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# POLARIZATION OF THE BINARY RADIO PULSAR 1913+16: CONSTRAINTS ON GEODETIC PRECESSION

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## ABSTRACT

Using polarization data, we determine the relative orientations of the line of sight and the pulsar's spin axis and magnetic moment. The data are sensitive enough to allow detection of spin-orbit geodetic precession in a few years if the spin axis is misaligned from the orbital angular momentum vector by more than a few degrees. From the secular pulse shape changes measured by Weisberg *et al.*, we conclude that the misalignment angle is no more than 15° but is probably more than 1°. We propose a specific beaming model to account for the pulse shape changes.

*Subject headings:* polarization — pulsars — stars: binaries — stars: individual (PSR 1913+16)

## I. INTRODUCTION

Geodetic precession of a freely falling, spinning body is a basic prediction of general relativity (Fokker 1920). The precession arises partly from the familiar Thomas precession of special relativity, but also probes the spatial geometry of curved spacetime (see Weinberg 1972; Misner, Thorne, and Wheeler 1973, p. 1118). The existence of geodetic precession (and its "weaker cousin"–gravitational spin-spin coupling) has not yet been directly demonstrated experimentally (Schiff 1960; Everitt 1974). Recent analysis of lunar laser-ranging data is consistent with the predictions of general relativity for the related precession of the orbital angular momentum of the Earth-Moon system as it orbits the Sun, the so-called de Sitter (1916) precession (Shapiro *et al.* 1988; Nortvedt 1988). Barker and O'Connell (1975*a*) discuss the differences between de Sitter and geodetic precession and derive the correct expression for the geodetic precession frequency for binary systems with comparably massive components.

The discovery of the binary pulsar PSR 1913+16 (Hulse and Taylor 1975) led many to suggest that geodetic precession could be observable in this system (Brecher 1975; Esposito and Harrison 1975; Barker and O'Connell 1975*b*; Hari Dass and Radhakrishnan 1975). Specific suggestions focused on the possibility that measurable changes in the pulsar's intensity waveform could result from the precession of its spin axis about the orbital angular momentum of the binary (Smarr and Blandford 1976). Weisberg, Romani, and Taylor (1989) have recently reported that the ratio of the twin intensity peaks in the 1.4 GHz intensity waveform of PSR 1913+16 changes monotonically between 1981 and 1987. The amplitude of the measured variations, about 1% per year, is consistent with the geodetic precession frequency of the system, $\Omega_{\rm prec} = 1.21$ deg yr$^{-1}$. However, this striking agreement is actually rather surprising. For smooth, circular pulsar beams, one would have expected that changes in the intensity ratio of the pulse components would be accompanied by comparable changes in their pulse phase spacing. This is not seen (Weisberg *et al.* 1989). It is possible that the observed changes may be enhanced by patchiness of the emission beam, similar to that recently proposed by Lyne and Manchester (1988).

Polarization changes provide an independent and perhaps cleaner signature of geodetic precession. Polarization position angles primarily reflect the magnetic field geometry of the radio-emitting region as projected onto the observer's "sky" (modulo the abrupt ~90° "mode changes" seen for some pulsars). As a result, the position angle waveform may carry more direct orientational information than does the intensity waveform, because the former depends sensitively on the "impact parameter" at which the observer's line of sight slices through the pulsar beam (e.g., Manchester and Taylor 1977). The dependence is especially sensitive if the impact parameter is small, as is probably required for PSR 1913+16 in order to account for the twin peaks in its intensity waveform. As the pulsar spin precesses, the time variability of the impact parameter should induce significant changes in the position angle waveform (Cordes and Wasserman 1984).

In an effort to detect geodetic precession, we have monitored the polarization of PSR 1913+16 at roughly six-month to 1 yr intervals since 1985. One purpose of this paper is to report on the status of that experiment. We also present a succinct description of the wealth of information that can be gleaned from polarization observations. The available data already can be used to constrain the relative orientations of the line of sight to the pulsar, its magnetic and spin axes, and the orbital angular momentum of the binary. This information, in turn, leads to limits on the mass and radius of the progenitor binary (e.g., Cordes and Wasserman 1984; Burrows and Woosley 1986; Bailes 1988). We also comment on some distinct peculiarities of the position angle waveform of PSR 1913+16, and mention some possible explanations that we shall treat in more detail in a subsequent paper.

## II. OBSERVATIONS

Observations at the Arecibo Observatory began in 1985 and are summarized in Table 1. Two different polarimeters were used to measure the Stokes parameters $I$, $Q$, $U$, $V$ at 1.4 GHz. The adding polarimeter and filter bank used in 1985 comprise a system nearly identical to that used by Stinebring *et al.* (1984) in a study of other pulsars. The total bandwidth was only 1 MHz, yielding waveforms that are severely limited in signal-to-noise ratio. In 1987, we developed a multiplying polarimeter system using the Observatory's 40 MHz correlator as a digital filter bank, thereby increasing the bandwidth by a factor of 20. A major advantage of the correlator method is that the result-

TABLE 1

JOURNAL OF OBSERVATIONS

| Epoch | $\nu$ (MHz) | $\delta\nu$ (kHz) | Sample Interval ($\mu$s) | Time Resolution ($\mu$s) | Total Time (hr) | Polarimeter | Comments | $S_\nu$[a] (mJy) |
|---|---|---|---|---|---|---|---|---|
| 1985.4........ | 1416 | 4 × 250 | 200 | 240 | 4.6 | adding | *I, Q, U, V* | 1.87 ± 0.10 |
| 1987.1........ | 1414 | 32 × 625 | 461 | 560 | 0.6 | correlator | *I* | 0.67 ± 0.10 |
| 1987.6........ | 1397 | 32 × 625 | 461 | 560 | 1.7 | correlator | *I, Q, U, V* | 0.85 ± 0.03 |
| 1988.3........ | 1415 | 32 × 625 | 231 | 510 | 1.7 | correlator | *I, Q, U, V* | 0.64 ± 0.06 |
| 1988.5........ | 1403 | 32 × 625 | 231 | 510 | 7.2 | correlator | *I, Q, U, V* | 1.86 ± 0.10 |

[a] Flux density averaged over a full pulse period.

ant polarization angles $\psi = (\frac{1}{2}) \tan^{-1} U/Q$ are completely independent of the gain calibration of the two receiver signals. With the adding polarimeter, the position angles are very strongly dependent on the gain calibration stability.

The pulse waveforms from 1985 and 1988 July are shown in Figure 1. The position angle origin is arbitrary but identical for the two epochs. The flux density varied by a factor of 3 over a time scale as short as two months. This may be due to (rather strong) refractive interstellar scintillation. Diffractive scintillations are quenched for our observing bandwidth and integration time. Given the radically different methods used to measure the polarization and the weakness of the pulsar, the agreement between the 1985 and 1988 waveforms is remarkable.

At 1.4 GHz, the pulse shows twin components that are commonly seen from pulsars and are thought to be associated with a "hollow cone" radio beam with an annular cross section sweeping through the line of sight. The waveform at 0.43 GHz (e.g., Taylor and Weisberg 1982) shows that the beam is not hollow, since a third "core" component appears between the pair of "conal" components. Core components in other pulsars also seem to have steeper radio spectra than do conal components (Rankin 1983; Hankins and Rickett 1988). The 1988 data in Figure 1 show that the maximum linear and circular polarization are ~36% and 17%, respectively. The position angle $\psi(\phi)$ rotates monotonically through a total of ~195° with maximum rotation rates on the trailing edge of the first component and the leading edge of the second component. Both the total swing and the shape of $\psi(\phi)$ are at odds with the curves expected from standard "dipole" models, as discussed below.

### III. EMISSION GEOMETRIES

The standard picture for interpreting pulsar polarization is the "rotating vector" model appropriate for dipole magnetic fields (Radhakrishnan and Cooke 1969). Let $\alpha$ be the angle between the pulsar spin axis $\hat{\Omega}$ and the dipole moment $\hat{\mu}$, and $\beta$ the angle between $\hat{\Omega}$ and the line of sight, $\hat{n}$. Figure 2 shows the

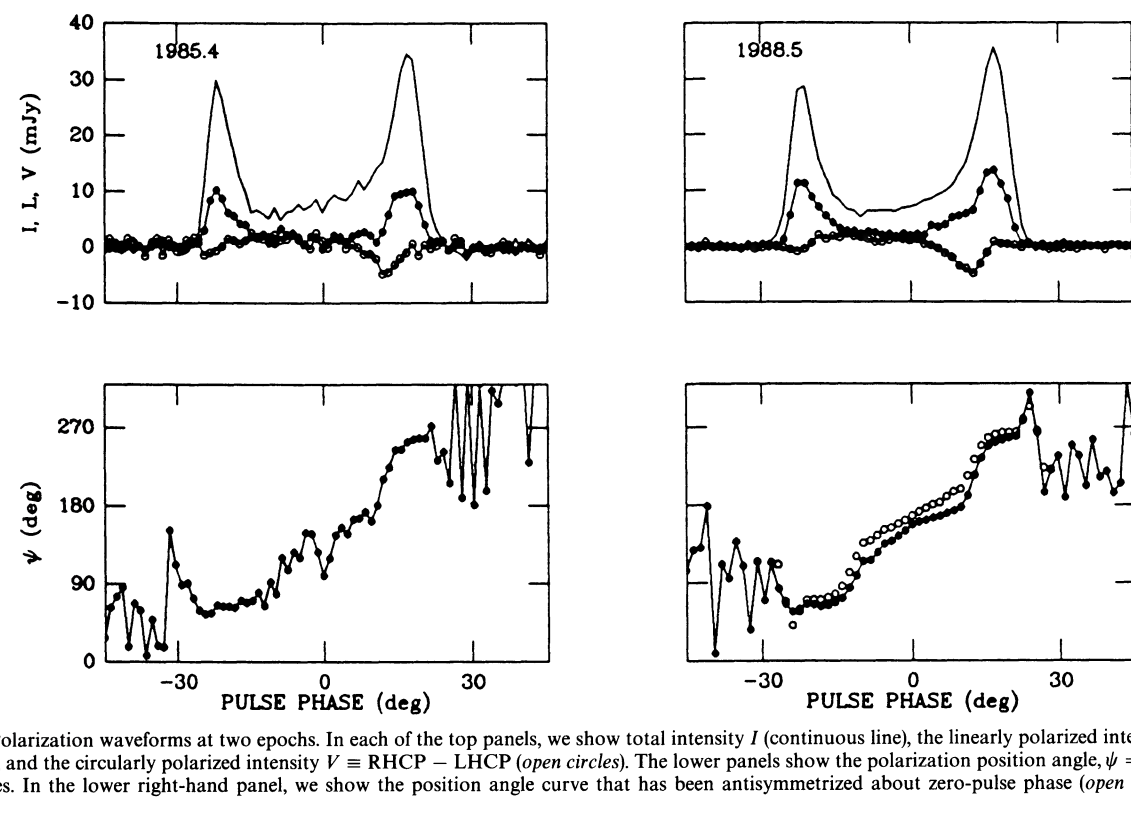


FIG. 1.—Polarization waveforms at two epochs. In each of the top panels, we show total intensity $I$ (continuous line), the linearly polarized intensity $\sqrt{Q^2 + U^2}$ (*filled circles*), and the circularly polarized intensity $V \equiv$ RHCP − LHCP (*open circles*). The lower panels show the polarization position angle, $\psi = (\frac{1}{2}) \tan^{-1} (U/Q)$ as filled circles. In the lower right-hand panel, we show the position angle curve that has been antisymmetrized about zero-pulse phase (*open circles* and offset by +10°).

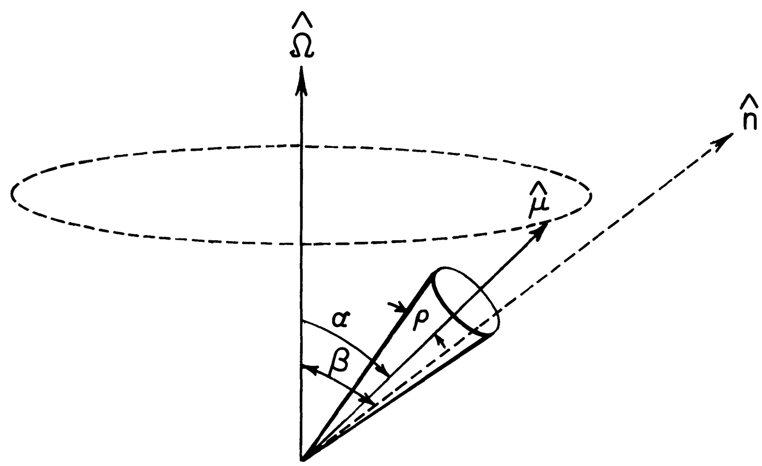


FIG. 2.—Geometry of the spin axis, magnetic moment, line of sight, and radiation beam

geometry along with a schematic radiation beam with an opening angle $2\rho$. Define the "impact parameter" $\sigma \equiv \beta - \alpha$; when line of sight, spin axis, and dipole moment are coplanar, $\sigma$ is the angle between $\hat{n}$ and $\hat{\mu}$. Assuming constant and sharply defined emission altitudes, strong relativistic beaming maps each pulse phase into a unique location in the magnetosphere. The polarization, in this model, is assumed to be either parallel to or perpendicular to the local magnetic field direction *at emission* and, of course, orthogonal to the line of sight; all magnetospheric propagation effects are ignored. It is then easy to show that the polarization angle at pulse phase $\phi$ is, up to an overall additive constant,

$$\psi(\phi) = \psi_\Omega + \tan^{-1}\left[\frac{\sin\alpha \sin\phi}{\sin\sigma + (1 - \cos\phi)\cos\beta\sin\alpha}\right], \quad (1)$$

where $\psi_\Omega$ is the position angle of the spin axis. This result holds for either of the two orthogonal emission modes assumed in the model. Equation (1) has a maximum slope at $\phi = 0°$

$$\left(\frac{d\psi}{d\phi}\right)_m = \frac{\sin\alpha}{\sin\sigma}. \quad (2)$$

The dipole model fits the polarization data of some pulsars *extremely* well (e.g., Lyne and Manchester 1988). For 1913+16, the dipole model fails, although the position angle waveform in Figure 1*b* shares some of the qualitative features of the dipole model, notably antisymmetry with respect to the pulse centroid. In Figure 1*b*, open circles (offset in the vertical direction by 10°) trace the antisymmetrical position angle curve (with zero phase defined by that phase about which the curve is most nearly antisymmetric). However, two aspects of the position angle waveform defy description in terms of the simple dipole model. First, the total position angle swing across the pulse is 195°, which is geometrically impossible in the model (for $\sigma > 0$). Moreover, the position angle jumps by $\approx 65°$ and $\approx 75°$ at phases $\approx +12°$. These jumps are apparently resolved temporally (although some of this may be instrumental), which contrasts with the nearly discontinuous switching seen in some pulsars. If these jumps correspond to "mode switching" the implied propagation modes are not orthogonal.

Under the assumption that the gross magnetic field structure of 1913+16 is at least approximately dipolar, we consider two alternative interpretations of the polarization waveforms:

1. The fastest rotations of $\psi$ (at $\phi \sim \pm 12°$) are caused by the superposition of different polarization modes while the rotation near the pulse centroid reflects field geometry. In this interpretation, the slope $d\psi/d\phi$ at the pulse centroid reflects the magnetic field geometry while the total swing of 195° does not. Using 1988 data, we obtain $d\psi/d\phi = 3.28$ which, using equation (2), yields a relation $\alpha(\sigma)$.

2. The net $\psi$ curve is a combination of conal emission that conforms to a dipole model and a core component whose emission deviates from the dipole model. In this interpretation, the total $\approx 195°$ position angle swing, although formally excluded in the dipole model, reflects the true impact parameter. Moderate distortions of the field from a dipole (or rotational aberration effects) may perhaps account for the total swing, suggesting that the impact angle is quite small. In this interpretation, the slope at pulse center should be much larger than actually seen. (The largest slope determinable with our instrumental time resolution would be $\approx 60$.)

We favor the second interpretation for reasons that unfold in the ensuing discussion. We have investigated a number of modifications of the simple dipole model in an effort to explain the anomalously small value of $d\psi/d\phi$. Quite generally, the addition of a toroidal component cannot produce the observed position angle jumps, even if the effect of the toroidal field is restricted to $|\phi| < 12°$. It is easy to show that a toroidal field in general displaces the position angle by a *symmetric* function of pulse phase. Additional quadrupolar fields cannot be ruled out, provided that the core emission region is significantly closer to the stellar surface than the cone, and the surface field is substantially quadrupolar. The flat slope of the position angle curve near the pulse center could, in principle, result if the core emission is spread over a significant altitude range (Deich 1986). Finally, we cannot rule out that some combination of nontrivial propagation effects and exotic magnetic field patterns is at work in 1913+16 (Arons and Barnard 1986; Barnard and Arons 1986; Barnard 1986). However, the near-perfect antisymmetry of the position angle curve leads us to believe that the magnetic field must be fairly well organized and additional effects must not be excessively (anti)symmetry breaking.

In any case, the observed slope $d\psi/d\phi$ at the pulse centroid sets a rather conservative lower bound on the true slope for the dipolar/conal component. As we shall see below, the values of $\sigma$ implied by the observed value of $d\psi/d\phi$ are rather large for "typical" values of $\alpha$. Indeed, the impact parameter for sin $\alpha \sim 1$ is generally comparable to reasonable estimates of the pulsar beam size itself (see eq. [3] and Lyne and Manchester 1988), which would be at odds with the appearance of sharp, twin peaks in the 1.4 GHz waveform. Consequently, we assert that the true value of $\alpha$ for fixed $\sigma$ is larger than that implied by blind application of equation (2).

Polarization data alone cannot uniquely determine both $\alpha$ and $\sigma$ because different sets of these angles yield essentially the same variation of position angle with pulse phase $\psi(\phi)$. However, other observables imply additional constraints on $\alpha$ and $\sigma$. A second relation (Lyne and Manchester 1988) results if we assume the radiation beam to be circular with opening angle $2\rho$ (Fig. 2), and use the geometrical relation

$$\alpha = \tfrac{1}{2}\{-\sigma + \cos^{-1}[\cos\sigma + 2(\cos\rho - \cos\sigma)/(1 - \cos\Delta\phi)]\}\,, \tag{3}$$

connecting the pulse width (at 10% of maximum intensity) $2\Delta\phi \approx 48^\circ.7$ and $\rho$. Lyne and Manchester (1988) propose that $\rho \approx 6^\circ.5 P^{-1/3}$, with $P$ the pulse period in seconds. This empirical scaling is based mostly on pulsars that have periods $\sim 10$ times larger than the 59 ms period of 1913+16, so the beam size for 1913+16 may differ from this relation by a considerable factor. We have used coefficients of $6^\circ.5$ and $7^\circ.5$ (implying beam sizes of $16^\circ.7$ and $19^\circ.3$ for 1913+16) to define two relations between $\alpha$ and $\sigma$. Rankin (1990) presents compelling evidence that $\rho \propto P^{-1/2}$ when pulsars of different empirical classes are properly identified. Extrapolating her results to the period of 1913+16 yields nearly the same range of $\rho$ that we find using the Lyne and Manchester scaling law.

If $\hat{\Omega}$ is parallel to the orbital angular momentum direction $\hat{J}$, then we get a third relation, $\alpha = i - \sigma$, where $i$ is the orbital inclination ($i = 0$ for a face on orbit). Note that this relation is a *necessary* but *not sufficient* condition for $\hat{\Omega} \parallel \hat{J}$. Weisberg and Taylor's (1984) timing solutions for orbital elements and post-Newtonian parameters imply $i = 46^\circ.5 \pm 1^\circ.3$.

A fourth constraint on $\alpha$ may be derived from the absence of an observed interpulse (at less than 2% of the pulse maximum). This implies that $\alpha$ cannot be in the range $90^\circ - (\rho + \sigma)/2 \leq \alpha \leq 90^\circ - (\sigma - \rho)/2$.

Figure 3 shows a portion of the $\alpha$, $\sigma$ plane. We restrict the plot to $\sigma \geq 0$ because $d\psi/d\phi > 0$ and to $0 \leq \alpha \leq 90^\circ$ because the general situation is one where the spin and orbital angular momenta seem to be nearly parallel (see below). The curve $\alpha(\sigma)$, based on the monimum slope $d\psi/d\phi = 3.28$, intersects the curves $\alpha(\sigma)$, based on the pulse and beam widths, at $\sigma \approx 10^\circ \pm 1$ and $\alpha \approx 35^\circ \pm 3$. Unless our estimated beam widths are grossly inaccurate, $\sigma < 10 \pm 1^\circ$ and $\alpha > 35 \pm 3^\circ$. If the total position angle swing actually reflects the true impact parameter, as in our favored interpretation of the data, then most likely $\sigma \ll 10^\circ$, implying $\alpha \approx 40^\circ$–$55^\circ$. Such values of $\alpha$ are also consistent with $\hat{\Omega}$ being nearly, if not exactly, parallel to $\hat{J}$.

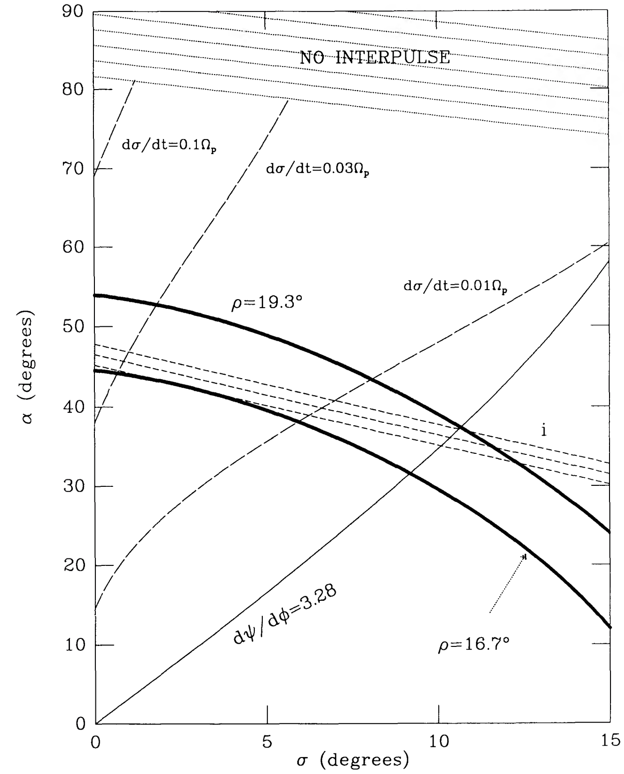


FIG. 3.—The $\alpha$, $\sigma$ plane, with constraints as discussed in the text. The heavy solid lines are loci for which the observed pulse width derives from beams with physical opening angles $\rho$. The long dashed lines denote the smallest (largest) values of $\alpha(\sigma)$ for which $\Omega_p^{-1}\, d\sigma/dt = 0.01$, 0.03, and 0.1 are permitted. Unless the beam of 1913+16 is fundamentally different from those of other pulsars, we believe that the solution for $\alpha$ and $\sigma$ must fall between these two curves.

## IV. SECULAR CHANGES AND GEODETIC PRECESSION

A comparison of our 1985 and 1988 data indicates that there is no statistically significant change in the waveforms, either in the separation of components, ratio of component amplitudes, or in the position angle curve. The component separation and amplitude ratio are $\Delta = 6.33 \pm 0.04$ ms and $R = 1.21 \pm 0.05$ for the 1985 data, while the 1988 data give $\Delta = 6.27 \pm 0.006$ ms and $R = 1.23 \pm 0.01$.

To further quantify secular changes in the waveforms, we computed two $\chi^2$ statistics. The first compares the intensity waveforms $I_1(\phi)$ and $I_2(\phi)$ while the second compares the $Q$ and $U$ waveforms (and, hence, the position angle waveforms). These are defined as

$$\chi_I^2 = \frac{\sum_\phi w(\phi)[I_1(\phi) - \gamma_I I_2(\phi)]^2}{(\sigma_{I_1}^2 + \gamma_I^2 \sigma_{I_2}^2)}\,,$$

$$\chi_{Q,U}^2 = [\![\textstyle\sum_\phi w(\phi)\{[Q_1(\phi) - \gamma_{Q,U} Q_2(\phi)]^2 + [U_1(\phi) - \gamma_{Q,U} U_2(\phi)]^2\}]\!]/ [(\sigma_{Q_1}^2 + \sigma_{U_1}^2 + \gamma_{Q,U}^2(\sigma_{Q_2}^2 + \sigma_{I_2}^2)]^{-1}\,,$$

where $w(\phi)$ is a weighting function that sums to unity (and was constant for pulse phases where the signal-to-noise ratio is greater than five and zero otherwise) and $\sigma_{I_{1,2}}$, etc. are off pulse rms variations which are due to radiometer noise. We evaluate $\chi_I^2$ and $\chi_{Q,U}^2$ for those values of the rescaling parameters $\gamma_I$, $\gamma_{Q,U}$ that minimize the numerators. Clearly, $\chi_{Q,U}^2 = \chi_I^2 = 1$ if the intensity waveforms are identical, apart from additive noise; significantly larger values imply nonnegligible changes in intensity, linearly polarized intensity, or position angle. We compared $Q$ and $U$ waveforms rather than position angles $\psi$ directly because errors in $\psi$ depend on the signal-to-noise ratio at each pulse phase, while errors are strictly additive to the Stokes parameters. Comparison of the 1985 and 1988 data leads to values $\chi_I^2 = 1.57$ and $\chi_{Q,U}^2 = 1.02$.

To assess the significance of $\chi_{Q,U}^2$ we generated perturbed waveforms by rotating $U(\phi)$ and $Q(\phi)$ for a single 1988 daily average through an additional position angle $\delta\psi(\phi) = r(\phi - \phi_0)$, where $r$ is the rate of rotation and $\phi_0$ is the phase about which the unperturbed position angle curve is antisymmetric. This is an arbitrary, antisymmetric perturbation, but should mimic some of the effects that would result from a real change in $\sigma$. To compute $\chi^2_{Q,U}$ as a function of $r$ we compared the simulated waveforms with the unperturbed average waveform from a different day. From these simulations we find that: (1) for data with the same signal-to-noise ratio (S/N) as a typical daily average for our 1988 data ($\sim 70$), $\chi^2_{Q,U}$ doubles for rates $r \approx 0.15$ (deg deg$^{-1}$); (2) for data with the S/N of our 1985 data ($\sim 34$), the rate must be $\sim 0.25$ (deg deg$^{-1}$) to double $\chi^2_{Q,U}$; (3) for small $r$, the statistic scales roughly as $\chi^2_{Q,U} \propto r^2 \times$ S/N ratio. With the S/N ratio of a waveform from several days observing, we expect that rates as small as 0.05 are detectable. Using equation (2), this suggests that changes in $\sigma$ as small as $0.^\circ1$ are probably detectable.

We conclude that our polarization data show no evidence for any change between 1985 and 1988, while the total intensity data indicate a possible change. With our new data acquisition system, which yields waveforms with time-invariant time resolution and S/N ratio much larger than that of the 1985 data, the prospects are good for measuring changes in all waveforms.

Although the polarization data at hand give no significant evidence for waveform changes, it is possible to assess what the magnitude of possible changes may be. We make use of the report by Weisberg, Romani, and Taylor (1989), that $R$ increased by $\sim 6\%$ between 1981 and 1987, while the component separation $\Delta$ changed by no more than $0.^\circ06$, or 0.15%. At first sight, the disparity between the changes in $R$ and $\Delta$ seems difficult to reconcile. Weisberg *et al.* (1989) correctly conclude that the observations exclude emission from a single smooth component of the pulsar beam. One possibility, favored by Weisberg *et al.* (1989) is that the conal component of the pulsar beam is patchy on angular scales much smaller than the beam width. Analogous, but somewhat less severe, patchiness has been inferred in a study of a large sample of pulsars by Lyne and Manchester (1988).

Another possibility, which we consider here, is that the required "patchiness" is due *solely* to the superposition of core and cone emission, so that the simplicity of smooth conal beams may be retained. We model the conal emission as a Gaussian profile of width $w_{\rm cone}$ about the angular radius $\theta_{\rm cone}$.

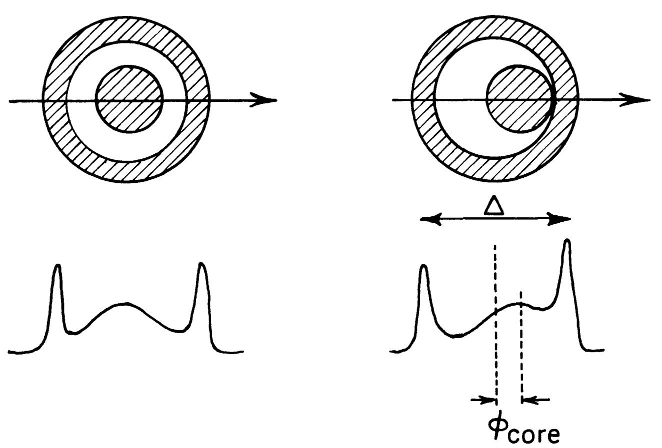


FIG. 4.—Schematic beam cross sections and intensity waveforms. The left-hand side shows the core and cone beam cross sections and waveform resulting if core and cone emission arise from the same altitude. The right-hand side shows the phase shift $\phi_{\rm core}$ that arises if core emission originates from a smaller altitude than conal emission.

We assume that the core emission is a Gaussian of width $w_{\rm core}$ centered on the dipole axis and take the conal intensity at the dipole axis to be $a_{\rm core}$ in units of the conal intensity. However, we suppose that the core and cone emission arise at different altitudes, which results in a phase shift $\phi_{\rm core}$ of the conal emission relative to the pulse centroid. Figure 4 shows the cross section of our beam model without and with the phase shift accounted for. At 0.43 GHz, the core component is clearly offset toward the second conal maximum, indicating that the core component may be at lower altitude than the cone (Taylor and Weisberg 1982).

If the core component was absent, the pulse components would be equally intense. The addition of the core perturbs the intensity ratio (second to first component) to

$$R \approx \frac{1 + a_{\rm core} \exp[-(\Delta/2 - \phi_{\rm core})^2/w^2_{\rm core}]}{1 + a_{\rm core} \exp[-(\Delta/2 + \phi_{\rm core})^2/w^2_{\rm core}]}$$

$$\approx 1 + a_{\rm core} \exp[-(\Delta/2 - \phi_{\rm core})^2/w^2_{\rm core}] ,$$

and the component separation becomes

$$\Delta \approx \Delta_0 - \frac{w^2_{\rm cone}(\Delta_0/2 - \phi_{\rm core})(R - 1)}{w^2_{\rm core}} ,$$

where $\Delta_0 = 2\sqrt{\theta^2_{\rm cone} + \sigma^2}/\sin\alpha \approx 2\theta_{\rm cone}/\sin\alpha$ is the component separation for $a_{\rm core} = 0$, and the last approximation holds when $\sigma \ll \theta_{\rm cone}/\sin\alpha$. Now let us suppose, as an extreme example, that the altitude of the core emission is a sensitive function of the impact parameter, and that $\sigma$ and, therefore, $\Delta_0$ changed very little between 1981 and 1987. The change in $\phi_{\rm core}$ induces the relative changes

$$\frac{\delta\Delta}{\delta R} \approx \frac{\partial\Delta/\partial\phi_{\rm core}}{\partial R/\partial\phi_{\rm core}} \approx \frac{-w^2_{\rm cone}[2(\Delta/2 - \phi_{\rm core})^2/w^2_{\rm core} - 1]}{(\Delta/2 - \phi_{\rm core})} ,$$

in $\Delta$ and $R$, where we have approximated $\Delta \approx \Delta_0$. For nominal parameters, $w_{\rm cone} \approx 3^\circ$, $w_{\rm core} \approx 12^\circ$, $\Delta \approx 39^\circ$, and $\phi_{\rm core} \approx 12^\circ$, we find $\delta\Delta/\delta R \approx 0.^\circ26$. Thus, a 6% change in $R$ would, in this model, be accompanied by only a $0.^\circ016$ change in the component separation, about four times smaller than the observational upper bound (Weisberg *et al.* 1989). Consequently, the observed change $\delta R$ and the upper limit on $\delta\Delta$ can be simultaneously accounted for in the context of a relatively smooth, multicomponent beam model. The success of this model in explaining the data of Weisberg *et al.*, lends support to our view that the impact parameter must be small, since core components are thought to appear only when this is true. Moreover, the model is consistent only for small changes in impact parameter, $|\delta\sigma^2| \ll (0.^\circ6 \sin\alpha)^2$.

More direct quantitative bounds on the characteristic amplitude of the precession may be established using the reported limit on changes in the component separation (Weisberg *et al.* 1989). We assume as before a circular beam centered on the magnetic moment. Figure 5 defines angles needed to discuss geodetic precession. The equation of motion for the precession of the pulsar spin axis $\hat{\boldsymbol{\Omega}}$ about the orbital angular momentum vector $\hat{\boldsymbol{J}}$ is

$$\frac{d\hat{\boldsymbol{\Omega}}}{dt} = \Omega_p \hat{\boldsymbol{J}} \times \hat{\boldsymbol{\Omega}}$$

where

$$\Omega_p = \frac{3\pi G m_2(1 + m_1/3M)}{ac^2(1 - e^2)P_{\rm orb}} \approx 1.21 \text{ deg yr}^{-1} ,$$

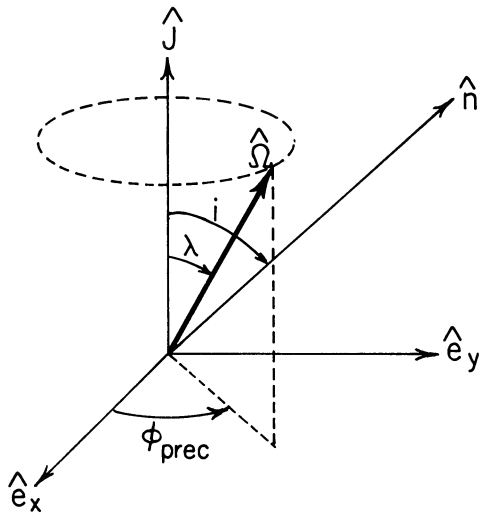


FIG. 5.—Geometry of geodetic precession

(Barker and O'Connell 1975*a*, *b*) where $m_1$, $m_2$ are the pulsar and companion masses, $M \equiv m_1 + m_2$, and we have used values for the masses and orbital elements from Taylor and Weisberg (1982). The amplitude of the precessional motion is determined by the conserved angle $\lambda = \cos^{-1} \hat{\boldsymbol{\Omega}} \cdot \hat{\boldsymbol{J}}$. As the pulsar spin axis precesses, $\alpha$ remains constant but $\sigma = \beta - \alpha$ changes. The rates of change of $\sigma$ and the pulse width $\Delta\phi$ (using eq. [3]) are

$$\frac{d\sigma}{dt} = -\frac{\Omega_p \sin i \sin \lambda \cos \phi_{\rm prec}}{\sin \beta}$$

and

$$\frac{d\Delta\phi}{dt} = -\left[\frac{\sin \sigma + \cos \beta \sin \alpha(1 - \cos \Delta\phi)}{\sin \alpha \sin \beta \sin \Delta\phi}\right] \frac{d\sigma}{dt} \quad (4)$$

where $\phi_{\rm prec} \equiv \Omega_p(t - t_0)$ is the precessional phase. Since $\delta\Delta\phi/\Delta\phi \le \epsilon = 1.5 \times 10^{-3}$ (Weisberg *et al.* 1989), equation (4) requires

$$|\sin \lambda \cos \phi_{\rm prec}| \le \left(\frac{\epsilon\Delta\phi}{\Omega_p T}\right) \times \left|\frac{\sin \alpha \sin^2 \beta \sin \Delta\phi}{\sin i[\sin \sigma + \cos \beta \sin \alpha(1 - \cos \Delta\phi)]}\right| . \quad (5)$$

The trigonometric factor on the right-hand side of equation (5) is largest for $\sigma = 0$ and $\alpha$ as large as possible. From Figure 3, the absence of interpulses requires $\alpha < 81°$. Using this value and $\epsilon\Delta\phi/\Omega_p T \approx 0.005$ gives an *absolute* upper bound $|\sin \lambda \cos \phi_{\rm prec}| < 0.2$; for $\alpha \approx 45°$ and $\sigma = 0$, we get a more stringent upper bound $|\sin \lambda \cos \phi_{\rm prec}| < 0.02$. Thus, either very little precession is occurring ($\lambda \ll 1$) or observations have been made at times with special values of $\phi_{\rm prec} \approx (n + \frac{1}{2})\pi$. For the nonspecial value $\phi_{\rm prec} \approx 45°$, the absolute upper bound becomes $\lambda < 15°$, while $\lambda < 2°$ for $\alpha \approx 45°$. In Figure 3, we have indicated the loci in $\alpha - \sigma$ phase space that bound regions in which $d\sigma/dt \ge 0.01\Omega_p$, $\ge 0.03\Omega_p$, and $\ge 0.1\Omega_p$ *are still permitted* by the *current* observational limits on $\Delta\phi$. More restrictive limits, hence smaller $\epsilon$, would move each bounding curve upward and to the left.

## V. DISCUSSION

Precession of the spin axis of PSR 1913+16 is very likely the cause of the change in intensity waveform seen by Weisberg *et al.* (1989) over a 6 yr period. While our polarization measurements do not yet show any secular changes, the vastly improved S/N ratio now available, combined with our models for the polarization signature, suggests that we should detect changes in polarization signature within a few years if $d\sigma/dt \sim 0.05\Omega_p$ or larger. Based on our polarization data, the absence of detectable change in pulse width (Weisberg *et al.* 1989), and other constraints, we argue that the angle $\alpha$ between the spin and magnetic axes is fairly well determined, and that the minimum "impact" angle $\sigma$ between the magnetic axis and the line of sight is very likely to be small (e.g., $\sigma \le 2°$). Moreover, either the spin-orbit angle $\lambda$ is very small or observations have been made at phases in the precession that are unfavorable for its measurement.

We are currently continuing polarization measurements at 1.4 GHz and extending the observations to other frequencies. Multifrequency observations should help us to better understand the magnetic field topology of 1913+16, and to explain its peculiar position angle curve. It is particularly important to determine the frequency dependence of the *I*, *Q*, *U*, and *V* waveforms in order to model the relative contributions of core and cone components at different pulse phases, $\phi$. Such information is crucial for any attempt to find the relative altitudes of the core and cone emission, a key ingredient for modeling the production of polarized radiation, and for unraveling the field configuration. From multifrequency measurements, we should be able to determine the rotation measure to the pulsar, which would allow us to fix absolute polarization angles. Independently interesting wavelength-dependent *magnetospheric* propagation effects will also be searched for in these data (e.g., Barnard and Arons 1986). If present, such effects could, of course, complicate any attempt to determine the rotation measure.

Ultimately, we shall attempt to determine the angle $\lambda$ between the pulsar spin and the binary orbital angular momentum. Combining this angle with the measured orbital elements (Weisberg and Taylor 1982) and proper motion (Taylor 1988) for 1913+16, we can constrain the presupernova mass and orbital radius of the progenitor binary, and the degree of asymmetry of the most recent supernova in the system (e.g., Cordes and Wasserman 1984; Burrows and Woosley 1986; Bailes 1988).

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