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CHROMOSPHERIC EXTENTS PREDICTED BY TIME-DEPENDENT ACOUSTIC WAVE MODELS

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ABSTRACT

In the present work I compute time-dependent acoustic wave models for the outer atmospheres of Arcturus (K2 IIIp), Aldebaran (K5 III), and Betelgeuse (M2 Iab). I discuss the propagation of short-period monochromatic shock waves as well as acoustic frequency spectra. My wave models show that the dissipation of the mechanical wave energy leads to an increase of the atmospheric temperatures up to chromospheric values. The combined effect of the higher temperatures and the momentum transfer of the waves increases the pressure scale heights of the atmospheres considerably. However, the chromospheric extents for the adopted stars, defined by the monotonic decrease with height of the time-averaged electron densities in the wave models, remain small compared with the stellar radii. For Arcturus and Aldebaran, I find a chromospheric extent of 1.12 and 1.13 R_*, respectively, corresponding to a time-averaged electron density of 10^7 cm^{-3}. For Betelgeuse, the chromospheric extent is 1.22 R_*. The small chromospheric extents in my wave models are in agreement with the observational result of Judge that the "extended chromospheres" of late-type giant and supergiant stars are due only to the low gravity of these stars. This result is further strong evidence that dissipating acoustic waves are an appropriate heating mechanism for the chromospheres of inactive late-type stars.

Subject headings: shock waves — stars: chromospheres — stars: late-type — wave motions

I. INTRODUCTION

An important property of late-type stars located to the right of the Linsky-Haisch dividing line in the H-R diagram is the apparent presence of extended chromospheres. That the atmospheric structures of solar-type and late-type giant stars are very different was discovered by Linsky and Haisch (1979) from an analysis of ultraviolet spectra of these stars. These spectra indicated two separate and distinct groups. The solar-type group shows emission lines which indicate the presence of chromospheres, transition regions, and, by implication, coronae. The nonsolar group shows emission lines formed at temperatures no hotter than 20,000 K, indicative of chromospheres only. This picture has essentially been confirmed by new observations of Simon, Linsky, and Stencel (1982) and Haisch (1987).

Stencel's (1981) preliminary estimates of the chromospheric sizes for noncoronal stars are very large. For Arcturus and Betelgeuse, he obtained 4–6 and 10–15 R_*, respectively. Brown and Carpenter (1984) derived chromospheric temperatures of various stars with a range between 7300 and 11,800 K. The revised computations of the chromospheric extents for different stars beyond the Linsky-Haisch dividing line by Carpenter, Brown, and Stencel (1985) are much smaller than those of Stencel: for Arcturus and Betelgeuse they found extents of 2.0 and 2.1 R_*, respectively. However, the C II density diagnostic method used by these authors has been strongly criticized by Schröder *et al.* (1988), who found discrepancies between the predictions of this diagnostic and eclipse measurements for cool giant stars which are members of ζ Aurigae systems. They concluded that the sizes of the chromospheres for stars beyond the Linsky-Haisch dividing line are significantly smaller than those obtained by Carpenter *et al.*

Judge (1987, 1989) summarized the oversimplifications made in the analysis of Carpenter, Brown, and Stencel (1985). He argued on the basis of model chromosphere calculations that the geometric extents of the C II λ2325 emitting regions do not change dramatically as a star crosses the Linsky-Haisch dividing line. Judge pointed out that, because the late-type giant and supergiant stars have lower gravities, the pressure scale heights increase as R_*^2, so that the geometric extents for the stellar atmospheres, measured in units of R_*, increase as R_*. He argued that the C II λ2325 emitting regions are not extended when measured in units of the pressure scale height. On the other hand, unambiguous evidence for warm, extended regions around several single cool giant and supergiant stars were presented by Drake and Linsky (1986), based on a sensitive VLA radio continuum survey at 6 cm. They found that these regions of free-free emission from a partially ionized chromosphere are extended, but more tenuous than the regions where the C II λ2325 line flux is formed. Clearly, the derived "geometric extent of the chromosphere" depends strongly on the diagnostic method which is used (cf. Judge 1987).

In the meantime, semiempirical chromospheric models, based on the line core shapes of the Mg II and the Ca II emission lines, were computed for various late-type giant stars. Linsky (1985) presented the present state of the treatment of radiative transfer, considering line formation with partial redistribution as well as different properties of the atmosphere such as geometrical extent and velocity fields. Ayres and Linsky (1975) constructed a homogeneous, hydrostatic and plane-parallel model for the photosphere and the lower chromosphere of Arcturus. They found a rise of temperature beginning with 3200 K in the temperature minimum layer up to 20,000 K in the chromosphere. The total geometric extent of their model is about seven thermal pressure scale heights, which is 0.1 R_*. Another semiempirical model which considers the influence of stellar wind flows and extended geometry was computed by Drake (1985). He found that the flow speed and electron temperature climb steeply to their maximum value of 40 km s^{-1} and 8400 K, respectively, in a very short distance above the photosphere. He also found that there is a broad, high-temperature plateau with a temperature of 7000 K extending from 1.2 to $\sim 13 R_*$ and an overlying region that has a rather cool temperature of typically 5000 K. Judge (1986*a*) tested the Ayres-Linsky Arcturus model by analyzing a number of UV lines present in *IUE* spectra. He found that for layers below $T = 8000$ K the mean thermal structure of the Ayres-Linsky model can account for many of the UV diagnostic lines observable by *IUE*, including the C II multiplet which is sensitive to electron density. Judge found a larger emission measure for somewhat higher temperatures than in the Ayres-Linsky model, but he also concluded that there is little or no gas at $T > 20{,}000$ K. Semiempirical chromospheric models for Aldebaran and Betelgeuse have been constructed by Kelch *et al.* (1978) and Basri, Linsky, and Eriksson (1981), respectively. For TX Psc, an N-type carbon star, a semiempirical chromospheric model was computed by Luttermoser *et al.* (1989). For late-type giant stars which are members of ζ Aurigae systems, the structure of the outer atmospheres was described by Che, Hempe, and Reimers (1983), Schröder (1985, 1986), Eaton (1988), and Hagen Bauer and Stencel (1989), who found that these stars have chromospheres and mass loss.

Theoretical models for the chromospheres of late-type stars have been computed by Schmitz and Ulmschneider (1981). They studied models with acoustic wave heating for a number of stars with effective temperatures between 4000 and 7000 K and gravities in the range of $\log g = 2$–5. These models were computed using a monochromatic, gray opacity. Ulmschneider, Schmitz, and Hammer (1979) computed a time-dependent acoustic wave model for the photosphere and the lower chromosphere of Arcturus. They found that the energy dissipation of the shocks leads to a chromospheric temperature which is significantly higher than the radiative equilibrium temperature. Subsequently, Cuntz, Hartmann, and Ulmschneider (1987) computed the propagation of shock waves in a much more extended atmosphere of Arcturus. They showed that monochromatic acoustic waves in the short-period range lead to chromospheric structures, but not to mass loss. Improved studies of acoustic wave models for chromospheric structures of late-type stars were given by Ulmschneider (1989). These models are motivated by new observational results of Schrijver (1987*a*, *b*), who identified slowly rotating "basal flux" stars, including single giant stars beyond the Linsky-Haisch dividing line. Schrijver suggested that the chromospheric heating of these stars is not dominated by magnetic fields and that acoustic waves appear to be an appropriate nonmagnetic heating mechanism.

Using a sample of 47 cool giant stars, Stencel and Mullan (1980) presented observational evidence that chromospheric structures are connected with mass motions. Previously, Stencel (1978) analyzed the ratio of the intensities of the self-reversed emission peaks in the center of the Ca II H and K emission lines to infer the presence of chromospheric mass flows. Subsequently, additional observational data have become available. Querci and Querci (1985), for instance, discussed the temporal variations in the UV spectra of TW Hor, a red giant C star. They concluded that the time-dependent behavior of the Ca II, Mg II, and Fe II emission lines shows the presence of chromospheric mass motions and waves.

In the present work time-dependent acoustic wave models for Arcturus (K2 IIIp), Aldebaran (K5 III), and Betelgeuse (M2 Iab) are calculated for monochromatic waves and acoustic frequency spectra determined by stochastically changing the wave periods. Therefore, it is possible to predict chromospheric extents for these stars, defined by the monotonic decrease with height of the time-averaged electron densities. The method, which is used to compute the wave models, is described in § II. Results are presented in § III. In § IV comparison with observations is discussed. Section V presents my conclusions.

II. METHOD

I have used for my wave computations an Eulerian time-dependent hydrodynamic code based on the method of characteristics. The code has been described in detail by Cuntz and Ulmschneider (1988). The characteristics method was chosen, because it permits an accurate treatment of the shock discontinuities. Since the radiation losses of the waves are usually strongly peaked behind the shocks, this method improves the shock treatment considerably.

Several methods are employed for the treatment of the boundary conditions depending on the relation between the flow speed and the sound speed. These are sub- and supersonic inflow and sub- and supersonic outflow. Shock waves are introduced at the inner boundary. At the outer boundary a transmitting boundary condition is used. This boundary condition allows the transmission of waves with sufficient efficiency and leads to physically acceptable results.

Radiation damping is considered in the thin plasma approximation using a Cox and Tucker type law. Judge (1990*a*), who studied the behavior of chromospheres with different gravities and activity levels, presented arguments that the effectively thin plasma approximation should work well for the chromospheric layers of inactive, low-gravity stars studied here. For the radiation loss function I use the version proposed by McWhirter, Thonemann, and Wilson (1975). For test calculations I have also used a temperature function fitted to computed radiation losses in the solar model of Vernazza, Avrett, and Loeser (1973). The run of the two radiation loss functions is shown in Figure 1. Unlike the radiation loss function proposed by Hearn and Vardavas (1981), mine does not consider radiative heating. Thus, if shock heating were missing in my

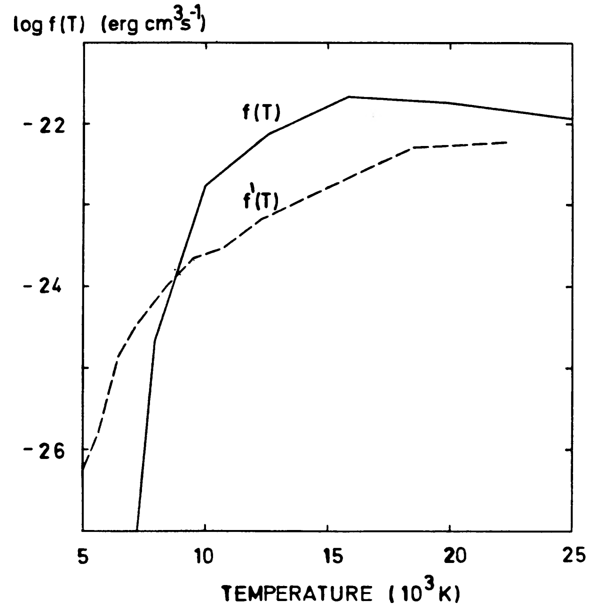

FIG. 1.—Radiation emission functions in the temperature range between 5000 and 25,000 K. The solid and dashed curves refer to the emission functions of McWhirter, Thonemann, and Wilson (1975) and Vernazza, Avrett, and Loeser (1973), respectively.

calculations, the radiative cooling would lead to unphysically low temperatures.

Ionization of hydrogen is explicitly taken into account. Except for very low temperatures, the electron density is determined by hydrogen ionization. Therefore, non–LTE effects in the hydrogen ionization equilibrium may be important. A simple formula for the departure coefficient of the hydrogen ground state, which assumes detailed balance in the Lyman lines, is given by Hartmann and McGregor (1980) and is used in the present work. For the computation of the internal energy of the gas, the entropy, the mean molecular weight, and the various thermodynamic derivatives, which are needed to solve the characteristics equations, I use the procedure of Wolf (1983, 1985).

III. RESULTS AND DISCUSSION

a) *Time-dependent Acoustic Wave Computations*

First I discuss the structure of an atmosphere supported by propagating shock waves. For the case of Arcturus, results are described in detail by Cuntz (1990), who studied the behavior of monochromatic as well as stochastic wave models. He found that chromospheres are formed as a result of the dissipation of the shocks. He found in the lower chromosphere an isothermal temperature layer of 6000 K, and beyond this layer a temperature increase up to 12,000 K. Strong shocks raise the temperature occasionally up to 20,000 K. For a monochromatic wave model with a period of 1.4×10^4 s and an initial velocity amplitude of 0.10 Mach, the time-averaged temperature reaches a maximum of 13,600 K. Moreover, both the monochromatic and the stochastic wave models show a strong increase of the pressure scale height due to the energy dissipation and momentum transfer of the waves.

For Aldebaran the results of my time-dependent acoustic wave computations are shown in Figures 2 and 3. Figure 2 shows a radiatively damped wave with an initial amplitude of 0.10 Mach and a period of 3.5×10^4 s after an elapsed time of 8.47×10^6 s. The assumed stellar parameters are given in Table 1. For my wave model I have started from an initial atmosphere assumed to be spherically symmetric and isothermal with a temperature of 5000 K. For the pressure at the inner boundary 5.0×10^{-1} dyn cm^{-2} was taken. This is the same value as in the previously discussed chromosphere models of Arcturus. At the time shown in Figure 2, more than 240 wave periods have already been introduced into the atmosphere. Presently, 24 shocks are contained in the model. I have selected

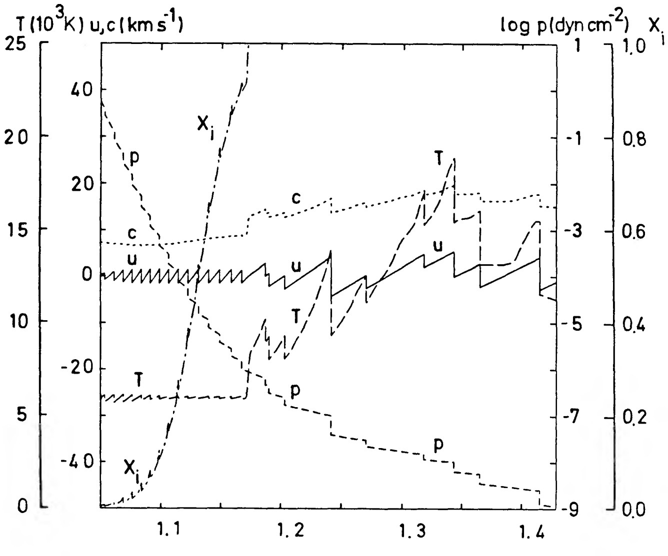

FIG. 2.—Monochromatic acoustic wave calculation with an initial velocity amplitude $M_0 = 0.10$ and a wave period $P = 3.5 \times 10^4$ s in an atmospheric model of Aldebaran. The flow speed u, the sound speed c, the temperature T, the pressure p, and the fraction of ionized hydrogen X_i are plotted as functions of the stellar radius.

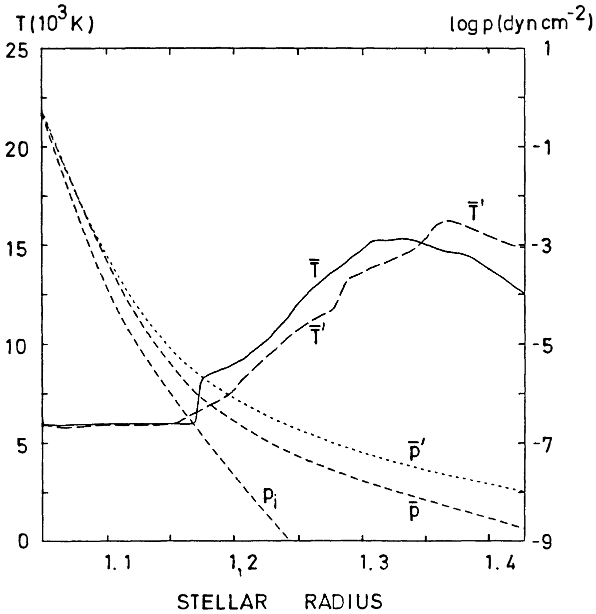

FIG. 3.—Time-dependent acoustic wave models for the chromosphere of Aldebaran. The time-averaged temperature and pressure due to monochromatic acoustic shock waves with an initial velocity amplitude $M_0 = 0.10$ and a period $P = 3.5 \times 10^4$ s are shown (*solid and short-dashed curves, respectively*). The long-dashed and dotted curves, labeled by primes, indicate the results for a stochastic wave calculation. The pressure of the initial atmosphere is indicated by p_i.

TABLE 1

STELLAR PARAMETERS AND HYDRODYNAMIC QUANTITIES FOR ADOPTED STARS

Quantity	α Boo	α Tau	α Ori
$T_{\rm eff}$ (K)	4250[a, b, c]	3970[c, d, e]	3800[f, g]
g_* (cm s^{-2})	50[b, h, i]	20[e]	0.4[g]
R_* ($R_\odot$)	26[c, j]	46[c, d]	1000[k]
H_p^T (cm)	7.67×10^9	1.92×10^{10}	9.59×10^{11}
$P_{\rm max}$ (s)	1.40×10^4	3.50×10^4	1.70×10^6
R_*/H_p^T	236	167	73
$R_*/\lambda_{\rm max}$	162	133	51

NOTE.—Mäckle *et al.* 1975 have found that the gravity of Arcturus is only $\log g_* = 0.90 \pm 0.35$. This low value has not been confirmed by other authors.

[a] Johnson *et al.* 1977.
[b] Martin 1977.
[c] Di Benedetto and Rabbia 1987.
[d] White and Kreidl 1984.
[e] van Paradijs and Meurs 1974.
[f] Tsuji 1976*b*.
[g] Tsuji 1976*a*.
[h] Ayres and Johnson 1977.
[i] Bell, Edvardsson, and Gustafsson 1985.
[j] Augason *et al.* 1980.
[k] Welter and Worden 1980.

the wave period corresponding to the maximum of the acoustic energy flux (cf. Bohn 1981, 1984). The initial wave amplitude has no essential influence on the structure of my wave model, however, because monochromatic shock waves attain a limiting shock strength after a distance of only a few wavelengths, independent of the initial wave amplitude (cf., e.g., Cuntz and Ulmschneider 1988).

The time-dependent behavior of the wave is similar to the case for the corresponding Arcturus model. The temperature increases starting from 6000 K in the lower part of the model and increasing to higher than 15,000 K. The maximum temperature of the wave, which is 18,900 K, is reached in the postshock region of a strong shock. The strength of the shocks in the wave model varies between 1.13 and 1.27. The shock strength M_S is defined as

$$M_S = \frac{U_{\rm SH} - u_1}{c_1}, \quad (1)$$

where $U_{\rm SH}$ is the shock speed and u_1 and c_1 are the gas and sound speed, respectively, in front of the shock.

In the present work I have also computed a chromospheric model for Aldebaran using an acoustic frequency spectrum, which is simulated by stochastically changing wave periods. The period distribution is assumed to be Gaussian with a standard deviation of 5.0×10^4 s centered at a peak value of 3.5×10^4 s (see Table 2). The spectrum is cut off at 1.0×10^3 s to avoid negative periods. For the initial wave amplitude I have used 0.10 Mach, which is the same amplitude as in the monochromatic wave model. I find extended chromospheric structures similar to those found for the monochromatic wave model. Also, interacting and overtaking of shocks occur as a result of the stochastic behavior of the flow. Strong shocks which lead to episodic mass loss are formed. The dynamics of the flow is quite similar to that described by Cuntz (1987).

Figure 3 shows the structure of the mean atmosphere for Aldebaran which was computed by time-averaging over many wave periods. Due to the energy dissipation of the shocks, the mean temperature increases up to 15,300 K at 1.28 R_* in the monochromatic wave model. In the stochastic wave model the temperature maximum reaches 16,200 K, and the position of the temperature maximum is now at 1.32 R_*. By averaging over many wave periods, I have also computed the mean flow speed due to the waves. In my monochromatic wave model I found a mean flow speed of 1.3 km s^{-1} at 1.42 R_*, corresponding to a very low time-averaged mass loss rate of 6.2×10^{-16} $M_\odot$ yr^{-1}. In the stochastic wave model, the time-averaged flow speed is 9.2 km s^{-1}, leading to a still low time-averaged mass loss rate of 2.0×10^{-14} $M_\odot$ yr^{-1}. It follows that propagating shock waves in the short-period range cannot produce a realistic mass loss rate. The same result was also obtained by Cuntz (1990) for his Arcturus model.

Although I have generally taken the radiation emission function given by McWhirter, Thonemann, and Wilson (1975), I have also done a test calculation with the radiation emission function of Vernazza, Avrett, and Loeser (1973). The run of both emission functions with temperatures between 5000 and 25,000 K is shown in Figure 1. The test calculation with the emission function of Vernazza *et al.* leads to a very similar behavior of the wave. As in the wave model computed with the McWhirter *et al.* function, I find a temperature plateau in the lower part of my chromospheric model, except that the temperature is only 5700 K instead of 6000 K. The general temperature structures of both wave models agree quite well, as a result of the adjustment of the net radiative cooling rate by changing the electron density. Since the emission function of McWhirter *et al.* is much smaller in the temperature range below 8000 K, the wave models computed with this emission function have higher temperatures in the radiation damping zone of the atmosphere, which in turn increase the hydrogen ionization degree. The higher electron density then readjusts the atmospheric temperature by the change of the net radiative cooling. Therefore, the difference between the two wave models is small. However, a reinvestigation of cooling rates at chromospheric temperatures is urgently needed (Judge 1990*b*).

TABLE 2

WAVE PARAMETERS USED IN STOCHASTIC WAVE COMPUTATIONS[a]

Star	P_0 (s)	$\sigma(P)$ (s)
α Boo	1.4×10^4	5.0×10^4
α Tau	3.5×10^4	5.0×10^4
α Ori	1.7×10^6	2.0×10^6

[a] P_0 and $\sigma(P)$ are the peak value and the standard deviation of the wave period distribution, respectively.

b) Pressure Scale Height in Wave-supported Atmospheres

An important property which I have found in my models is the increase of the pressure scale height due to the waves. In the outer part of my monochromatic wave model of Aldebaran, I find a pressure scale height of 1.85×10^{11} cm, which is about 6 times larger than that obtained in the undisturbed atmosphere. The increase in the pressure scale height in the wave models results from two contributions. First, the energy dissipation by the shocks increases the mean temperature of the atmosphere, which in turn increases the pressure scale height. The second contribution is due to wave pressure provided by the momentum transfer of the waves. Unfortunately, it is not possible to separate these two effects precisely, except for isothermal wave calculations. Such a model, however, is only appropriate to atmospheric layers in which strong radiation damping occurs. A detailed discussion of the wave pressure and its influence on the structure of a wave-supported atmosphere will be given in a future work (Gail, Cuntz, and Ulmschneider 1990).

One possible method of deducing whether the increase of the local pressure scale height in a wave-supported atmosphere is due mainly to the increase of the atmospheric temperature (and the change of the mean molecular weight) or to the momentum transfer of the waves is to compare the actual pressure scale height of the atmosphere with the thermal pressure scale height. The actual pressure scale height H_p is given by

$$H_p = -\bar{p}\left(\frac{d\bar{p}}{dr}\right)^{-1}, \quad (2)$$

where $\bar{p}$ is the time-averaged atmospheric pressure and r is the radius. The thermal pressure scale height H_p^T is given by

$$H_p^T = \frac{\mathcal{R}\bar{T}}{\bar{\mu} g_*}\frac{r^2}{R_*^2}, \quad (3)$$

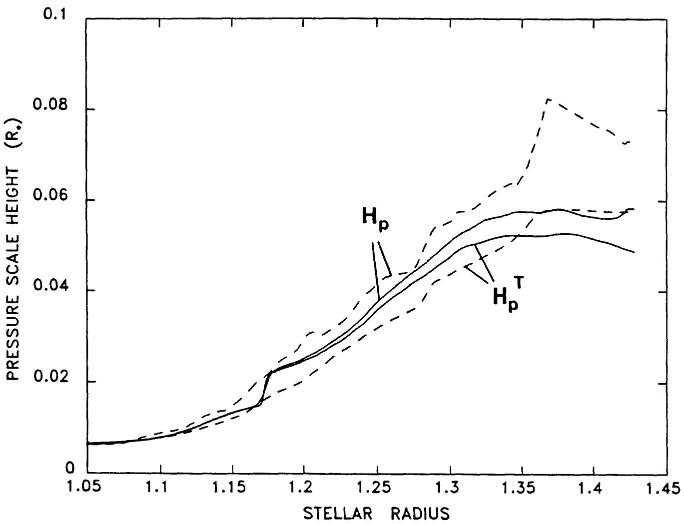

FIG. 4.—Two versions of the local pressure scale height in time-dependent acoustic wave models for the chromosphere of Aldebaran as shown in Fig. 3. The actual pressure scale height is labeled as H_p, and the thermal pressure scale height is labeled as H_p^T. The solid and dashed curves indicate the results for monochromatic and stochastic wave models, respectively.

where $\mathscr{R}$ is the gas constant, $\bar{T}$ the time-averaged temperature, $\bar{\mu}$ the time-averaged mean molecular weight, g_* the gravity of the star, and R_* the stellar radius.

The difference between these two definitions is the following: The actual pressure scale height H_p, which refers to the run of the time-averaged atmospheric pressure, contains the entire influence of the waves on the atmospheric structure (especially the momentum transfer of the waves), while the thermal pressure scale height H_p^T depends only on the time-averaged atmospheric temperature and the time-averaged mean molecular weight. It refers mainly to the energy input by the waves.

Figure 4 shows the run of these two versions of the local pressure scale height in time-dependent acoustic wave models for the chromosphere of Aldebaran. Figure 4 shows that in the short-period monochromatic wave model the difference between the actual pressure scale height and the thermal pressure scale height increases with height but remains small. Hence it follows that the momentum transfer by the waves does not change the atmospheric structure very much. The mean thermal structure of this wave model is close to hydrostatic equilibrium. On the other hand, in the stochastic wave model, the actual pressure scale height increases significantly faster than the thermal pressure scale height. The reason for this behavior is that the momentum transfer by strong shocks formed by merging is more important.

To determine the chromospheric extents, I have computed the time-averaged electron number densities n_E in my wave models. The heights, which correspond to specified values of n_E, are listed in Table 3. For example, for an electron number density of 10^7 cm^{-3}, I find that the chromosphere of Aldebaran has an extent of 1.13 R_*. This result is found both for the case of the monochromatic wave model and for the case of the wave model with an acoustic frequency spectrum. For the chromospheres of Arcturus and Betelgeuse, I find extents of 1.11 and 1.22 R_*, respectively.

These chromospheric extents can be compared with predictions of a scaling law assuming that the "extended chromospheres" of cool giant and supergiant stars are due only to the low gravities of these stars. This behavior was first suggested by Judge (1987, 1989), who argued on the basis of model chromosphere calculations that the geometric extents of the C II λ2325 emitting regions do not change dramatically as a star crosses the Linsky-Haisch dividing line. Judge pointed out that because the late-type giant and supergiant stars have lower gravities, the pressure scale heights increase as R_*^2 (if the change of the mass is negligible), so that the geometric extents for the stellar atmospheres, measured in units of R_*, increase as R_*. For stars with different masses M_*, the chromospheric heights $H_{\rm Ch}$, measured in units of R_*, behave as

$$\frac{H_{\rm Ch}}{R_*} \sim \frac{R_*}{M_*} \sim \frac{1}{g_* R_*} . \quad (4)$$

Table 4 compares the results for the chromospheric extents of Arcturus, Aldebaran, and Betelgeuse with predictions of the scaling law, which are 1.10, 1.11, and 1.20 R_*, respectively. I have used the Sun as a reference star in all three cases. For the solar chromospheric height, I have taken $H_{\rm Ch} = 1.5 \times 10^8$ cm (Vernazza, Avrett, and Loeser 1981). The chromospheric extents found by this simple scaling law agree nicely with the results given by time-dependent acoustic wave computations. These results are consistent with the picture described by Judge (1987, 1989). The chromospheres of the three stars thus consist of the same number of pressure scale heights.

I have studied the influence of acoustic frequency spectra on the chromospheric extents for all of the adopted stars. The wave parameters used for the different wave period distributions are given in Table 2. I find that the chromospheric extents are larger in the stochastic wave models than in monochromatic wave models. However, as shown in Table 3, this behav-

TABLE 3

COMPUTED CHROMOSPHERIC EXTENTS (in R_*)

	α Boo		α Tau		α Ori	
n_E	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
10^9.........	1.06	1.06	1.06	1.06	1.06	1.06
10^8.........	1.08	1.09	1.09	1.09	1.13	1.13
10^7.........	1.11	1.12	1.13	1.13	1.22	1.22
10^6.........	1.13	1.16	1.16	1.18	1.32	1.34
10^5.........	1.16	1.21	1.20	1.23	1.45	1.49

NOTE.—The inner boundary of the models lies at 1.05 R_*. Table 3 shows the chromospheric extent related to a fixed time-averaged electron number density n_E. Model 1 and model 2 indicate the results for monochromatic and stochastic wave models, respectively.

TABLE 4

COMPARISON OF COMPUTED CHROMOSPHERIC EXTENTS (in R_*) WITH OBSERVATIONS

Source	α Boo	α Tau	α Ori
Theory	1.12	1.13	1.22
Scaling law	1.10	1.11	1.20
Stencel 1981	4–6	...	10–15
Carpenter, Brown, and Stencel 1985	2.0	1.4	2.1
Judge 1986*a*, *b*	<1.2	<1.2	...
Byrne *et al.* 1988	<1.3	...	...

NOTE.—The computed chromospheric extents (designated "Theory") are given by time-dependent stochastic wave computations corresponding to a fixed time-averaged electron number density of $n_E = 10^7$ cm^{-3}. The position of the inner boundary of the atmospheric shells is at 1.05 R_* in all wave models.

ior is significant only for time-averaged electron number densities of 10^6 cm^{-3} or lower.

I have also investigated in what way the shape of an acoustic frequency spectrum changes the pressure scale height of the atmosphere and therefore the chromospheric extent. Figure 5 shows the run of the actual pressure scale height in time-dependent acoustic wave models for Arcturus. For the stochastic wave model of Arcturus, I have taken two different wave period distributions. Both are assumed to be Gaussian, but with two different standard deviations, one 2.0×10^4 s and the other 5.0×10^4 s. Figure 5 shows that for both stochastic wave models the actual pressure scale height is the same at each height. This agreement is a consequence of the stellar atmosphere being supported by a stochastic wave field, indicating that the atmospheric structure is determined by very strong shocks formed by merging. This behavior occurs in every wave model into which shock waves with stochastically changing wave periods are introduced, largely independent of the shape of the wave period distribution.

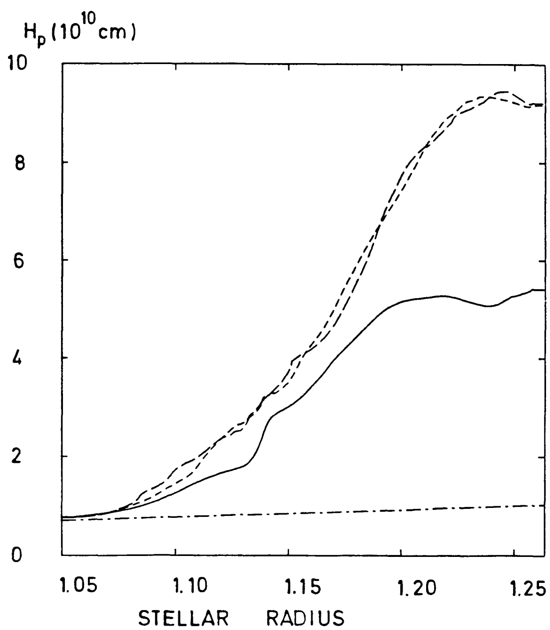

FIG. 5.—Behavior of the actual pressure scale height in time-dependent acoustic wave models for Arcturus. The solid curve shows the result for a monochromatic wave. The short-dashed and long-dashed curves correspond to stochastic waves. The standard deviation which is used for wave period distribution in these models are 2.0×10^4 and 5.0×10^4 s, respectively. The pressure scale height of the initial atmosphere is indicated by the dash-dot curve.

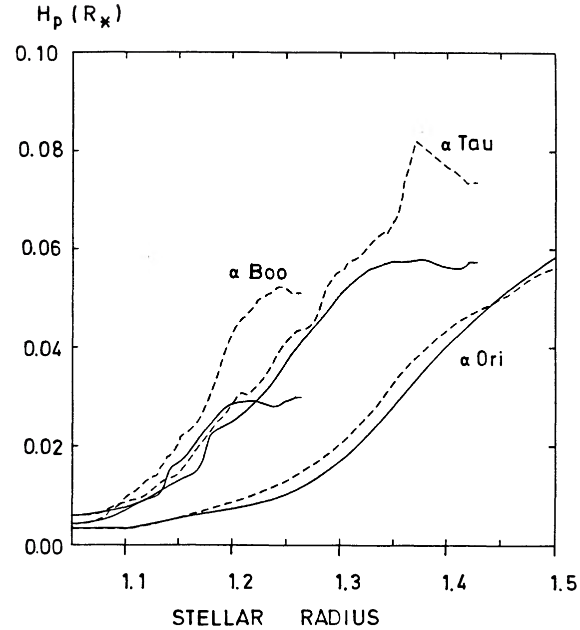

FIG. 6.—Behavior of the actual pressure scale height in time-dependent acoustic wave models for Arcturus (α Boo), Aldebaran (α Tau), and Betelgeuse (α Ori). The solid and dashed curves indicate the results for monochromatic and stochastic wave models, respectively. The wave parameters are given in Table 2.

Figure 6 shows the behavior of the actual pressure scale height in units of the stellar radius for time-dependent wave models of Arcturus, Aldebaran, and Betelgeuse. The results for monochromatic as well as stochastic wave models are given. Figure 6 shows that for each star the actual pressure scale height increases with height. This effect is stronger for stochastic wave models, as expected. For Arcturus and Aldebaran, I find that the actual pressure scale heights are everywhere smaller than one-tenth of the stellar radius for both monochromatic and stochastic wave models. The reason for this behavior is that the waves dissipate their energy immediately beyond the temperature minimum layers of the stars. This result predicts that for stars for which acoustic heated chromosphere models are appropriate, the major part of the emission line cooling should occur only in a very narrow atmospheric layer. In the case of Aldebaran, for instance, the acoustic energy flux drops by 2 orders of magnitude between 1.05 and 1.07 R_*. My wave models show that the damping length for the acoustic wave energy flux is typically a few wavelengths. In addition, as shown in the Appendix, short-period acoustic waves have nearly the same wavelength as the photospheric pressure scale height of the star.

In the case of Betelgeuse, the actual pressure scale height for short-period acoustic waves could be somewhat larger than 0.1 R_*, especially at atmospheric heights beyond 1.5 R_*. However, in this region, the particle densities in my wave models are lower than 10^4 cm^{-3}. This value is far smaller than for typical chromospheres. I anticipate, however, that higher particle densities can be obtained at this height by considering waves with longer wave periods.

IV. COMPARISON WITH OBSERVATIONS

In this section I confront the results obtained by time-dependent wave computations with observations. Semiempirical chromospheric models for the adopted stars, based on observations of the Ca II H, K, and IR triplet lines and the Mg II *h* and *k* lines, have been derived by several authors. The results are described in detail by Ayres and Linsky (1975), Kelch *et al.* (1978), Basri, Linsky, and Eriksson (1981), Drake (1985) and Judge (1986*a*, *b*). I note, however, that all of these chromospheric models assume a mean chromosphere which is smooth and shows no time-dependent behavior. Computed time-dependent shock wave models for such layers, on the other hand, always show a very bumpy atmosphere in which the chromospheric radiation occurs mainly behind shocks (cf., e.g., Ulmschneider, Muchmore, and Kalkofen 1987). Hence it follows that smooth semiempirical chromospheric models are not realistic and may produce misleading results. A detailed comparative discussions of theoretical and semiempirical chromospheric models will be given in a future work.

Semiempirical chromospheric models have been constructed for Aldebaran by Kelch *et al.* (1978) and Judge (1986*b*). Kelch *et al.* used high-resolution line spectra to study the shape of the Ca II K line and the Mg II *h* and *k* lines assuming a hydrostatic, plane-parallel atmosphere model. They derived a run of the temperature and electron density as functions of the column mass density consistent with the data. The revised chromosphere model of Judge (1986*b*) is based on the interpretation of emission line fluxes using opacity-sensitive line ratios and electron density measurements. He found that the model of Kelch *et al.* might be correct below $\sim$8000 K. Above 8000 K, the observed fluxes in all emission lines except Lyα can be accounted for in his model simply by reducing the maximum temperature from 20,000 to 12,000 K. Judge pointed out that there is no evidence from *IUE* spectra for emission regions which are geometrically extended beyond $\sim 0.2\ R_*$, similar to this semiempirical chromosphere model of Arcturus (Judge 1986*a*). This result is in disagreement with the work of Carpenter, Brown, and Stencel (1985), who used the emission line flux of the C II UV 0.01 multiplet near 2325 Å in *IUE* spectra. He derived for α Tau a chromospheric thickness of $0.4\ R_*$.

The method of Carpenter, Brown, and Stencel (1985) has been criticized by Schröder *et al.* (1988), who studied two cool giant stars which are members of ζ Aurigae systems, ζ Aurigae itself, and 32 Cygni. They calculated that Carpenter *et al.* overestimated the chromospheric extent in cool giants by a factor of 10, and concluded that the C II emission line flux can be reproduced by a chromosphere with a thickness of only $0.1\ R_*$. This result agrees very well with the conclusions of Judge (1987, 1989), who argued that the "extended chromospheres" of late-type giant and supergiant stars are due only to the low gravity of these stars. Moreover, Byrne *et al.* (1988) have published a new study for the electron densities in the chromospheres of late-type stars. They found that the electron densities in the C II emitting regions are substantially larger as in the models of Carpenter *et al.*, and the thicknesses of the chromospheres are correspondingly reduced. In the case of Arcturus, the chromospheric thickness is smaller by at least a factor of 3.5 than found by Carpenter *et al.*

The chromospheric extents computed in my time-dependent acoustic wave models are listed in Table 3. Referring to an electron number density of 10^7 cm^{-3}, I find that the chromosphere of Aldebaran has an extent of $1.13\ R_*$. This result is found both for the monochromatic wave model and the wave model with an acoustic frequency spectrum. For the chromospheres of Arcturus and Betelgeuse, I find extents of 1.11 and $1.22\ R_*$, respectively. The reason for these small chromospheric extents is that the waves dissipate their mechanical energy immediately beyond the temperature minimum layers of the stars (cf. § III*b*). The results given by my time-dependent wave models are not in agreement with the work of Stencel (1981) and Carpenter, Brown, and Stencel (1985), who have derived much higher values (see Table 4), but my results confirm the results of Judge (1986*a*, *b*) and Byrne *et al.* (1988). However, I note that the several authors use different methods to derive chromospheric extents.

The chromosphere of Betelgeuse has been studied by Basri, Linsky, and Eriksson (1981), who constructed a semiempirical model matching high-resolution absolute flux profiles of the Ca II K and Mg II *h* and *k* lines. They found that the flux in these emission lines is generated by extended chromospheric structures with temperatures between 5500 and 7000 K. Basri *et al.* found no evidence for temperatures larger than 10,000 K. Radio emission observations for the extended chromosphere of Betelgeuse were described by Newell and Hjellming (1982). They found a partially ionized, chromospheric region which extends from 1 to 4 R_*. The corresponding electron temperature was given as roughly 10,000 K. Subsequently, Brown and Carpenter (1984) deduced a temperature of 8900 K for the C II line formation region. The derived chromospheric extent is $2.0\ R_*$ (Carpenter, Brown, and Stencel 1985), but as mentioned above, their method might be wrong.

A further property of the atmospheric structures of cool giant and supergiant stars is the presence of outflow variabilities which provide strong evidence for wave motions. For Arcturus, outflow variabilities were first found by Chiu *et al.* (1977) from their study of high-resolution spectra of the Ca II K line. They determined flow speeds of 13 km s^{-1} at the Ca II line center and 16 km s^{-1} at the Mg II line center. For Betelgeuse, evidence for variable inflows and outflows exist also. Van der Hucht *et al.* (1979) found that several emission lines indicate outflows both in various regions in the chromosphere and in circumstellar regions. On the other hand, inward accelerating wind flows are described by Boesgaard and Magnan (1975) and by Boesgaard (1979). It seems that different types of mass loss phenomena are indicated.

Variable outflows and inflows can also be obtained from time-dependent acoustic wave models, especially when acoustic frequency spectra are considered. These spectra can be simulated by stochastically changing wave periods (cf. § III*a*). In the case of Arcturus, a detailed study has been undertaken by Cuntz (1987). This type of wave model behaves stochastically, leading to strong inflows and outflows. He calculated flow speed values between 20 and 50 km s^{-1}, which agree nicely with the observational constraints.

For Betelgeuse and other supergiant stars with dust shells, appropriate atmospheric models must include the influence of dust formation (see, e.g., Sutton *et al.* 1977; McMillan and Tapia 1978). However, a time-dependent wave model which describes self-consistently the chromospheric structures of the stars as well as the formation and dynamics of the dust is still missing. First results which deal with the possible interactions of the chromospheric component and the dust shell as well as with the role of the various structures for the mass loss of the star are given by Stencel, Carpenter, and Hagen (1986), Gail and Sedlmayr (1987), and Wirsich (1988).

A further property of Betelgeuse and other semiregular giants and supergiants is the presence of long-period variability (see, e.g., Querci 1986). This result is strong evidence that long-period waves must be considered to obtain more realistic hydrodynamic models for the outer atmospheres of these stars. Wood (1979), Willson and Bowen (1985), Bowen (1988*a*, *b*), and some others have computed theoretical models for regular Mira variables with long-period pulsation modes. They found that the deposition of shock wave energy greatly distends the outer atmospheres of the stars. The wave models are characterized by a small monotonic decrease with height of the time-averaged atmospheric pressure. The actual pressure scale heights are much larger than the thermal pressure scale heights, which are produced by the mean temperature structures.

V. CONCLUSIONS

I have computed theoretical models for chromospheric structures of late-type giant stars including the time-dependent propagation of acoustic waves. I have discussed models with short-period monochromatic shock waves as well as a spectrum of acoustic waves, and have applied my method to the stars Arcturus (K2 IIIp), Aldebaran (K5 III), and Betelgeuse (M2 Iab).

I define "chromospheric extents" by the monotonic decrease with height of the time-averaged electron densities. The chromospheric extents, which I have found in my wave models, are 1.12, 1.13, and 1.22 R_*, respectively, corresponding to a time-averaged electron density of 10^7 cm^{-3}. These extents are significantly smaller than those derived in the semi-empirical studies of Stencel (1981) and Carpenter, Brown, and Stencel (1985), and in the case of Arcturus and Aldebaran they confirm the results of Judge (1986*a*, *b*) and Byrne *et al.* (1988). However, I note that the methods for deriving chromospheric extents are not the same. The reason for the small chromospheric extents in my wave models is that the waves dissipate their mechanical energy immediately above the photospheres of the stars. This result predicts that, for those stars for which acoustic heated chromosphere models are appropriate, the major part of the emission line cooling should take place only in a very narrow atmospheric layer.

I have applied a simple scaling law to predict the chromospheric extents of the adopted stars, assuming that the "extended chromospheres" of cool giant and supergiant stars are due only to the low gravities of these stars (cf. Judge 1987, 1989). I find that the chromospheric extents given by the scaling law agree nicely with the results of my time-dependent wave models. This result means that the chromospheres of all stars for which the scaling law is valid consist of the same number of pressure scale heights.

For the time-averaged temperature structure in my wave models, I find a region at 6000 K above the temperature minimum layer and then an increase in temperature to about 15,000 K at the temperature maximum. The actual pressure scale height in the wave models, related to the time-averaged atmospheres, shows the following behavior: In short-period monochromatic wave models, the actual pressure scale height is essentially given by the thermal pressure scale height due to the mean temperature structure. In these wave models the mean thermal structure is thus close to hydrostatic equilibrium. On the other hand, in stochastic wave models, the actual pressure scale height increases significantly faster than the thermal pressure scale height. The reason for this behavior is that the momentum transfer by strong shocks formed by merging is more important. However, this effect changes the structure of the mean atmosphere substantially only at large atmospheric heights where the time-averaged electron number densities are lower than 10^6 cm^{-3}. In these layers the actual pressure scale height is not consistent with the prediction of the simple scaling law, which is derived solely from the condition of hydrostatic equilibrium.

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APPENDIX

HYDRODYNAMIC QUANTITIES RELATED TO THE ADOPTED STARS

In this section I derive the hydrodynamic quantities that are related to the atmospheric structures of Arcturus, Aldebaran, and Betelgeuse. The stellar parameters used for these stars are given in Table 1. The various hydrodynamic quantities are important for the structure of wave-supported atmospheres.

As shown by Bohn (1981, 1984), the maximum of the acoustic energy flux occurs near the wave period

$$P_{\max} = \frac{P_c}{10} \tag{A1}$$

where P_c is the acoustic cutoff period, given by

$$P_c = \frac{4\pi c_z}{\gamma g_*} . \tag{A2}$$

Here c_z is the sound speed at the top of the convection zone, γ is the ratio of the specific heats, and g_* is the gravity of the star.

The wavelength $\lambda_{\max}$ for a wave with the period $P_{\max}$ is given by

$$\lambda_{\max} = \mathrm{c}\mathrm{P}_{\max} \simeq \frac{4\pi}{10} H_p^T \qquad \text{with} \qquad c = \frac{\mathcal{R}}{\mu} T , \tag{A3}$$

where c is the sound speed, T the temperature, $\mathcal{R}$ the gas constant and μ the mean molecular weight (cf. Ulmschneider 1989). The thermal pressure scale height H_p^T is defined by equation (3).

For the computation of the thermal pressure scale height H_p^T, I use $T = 6000$ K. For γ and μ, I take 5/3 and 1.3, respectively, appropriate to the equation of state for a neutral gas. The temperature is chosen with respect to typical properties of stellar chromospheres. Table 1 shows the thermal pressure scale height H_p^T and the period P_{max} for the stars which I have studied. I also compare the thermal pressure scale height and the acoustic wavelength λ_{max} with the stellar radii for these stars.

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