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THE STRUCTURE OF THE CEPHEID INSTABILITY STRIP

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ABSTRACT

About 100 classical Cepheids having color excesses on a homogeneous system with standard errors ≤ 0.02 mag are used with the Feast-Walker period-luminosity-color relation to study the distribution of such stars in the instability strip. It is found that $\langle B-V \rangle_{\rm mag}$ is a better indicator of mean effective temperature than is $\langle B \rangle_i - \langle V_i \rangle_i$. The blue edge of the color-magnitude distribution is consistent with the theoretical blue edge for $Y = 0.28$ and $Z = 0.02$. Although the highest amplitude stars are found near the center of the period-color array, high- and low-amplitude stars can intermingle, and both kinds are to be found near the edges of the distribution. The same is true on the *C-M* array. Finally, it is pointed out that the Cepheids do not populate the instability strip uniformly if the red edge is taken to be parallel to the theoretical blue edge. Rather, the local instability region runs as a parallelogram in the *C-M* array from the theoretical blue edge upward and to the red.

Subject headings: stars: Cepheids — stars: evolution — stars: pulsation

I. INTRODUCTION

The structure of the Cepheid instability strip in the H-R diagram has been discussed before, for example, by Sandage and Tammann (1971), Yakimova, Nikolov, and Ivanov (1975), Pel and Lub (1978), and Ivanov and Nikolov (1979). Results, however, have been less than satisfactory and conclusions sometimes contradictory, a situation doubtless arising from an inadequate supply of accurate data. In the last decade, however, greatly improved Cepheid data—improved both in quality and quantity—have become available. In particular, the two observational quantities, absolute magnitude and reddening-free color, are now much better known than they were a decade ago. In some quarters, absolute magnitudes of Cepheids are now considered known to 0.1 or 0.2 mag (Walker 1988; Gieren 1988), while a recent study (Fernie 1990) has provided color excesses for several hundred Cepheids on a uniform scale with precisions of about 0.03 mag. It therefore seems appropriate now to reconsider the structure of the instability strip.

II. DATA

Of the several hundred stars in the color excess study of Fernie (1990, hereafter F90), 98 have excesses determined from at least three photometric systems with the standard error of the mean ≤ 0.02 mag. Nearly half of these (47) have been determined from at least four photometric systems and have $\sigma \leq 0.012$ mag. These 98 stars form the basis for this study and are listed in Table 1. The excesses listed there are on the scale defined by Cepheids in clusters and associations as discussed in F90. The mean $B-V$ values and visual light curve amplitudes have been taken from various sources in the recent literature. Unless stated otherwise, all colors quoted hereafter are taken to be intrinsic colors.

III. WHICH MEAN COLOR INDEX?

A perennial problem in this kind of work is how to define the mean color index of a star over its pulsation cycle. It has long been tacitly accepted that an intensity mean is most appropriate, so that for $B-V$ one first converts all B magnitudes to intensities, finds the average intensity over the cycle, converts it to a magnitude, does the same in the case of V, and reconstitutes the color index as $\langle B \rangle - \langle V \rangle$. In F90, however, it was found that better results were obtained when the mean color index was taken simply as the straightforward average of (say) $B-V$ over the cycle without any conversions to intensity involved, i.e., $\langle B-V \rangle$. Similar findings have been reported by Gray (private communication) and Sandage (1989) at IAU Colloquium 111. In this section, I pursue the question by making three tests.

First, however, note that $\langle B \rangle - \langle V \rangle$ and $\langle B-V \rangle$ cannot both represent the mean effective temperature equally well, i.e., they are not simple fractions or minor offsets of one another; instead, their relationship depends on another physical quantity which varies from Cepheid to Cepheid, viz. the amplitude of the light curve. This arises because light curves of larger amplitude are less sinusoidal, and the nonlinear logarithmic relation between intensity and magnitude then causes an increasing divergence between the two kinds of mean. Figure 1 shows the difference between the two kinds of mean as a function of V amplitude, using data from Table 1. Least-squares yields

$$\langle B \rangle - \langle V \rangle - \langle B-V \rangle = -0.003 + 0.010(V \text{ amp}) - 0.072(V \text{ amp})^2 .$$

Thus, it is easy to convert between the two indices, but if one of them is a good representative of temperature, the other must be not as good (unless an amplitude term is invoked with it.)

The first test between the two is as follows. The period-mean radius relation for Cepheids is now known quite well; Moffett and Barnes (1987, Table 2) give a summary of recent determinations, showing accord between theory and observation. I have adopted the P-R relation of Gieren, Barnes, and Moffett (1989), which is based on 101 Cepheids:

$$\log R = 1.108 + 0.743 \log P .$$

This was used to compute the radii of the Cepheids in Table 1.

Absolute magnitudes of these Cepheids were then found from the period-luminosity (P-L) relation of Walker (1988),

TABLE 1

INPUT DATA

Star	log P	E_{B-V}	$(Ampl)_V$	$\langle B-V\rangle_o$	$(\langle B\rangle-\langle V\rangle)_o$
U Aql	0.847	0.399	0.757	0.660	0.627
SZ Aql	1.234	0.641	1.163	0.853	0.746
TT Aql	1.138	0.495	1.082	0.874	0.799
FF Aql	0.650	0.224	0.321	0.538	0.531
FM Aql	0.786	0.646	0.724	0.663	0.630
FN Aql	0.977	0.510	0.564	0.722	0.703
V496 Aql	0.833	0.413	0.349	0.743	0.734
η Aql	0.856	0.149	0.799	0.679	0.640
RT Aur	0.571	0.051	0.803	0.584	0.542
RX Aur	1.065	0.276	0.664	0.712	0.683
SY Aur	1.006	0.454	0.638	0.569	0.545
RW Cam	1.215	0.649	0.888	0.720	0.700
RX Cam	0.898	0.569	0.729	0.655	0.623
RY CMa	0.670	0.248	0.726	0.630	0.599
SS CMa	1.092	0.549	0.981	0.724	0.676
U Car	1.588	0.283	1.194	0.993	0.895
V Car	0.826	0.174	0.596	0.723	0.699
VY Car	1.279	0.243	1.092	0.999	0.921
WZ Car	1.362	0.384	1.23:	0.878	0.765
XX Car	1.196	0.349	1.303	0.808	0.713
XY Car	1.095	0.417	0.879	0.845	0.798
XZ Car	1.221	0.367	1.074	0.958	0.888
YZ Car	1.259	0.396	0.805	0.773	0.727
AQ Car	0.990	0.161	0.613	0.791	0.767
ER Car	0.888	0.101	0.572	0.748	0.716
GI Car	0.646	0.175	0.334	0.572	0.564
IT Car	0.877	0.193	0.326	0.793	0.785
ℓ Car	1.551	0.170	0.725	1.136	1.102
RW Cas	1.170	0.420	1.190	0.903	0.807
SU Cas	0.290	0.287	0.414	0.423	0.414
SZ Cas	1.135	0.819	0.416	0.614	0.600
CF Cas	0.688	0.566	0.603	0.631	0.610
DL Cas	0.903	0.533	0.571	0.638	0.621
V Cen	0.740	0.289	0.811	0.627	0.583
XX Cen	1.040	0.260	0.905	0.768	0.722
V339 Cen	0.976	0.428	0.614	0.799	0.779
δ Cep	0.730	0.092	0.838	0.616	0.566
R Cru	0.765	0.152	0.774	0.662	0.620
S Cru	0.671	0.163	0.709	0.633	0.599
T Cru	0.828	0.193	0.493	0.745	0.728
SU Cyg	0.585	0.096	0.766	0.504	0.474
SZ Cyg	1.179	0.631	0.879	0.901	0.843
TX Cyg	1.168	1.181	1.221	0.679	0.595
VZ Cyg	0.687	0.289	0.674	0.616	0.587
DT Cyg	0.398	0.039	0.288	0.505	0.499
TX Del	0.790	0.023	0.628	0.752	0.714
β Dor	0.993	0.044	0.605	0.791	0.763
W Gem	0.898	0.283	0.822	0.668	0.626
ζ Gem	1.006	0.018	0.480	0.800	0.780
X Lac	0.736	0.362	0.405	0.550	0.539
Y Lac	0.636	0.217	0.705	0.543	0.513
Z Lac	1.037	0.404	0.968	0.741	0.692
RR Lac	0.807	0.353	0.764	0.569	0.532
BG Lac	0.727	0.336	0.611	0.635	0.614
GH Lup	0.968	0.364	0.174	0.855	0.853
T Mon	1.432	0.209	1.028	1.045	0.959
SV Mon	1.183	0.249	1.105	0.906	0.813
R Mus	0.876	0.120	0.819	0.679	0.639
UU Mus	1.066	0.413	1.090	0.798	0.734
S Nor	0.989	0.189	0.614	0.784	0.757
Y Oph	1.234	0.655	0.483	0.733	0.716
BF Oph	0.609	0.247	0.636	0.650	0.622
RS Ori	0.879	0.389	0.812	0.598	0.557
GQ Ori	0.935	0.279	0.685	0.728	0.695
κ Pav	0.959	0.045	0.846	0.633	0.582
VX Per	1.037	0.515	0.684	0.666	0.641
AW Per	0.810	0.534	0.812	0.549	0.520
VZ Pup	1.365	0.471	1.274	0.789	0.687
AT Pup	0.824	0.183	0.971	0.654	0.593
MY Pup	0.755	0.064	0.184	0.588	0.585
S Sge	0.923	0.127	0.718	0.710	0.673

TABLE 1—*Continued*

Star	log P	E_{B-V}	$(Ampl)_V$	$<B-V>_o$	$(<B>-<V>)_o$
U Sgr	0.829	0.403	0.717	0.728	0.695
W Sgr	0.881	0.111	0.805	0.682	0.638
X Sgr	0.846	0.197	0.590	0.582	0.557
Y Sgr	0.761	0.205	0.725	0.687	0.652
WZ Sgr	1.339	0.467	1.105	1.015	0.926
YZ Sgr	0.980	0.292	0.674	0.761	0.730
AP Sgr	0.704	0.192	0.832	0.656	0.615
BB Sgr	0.822	0.284	0.597	0.725	0.703
V350 Sgr	0.712	0.312	0.705	0.623	0.593
RV Sco	0.783	0.342	0.824	0.657	0.613
V482 Sco	0.656	0.360	0.652	0.642	0.616
V500 Sco	0.969	0.599	0.715	0.705	0.676
V636 Sco	0.832	0.217	0.532	0.740	0.721
Z Sct	1.111	0.542	0.986	0.845	0.792
SS Sct	0.565	0.337	0.523	0.628	0.608
EV Sct	0.490	0.679	0.300	0.482	0.477
ST Tau	0.606	0.355	0.778	0.528	0.492
SZ Tau	0.498	0.294	0.330	0.558	0.550
R TrA	0.530	0.127	0.550	0.616	0.593
S TrA	0.801	0.100	0.735	0.691	0.652
α UMi	0.599	-0.007	0.050	0.584	0.586
T Vel	0.666	0.281	0.612	0.666	0.638
RZ Vel	1.310	0.335	1.20:	0.906	0.794
SW Vel	1.370	0.349	1.274	0.915	0.802
T Vul	0.647	0.064	0.643	0.600	0.572
U Vul	0.903	0.654	0.718	0.654	0.621
SV Vul	1.653	0.570	1.054	0.969	0.881

which is

$$\langle M_V \rangle = -2.91 \log P - 1.21 ,$$

and which comes from Cepheids in clusters and associations. Each $\langle M_V \rangle$ was converted to $\langle M_{bol} \rangle$ through the bolometric correction scale of Flower (1977) applied iteratively.

Having the mean radius and luminosity for each Cepheid permitted the calculation of $\langle \log T_e \rangle$. The test between $\langle B \rangle - \langle V \rangle$ and $\langle B-V \rangle$ then comprised plotting $\langle \log T_e \rangle$ first against one and then the other for the 98 stars in the sample and seeing which provided the fit with least scatter. The clear decision was in favor of $\langle B-V \rangle$, for which the scatter was 20% less than for $\langle B \rangle - \langle V \rangle$.

Figure 2 shows this plot for the $\langle B-V \rangle$ case, together with the calibration of log T_e versus $B-V$ for supergiants from Flower (1977). Pel (1985), in reviewing such calibrations, noted that there is good agreement among various workers as to the slope of this calibration, even though zero points differ. Flower's scale lies near the middle of these calibrations. It is clear from Figure 2, however, that the present calculations give a distribution whose slope does not agree with Flower's scale. The same is true of the $\langle B \rangle - \langle V \rangle$ case. Numerical experi-

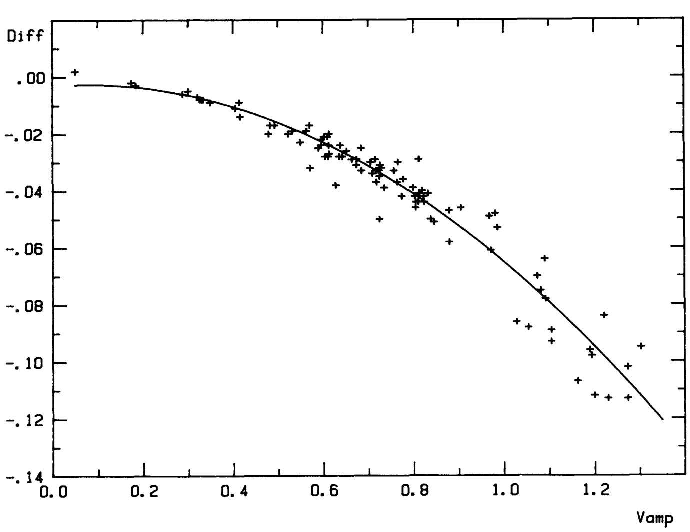

FIG. 1.—The difference $(\langle B \rangle - \langle V \rangle) - \langle B-V \rangle$ as a function of the amplitude of a Cepheid's V light curve. The line has the equation $(\langle B \rangle - \langle V \rangle) - \langle B-V \rangle = -0.003 + 0.010(V\text{ amp}) - 0.072(V\text{ amp})^2$.

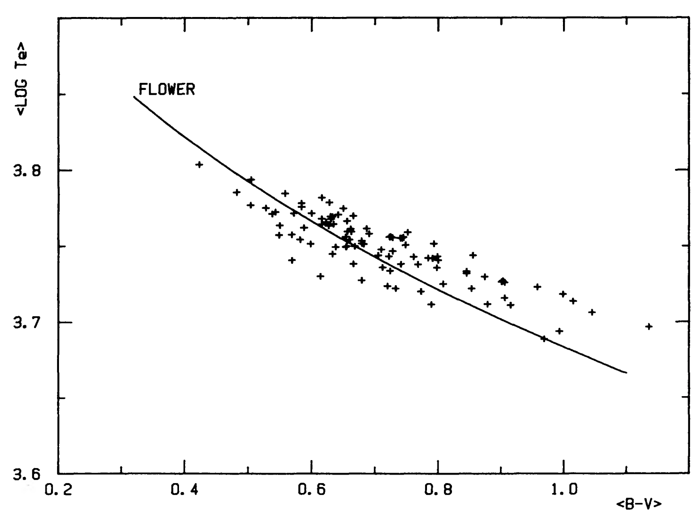

FIG. 2.—Log T_e for the stars of Table 1 computed from current period-radius and period-luminosity relations, plotted against $\langle B-V \rangle$. The line is Flower's scale for nonpulsating supergiants. Better agreement could be achieved by modest changes to the P-R and/or P-L relations.

ments quickly reveal, though, that either reducing the slope of the P-L relation from −2.91 toward −2.6 or increasing the slope of the P-R relation from 0.743 toward 0.8 would give a distribution with a slope in much better agreement with Flower. Lesser corrections to each relation, and possibly small changes to the BC function, would fine-tune the distribution quite satisfactorily without violating present uncertainty estimates, but no attempt to do so is made here, since there is no way of knowing how to distribute the changes among the various quantities. The point is only that when temperatures are calculated in this way, $\langle B-V \rangle$ represents them better than does $\langle B \rangle - \langle V \rangle$.

The second test is more straightforward and consists of determining which form of the color gives a period-color relation with the least scatter Discussion of this relation is given in the next section; here I note only that $\langle B-V \rangle$ gives a 3% smaller standard error than does $\langle B \rangle - \langle V \rangle$ and the square of its correlation coefficient (a standard indicator of goodness of fit) is 12% better.

The third and most obvious test is to see which kind of mean color gives the better period-luminosity-color (PLC) relation. I have based the input data for Cepheids in clusters and associations on Table 2 of Feast and Walker (1987; hereafter FW), but replacing their color excesses with ones from Table 1 and therefore recalculating $\langle M_V \rangle$ and the intrinsic colors. I also omitted EV Sct, TW Nor, and SV Vul, for which F90 found significant discrepancies in color excess compared to the FW listing, and corrected the period of SZ Tau from 4.03 to 3.15 days.

Redetermining the PLC relation from these data yields unsatisfactory results. The coefficient of the color term in the PLC relation has values of about 0.6 ± 0.5, compared to values in the range 2.1–2.7 adopted by FW. In fact, it seems FW did not use the cluster Cepheids to determine this number but instead relied on previous work on Magellanic Cloud Cepheids. Earlier attempts to determine it from Galactic Cepheids (e.g., Fernie and McGonegal 1983; Hindsley and Bell 1989) have also failed. For what it is worth, however, the PLC solution using $\langle B-V \rangle$ gave a σ of 0.236 mag against 0.239 for $(\langle B \rangle - \langle V \rangle)$.

This inability to find the color term in the PLC from Galactic Cepheids is puzzling, inasmuch as its existence is required by theory and it certainly seems present in Magellanic Cloud Cepheids (Martin, Warren, and Feast 1979; Caldwell and Coulson 1986). Because of the improvement it offered in these studies, I have continued to use the PLC relation of FW for computing absolute magnitudes in this paper.

The conclusion of this section, then, is that all available evidence suggests that $\langle B-V \rangle$ is a better representative of effective temperature than is $\langle B \rangle - \langle V \rangle$. Accordingly, the remaining discussion is carried out in terms of $\langle B-V \rangle$ only. In fact, though, the work has been carried through with both indices, and no major conclusions would be altered by using $\langle B \rangle - \langle V \rangle$ instead.

IV. THE PERIOD-COLOR RELATION

Figure 3 displays the period-color relation, the line being a least-squares fit. It has the equation

$$\langle B-V \rangle = 0.311 + 0.438 \log P$$
$$\pm 0.025 \pm 0.026 ,$$

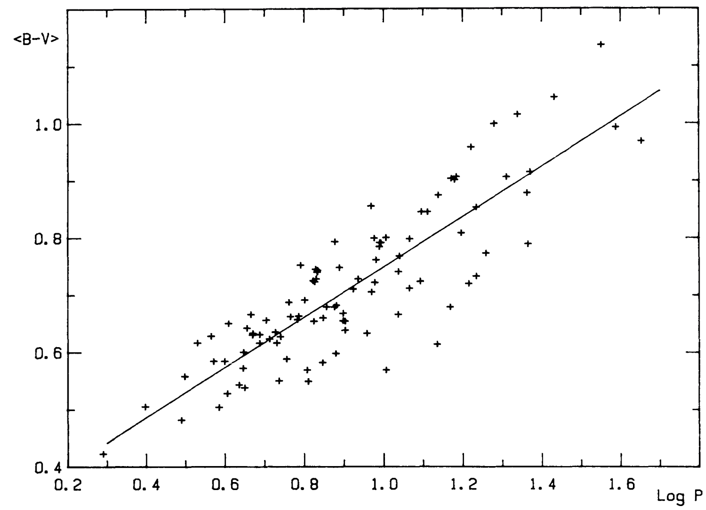

FIG. 3.—The period-color relation. The line is a least-squares fit.

which is very similar to previous determinations (Dean, Warren, and Cousins 1978, Table 3). The standard deviation of an individual point is 0.070 mag, and the overall width at constant period is about 0.25 mag. The latter is more than 10 times the uncertainty in an individual $\langle B-V \rangle$, testifying to the reality of a spread in color at a given period, even though some small part of the spread might be due to unresolved binaries.

However, there is a fairly well defined upper (red) edge to the distribution, and this is steeper than the least-squares line. Similarly, the less well defined lower (blue) edge is probably also steeper. This suggests a nonuniform distribution of points, and indeed there does seem to be a bunching of points toward the red edge, coupled with a paucity of bluer, shorter period Cepheids, e.g., at around $\log P = 0.7$, $\langle B-V \rangle = 0.43$.

Figure 4 addresses the question of whether the pulsation amplitude depends on position in the period-color plot. Previous work has led to disagreements as to whether the higher

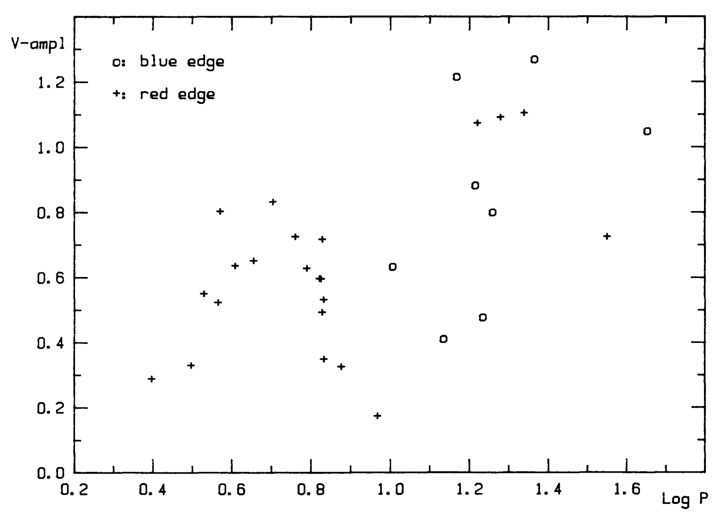

FIG. 4.—The amplitudes of stars near the blue edge of the distribution in Fig. 3 (*open circles*) compared to the amplitudes of stars near the red edge (*plusses*). Apart from the lack of short-period stars near the blue edge, and amplitudes generally increasing with period, there is no obvious dependence of amplitude on position. At a given period (e.g., $\log P = 0.8$ and 1.2), stars at the same edge can have widely differing amplitudes.

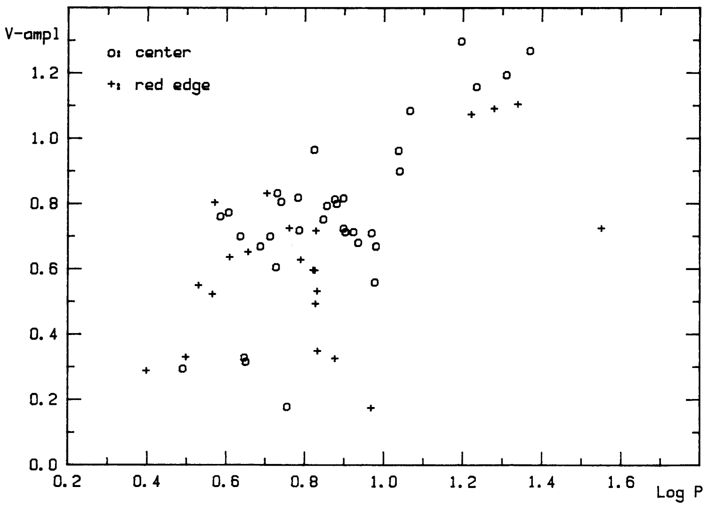

FIG. 5.—Same as Fig. 4, but now comparing stars near the center of the distribution with those at the edge. In a general way, the highest amplitude stars at a given period are found at the center of the strip, but not all center stars have high amplitude.

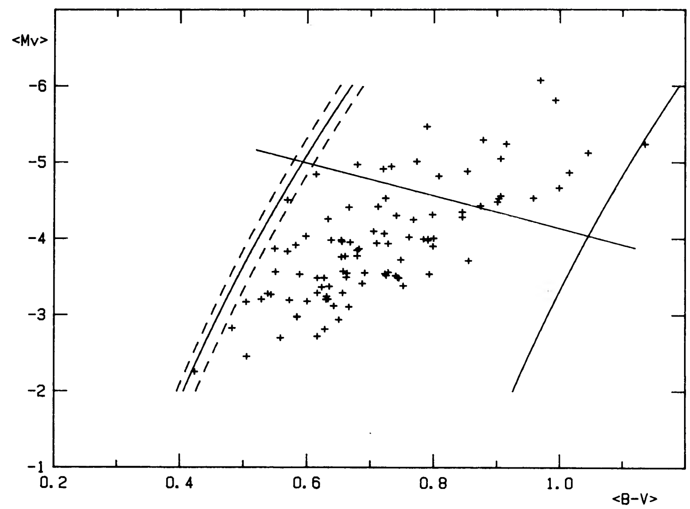

FIG. 6.—The instability strip in the color-magnitude array. The solid line flanked by two dashed lines is the theoretical blue edge for $Y = 0.28$, $Z = 0.02$. The bluer dashed line corresponds to $Y = 0.32$, $Z = 0.02$; the redder dashed line corresponds to $Y = 0.28$, $Z = 0.03$. The red edge is arbitrarily drawn parallel to the blue edge and to fit the reddest Cepheid. The near-horizontal line is a line of constant period ($= 15$ days).

amplitude stars lie at the red edge or the blue edge. In Figure 4, the plusses represent stars that lie close to the red edge in Figure 3; open circles represent the stars that are close to the blue edge. The lack of blue edge stars at shorter periods has been remarked on above, but the point of interest here is that high- and low-amplitude stars can be found at either edge. Near log $P = 0.8$, for instance, the highest and lowest amplitudes are both at the red edge, while near log $P = 1.2$, the same is true of blue edge stars. In Figure 3, therefore, neighboring points can represent stars of quite different amplitudes. Figure 5, however, compares red edge stars with stars lying close to the center of the distribution in Figure 3, and now it is seen that in general (but with notable exceptions), the center stars have higher amplitudes than do the edge stars.

I conclude that while in a rough way the stars at the center of the strip have the highest amplitudes (as one might intuitively expect), there is no very clear mapping of amplitude with position in the strip. This is perhaps not surprising, since a Cepheid near the edge of the strip might be evolving either into the strip or out of the strip, and its amplitude may depend on that direction. When enough data are eventually collected, it would be interesting to see whether neighboring edge stars of different amplitude have period changes of opposite sign.

V. THE M_V-$\langle B-V \rangle$ PLANE

Figure 6 shows the instability strip in the color-magnitude plane. The absolute magnitude of each star has been calculated from the PLC relation of FW, viz.

$$\langle M_V \rangle = -3.53 \log P + 2.13(\langle B \rangle - \langle V \rangle) - 2.11 .$$

The solid line with dashed lines on each side of it is the theoretical blue edge of the instability strip as determined by Iben and Tuggle (1975, hereafter IT) for the choice of $Y = 0.28$, $Z = 0.02$, and $\beta = 0.29$. The latter is defined by the equation

$$\log (\text{mass}) = \alpha + \beta \log L ,$$

and, as can be seen from Figure 1 of IT, at luminosities below $M_V = -6$ the position of the blue edge is quite insensitive to the choice of β in the range $0.20 < \beta < 0.33$. Following Carson and Stothers (1984), I have used $\beta = 0.29$ throughout. Conversion of the IT theoretical log T_e, log L to $\langle B-V \rangle$, $\langle M_V \rangle$ has been done using Flower's (1977) scales.

The effect of varying either Y or Z is shown by the dashed lines. The leftmost dashed line corresponds to $Y = 0.32$, $Z = 0.02$, while the rightmost dashed line is for $Y = 0.28$, $Z = 0.03$. It is seen that the solid line ($Y = 0.28$, $Z = 0.02$) fits the observations quite well, but that given even small errors in the observations, conversion scales, etc., one could not claim to derive Y to anything better than say ± 0.05, or Z to better than ± 0.02, especially if one allowed larger Y *and* larger Z values together. But theory and observation are at least consistent when currently acceptable values are used.

The red edge of the instability strip is drawn on the assumption that it is parallel to the blue edge (since the strip extends far below the limits of Fig. 6 toward the δ Scuti stars and even the ZZ Ceti stars) and by placing it to fit the reddest star. The latter, although somewhat isolated in the figure, is ℓ Car, a third magnitude star of low reddening and with many reliable observations, so its position is probably well fixed.

As drawn, the instability strip has a width in log T_e at $\langle M_V \rangle = -4$ of about 0.11, which is significantly larger than the 0.034 to 0.072 estimated by Deupree (1980) from numerical calculations. Other estimates, discussed by Buchler, Moskalik, and Kovács (1990), fall within this range too, and these authors, in presenting a novel method of finding the strip's width, place an upper limit at 0.057. If ℓ Car is disregarded and the red edge in Figure 6 moved blueward to the next three stars, the width becomes 0.094, which is still high. In any case, there seems no reason to discard ℓ Car. It appears a perfectly normal well-behaved Cepheid of healthy amplitude (0.7 mag in V), and I have checked the observational data among different observers and find them consistent. Even in the unlikely event it is a first overtone pulsator, its position in the figure would move upward only to -5.7. Finally, even if the red edge is not parallel to the blue edge but has a shallower slope, there seems no escape from the quoted width of the strip at $\langle M_V \rangle = -5$.

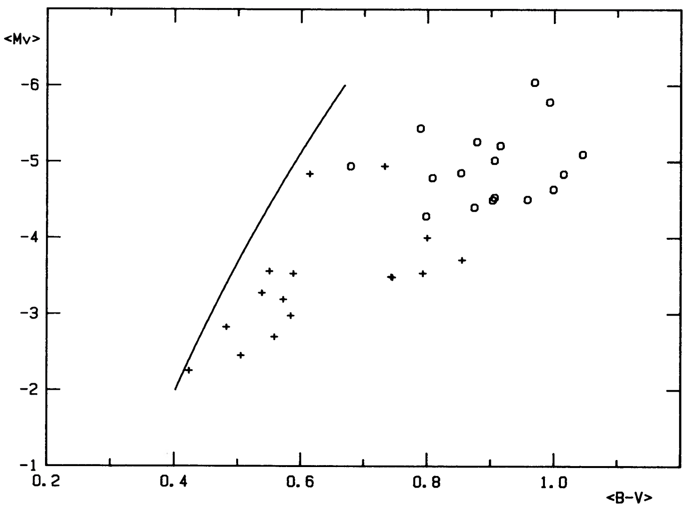

FIG. 7.—The *C-M* instability strip showing only stars of extreme amplitude. Open circles are stars with V amp > 1 mag, plusses have V amp < 0.5 mag. Close comparison with Fig. 6 shows that not all stars near the blue edge have low amplitude and not all stars near the red edge have high amplitude, while at some points within the strip, low and high amplitudes are contiguous.

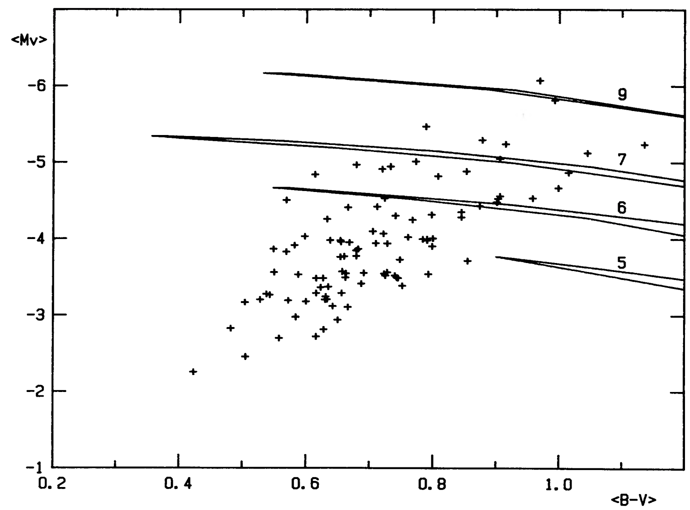

FIG. 8.—Theoretical evolutionary tracks for stars of 5–9 solar masses shown crossing the instability strip for the second and third times. These emphasize that the Cepheids bunched in the lower left of the strip (see Fig. 6) can only have reached there by passing through the empty region in the lower right, making the lack of Cepheids there surprising.

The near-horizontal line in Figure 6 is a line of constant period for $P = 15$ days and is shown only for reference.

Figure 7 returns to the question of pulsational amplitudes as a function of position within the strip, this time on the color-magnitude array. Open circles in the figure represent stars with V light curve amplitudes exceeding 1 mag; crosses are stars with amplitudes under 0.5 mag. Clearly, there is a general separation of the two, with mostly crosses to the left and circles to the right, but this only shows that short-period Cepheids do not have large amplitudes while many long-period ones do. In fact, a close comparison of Figures 6 and 7 reveals several stars close to the theoretical blue edge in Figure 6 that are not on Figure 7; i.e., there are stars close to the blue edge with considerable amplitudes, while in Figure 7 we find low-amplitude stars halfway across the distribution. Also, at a given color, both high and low amplitudes may occur at the top, middle, or bottom of the distribution. I again conclude that apart from high amplitudes (> 1 mag) occurring only among long-period Cepheids, there is no clear mapping of amplitudes within the instability strip; stars of very dissimilar amplitude may lie close together there.

A striking feature of Figures 6 and 7 is the parallelogram-like distribution of points sloping up within the instability strip from lower left to upper right. If the edges of the strip are at least roughly as shown, why are there no stars in the region typified by $\langle B-V \rangle = 0.8$, $\langle M_V \rangle = -2.5$ or $\langle B-V \rangle = 0.7$, $\langle M_V \rangle = -6$? The empty lower right region is particularly puzzling. Most Cepheids bordering it are of considerable amplitude, say greater than 0.7 mag; there is no hint that amplitudes are rapidly diminishing with decreasing luminosity in this region, so that one cannot argue that the Cepheids which are really there are of too low amplitude to have been discovered as such. Moreover, these would be shorter period Cepheids, in which low amplitudes are more easily detected than in long-period Cepheids. That not a single star is found in this region makes such a selection effect improbable as an explanation. Furthermore, since virtually every Cepheid has already been a red giant, the stars that crowd to the lower left of the strip must all have passed through the empty region to get there, yet we catch none in the act!

Figure 8 again shows the 98 Cepheids in the color-magnitude array, now with the theoretical evolutionary tracks of Bertelli *et al.* (1986) superposed. These show the second (lower branch) and third (upper branch) crossings for 5, 6, 7, and 9 $M_\odot$ having $Y = 0.28$, $Z = 0.02$. Ironically, the 5 $M_\odot$ track suggests it is the lower right part of the strip that should be populated and the lower left empty, just the opposite of what is observed. However, it is well known from the similar tracks of Becker, Iben, and Tuggle (1975) that a very slight reduction in the assumed value of Z leads to a much greater blueward penetration of the loop, so the discrepancy is not serious. What the tracks do emphasize, though, is that the stars bunched in the lower left must have got there directly through the "empty quarter," and, since 4 and 5 $M_\odot$ stars evolve more slowly than the 7 $M_\odot$ stars we see spread across the strip, we would expect to see considerable numbers of them in the lower right. Instead there are none. It is difficult to avoid the conclusion that the stars that must be there are not pulsating; i.e., that locally at least, the red edge of instability is not far from the lower part of the distribution in Figures 6–8.

VI. SUMMARY AND CONCLUSIONS

Ninety-eight classical Cepheids having color excesses believed known to better than 0.02 mag have been used to study the structure of the instability strip. The magnitude mean color $\langle B-V \rangle$ is found preferable to the intensity mean $(\langle B \rangle - \langle V \rangle)$, because it yields more consistent values of E_{B-V}, gives a better representation of $T_{\rm eff}$ as calculated through the P-R and P-L relations, produces less scatter in the P-C relation, and (marginally) less scatter in the PLC relation. A relation is given for converting from one kind of mean color to the other.

The present P-C relation is similar to previous determinations, although it seems Cepheids are not uniformly distributed within the P-C strip but tend to bunch toward the red edge. The highest amplitude Cepheids are generally found near the center of this strip, but there is no clear mapping of amplitude with location in the strip.

In the color-magnitude plane, the blue edge of the instability strip is found to be consistent with the theoretical blue edge having $Y = 0.28$, $Z = 0.02$. If the red edge is parallel to the blue edge, then to accommodate the present data, the strip must have a width in log T_e of about 0.11, which is significantly wider than found in previous studies, either theoretical or observational.

In addition, if the red and blue edges are parallel, the Cepheids are found in a distribution within the strip that avoids lower luminosity red and higher luminosity blue regions. This is despite the fact that the many Cepheids found in the lower, blue region must have evolved through the lower, red region. It appears unlikely that this is a selection effect, and no satisfactory explanation is found beyond the possibility that the edges of the strip are not parallel.

Finally, as in the case of the P-C strip, there is no clear correlation between amplitude and position within the strip, other than the well-known fact that higher amplitudes are found among longer period Cepheids.

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