THE ASTROPHYSICAL JOURNAL, 353:118–122, 1990 April 10

CONSTRAINTS ON THE MASS DISTRIBUTION NEAR THE CENTERS OF M31 AND M32

DOUGLAS RICHSTONE AND GARY BOWER
Department of Astronomy, University of Michigan

AND

ALAN DRESSLER
Mount Wilson and Las Campanas Observatory
Received 1989 July 3; accepted 1989 October 3

ABSTRACT

We have reinterpreted spectroscopic data on the motions of stars near the centers of M31 and M32. The mass distribution near the centers of each of these galaxies must be more concentrated than the light. We have constructed models containing black holes in the mass range 4–5 × 10^7 $M_\odot$ for M31 and 0.7–3 × 10^6 $M_\odot$ for M32. Extended models of the unseen mass must have core radii less than 0"4 and must contain $\sim 10^8$ $M_\odot$ for M31 and $\sim 10^7$ $M_\odot$ for M32.

There are two astrophysical constraints on extended models. First, the objects that comprise them must not collide frequently. Second, the two-body relaxation time must be long enough that the nuclei have not already collapsed. These constraints are not particularly interesting at present but are likely to become so if Space Telescope observations reduce the maximum core radii of the mass distributions in the two galaxies.

Subject headings: black holes — galaxies: individual (M31, M32) — galaxies: nuclei

I. INTRODUCTION

The nearby galaxies M31 and M32 have been shown to have concentrations of mass near their centers which have higher mass-to-light ratios than the stars that appear to dominate the mass distribution of the galaxies on scales of tens to hundreds of parsecs. In each case two independent sets of investigators have argued on different grounds for the presence of this mass and tentatively identified it as a black hole. In the case of M31, Dressler and Richstone (1988, hereafter DR) have argued that the light distribution, dispersion profile, and central rotation curve mandate a mass of about 5 × 10^7 $M_\odot$, while Kormendy (1988) has argued for an object about a factor of 2 less massive. In M32, DR obtained a point mass of about 5 × 10^6 $M_\odot$, while Tonry (1987) has suggested a similar mass. The DR study was unique in its ability to role out the possibility of accounting for the observations with a constant M/L spherical but very anisotropic distribution function. This was accomplished through the use of a maximum entropy modeling technique (Richstone and Tremaine 1988, hereafter RT).

It is, however, well known to all investigators in this area that the observational data do not point unambiguously to a central *point* mass. Rather, some additional mass is required on a rather small spatial scale near the center of the galaxy. In this paper, we use the same observational data presented in DR to constrain that mass distribution. In particular, a two-component model is constructed in which the galaxy mass distribution consists of the light distribution times an unspecified M/L plus a mass distribution of specific radial profile but unspecified scale size and mass (and hence, central density). In M31, we find that the central dark mass distribution must contain 10^8 $M_\odot$ within 3.4 pc of the center. In M32, the distribution must contain about 10^7 $M_\odot$ within 3.4 pc of the center.

The method we used is described in § II. In § III, we explore point mass models with differing central masses to find the range of central point masses consistent with the data. In § IV, we report on dark mass *distributions* consistent with the observations.

II. METHOD

Most key elements of the method employed here have been described in DR and RT. The observational data consist of a light distribution $I(R)$, a projected dispersion profile $\sigma_p(R)$, and a projected major axis rotation curve $v_p(R)$ along the major axis, all as a function of central distance projected on the sky (R). The method we use consists of five steps.

1. Convert $I(R)$ into the spatial luminosity density $\epsilon(r)$ (in $L_\odot$ pc^{-3}) by assuming spherical symmetry.

2. Assume a mass distribution of the form

$$\rho(r) = \alpha\epsilon(r) + \mu(r)\,, \tag{1}$$

where α is the unknown M/L, and μ is the specified density distribution of the dark mass. For M32, we took the model for $\epsilon(r)$ specified by Tonry (1987, eq. [2]). For M31, we fitted a two-component analytic King model to the profile derived by Light, Danielson, and Schwarzschild (1974). Each component has a luminosity density distribution of the form

$$\begin{aligned} \epsilon(r) &= \epsilon_0[1 + (r/a)^2]^{-3/2}\,, \quad && r < r_{\max} \\ \epsilon(r) &= 0, && r > r_{\max} \end{aligned} \tag{2}$$

where $r_{\max}$ was set at 100". We refer to a as the core radius of the distribution. The nuclear component had $\epsilon_0 = 6.5 \times 10^4$ $L_\odot$ pc^{-3} (based on B magnitudes) and $a = 0\rlap{.}''4$. The bulge component had $\epsilon_0 = 35$ $L_\odot$ pc^{-3} and $a = 30''$.

To model a central point mass, $\mu(r) = C\,\delta(r)$. For the extended mass models we chose a density distribution of the form given by equation (2) and varied the central density and core radius with $r_{\max} = 10''$.

3. Construct a complete set of spherically symmetrized orbits in the mass distribution defined by equation (1).

4. Using that set of orbits, use a constrained maximum entropy technique (RT) to construct a dynamical model by

superimposing as many of the orbits as needed to match the light distribution $\epsilon(r)$ and to reproduce the observed $\sigma(R)$ profile [in fact, we match a fake $\sigma(R)$ profile constructed by trial and error, so that after step 5, the observed $\sigma(R)$ profile is well matched]. The orbits are assumed to be populated always in the prograde direction so that the galaxy is the maximally rotating version of the model. The maximum rotation speed of the model and the quality of the fit to the $\sigma(R)$ profile can be increased or decreased by varying the multipliers β_1 and β_2 in the modified entropy that is being maximized:

$$S' = S - \beta_1 \chi^2 + \beta_2 J^2 , \tag{3}$$

where S is the usual thermodynamic entropy $\int f \ln f \, d^3x \, d^3v$, χ^2 measures the goodness of fit of the model to the dispersion observations, and J^2 is the sum of the squared specific angular momentum of the orbits which pass near the center of the galaxy, weighted by their contribution to the light in that region.

5. The model so constructed is compared to the observations by projecting the contributions to the first two velocity moments from each orbit onto lines of sight and integrating the luminosity-weighted moments along those lines of sight through the galaxy at all R (see Richstone and Tremaine 1984). These moments are then convolved with a seeing profile (assumed Gaussian[1]) to mimic the effect of Earth's atmosphere on the observations. The resulting rotation and dispersion profiles are compared to the observations. The dispersion profiles of the model must fit the observations at an acceptable χ^2 while the rotation curves must equal or exceed the observed rotation curves everywhere (the model's rotation curve can always be lowered at a given radius by reversing some fraction of the orbits with pericenters near that radius). A failure to match the dispersion profile to an acceptable χ^2 and to meet or exceed the observed rotation rate implies that the model is unacceptable.

III. POINT MASS MODELS

Point mass models are constructed for each galaxy by assuming that $\mu(r) = \tilde{m}\,\delta(r)$ in equation (1). Although the mass of the black hole cannot be specified in advance, the velocity dispersion data from a few to nearly 100″ from the center of both galaxies constrains the mass-to-light ratio of the stellar population quite well under the assumption of equation (1).

For a range of point masses, models were constructed with a value of the weighting factors β_1 and β_2 chosen in advance. The dispersion profiles and rotation curves of these models were computed by projecting their rotation velocities and velocity dispersions onto the line of sight and convolving them with seeing as described above. This yielded a range of masses with acceptable fits to both the dispersion and rotation curves. At the edges of the acceptable range, each of the weighting factors was adjusted to see if the range could be extended. Usually the acceptable range could not be enlarged in this way. The results of this procedure can be seen in Figure 1 for M31 and Figure 2 for M32. For M31 the point mass must lie in the range $4 \times 10^7 \leq M \leq 5 \times 10^7\, M_\odot$. For M32 the point mass must lie in the range $7 \times 10^5 \leq M \leq 3 \times 10^6\, M_\odot$. Note that the requirement that the model rotation curve match or exceed the observed curve sets the lower limit of the point mass, while the requirement that the model velocity dispersion profile match the observations tends to set the upper limit of the range.

Some understanding of this phenomenon can be gained by considering circular orbits near the galaxy center. A shell model consists entirely of circular orbits at each radius. Hence $\sigma_r = 0$, $\overline{v_t^2} = GM_r/2r = \frac{1}{2}\, v_c^2$ (the factor of 2 arises because the orbits must form a spherical shell and $\overline{v_t^2}$ is the one-dimensional dispersion) and $v_{\rm rot,max} = (2/\pi)(\overline{v_t^2})^{1/2}$. Thus, the squared circular velocity must exceed the maximum internal rotation rate by nearly a factor of 5,

$$v_c^2 \geq \pi^2 v_{\rm rot}^2/2 . \tag{4}$$

So an observed rotation rate *tends* to set a lower limit on the mass distribution near the center of the galaxy.

Next we consider the contribution to the projected second moment of the orbits in the shell model. It is easily shown to be $\langle v_t^2 \rangle$. Intuition suggests that the addition of nearly radial orbits can only increase the observed velocity dispersion at small radii for a fixed mass distribution since they must travel faster than v_c when they are near their pericenters. So, at fixed mass, the shell model seems to have the lowest possible dispersion near the center with $\sigma^2 \sim v_c^2/2$. Thus, the dispersion sets an upper limit on the mass distribution

$$v_c^2 \leq 2\sigma^2 . \tag{5}$$

This argument is confirmed by the models constructed in Richstone and Tremaine (1984) in which the lowest central dispersions at fixed mass appear in the models which are dominated by nearly circular orbits.

IV. DISTRIBUTED MASS MODELS

Distributed mass models are investigated by the following procedure. First, using equations (1) and (2), a variety of mass distributions are constructed by varying the core radius a of the dark matter distribution and, at each value of a, varying the central density μ_0. The mass inside any radius is defined by a, μ_0, and by the M/L of the stellar material, determined by the modeling process. The central density is varied until the upper and lower limits which produce acceptable fits to the dispersion profile and rotaton curve are found. At any fixed core radius, this procedure is the same as the one used to define acceptable point mass ranges in § III. The results obtained from this procedure are displayed in Tables 1 and 2 and Figures 3 and 4. They are most simply described in terms of the mass enclosed inside 1″.

For M31 and M32, solutions are found only for $a \leq 0\farcs4$ (or 1.3 pc). For M31, all solutions enclose about $10^8\, M_\odot$ inside $r = 1''$. The mass-to-light ratio inside this radius is about 100. For M32, all solutions contain $10^7\, M_\odot$ within 1″, and the M/L inside this radius is about 30.

These distributed mass models must be collections of

TABLE 1

M31 Model Ranges

a	$M(1'')$ $(M_\odot)$	$\bar{\rho}(0\farcs1)$ $(M_\odot\,{\rm pc}^{-3})$	$\bar{\rho}(0\farcs4)$ $(M_\odot\,{\rm pc}^{-3})$
0″.2.........	9.5×10^7	1.2×10^7	3.5×10^6
	1.2×10^8	1.7×10^7	4.8×10^6
0.4.........	1.1×10^8	4.8×10^6	2.6×10^6
	1.3×10^8	5.6×10^6	3.1×10^6

[1] The use of a Gaussian point-spread function is not ideal. Various authors have shown that the sum of two Gaussians or a Gaussian plus a skirt is much more appropriate. Our idealization is conservative; the use of a more appropriate function would shrink the range of acceptable models further.

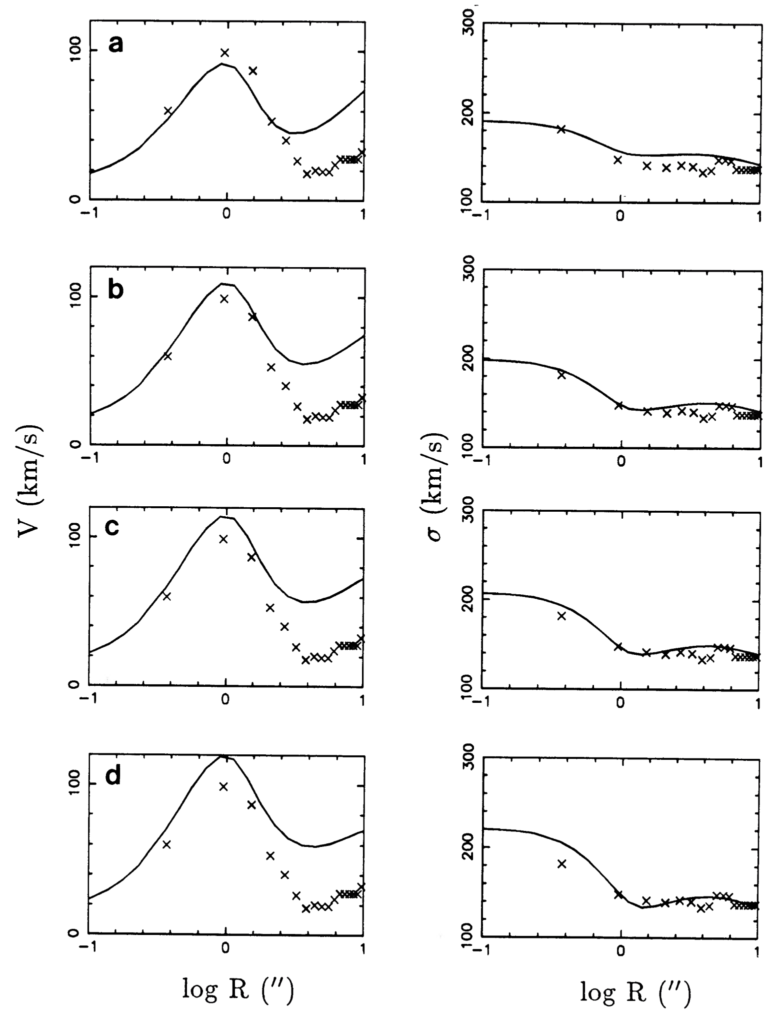

FIG. 1

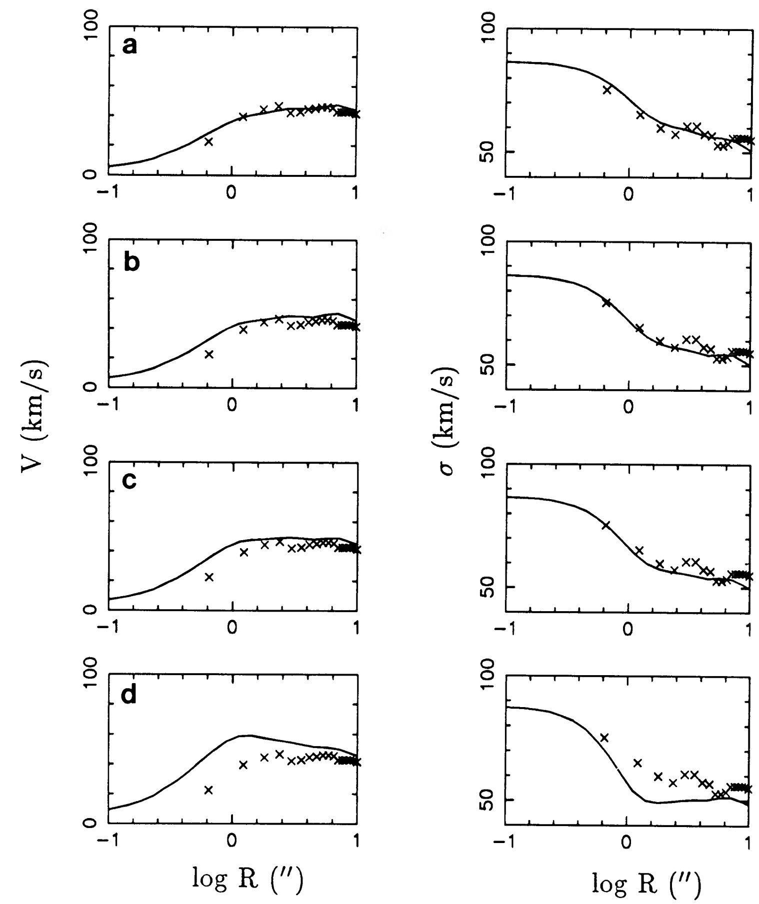

FIG. 2

FIG. 1.—Rotation velocity (v) and dispersion (σ) for maximally rotating spherical galaxy models of constant M/L with an additional central point mass. The data points are for M31. The topmost curve clearly rotates too slowly, while the lowest curve has too large a central velocity dispersion. Masses of the point mass were (*a*) $-2.5 \times 10^7\ M_\odot$, (*b*) $-4.1 \times 10^7\ M_\odot$, (*c*) $-5.1 \times 10^7\ M_\odot$, and (*d*) $-6.1 \times 10^7\ M_\odot$.

FIG. 2.—Same as Fig. 1 for M32. The topmost curve probably rotates too slowly, while the lowest curve fails to match the dispersion. The masses of the central mass point were (*a*) -0, (*b*) $-1.4 \times 10^6\ M_\odot$, (*c*) $-2.4 \times 10^6\ M_\odot$, and (*d*) $-6.4 \times 10^6\ M_\odot$.

TABLE 2

M32 Model Ranges

a	$M(1'')$ ($M_\odot$)	$\bar{\rho}(0\rlap{.}''1)$ ($M_\odot$ pc^{-3})	$\rho(0\rlap{.}''4)$ ($M_\odot$ pc^{-3})
0\.″2..........	1.4×10^7	1.4×10^6	4.4×10^5
0.4..........	1.3×10^7	1.0×10^6	3.7×10^5
	1.4×10^7	1.0×10^6	3.7×10^5

objects, so at least two astrophysical constraints may be applicable. If the average star physically collides with another in a sufficiently short time, stellar mergers may lead to the build-up of a very massive object in a time scale less than the age of the galaxy. Second, if the two-body relaxation time is sufficiently short, the nucleus may core collapse.

Both of these processes have been studied in connection with globular clusters and in the context of M31 and M32 by Goodman and Lee (1989). First, we consider M31.

The stellar collision rate for each star may be written as

$$1/t_{\rm col} = 16\pi^{1/2} n\sigma r_*^2(1 + \Phi) \tag{6}$$

(Binney and Tremaine 1987), where Φ is the Safronov number defined by $\Phi = Gm_*/(2\sigma^2 r_*)$, n is the number density of stars, r_* and m_* are the stellar radii and masses, and σ is the velocity dispersion (taken as isotropic for this estimate). If this collision time is less than a Hubble time, it seems likely that a runaway merger process will occur, leading to a supermassive star or a black hole. The lower limit on the average density inside 0\.″1 from Table 1 is $\sim 5 \times 10^6$ $M_\odot$ pc^{-3}, and we adopt a value of $\sigma = 250$ km s^{-1}. For these parameters, the collision time scale for main-sequence stars near 1 $M_\odot$ in M31 is

$$t_{\rm col} = 2.2 \times 10^{10}\ {\rm yr}\left(\frac{m_*}{M_\odot}\right)^{-1} \tag{7}$$

(the m_*^{-1} dependence appears because we have approximated the mass radius relation near the Sun as $m \sim r$). So, if the mass in the nucleus is carried by main-sequence stars, they must be less massive than 1 $M_\odot$ to avoid merging with each other. A more severe constraint is derived from the central M/L of

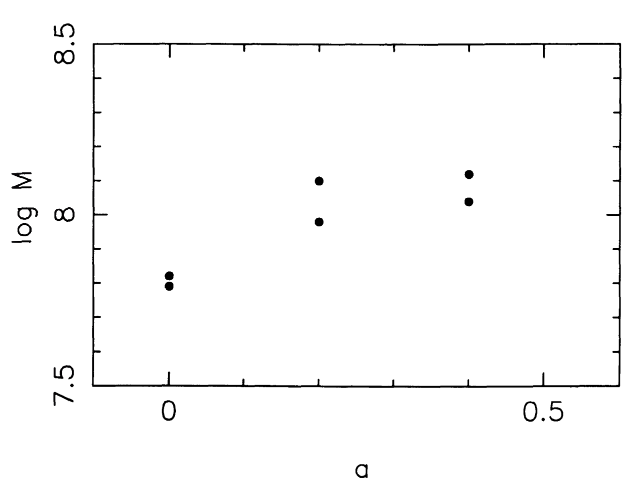

Fig. 3.—Locus of acceptable models as a function of mass inside 1″ (M) and core radius (a) in arcseconds for M31. For core radii larger than 0, all acceptable models had roughly 1×10^8 $M_\odot$ within 1″ of the galaxy center. No models had core radii larger than 0\.″4.

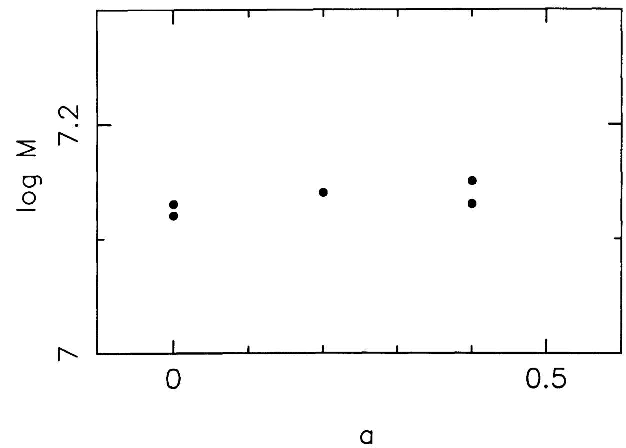

Fig. 4.—Same as Fig. 3 for M32. No satisfactory models were found with core radii larger than 0\.″4. All distributed mass models had $\sim 1 \times 10^7$ $M_\odot$ inside a 1″ radius.

around 100, which suggests that if the mass is main-sequence stars, they must be less massive than 0.25 $M_\odot$.

The central relaxation time may be written as

$$t_r = 0.34\,\frac{\sigma^3}{G^2 m_*^2 n \ln(\Lambda)} \tag{8}$$

(see Binney and Tremaine 1989). For the parameters given above and $\ln(\Lambda) = 10$, the central relaxation time in M31 is

$$t_r = 5.1 \times 10^9\ {\rm yr}\left(\frac{m_*}{M_\odot}\right)^{-1}, \tag{9}$$

(the m_*^{-1} dependence arises because the central mass density $\rho = m_* n$ is constrained by the observational data much better than n).

Although the evolution of dense stellar systems is more properly parameterized in terms of the half-mass relaxation time, Cohn's (1985) simulations core-collapse after a few hundred central relaxation times (the "few" approaches one for reasonable mass spectra). So, since M31 has a resolved core, it seems most unlikely that the objects which dominate the mass are heavier than 50 $M_\odot$, regardless of their size.

In the case M32, we adopt $\rho(0\rlap{.}''1) = 1 \times 10^6$ $M_\odot$ pc^{-3} and $\sigma = 100$ km s^{-1}. The corresponding limit for physical collisions is

$$t_{\rm col} = 6.0 \times 10^{10}\ {\rm yr}\left(\frac{m_*}{M_\odot}\right)^{-1}, \tag{10}$$

so stars more massive than the Sun have a very good chance for survival in M32. As in M31, the central M/L provides a more severe constraint on the possibility of carrying the mass in the form of main-sequence stars. The central relaxation time in M32 is given by

$$t_r = 1.8 \times 10^9\left(\frac{m_*}{M_\odot}\right)^{-1} {\rm yr}, \tag{11}$$

yielding a limit ~ 10 $M_\odot$ due to two-body relaxation of putative degenerate or point mass objects.

Although these astrophysical arguments are unimpressive given our present upper limits for the core radii of aggregate models, it is of great interest to consider the impact of Space Telescope data with much greater angular resolution. Current

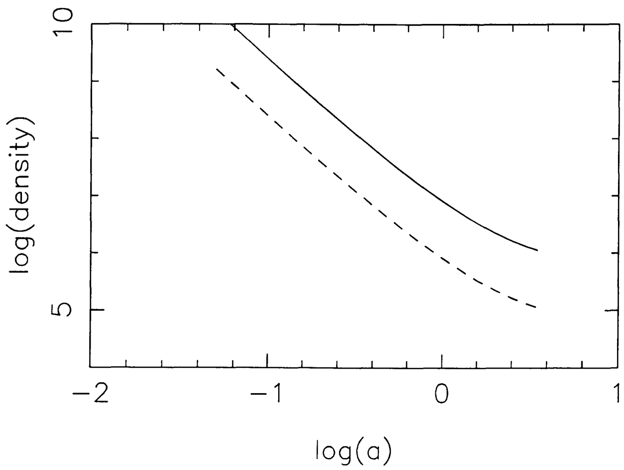

FIG. 5.—Relationship between central density in $M_\odot$ pc^{-3} and core radius a in parsecs for the mass model given in eq. (2) and our analysis of the M31 and M32 data. For M31 (*solid line*), we assumed the mass within 1″ (3.4 pc) was fixed at $10^8\ M_\odot$. For M32 (*dashed line*), we assumed that the mass inside 1″ was $10^7\ M_\odot$.

spectroscopy obtained in seeing with a FWHM of 0″.5 yields an upper limit to the core radii of the *mass* distribution in both galaxies of about 0″.4 (~ 1.5 pc). If the mass is *in fact* a single black hole, then Space Telescope data with a spatial resolution of $\sim$0″.05 will reduce the upper limit core radii by a factor of 10. Examination of Figure 5 shows that the central density of our straw man model will go up by a factor of > 100 for each galaxy. For M31, the physical collision time is reduced to

$$t_{\rm col} = 2.3 \times 10^8 \left(\frac{m_*}{M_\odot}\right)^{-1} \text{ yr} ,$$

and main-sequence stars cannot carry the bulk of the mass unless they are brown dwarfs. Also, in M31, the central relaxation time would drop to

$$t_r = 5.8 \times 10^7 \left(\frac{m_*}{M_\odot}\right)^{-1} \text{ yr} .$$

Even degenerate objects would have to be less massive than 0.5 $M_\odot$ in order for the core to have not evolved through core collapse. The limit for collisions in M32 would restrict main-sequence stars to $m_* < 0.1\ M_\odot$. Even better, the limit based on the relaxation time is also 0.1 $M_\odot$, regardless of the size of the objects. Thus it appears that Space Telescope will be able to significantly strengthen the case for a massive black hole in M31 by restricting the alternative models to rather incredible parameters.

We are grateful to Jeremy Goodman and John Tonry for helpful discussions. This research was supported by NSF 87-20028.

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GARY BOWER and DOUGLAS RICHSTONE: Department of Astronomy, Dennison Hall, University of Michigan, Ann Arbor, MI 48109-1090

ALAN DRESSLER: The Observatories of the Carnegie Institution of Washington, 813 Santa Barbara Street, Pasadena, CA 91101-1292