into the halo is characterized by the two-dimensional "snowplow" behavior with $R \propto t^{1/3}$. We assume that the latter characterizes the whole expansion, which overestimates Q_{2D} but only by a small factor for large breakthrough bubbles. With this approximation, the contribution of the expansion is equal to $\frac{3}{5}$ that of the contraction phase. Adding these together gives

$$Q_{2D} = 277 \langle L({\rm wind})_{38}^{1/2} \rangle \Sigma v_{\rm rms}^{-5/2} h_{100}^2 n_0^{-1/2} . \quad (8)$$

In Paper I, the factor 277 was instead 520, and was thus too large by a factor of 1.9.

The mechanical luminosity of the superwind, L(wind), is equal to the total energy released divided by the time interval in which it is released, $\tau_{\rm SN}$. As in Paper I, the total energy is derived from supernovae, at 10^{51} ergs apiece, and winds from O stars (which constitute a small fraction of the supernova progenitors), at 10^{51} ergs each; combining all these, we obtain a grand average of 1.18×10^{51} ergs per supernova. Paper I assumed that the energy superwind blows for 30 Myr. However, McCray and Kafatos (1987) use a better value, 50 Myr, and in this paper we will increase this to 60 Myr to account for the fact that not all the stars in a cluster are formed simultaneously. Keeping $\tau_{\rm SN}$ as a free parmeter, this gives

$$L({\rm wind})_{38} = 3.72 \times 10^{-2} \, \tau_{{\rm SN},7}^{-1} \, N . \quad (9)$$

Using equation (1b), this is equivalent to

$$L({\rm wind})_{38} = 8.3 \, \frac{\tau_{\rm H\,II}}{\tau_{\rm SN}} \, L({\rm obs})_{38} . \quad (10)$$

After the present paper had been submitted, Palous (1989) pointed out that the stretching of supershells by differential galactic rotation reduces the total area that they occupy (Tenorio-Tagle and Palous (1987, hereafter TTP); Palous, Franco, and Tenorio-Tagle (1989, hereafter PFTT). These effects begin to become important after $\sim 0.5 t_{\rm ref}$, where $t_{\rm ref} \approx 100$ Myr for the Galaxy. This is comparable to the total time that a large bubble exists. Thus, our value of Q_{2D} given by equation (8) is an overestimate.

b) *Breakthrough Dynamics*

Paper I argued that at least 12 SN are required for breakthrough. This corresponds to $L({\rm wind})_{38} > 0.45 \tau_{{\rm SN},7}^{-1}$, or 0.15 for $\tau_{{\rm SN},7} = 3$ as assumed in Paper I. MM's more recent detailed calculations show that breakthrough occurs when their parameter D (their eq. [29]),

$$D \approx 940 \, L({\rm wind})_{38} \, h_{100}^{-2} \, P_{04}^{-3/2} \, n_0^{1/2} , \quad (11)$$

exceeds a value somewhat smaller than 100. For our adopted ISM parameters (§ III), this occurs when $L({\rm wind})_{38} > 0.12$, or $L({\rm obs})_{38} > 0.0415 \tau_{\rm SN}/\tau_{\rm H\,II}$. The near equality of this more rigorous limit with Paper I's limit is purely fortuitous.

c) Evaluation of Q_{2D}

Combining equations (9), (10), and (5), together with $L(\mathrm{obs})_{\mathrm{min},38} = 0.0145\tau_{\mathrm{SN}}/\tau_{\mathrm{H\,II}}$, we obtain $\langle L(\mathrm{wind})_{38}\rangle^{1/2}\Sigma = 0.53\tau_{\mathrm{H\,II},7}^{0.14}\tau_{\mathrm{SN},7}^{-1.14}$. Inserting this into equation (8), and using the interstellar parameters of § III, we find $Q_{2D} = 2.1\tau_{\mathrm{H\,II},7}^{0.14}\tau_{\mathrm{SN},7}^{-1.14} = 0.30$. This corresponds to an area filling factor for the hot bubbles of $Q/(1+Q) = 0.23$. Our value $Q_{2D} = 0.30$ is nearly two orders of magnitude smaller than the values derived in Paper I. This is fortunate, because the extremely large values of Paper I are not supported by observational data.

In comparing this prediction with observations of the solar neighborhood, we recall that our derived area filling factor is a Galaxy-wide average because we used the Galaxy-wide average value of Σ, denoted below as $\langle\Sigma\rangle$. However, the supernova rate is a function of Galactocentric radius. We will express $\Sigma_\odot$, the solar-neighborhood value, as a fraction of $\langle\Sigma\rangle$, and then derive Q_{2D} for the solar neighborhood.

The radial dependence of σ was given in Figure 1 of Paper I. Paper I defined σ to be the *supernova* birthrate per kpc^2, while our Σ is the *cluster* birthrate per kpc^2; the two are proportional, because Paper I assumed a constant number of supernovae per cluster. $\langle\Sigma\rangle$ was obtained from KEH's Figure 6; KEH averaged over the radius of "the star-forming disk." From Figure 1 of Paper I, it appears that this is just the same as the Sun's Galactocentric radius. We define $\langle\sigma\rangle$ as the average supernova rate per kpc, equal to the total supernova rate divided by the area of the star-forming disk (and thus commensurate with $\langle\Sigma\rangle$). For the parameters of Paper I, $\langle\sigma\rangle = 73$ kpc^{-2} supernovae kpc^{-2} Myr^{-1} and $\sigma_\odot \approx 25$ supernovae kpc^{-2} Myr^{-1} $= \langle\sigma\rangle/3$. Thus $\Sigma_\odot = \langle\Sigma\rangle/3$.

In passing, we adjust Figure 1 for changes in two parameters, the total Type II supernova rate and the radius of the solar circle. In Paper I, we assumed the total Type II supernova rate to be 0.023 yr^{-1}, while in the present paper we assume 0.39 of this value, 0.009 yr^{-1}. In Paper I, we assumed the radius of the solar circle to be 10 kpc, while here we assume 8.5 kpc. Thus, for commensurability with the present paper, the vertical scale of Figure 1 must be scaled by $(0.39/0.85^2) = 0.54$. Thus, for the parameters of the present paper, $\langle\sigma\rangle = 39$ kpc^{-2} supernovae kpc^{-2} Myr^{-1} and $\sigma_\odot \approx 13$ supernovae kpc^{-2} Myr^{-1}.

The above relation, $\Sigma_\odot = \langle\Sigma\rangle/3$, yields $Q_{2D} = 0.10$ for the solar neighborhood. Following Paper I, we apply the factor s, equal to (2.1, 0.7) for (inside, outside) spiral arms; this gives $Q_{2D} = (0.21, 0.07)$, corresponding to area filling factors of (0.17, 0.07). The former value is in excellent agreement with the volume filling factor of large H I holes in the solar neighborhood (Heiles 1980). For M31, another Sb galaxy, the observed area filling factor can be derived from Figures 21 and 22 of Brinks and Bajaja (1986). It peaks at ~ 0.09, corresponding to $Q_{2D} = 0.10$, for galactocentric radius ~ 10 kpc; presumably the M31-wide average $Q_{2D} \approx 0.05$. Our calculated value should also apply roughly to M31, and it is too large. M33 is an Sc galaxy and has an observed area filling factor less than 0.4 (Deul and den Hartog 1989), corresponding to $Q_{2D} < 0.67$. KEH's data cause us to predict that Sc galaxies have 3 to 5 times higher values of Q_{2D} than do Sb's, and this again suggests that our predicted value is too high. We conclude that our predicted values of Q_{2D} are roughly correct, probably too large, but emphasize that there seems to be considerable variation from one galaxy to another.

V. $Q_{2D}^{\mathrm{H\,II}}$ FOR H II REGIONS

The basic theory of the Strömgren sphere (see Spitzer 1978), together with the approximate ratio of L(obs) to L(uv), allows us to write

$$R_{\mathrm{H\,II}} < 0.12\,L(\mathrm{obs})_{38}^{1/3} n_0^{-2/3}\ \mathrm{kpc}\,. \tag{12}$$

The inequality results from clumping, which should be minimal (McKee *et al.*). $Q_{2D}^{\mathrm{H\,II}}$ is equal to $\Sigma^{\mathrm{H\,II}}\pi R_{\mathrm{H\,II}}^2\tau_{\mathrm{H\,II}}$, or

$$Q_{2D}^{\mathrm{H\,II}} < 0.45\langle L(\mathrm{obs})_{38}^{2/3}\rangle\Sigma^{\mathrm{H\,II}} n_0^{-4/3}\tau_{\mathrm{H\,II},7}\,, \tag{13}$$

where $\Sigma^{\mathrm{H\,II}}$ is the formation rate per Myr per unit area of disk of clusters that produce the H II regions of interest.

We assume that the only H II regions to produce observable holes are those that attain breakthrough, i.e., $R_{\mathrm{H\,II}} > h$. For the ISM parameters values of § III, and assuming equality in equation (12), this requires $L(\mathrm{obs})_{38} > 0.51$. Applying equation (5) yields $Q_{2D}^{\mathrm{H\,II}} = 0.032$, about 9 times smaller than Q_{2D}. The main reason is that the H II region produced by a cluster's massive stars is smaller than the bubble produced by its supernovae.

The partial correlation of observed H I holes with OB associations and H II regions in M31 found by Brinks and Bajaja (1986) and in M33 by Deul and den Hartog (1989) may be consistent with our results. The H II regions are not easily visible on broad-band optical photographs because the emission measures are small. The emission measure of an H II region that breaks through is $2hn_0^2$, or 68 cm^{-6} pc for the ISM parameters of § III.

VI. BLOWOUT: Q_{2D} AND $\dot{M}$

Blowout requires a more stringent condition on D in equation (11), because we must use the scale height of the high-$|z|$ component (and, according to MM, the total density at $z = 0$). There are two extended components, the Lockman and the Reynolds. Here we concentrate on the Lockman component, because the numerical calculations have been performed for this case. The scale height of the Reynolds component is 3 times that of the Lockman component (Reynolds 1989), and the Reynolds component is clumped, at least near $|z| = 0$ (Kulkarni and Heiles 1988); it would be useful to have numerical simulations of this case.

With $h_{100} = z_{L,100} = 5$, $D > 100$ requires $L(\mathrm{wind})_{38} > 1.036$. This corresponds to 167 supernovae, or $L(\mathrm{obs})_{38} > 0.125\tau_{\mathrm{SN}}/\tau_{\mathrm{H\,II}} = 0.375$. We assume that equation (8) applies for these bubbles, but use $h_{100} = h_{c,100} = 1.7$ instead of $h_{100} = z_{L,100} = 5$ because the dynamics of the classical dense disk gas should not be affected much by the evolution of the bubble after breakthrough.

We obtain $Q_{2D} = 0.53\tau_{\mathrm{H\,II},7}^{0.14}\tau_{\mathrm{SN},7}^{-1.14} = 0.076$. Thus, about $\frac{1}{3}$ of the H I hole area is occupied by blowout bubbles. Associated with these blowout shells are two forms of mass injected into the halo, cold shell fragments and hot gas.

These numbers are all considerably smaller if the same theory applies to the Reynolds component, which has $h_{100} = 15$. Blowout requires $L(\mathrm{wind}) > 9.32$. This gives $Q_{2D} = 0.0186$, only about 8% of the total hole area.

a) $\dot{M}_{\mathrm{cold}}$: Cold Shell Fragments

The supernovae drive a radiative shock into the ambient ISM. The matter in this cold radiative shell moves up in $|z|$ through the negative density gradient, accelerating and undergoing Rayleigh-Taylor instability, which makes it break up into fragments of cold neutral gas (McCray and Kafatos 1987).

If the supernovae are infrequent and do not approximate a continuous wind, there may perhaps be further instabilities (Tenorio-Tagle, Bodenheimer, and Różyczka 1987).

These fragments are "missiles," launched into the halo in ballistic trajectories with $|z|$-velocities ~ 100 km s^{-1} from heights equal to several scale heights of the gas (MMN), and have been discussed by Cioffi (1986) and Charlton and Salpeter (1989). Whether a cloud can actually be regarded as pursuing ballistic trajectories depends on whether the cloud remains a self-contained unit and, if so, whether friction with and mass accretion from the ambient halo gas are significant forces. We argue below in § VI*b* that the pressure of diffuse halo gas is likely to be sufficient to contain the clouds. A ballistic trajectory beginning at $|z| = 4z_L \approx 2$ kpc with a $|z|$-velocity of 100 km s^{-1} would rise to $|z| \approx 3.7$ kpc and fall back to $|z| = 0$ after about 80 Myr.

For the particular models treated numerically by MMN, about 0.075 of the total mass of ambient ISM in the cylinder of height h and radius R_f was injected into the halo as cold clouds. If this applies generally, then the rate of injected mass of cold cloud fragments is

$$\dot{M}_{\rm cold} = 5.0 \times 10^5 \langle L({\rm wind})_{38}^{1/3} \rangle \times \Sigma h_{100}^{7/3} n_0^{2/3} v_{\rm rms}^{-1}\ M_\odot\ {\rm kpc}^{-2}\ {\rm Myr}^{-1}\ . \quad (14)$$

For the Galactic parameters in § III, this is $10300 \tau_{\rm H\,II,7}^{0.14} \tau_{\rm SN,7}^{-1.14}$ $M_\odot$ kpc^{-2} Myr^{-1}. If this occurs uniformly over a disk of radius 8.5 kpc, it becomes $\dot{M}_{\rm cold} = 0.33\ M_\odot$ yr^{-1}. This is split equally between the "northern" and "southern" halo hemispheres. For the Reynolds-component parameters, $\dot{M}_{\rm cold}$ is only about 0.06 $M_\odot$ yr^{-1}.

The state of matter in these clouds depends on the ambient halo pressure, the photon heating rate (Bregman and Harrington 1986), and the degree to which the cloud remains an independent entity in the halo environment. If clouds remain well-defined and follow ballistic trajectories, the amount of gas resident in the halo in the form of these clouds is $\dot{M}_{\rm cold}$ multiplied by the residence time for a ballistic trajectory, or about 2.6×10^7 $M_\odot$ (0.5×10^7 $M_\odot$ for Reynolds-component parameters).

Clouds of relatively highly ionized gas are routinely detected in *uv* absorption lines observed against stars located at high $|z|$ (Savage and Massa 1987; Danly 1989). These may arise from the environmentally modified cold gas clouds discussed here. The simplest estimate for the column density of the injected clouds is obtained by assuming that all of the injected cold gas is uniformly spread over the area of the cylinder having area πR_f^2. This estimate neglects the Rayleigh-Taylor induced clumping, which raises the column density, and probable expansion of the cloud during its voyage through the halo, which lowers the density. This estimate gives an H I column density of about 2×10^{19} cm^{-2}, which is larger than typical values found by the *uv* observers. This may imply that the clouds expand substantially in the halo.

b) $\dot{M}_{\rm hot}$: *Hot, Diffuse Gas*

Hot gas is produced by evaporation of gas from clouds and the inside of the cold shell. This gas is important for the halo, because it is injected at high temperature, can travel to high $|z|$, and can spread out into a large volume. Using equations (9) and (11) from Paper I and equation (10) above, we obtain

$$\dot{M}_{\rm hot} = 9.9 \times 10^3 \langle L({\rm wind})_{38}^{8/21} \rangle \Sigma n_0^{1/3} h_{100}^{41/21} + 1.65 \times 10^4 \times \langle L({\rm wind})_{38}^{5/7} \rangle \Sigma h_{100}^{2/7} \tau_{\rm SN,7}\ M_\odot\ {\rm kpc}^{-2}\ {\rm Myr}^{-1}\ . \quad (15)$$

Here the first term corresponds to the gas evaporated before breakthrough and the latter term to afterward. For h we use $h_{c,100} = 1.7$, because after the bubble breaks through the thin dense disk the interior hot gas expands very rapidly, so evaporation becomes nearly as ineffective as it would be if the bubble had blown out (see Fig. 8 of Mac Low and McCray 1988).

With our adopted parameters, we obtain $\dot{M}_{\rm hot} = 3650\ M_\odot$ kpc^{-2} Myr^{-1}. If this occurs uniformly over a disk of radius 8.5 kpc, it becomes $\dot{M}_{\rm hot} = 0.83\ M_\odot$ yr^{-1}. Again, this is split equally between the "northern" and "southern" halo hemispheres, and the value would be much smaller for the Reynolds-component parameters.

Our estimate of $\dot{M}_{\rm hot}$ neglects the supernova ejecta itself (Tenorio-Tagle 1989). This is justified because the ejecta mass is much smaller than the mass of evaporated gas. The relative proportion of supernova ejecta is larger for the clusters having fewer supernovae. For the smallest cluster capable of producing a blowout bubble, $L({\rm wind})_{38} = 1.036$, and this produces a total mass of evaporated gas of 1.4×10^5 $M_\odot$. Even if each of the 167 supernovae add 50 $M_\odot$ to this evaporated gas, the ejecta adds only 6%. Although this is negligible in terms of added mass, it may be important because this added gas should be enriched with heavy elements. This may increase the cooling rate of the diffuse halo gas by a significant factor, with the possible result of affecting the gas dynamics of the halo gas in a qualitative way. In particular, the increased cooling may prevent the hot gas from leaving as a galactic wind.

Most clustered supernovae are not members of clusters that are large enough to produce blowout bubbles. Thus, most of the supernova-produced heavy elements are not injected into the halo. Rather, they are injected into a hot bubble that is confined within the gaseous disk, and they cannot easily migrate over long distances in the galaxy.

What is the fate of the diffuse hot gas that is injected into the halo? It is injected at a high temperature and is heated further by the Type I supernovae. In Paper I we took the scale height of Type I supernovae, $h_{\rm SNI}$, to be 325 pc, larger than the scale height of the gas, h. Thus the Type I supernovae were very effective in heating the diffuse halo gas. However, Lockman's disk component has $h_L = 500$ pc, which is larger than $h_{\rm SNI}$; Reynolds's component is 3 times thicker. If all of the Type I supernova energy is absorbed by these components, then the Type I supernovae will not be an effective agent for the diffuse halo gas.

The question is very important. Paper I showed that, for negligible radiative cooling and $\dot{M}_{\rm hot} \lesssim 2.1\ M_\odot$ yr^{-1}, the energy input from the Type I supernovae would heat the gas so much that it would exit as a wind. With $h_{\rm SNI}$ smaller than the $|z|$-extended components, a smaller fraction of the Type I supernova energy will heat the halo gas, and the gas might survive without leaving as a wind. We cannot go further here, for several reasons. Our theory is only a crude approximation, particularly with regard to the Reynolds component; radiative cooling and other heating processes may be important; and the extended components are probably clumpy, which would allow a larger fraction of the Type I supernova energy to reach the halo gas.

We conclude that a significant fraction of $\dot{M}_{\rm hot}$ might exit the Galaxy as a wind, depending on these details. If the gas exits as a wind it should do so with a velocity of order 200 km s^{-1}, which would make its residence time of order 50 Myr. If the gas does not exit as a wind, it will fall to the Galactic plane after it cools. The cooling time should be smaller than this value. Thus the amount of hot gas in the halo, which is equal to

$\dot{M}_{\rm hot}$ multiplied by the residence time, should be less than $4 \times 10^7 \, M_\odot$ (much less for Reynolds-component parameters).

If the mass of diffuse halo gas is equal to $4 \times 10^7 \, M_\odot$ and the gas is spread uniformly over a volume of radius 8.5 kpc, the volume density is 4×10^{-4} cm^{-3}. If $T = 10^7$ K, then the gas pressure is 4000 cm^{-3} K. This is close to the gas pressure in the disk and would be enough to confine the cold fragments discussed above in § IV*a*. If instead the diffuse halo gas returns to the disk in a shorter time, the temperature would be lower and the total volume smaller; the pressure is likely to remain reasonably high, and the cold clouds could still be contained by the diffuse gas. However, for the Reynolds-component parameters, the halo pressure would be much smaller and the cold clouds could not be so contained.

VII. Q_{3D} FOR SMALL CLUSTERS

Only some clusters ("large" clusters) are large enough to produce breakthrough bubbles. We estimate the fraction of supernovae in small clusters by subtracting the supernova birthrate in all large clusters in a galaxy from the galactic-wide supernova birthrate. The galactic-wide supernova birthrate in large clusters can, in turn, be obtained from the galactic-wide production rate of Hα luminosity associated with large clusters.

From § IV*b* breakthrough requires $L(\text{obs})_{38} > 0.0145\tau_{\rm SN}/\tau_{\rm H\,II}$. The Galactic formation rate of Hα luminosity associated with large clusters can be obtained by setting $y = 1$ and $L(\text{obs})_{\rm min,38} = 0.0435$ in equation (5). The formation rate of Hα luminosity is related to the supernova rate by equation (1b). For the Galaxy, this calculation gives the rate of supernova in large clusters to be 19 kpc^{-2} Myr^{-1}. For the parameters of the present paper, $\langle\sigma\rangle = 39$ kpc^{-2} Myr^{-1} (recall that $\langle\sigma\rangle$ is the average rate of all Type II supernova per kpc^2; see § IV*c*). Thus, about half of all Type II supernovae reside in large clusters that produce breakthrough.

The remaining Type II supernovae are located in small clusters. Supernovae in a small cluster act more as individual supernovae with an energy equal to the combined energy of all N supernovae in the cluster. This was discussed in Paper I: for clusters containing N supernovae, Q_{3D} is equal to the value it would have if the supernovae were independent, multiplied by $N^{0.028}$. Since $N < 11$ for small clusters, this factor is less than 2. Thus, the effect of small clusters on the porosity of the ISM is roughly the same as if the associated supernovae were uncorrelated. These Type II supernovae add to the effect of uncorrelated Type I supernovae, whose effects were discussed in Paper I.

VIII. THE LARGEST SHELLS

Detailed properties of Galactic shells were given by Heiles (1979). There were errors for two shells in Table 2 of that paper. For GS 123+07−127 and GS 139−03−69, the values of log $R_{\rm sh}$ should be 3.0 and 1.9, smaller by factors of 1.6 and 2.5 than the values given; listed values for log n_0, log M, and log E_k are also erroneous.

Shell radii in the Galaxy range up to 1300 pc, if we include only those shells with maximum confidence. Can such large shells be produced by the largest clusters? As originally emphasized by Tenorio-Tagle (1979), it is difficult for clustered supernova to produce a shell radius very much larger than h, and in addition there are other observational reasons for suspecting a different mechanism might operate (Mirabel 1982).

The ratio of the largest radius attained by a shell to the disk height can be calculated from the formulae given in Paper I, and is given by

$$\frac{R_f}{h} = 5.5 h_{100}^{-1/3} L(\text{wind}, 38)^{1/6} n_0^{-1/6} v_{\rm rms}^{-1/2} \, . \tag{16}$$

The dependence of R_f/h on the ISM parameters is very weak. Enormous values of L(wind) and n_0^{-1} are required to produce large values of R_f. A small $v_{\rm rms}$ would help, but anywhere supernovae significantly stir the ISM we expect $v_{\rm rms} \approx 10$ km s^{-1}. This leaves h as the only parameter able to significantly affect R_f.

For the Galaxy, $L(\text{obs})_{\rm max,38} \approx 20$. Inserting this together with the other parameters in § III yields $R_f = 333 h_{100}^{2/3}$ pc. To obtain $R_f = 1300$ pc, we require $h \approx 800$ pc. Such large values of h are attained only at the very outermost reaches of the Galaxy. Not all of the observed large supershells are located at such extreme Galactocentric distances.

We must appeal to other possibilities. One is that the radii of large supershells is increased in the tangential direction by galactic differential rotation (TTP, PFTT), and this larger radius might mistakenly be used to estimate the energy required to create the supershell (Palous 1989). Another is that there are fluctuations in h; a large cluster, located in a region where h happens to be large (perhaps because of a previous cluster's supernovae), will be "lucky" and make a bigger splash than usual. We might have "clusters of clusters": the effects of multiple large clusters, located nearby in space and time, can be additive, not only in terms of L(wind) but also, probably more importantly, in terms of the earlier clusters modifying the ambient ISM for later ones, for example, by increasing the local value of h. Occasionally, several strategically located clusters might create neighboring holes that look like one large hole.

Apart from such effects, the most likely mechanism is completely different. Tenorio-Tagle (1979) and Tenorio-Tagle *et al.* (1987) have suggested that infalling high-velocity clouds can impart large energies to the disk ISM and cause the very largest supershells. They, and MMN, have suggested ways by which this process can be observationally distinguished from clustered supernovae. One important difference is that shells produced by colliding clouds should not contain diffuse X-ray emitting hot gas. Another is direct observational evidence, in the form of 21 cm line maps, for collisional interaction this in the Galaxy (see review by Mirabel 1989) and in external galaxies (Brinks 1989). In our Galaxy, some large shells are morphologically associated with high-velocity gas. The only external galaxies that exhibit very large holes have been observed are those with high-velocity gas. These include our Galaxy (Heiles 1984) and M101 (van der Hulst and Sancisi 1988). M31 contains no very large holes, and high-velocity gas at the level seen in our Galaxy is absent in M31 (Brinks 1989).

IV. SUMMARY, DISCUSSION, AND CAVEATS

a) Reconciliation with Paper I

Q_{2D}, as calculated in the present paper, is much smaller than that calculated in Paper I. For the solar neighborhood, Paper I obtained $Q_{2D} = (13, 4.4)$ and we now obtain (0.21, 0.07) for (inside, outside) spiral arms. The present values are in reasonably close agreement with observations. The reduction, by nearly two orders of magnitude, comes from three factors. First, the present expression for Q_{2D} is 8/15 of that in Paper I.

Second, the present ISM parameters provide another reduction by 0.64.

Third, the introduction of a realistic distribution for the number of supernovae per cluster makes a huge difference. A small portion of this difference comes from the factor $\langle L(\text{wind})^{1/2}\rangle$. In Paper I we assumed 40 supernovae per cluster, so that $\langle L(\text{wind})_{38}^{1/2}\rangle = 0.70$. Here, the power-law distribution tends to raise $\langle L(\text{wind})^{1/2}\rangle$, but this is compensated for by our twice-larger assumed value of τ_{SN}; the net effect is that $\langle L(\text{wind})_{38}^{1/2}\rangle = 0.62$, almost as large as the value in Paper I.

The lion's share of the difference comes from the value of Σ. In Paper I, the present parameter Σ was replaced by the combination σN^{-1}, equal to the total supernova rate divided by the number of supernovae per cluster. As discussed above in § IV*c*, $\langle\sigma\rangle = 73$ kpc^{-2} Myr^{-1} in Paper I, yielding $\langle\Sigma\rangle = 1.88$ clusters kpc^{-2} Myr^{-1}. In the present paper, $\langle\Sigma\rangle = 0.122$ clusters kpc^{-2} Myr^{-1}, 0.065 the value in Paper I. This is a direct result of the steep power law for the cluster formation rate: only a fraction of all clusters are large enough to achieve breakthrough.

b) Summary and Discussion

We have calculated the rate at which clustered Type II supernovae (i.e., supernovae whose progenitors are massive stars) produce breakthrough bubbles, which are bubbles that break through the dense, thin layer of disk gas. Owing to the fact that there is also a thicker, less dense component of disk gas, only clusters with large numbers of supernovae can produce "blowout bubbles," which break through *all* the disk gas and inject mass into the gaseous halo. We derived the distribution of N, the number of supernovae per cluster, from observational data on extragalactic H II regions and an empirically adjusted, theoretically derived relation between H II region brightness and number of supernovae per cluster. Considering only the neutral Lockman extended-$|z|$ component, about $\frac{1}{3}$ of the breakthrough bubbles also achieve blowout; the fraction may be very much smaller if the ionized Reynolds extended-$|z|$ component governs blowout. Both cold clouds and hot diffuse gas are injected into the halo, in roughly equal amounts. For the Lockman-component parameters, diffuse hot gas in the halo probably provides enough pressure to contain the cold gas clouds until they fall back to the disk.

The dependence of supernova rates on galactocentric radius was discussed in detail in Paper I, and updated in § IV*c*. To a first approximation, rates of clustered supernovae follow the distribution of starlight and star formation. Thus, within an individual galaxy, the fraction of the disk area occupied by the "holes" produced by breakthrough bubbles should vary considerably.

Comparison of our predictions with observational data yield discrepancies, although they are not unreasonable given the uncertainties and approximations in the approach and in the data. Different types of galaxy have different rates of clustered supernovae: rates increase in later type galaxies, so that Sb, Sc, and Irregular galaxies have progressively higher rates.

For our Galaxy, which is an Sb, we obtained a Galaxy-wide average $Q_{2D} = 0.30$, which corresponds to an area filling factor for breakthrough bubbles of 0.23, which is probably in agreement with the situation. In M31, the observed Q_{2D} peaks at about 0.10, implying a "galaxy-wide average" of perhaps 0.05, which is about 7 times smaller than the predicted value if M31 is described by the same parameters as the Galaxy. Values for Sc and Irregular galaxies should be higher by factors of 3–5 in typical cases. However, the Sc galaxy M33 has an observed average $Q_{2D} < 0.67$, about twice the value predicted for the Galaxy.

All of this indicates that our predicted values of Q_{2D} are at least sometimes too large by a factor of 2 or so. As discussed in § IV*a*, stretching of the supershells by galactic differential rotation was not included in our calculation of Q_{2D} and would reduce our derived values somewhat. Our results are affected by the factor used to adjust equation (1*a*) to obtain equation (1*b*); we could have obtained better agreement by using a smaller adjustment factor, but the need to use a factor at least as large as that actually used seems compelling. Also, our results are directly affected by the value of $L_{\min}$, which may be somewhat uncertain.

There is observational evidence in support of our fundamental approach. The evidence is the H I supershells observed in our Galaxy (Heiles 1984) and H I holes observed in M31 (Brinks and Bajaja 1986) and M33 (Deul and den Hartog 1989). Further, in external galaxies the beautiful Hα photographs of M33 by Courtès *et al.* (1987) and of the LMC by Meaburn (1980) exhibit many large ringlike H II regions. The large rings can hardly be anything else but evolved shells produced by the superwinds of the central clusters. It takes about 5 Myr for shells to expand to the typical size of these rings, and because stars in a cluster form over a longer period of time a fraction of the shells should be ionized.

In the classification scheme of Norman and Ikeuchi (1989), we predict our Galaxy to be in the chimney phase, while we predict the later type galaxies to have $Q_{2D} > 1$ (but only marginally) and thus be in the three phase domain. Because the supernova rate varies systematically with galactocentric radius, there frequently may be portions of a galaxy that lie either well within or far from the three-phase domain. An exception to this is "starburst" galaxies, which have extremely high supernova rates over large portions of their disk areas.

Central to our ideas is the fact that an observationally derived quantity, Q_{2D}, and a desired but observationally elusive quantity, $\dot{M}$ (the mass injection rate into the halo), are inextricably related, although the quantitative details depend on the ISM parameters, and in particular the thickness of the extended-$|z|$ gas component. To evaluate $\dot{M}/Q_{2D}$ we use equation (14) for cold clouds and the dominant term in equation (15) for diffuse hot gas, and we use equation (9) for Q_{2D}. In performing the calculations it is important to include the dependence on the ISM parameters of the minimum L(wind)'s required for blowout and for breakout, using equation (11), and to adopt a value of a (see eq. [2] and its discussion). Here we use the value appropriate for the Galaxy, $a = -2.14$; other values of a change the quantitative relationship but not the overall conclusion. Setting $P \propto n_0 v_{\text{rms}}^2$ in equation (11), we obtain

$$\frac{\dot{M}_{\text{cold}}}{Q_{2D}} \propto n_0\, v_{\text{rms}} \tag{17a}$$

and

$$\frac{\dot{M}_{\text{hot}}}{Q_{2D}} \propto h^{-1.3} n_0^{0.7} v_{\text{rms}}^{3.1} \tag{17b}$$

There is some room for variation, particularly for $\dot{M}_{\text{hot}}/Q_{2D}$, with the ISM parameters.

The important point is that mass cannot be injected into the halo unless there is a corresponding Q_{2D}. Q_{2D} can be observationally derived, either relatively directly by observing the H I holes, or much less directly (and with more uncertainty) by observing the large H II regions and applying equations (9) and (13).

The inner portions of spiral galaxies have higher supernova rates and higher rates of mass injection into the halo $\dot{M}$. However, most of the supernova-produced heavy elements are not directly injected into the halo without first mixing with the ambient ISM, because most supernovae are located in clusters that are too small to produce blowout. Direct mass injection into the halo should be stronger in Sc and Irregular galaxies, which have higher supernova rates, and thus higher values of $\dot{M}$ than Sb galaxies.

c) *Caveats*

Our approach has many weaknesses, both theoretical and observational. Theoretically, we use a crude theory for bubble dynamics in the stratified disk of a galaxy and a crude two-component model for the stratified disk. We have assumed the gas layer to be uniform. We have neglected the Reynolds component because it is highly clumped at $|z| = 0$, but its clumping probably decreases at high $|z|$.

Observationally, we adopt a relation between the Hα luminosity of the H II regions associated with a young cluster and the number of supernovae that will explode in the cluster. This relation can be derived theoretically under at least three assumptions, all of which must apply at least to an ensemble average of clusters. One is that the IMF for all clusters should be the same, which is equivalent to assuming that the number of supernovae in a cluster is directly proportional to the total number of ionizing photons emitted by stars in the cluster. Another is that the time over which stars form in clusters should be the same, which is equivalent to assuming that all clusters have the same relation between the Hα luminosity and the total number of ionizing photons emitted. The third is that the cluster lifetime is independent of the cluster luminosity. These, and other unstated, assumptions may not apply within an individual galaxy, and they also may not apply when comparing different types of galaxy. KEH have elaborated on these and other assumptions.

One disturbing point was our necessity to adjust the theoetically derived relation between the total number of ionizing photons and the number of supernovae in a cluster. An adjustment was required only for external galaxies, not the Galaxy. The basic reason for the required adjustment is that the integrated Hα luminosities of external galaxies are lower than the equivalent observationally derived quantity for the Galaxy. We do not understand the origin of this discrepancy.

Finally, our approach neglects the possibility self-propagating star formation in the supershell shock, discussed explicitly by McCray and Kafatos (1987), TTP, and PFTT. This seems to be occurring in the large supershells in the LMC and other Magellanic-type irregulars, and possibly in NGC 4449. Perhaps this is the cause of the extremely large Hα luminosities for some of the giant H II regions in these types of galaxy. This process is similar to a detonation front and can produce star clusters that are themselves correlated in space and time; it could lead to extremely large-scale phenomena, including injection of the entire disk mass into the halo.

It is a pleasure to acknowledge discussions with Elias Brinks, Robert Kennicutt, Bon-Chul Koo, Richard McCray, Jan Palous, Guellermo Tenorio-Tagle, and Rene Walterbos. Chris McKee provided many helpful, significant suggestions and criticisms, and Mordecai-Mark Mac Low contributed both helpful discussions and, while refereeing the paper, caught a significant error. This material is based upon work supported by the National Science Foundation under award No. AST-8818544.

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