THE ASTROPHYSICAL JOURNAL, 349:L55–L58, 1990 February 1

SPIN-UP OF YOUNG PULSARS DUE TO RAPID COOLING BY NEUTRINO EMISSION

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Received 1989 June 14; accepted 1989 November 3

ABSTRACT

A young pulsar cools rapidly by neutrino emission. The resulting thermal contraction leads to spin-up in the earliest epochs when the effect dominates over spin-down by magnetic dipole radiation. This initial spin-up may last several years. The characteristic time dependence of the neutrino luminosity can also dominate in the second derivative of the rotation rate for as long as thousands of years with favorable initial conditions.

Subject headings: pulsars — stars: neutron

I. INTRODUCTION

A young pulsar cools by neutrino emission processes during the first 10^4–10^5 yr of its life (e.g., Tsuruta 1986). Eventually, the photon luminosity from the star's surface dominates. In the first decades after birth, the neutrino luminosity rapidly diminishes the star's internal energy, and thermal contraction changes its structure. This is negligible for cooling calculations: it is a good approximation to use a zero-temperature neutron star structure, evolving the temperature as required by thermal emission rates, conductivities, and heat capacities, while keeping the density and pressure distributions constant (Baym 1981).

Whether thermal contraction plays a role in the rotational dynamics of the star is a different issue. This is determined by a comparison of the ensuing spin-up rate with the deceleration caused by the canonical dipole magnetic torque responsible for the spin-down of radio pulsars. We show that pulsars should initially spin up because of the thermal contraction caused by their very substantial neutrino luminosities.

II. CONTRACTION CAUSED BY NEUTRINO EMISSION

For our purposes, it is sufficient to consider a uniform density isothermal model neutron star. The pressure is taken to be that of degenerate noninteracting neutrons. The heat capacity is provided by the neutrons or, if the neutrons are in the superfluid phase, by the degenerate relativistic electrons. The generic cooling equation is

$$c_0 T \frac{dT}{dt} = -L_\nu = -aT^\beta \,, \tag{1}$$

where $c_0 T$ is the total heat capacity, and L_ν is the neutrino luminosity, parameterized by the coefficient a and the temperature dependence T^β. This gives

$$L_\nu(t) = L_\nu(0)\left(\frac{t_{\rm th}}{t + t_{\rm th}}\right)^{\beta/(\beta-2)} . \tag{2}$$

With an initial temperature T_0^2 the cooling time scale $t_{\rm th}$ is

$$t_{\rm th} = \frac{c_0 T_0^2}{(\beta - 2)L_\nu(0)} . \tag{3}$$

To estimate the thermal contraction, we follow Baym's reasoning (1981), generalizing it to include rotation. For a star near thermal equilibrium, the pressure adjusts to changes of volume affecting the gravitational and the (rotational) kinetic energy such that the total energy is stationary as a function of volume, and the only changes are caused by entropy and angular momentum changes due to the emission processes (the virial theorem):

$$3PV + E_g + 2E_r = 0 \,, \tag{4}$$

where P and V are pressure and volume and E_g and E_r are the gravitational and rotational energies. The pressure of the degenerate neutrons is

$$P = P_0[1 + \alpha(T/T_{\rm F})^2] \,, \tag{5}$$

where $P_0 = 2nE_F/5$, $\alpha = 5\pi^2/8$ for normal neutrons (Chandrasekhar 1958), n is the neutron number density, and $E_{\rm F}$ and $T_{\rm F}$ are the neutron Fermi energy and temperature. In the superfluid phase, α is approximately $5\pi^2/16$ (Bardeen, Cooper, and Schrieffer 1957). A local change in pressure reflects entropy changes due to cooling and adiabatic density changes:

$$\frac{\delta P}{P} = \frac{1}{P}\left[\left(\frac{\partial P}{\partial \sigma}\right)_n \delta\sigma + \left(\frac{\partial P}{\partial n}\right)_\sigma \delta n\right] = \frac{2\alpha T}{T_F^2}\,\delta T + \Gamma\,\frac{\delta n}{n} \,, \tag{6}$$

where Γ is the adiabatic index. An exact calculation of the thermal contraction should analyze the response of the stellar structure equations in a particular neutron star model. Instead, we obtain an estimate by varying the global virial relation, equation (4). Using

$$\delta E_g = \frac{1}{3}\, E_g\, \frac{\delta n}{n} \tag{7}$$

and

$$\delta E_r = \Omega\,\delta J - E_r\,\frac{\delta I}{I} = \Omega\,\delta J + \frac{2}{3}\,E_r\,\frac{\delta n}{n} \,, \tag{8}$$

where I is the moment of inertia, we find

$$\frac{\delta P}{P} = \left(\frac{4}{3} - \frac{2}{9}\,\frac{E_r}{PV}\right)\frac{\delta n}{n} - \frac{2}{3}\,\frac{\Omega\,\delta J}{PV} . \tag{9}$$

The fractional change in the moment of inertia is then

$$\frac{\delta I}{I} = \frac{2}{3}\frac{\delta V}{V} = -\frac{2}{3}\frac{\delta n}{n} = \frac{(4\alpha T\,\delta T/T_F^2) + (4\Omega\,\delta J/3PV)}{3\Gamma - 4 + (2E_r/3PV)} . \quad (10)$$

The cooling in a time dt is given by equation (1), $c_0 T\,\delta T = -L_\nu dt$. For a neutrino emissivity ϵ_ν, the angular momentum carried by neutrinos emitted from unit volume is $\epsilon_\nu r^2\Omega/c^2$, where r is the distance of the volume element from the rotation axis. The angular momentum loss from a spherical core region of radius R_ν with uniform emissivity ϵ_ν is

$$\delta J = -\frac{2}{5}\frac{L_\nu dt R_\nu^2\Omega}{c^2} = -\frac{L_\nu dt R_\nu^2}{Mc^2R^2} J . \quad (11)$$

From equations (1), (10), and (11), the spin-up due to cooling is

$$\frac{\dot\Omega dt}{\Omega} = \frac{\delta J}{J} - \frac{\delta I}{I} = AL_\nu dt \quad (12)$$

where

$$A = \frac{4\alpha}{c_0 T_F^2}\left[\frac{1 + (2/15)(c_0 T_F^2/\alpha PV)(R_\nu^2\Omega^2/c^2)}{3\Gamma - 4 + (2/15)(MR^2\Omega^2/PV)} - \frac{c_0 T_F^2}{4\alpha Mc^2}\frac{R_\nu^2}{R^2}\right] . \quad (13)$$

For normal neutrons c_0 is $N_n\pi^2k_B^2/(2E_F)$, where N_n is the number of neutrons in the star (Chandrasekhar 1958), so that $4\alpha/c_0 T_F^2$ is $5(N_n E_F)^{-1}$. Well below the superfluid transition temperature, the heat capacity is provided by relativistic electrons and c_0 is $N_e\pi^2k_B^2/E_F^e$, where N_e is the number of electrons and E_F^e is the electron Fermi energy. With the appropriate α for the superfluid phase $4\alpha/c_0 T_F^2$ is approximately $5(4N_e E_F)^{-1}$. The electron number density is about 0.01 times that of neutrons for beta equilibrium between noninteracting particles in a uniform density neutron star of 1 $M_\odot$. PV is equal to $2N_n E_F/5$ in either phase, and Γ is 5/3. The corrections in the square brackets are negligible except for $2MR^2\Omega^2/15(PV)$ which is important if the star is close to centrifugal break-up.

III. SPIN-UP BY NEUTRINO EMISSION

The spin-up resulting from neutrino cooling is found using equations (2), (3), and (12). The evolution of the rotation rate due to cooling alone is

$$\Omega(t) = \Omega(0)\exp\left[A\int L_\nu(t)dt\right] . \quad (14)$$

The exponent is of the order of T_0^2/T_F^2, $O(10^{-4})$ for $E_F = 100$ MeV and $T_0 = 1$ MeV. Since cooling draws on only the thermal part of the internal energy of degenerate matter, its cumulative contribution to the evolution of the rotation rate is utterly negligible. However, the spin-up rate and its derivative can be conspicuous in comparison to dipole spin-down. With $\Omega = \Omega_0$ in equation (12),

$$\dot\Omega \simeq \Omega_0 AL_\nu(0)\left(\frac{t_{th}}{t}\right)^{\beta/(\beta-2)} , \quad (15)$$

where $t \gg t_{th}$ at an age of a few years or more. The second derivative is

$$\ddot\Omega \simeq \frac{\beta}{\beta-2}\frac{\Omega_0 AL_\nu(0)}{t_{th}}\left(\frac{t_{th}}{t}\right)^{(2\beta-2)/(\beta-2)} . \quad (16)$$

For comparison, in the early years when the age t is much less than the dipole spin-down time scale $t_d = (2k\Omega_0^2)^{-1}$, the dipole spin-down and its derivative are $\dot\Omega_d \simeq -k\Omega_0^3$, $\ddot\Omega_d \simeq 3k^2\Omega_0^5$ [$k = 2B^2R^6/(3c^3I)$, and B is the surface magnetic field]. The ratio of the spin-up rate due to cooling to the dipole spin-down rate is, from equation (15),

$$\frac{\dot\Omega}{|\dot\Omega|_d} = \frac{AL_\nu(0)}{k\Omega_0^2}\left(\frac{t_{th}}{t}\right)^{\beta/(\beta-2)} \equiv \left(\frac{t_1}{t}\right)^{\beta/(\beta-2)} . \quad (17)$$

Spin-up prevails until a time

$$t_1 \equiv \left(\frac{AL_\nu(0)}{k\Omega_0^2}\right)^{(\beta-2)/\beta} t_{th} , \quad (18)$$

which is longer than a few years for several interesting combinations of initial conditions and neutrino processes (see below and Table 1). The effect of cooling will prevail for a much longer period in the second derivative of the rotation rate. From equation (16),

$$\frac{|\ddot\Omega|}{\ddot\Omega_d} = \left(\frac{t_2}{t}\right)^{(2\beta-2)/(\beta-2)} , \quad (19)$$

where

$$t_2 = \left[\frac{\beta}{3(\beta-2)}\frac{AL_\nu(0)}{k^2\Omega^4}\right]^{(\beta-2)/(2\beta-2)} t_{th}^{\beta/(2\beta-2)} \quad (20)$$

and t_2 is thousands of years or longer in some cases of interest. A younger pulsar will have a large and negative second derivative.

IV. RESULTS AND DISCUSSION

We calculate the effect for a 1 $M_\odot$ uniform density neutron star, $R = 10$ km, and $T_0 = 10^{10}$ K. The dependence of the above discussion on mass and radius is quite weak. Two major possibilities for the source of neutrino emission are the following:

Case (i).—URCA processes in a normal (i.e., nonsuperfluid) star. The heat capacity is that of the normal neutrons. The neutrino luminosity scales with the eighth power of the temperature ($\beta = 8$), and $a = 1.9 \times 10^{-33}(M/M_\odot)^{2/3}R_6$ ergs s^{-1} for emission throughout a uniform density star of mass M (Friman and Maxwell 1979). The initial temperature is taken to be 10^{10} K. Evaluating the coefficient in equation (15), we find the spin-up rate

$$\dot\Omega = 1.73 \times 10^{-3}\Omega_0 t^{-4/3} \text{ rad s}^{-1} . \quad (21)$$

The crossover times are $t_1 = 2.4 \times 10^{10}(B_{12}\Omega_0)^{-3/2}$, $t_2 = 8 \times 10^{12}(B_{12}\Omega_0)^{-12/7}$ s.

Case (ii).—Neutrino bremsstrahlung from the crust in the case of a neutron star with a superfluid core. Starting with T_0 in the 10^{10} K range, the star would initially cool in the normal phase as in case i, and switch to cooling through crust neutrino bremsstrahlung when it reaches the transition temperature T_c for the core to become superfluid. For simplicity we take $T_0 = 10^{10}$ K and apply the cooling law for the superfluid case from the start. This simple model is adequate to describe the rotational dynamics to be expected if the star is currently superfluid, and avoids the uncertainty in T_c and the complication of going through the transition. Neutrino luminosity by bremsstrahlung in the crust is characterized by $\beta = 6$ (Maxwell 1979; Soyeur and Brown 1979). We adopt $a = 5 \times 10^{-16}(M_c/0.1M)$, where M_c is the mass of the neutrino-emitting crust. The spin-up rate is

$$\dot\Omega = 8.9 \times 10^{-3}\Omega_0 t^{-3/2} \text{ rad s}^{-1} , \quad (22)$$

TABLE 1

COMPARISON OF SPIN-DOWN BY DIPOLE ELECTROMAGNETIC RADIATION AND SPIN-UP BY NEUTRINO EMISSION

B (G) (1)	Ω_0 (rad s^{-1}) (2)	$\dot{\Omega}_d$ (rad s^{-2}) (3)	$\ddot{\Omega}_d$ (rad s^{-3}) (4)	t_1(s) / t_2(s) Mode i[e] (5)	Mode ii[f] (6)
10^9..........	60	-5.3×10^{-18}	1.4×10^{-36}	3.9×10^{11}	2.2×10^{12}
				9.9×10^{14}	2×10^{14}
	1.2×10^4	-4.3×10^{-11}	4.6×10^{-25}	4.1×10^8	1.4×10^9
				9.2×10^{10}	3.5×10^{10}
10^{12}.........	60	-5.3×10^{-12}	1.4×10^{-24}	5.2×10^7	2.2×10^8
				7.1×10^9	3.2×10^9
	400	-1.6×10^{-9}	1.9×10^{-20}	2.1×10^6	1.3×10^7
				2.2×10^8	1.3×10^8
	1.2×10^4	-4.3×10^{-5}	4.6×10^{-13}	1.3×10^4	1.4×10^5
				6.6×10^5	5.5×10^5

NOTE.—Cols. (1)–(4) give the magnetic field, the rotation rate, and its derivative due to dipole radiation. Cols. (5) and (6) give the crossover times t_1 and t_2 for two neutrino emission modes. Spin-up by cooling, eq. (15), will prevail over dipole spin-down until a time t_1 after the star's birth, and the second derivative will be determined by cooling, eq. (16), until the time t_2. Mode i is neutrino emission from a normal (nonsuperfluid) star by URCA processes. Mode ii is neutrino bremsstrahlung from the crust of a superfluid star.

prevailing until $t_1 = 5.1 \times 10^{10}(B_{12}\Omega_0)^{-4/3}$ s. The crossover time for the second derivative is $t_2 = 2.2 \times 10^{12}(B_{12}\Omega_0)^{-8/5}$ s.

We shall not discuss the alternative of neutrino emission from a pion condenstate. Recent calculations (Tatsumi 1983) yield pion condensate cooling rates an order of magnitude smaller than the earlier results (e.g., Maxwell *et al.* 1977). The condensate, which has $\beta = 6$, gives t_1 and t_2 that are shorter than the crossover times in case (*ii*) by a factor of about 2.

Table 1 gives the crossover times t_1 and t_2 for five sets of initial conditions, given in the first two columns. Strong (10^{12} G) and weak (10^9 G) fields and rapid (1.2×10^4 rad s^{-1}) to Crab-like (400 rad s^{-1}) and slow (60 rad s^{-1}) initial rotation rates are considered representing a range of birth conditions including the rotation (Kristian *et al.* 1989) and the vibration (Wang *et al.* 1989) interpretations of the possible 0.5 ms pulsar in SN 1987A. The next two columns give the dipole spin-down rate and its derivative, constants for the first years of the pulsar's life. Spin-up due to cooling, as given in equations (21)–(22), prevails over dipole spindown from the birth of the star until time t_1, while the second derivative of the rotation rate is determined by cooling for the longer period t_2.

Spin-up due to cooling will be observable for at least a few years, and possibly much longer after the pulsar's birth if the dipole power is weak. This is true in the case of a pulsar born with a weak (10^9 G) field, regardless of its rotation rate. If B is in the 10^{12} G range, but Ω is 60 rad s^{-1}, as indicated in the vibration interpretation of the 0.5 ms pulsations from SN 1987A, the spin-up effect again dominates for about 2 yr in case (*i*), and 7 yr in case (*ii*). If $B = 10^{12}$ G and $\Omega_0 = 400$ rad s^{-1}, similar to the inferred Ω_0 of the Crab pulsar, spin-up will last for a few months.

In general, $t_2 > t_1$, and with all combinations of Ω_0 and B but one, a negative $\ddot{\Omega}$ will be observed for at least the first 3 yr or possibly much longer. This means that the effect of cooling will still be detected through a negative $\ddot{\Omega}$ in the case of a pulsar that becomes observable at a time later than t_1.

If spin-up or a negative $\ddot{\Omega}$ is observed, the time dependence will distinguish between the different modes of neutrino emission. Case (*i*) will yield a $t^{-4/3}$ dependence of the spin-up rate. This will indicate that the neutron star core is still normal. L_ν and T can then be inferred from the rotational behavior. The inferred temperature will be an upper limit on T_c for superfluidity in the star's core. Crust neutrino bremsstrahlung (case [*ii*]) gives a spin-up rate with a $t^{-3/2}$ dependence. This will indicate a superfluid star, and the temperature inferred would furnish a lower bound to the superfluid's transition temperature.

We conclude that the huge neutrino luminosities of young neutron stars will lead to effects of an observationally interesting magnitude in the star's rotational behavior. If observed, these effects will yield information about neutrino luminosities and the neutron star core, on the presence of superfluids, on the transition temperatures and perhaps on pion condensates. The present calculation illustrates the order of magnitude and compares the effect in the context of different scenarios. No other known mechanism predicts a secular spin-up in an isolated pulsar.

The same physical conclusions should hold in a general relativistic treatment. The star is then losing mass-energy also (Thorne 1971), but $\delta M/M$ should be about NE_F/Mc^2 smaller than the thermal contraction $\delta R/R$. By continuity from the Newtonian arguments, $\delta R/R$ is expected to be of similar magnitude. Treatment of rotation (Bardeen 1970) and slow rotation expansions of the angular momentum in terms of a moment of inertia and a rotation rate at infinity (Abramowicz and Wagoner 1976) are available only for isentropic stars. Even in the Newtonian case, investigation of specific stellar models is beyond our present aims of demonstrating the qualitative effects, using global virial and thermodynamic arguments. The validity of these same principles in general relativity encourages the conjecture that the effects discussed will be exhibited by a relativistic neutron star, though they may have a more complicated signature, in particular for the cases of rapid rotation.

We thank Marek Abramowicz, Metin Gürses, Rahmi Güven, John Middleditch, Yavuz Nutku, David Pines, Mal

Ruderman, and Hugh Van Horn for useful discussions, and the referee for helpful suggestions. M. A. A. acknowledges support from NSF grant PHY 86-00377 and NASA grant NSG-7653 at the University of Illinois. H. O. acknowledges support from Deutsche Forschungsgemeinschaft project Mo 416/3-3 and thanks the Research Institute for Basic Sciences of Turkey for hospitality and the TOKTEN program of the UN, which sponsored his visit to Gebze.

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