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# THE PERIODICITIES IN THE INFRARED EXCESS OF G29–38: AN OSCILLATING BROWN DWARF?

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## ABSTRACT

We have investigated the oscillatory behavior of brown dwarfs. The observed periodicities in the infrared excess of the white dwarf Giclas 29–38 are consistent with low-degree, intermediate radial order *p*-mode oscillations of a brown dwarf companion to the white dwarf. These oscillation modes have the correct frequencies, act on observable layers of the atmosphere, and may be excited to sufficient amplitudes to explain the observations.

*Subject headings:* infrared: sources — stars: brown dwarfs — stars: pulsation

## I. INTRODUCTION

A continuing controversy surrounds the discovery by Zuckerman and Becklin (1987) of an infrared excess from the white dwarf star Giclas 29–38. The infrared color temperature of the excess radiation is 1200 ± 200 K, with a total luminosity estimated at $5 \times 10^{-5}\ L_\odot$; this corresponds to an object with radius roughly $0.15\ R_\odot$. Although these parameters are fairly well characterized (Greenstein 1988), the interpretation is far from clear. The primary interpretation of Zuckerman and Becklin (1987) is that they detected a brown dwarf (substellar mass object) with a mass from 0.04 to 0.08 $M_\odot$, based on published theoretical models. However, they allow that an alternative explanation is a disk of particulate matter in close orbit around the white dwarf.

To determine if the infrared excess might be due to dust surrounding the white dwarf, Graham *et al.* (1989) searched for variability in the infrared excess. Two significant periodicities at $K$ (2.2 $\mu$m) with frequencies of $2.59 \times 10^{-2}$ rad s$^{-1}$ and $3.47 \times 10^{-2}$ rad s$^{-1}$ (periods of 181 and 243 s) were seen. Since the periodicities were not seen at shorter wavelengths, they concluded the variations were in the source of the infrared excess, and not in the emission from the white dwarf. Graham *et al.* attributed the signal to variability in the heating of a dust ring due to *g*-mode nonradial oscillations of the white dwarf. For this model to be successful, only certain orders of two particular oscillation modes of the white dwarf must be excited. These modes, if the viewing geometry is correct, will then not be seen from Earth but could produce differential heating to a dust band which then produces the variable IR excess. Since the lifetime of the required dust against Poynting-Robertson drag is very short ($\sim 2 \times 10^6$ yr), the model requires a fortuitously recent asteroid collision to provide the necessary material. Graham *et al.* also suggested pulsations of a brown dwarf as another possible source of the infrared variations but did not investigate the possibility. In this *Letter* we report results of calculations indicating that an oscillating brown dwarf can explain the observed periodicities in the infrared excess of G29–38 while not requiring special viewing geometries or times.

## II. PROPERTIES OF BROWN DWARF OSCILLATIONS

Stellar oscillation modes are classified by their degree $l$, order $m$, and radial order $n$. For simple stellar interiors (like brown dwarfs), the radial order counts the number of nodes in the radial displacement eigenfunction. If the periodicities in the infrared excess are due to oscillations of a brown dwarf companion to G29–38, then these must be very low degree oscillation modes ($l \lesssim 3$), since higher degree oscillations cannot be detected from full disk observations (Christensen-Dalsgaard 1988). Assuming a fully convective, adiabatic interior, the only expected brown dwarf oscillations which might be observed are the $l = 2$ or 3 nonradial *f*-modes, the $l = 1$, 2, or 3 *p*-modes, and the $l = 0$ radial pulsations. Calculation of brown dwarf oscillation frequencies (below) reveals that only the intermediate radial order ($n \sim 4$–10) *p*-mode nonradial oscillations and similar overtone radial oscillations have periods in the 3–4 minute range. Since the oscillation spectrum of the nonradial modes is richer and since these modes may be more easily excited by turbulent convection, we do not further consider the radial oscillation modes (except to list their frequencies in Table 1B).

Acoustic-gravity waves propagating in an isothermal atmosphere produce a temperature perturbation $\Delta T$ to the background temperature $T$ (Mihalas and Mihalas 1984, eq. [53.33d]):

$$\left|\frac{\Delta T}{T}\right| = \frac{(\gamma - 1)\omega c k_z}{|\omega^2 - c^2 k_x^2|}\left[1 + \left(\frac{1}{2k_z h}\right)^2\left(1 - \frac{2c^2 k_x^2}{\gamma\omega^2}\right)^2\right]^{1/2}\left|\frac{W_1}{c}\right|, \tag{1}$$

where $c$ is the sound speed; $\gamma$ is the ratio of specific heats; $k_z$ and $k_x$ are the real components of the vertical and horizontal wavenumbers, respectively; $\omega$ is the oscillation frequency; $h$ is the scale height; and $W_1$ is the vertical component of the wave velocity. In the limit appropriate to low degree, intermediate radial order brown dwarf oscillations [$\omega^2 \gg c^2 k_x^2$ and $k_z \ll (1/h)$] equation (1) reduces to

$$\frac{\Delta T}{T} \sim (\gamma - 1)\frac{v_p}{2h\omega}, \tag{2}$$

where $v_p$ is the surface *p*-mode velocity amplitude. For observations at 2 $\mu$m, the intensity fluctuation $\Delta I/I \sim 6\Delta T/T$. At G29–38 the amplitudes of the 181 and 243 s periodicities are 0.028 ± 0.001 and 0.027 ± 0.001 mag, respectively (Graham *et al.* 1989), implying $\Delta I/I = 0.026$. For a brown dwarf atmosphere $\gamma = 1.4$ and $h \approx 6$ km; the required *p*-mode velocity amplitudes are thus $\sim 3 \times 10^2$ cm s$^{-1}$. While large, these velocities may be obtainable since the product of the oscillation frequency $\omega$ and the convective turnover time $\tau_H$ is about 1. This is the optimum circumstance for the excitation of acoustic modes by convection (Unno *et al.* 1979). *p*-mode excitation is discussed further below.

Acoustic waves cannot propagate in regions of an atmosphere where the acoustic cutoff frequency $\omega_{ac} > \omega$. In a nonisothermal atmosphere,

$$\omega_{ac}^2 = \frac{c^2}{4h^2} + \frac{\gamma g}{2T}\frac{\partial T}{\partial z}, \quad (3)$$

where $z$ is height and $h$ is now specifically the density scale height. The reflection point for upward-propagating acoustic waves of frequency $\omega$ (Christensen-Dalsgaard 1986) is at the level in the atmosphere where

$$\omega \approx \omega_{ac}. \quad (4)$$

The energy density perturbation varies periodically below the reflection level and declines exponentially with height (evanescent region) above (Unno *et al.* 1979). The length scale of this decline is of the order of the vertical wavelength, which for intermediate radial order *p*-modes may still be quite long. Nevertheless, to assure that the *p*-modes produce adiabatic heating via equation (1) at a level in the atmosphere which can be directly observed, the level at which equation (4) holds should be near optical depth $\frac{2}{3}$. Modes with lower frequencies will reflect deeper in the atmosphere and, depending on the evanescent wave wavelength, may not produce observable effects. To demonstrate that the variations in the IR excess of G29–38 may be due to a companion brown dwarf, we must establish that brown dwarf oscillation frequencies are in the correct range, that the modes propagate where they can be observed, and that the expected mode velocities are sufficiently large.

TABLE 1A

POLYTROPE NONRADIAL OSCILLATION FREQUENCIES FOR $\nu = 1.5$, $\Gamma = 5/3$

| Mode | $\omega^2R^3/GM$ $l = 1$ | $l = 2$ | $l = 3$ |
|---|---|---|---|
| $p_{10}$ | 258.81 | 278.89 | 298.69 |
| $p_9$ | 215.28 | 233.50 | 251.17 |
| $p_8$ | 175.55 | 191.95 | 207.78 |
| $p_7$ | 139.47 | 154.09 | 168.17 |
| $p_6$ | 107.14 | 119.93 | 132.24 |
| $p_5$ | 78.699 | 89.641 | 100.14 |
| $p_4$ | 54.300 | 63.368 | 72.042 |
| $p_3$ | 34.064 | 41.272 | 48.134 |
| $p_2$ | 18.111 | 23.508 | 28.595 |
| $p_1$ | 6.138 | 10.286 | 13.657 |
| $f$ | ... | 2.1198 | 3.7421 |

TABLE 1B

POLYTROPE RADIAL OSCILLATION FREQUENCIES FOR $\nu = 1.5$, $\Gamma = 5/3$

| Mode | $\omega^2R^3/GM$ ($l = 0$) |
|---|---|
| 10*H* | 283.66 |
| 9*H* | 238.09 |
| 8*H* | 196.47 |
| 7*H* | 158.55 |
| 6*H* | 124.29 |
| 5*H* | 93.842 |
| 4*H* | 67.328 |
| 3*H* | 44.872 |
| 2*H* | 26.584 |
| 1*H* | 12.547 |
| *F* | 2.7228 |

## IV. CALCULATIONS

To calculate accurately the oscillation frequencies for low-degree and low radial order oscillations requires numerical solution of the full fourth-order differential equations governing the problem (Unno *et al.* 1979). Asymptotic solutions or the Cowling approximation, in which perturbations to the internal gravitational field of the star are neglected, are not appropriate in this limit. Instead we use the formulation of Unno *et al.* (1979) for adiabatic oscillations and a finite difference relaxation technique described by Press *et al.* (1986). The oscillation eigenfunctions and eigenfrequencies are found using a grid of 300 points spaced equally in radius, giving results accurate to $< 1\%$ through radial order $n = 10$ when compared with tabulations for polytropic stars (e.g., Robe 1968).

The equation of state for brown dwarfs, especially those in the higher mass range, may be well approximated by a $\nu = 1.5$ polytrope. The calculated nonradial oscillation frequencies for fully convective $[\Gamma = (d \ln P/d \ln \rho)_{ad} = 5/3]$ polytropes are listed in Table 1A, where $G$ is the gravitational constant, $R$ is the brown dwarf radius, and $M$ is the mass. Radial oscillation frequencies are listed in Table 1B for the fundamental mode $F$ and the overtones. The advantage of considering polytropic models is that $R$ and $M$ may be chosen independently without the need of calculating a new interior model. We have also calculated the oscillation frequencies for several specific brown dwarf models to compare with the polytropic results. The interior models are those of Lunine *et al.* (1989). We find for the examples tested ($M = 0.05$, $0.075\ M_\odot$; $T_e \approx 1100$ K) in the frequency range of interest, the difference in oscillation frequencies from those in Tables 1 was never greater than 10%. Since the uncertainties in the mass and radius of the putative brown dwarf are sizable, use of the polytrope results introduces no larger uncertainty. We thus employ the polytropic frequencies as we further consider brown dwarf oscillations.

The calculated polytropic oscillation frequencies shown in Tables 1A and 1B confirm that frequencies near $3 \times 10^{-2}$ rad s$^{-1}$ are produced by radial and low $l$, intermediate $n$ nonradial *p*-mode oscillations of brown dwarfs in the mass range $M = 0.04$–$0.08\ M_\odot$. Evaluation of exactly which modes are responsible for the observations would require use of the detailed brown dwarf models, which in turn would only be justified by discovery of additional periodicities in the infrared excess.

To determine the level at which upward-propagating acoustic waves reflect in the atmosphere, we use equation (3) for the acoustic cutoff frequency and a suite of brown dwarf atmosphere models listed in Table 2. The variation with pressure level of acoustic cutoff frequency is shown in Figure 1 for model B2; the other models produce similar results. In all cases there is a broad band reflecting boundary near the 6 bar level which reflects waves with frequencies $\lesssim 0.1$ rad s$^{-1}$. The dashed line in the figure represents the pressure level of optical depth $\frac{2}{3}$. The *p*-modes under consideration, with frequencies

TABLE 2

BROWN DWARF MODEL ATMOSPHERES

| Model | $T_e$ (K) | $g$(cm s$^{-2}$) | $l_M$[a] | $h_{pr}$(km) | $v_H$(cm s$^{-1}$) | $\tau_H$(minutes) | $V_p$(cm s$^{-1}$) |
|---|---|---|---|---|---|---|---|
| A1......... | 1200 | $0.8 \times 10^5$ | 0.1 | 7.6 | $3 \times 10^3$ | 0.4 | $2 \times 10^2$ |
| A2......... | 1400 | $0.8 \times 10^5$ | 0.1 | 8.6 | $4 \times 10^3$ | 0.4 | $3 \times 10^2$ |
| B1......... | 1200 | $0.8 \times 10^5$ | 1.6 | 7.5 | $2 \times 10^4$ | 1.0 | $1 \times 10^4$ |
| B2......... | 1400 | $0.8 \times 10^5$ | 1.6 | 9.5 | $3 \times 10^4$ | 0.8 | $2 \times 10^4$ |
| C1......... | 1400 | $1.6 \times 10^5$ | 1.6 | 3.8 | $2 \times 10^4$ | 0.5 | $4 \times 10^3$ |

[a] Mixing length in units of the pressure scale height.

near $3 \times 10^{-2}$ rad s$^{-1}$, will be reflected within one scale height of optical depth $\frac{2}{3}$. Thus infrared radiation from adiabatic heating produced by these modes should easily escape. We note that equation (3) is strictly valid only for atmospheres with a slowly varying temperature profile; the atmosphere models considered here violate that criterion (Beer 1974) by a factor of 2. The resultant uncertainty in the exact location of the reflecting boundary is not of great importance, however, since the vertical wavelength of the waves is large.

The dominant heat transport mechanism throughout the interior of brown dwarfs with effective temperatures near 1200 K is convection, not radiation. We thus expect that the well-known $\kappa$ and $\gamma$ mechanisms of $p$-mode oscillation excitation (Unno *et al.* 1979) are unimportant to brown dwarfs, although we have not completed a detailed calculation. Instead we assume that the oscillations must be excited by interaction with turbulence. There is as yet no theory which rigorously estimates $p$-mode amplitudes excited by turbulent convection. Goldreich and Kumar (1988, eqs. [49]), however, have estimated the energy $E$ contained in acoustic modes of frequency $\omega$ in equilibrium with "turbulent pseudo-convection":

$$E(\omega) \sim \rho_0 H^3 c^2 \qquad \text{for } \omega\tau_H \lesssim 1 ,$$
$$E(\omega) \sim \rho_0 H^3 c^2 (\omega\tau_H)^{-13/2} \quad \text{for } 1 \lesssim \omega\tau_H \lesssim M^{-2} , \qquad (5)$$

where $H$ is the size of the largest eddies, $\rho_0$ is the density, $M$ is the Mach number for eddy velocity $v_H$, and the eddy lifetime $\tau_H \sim H/v_H$ where the modes are driven. Note that the energy per mode falls off dramatically when the product $\omega\tau_H \gtrsim 1$. Modes for which $\omega \gtrsim (1/\tau_H)$ will contain negligible energy and likely to be unimportant to the oscillation spectra of the star.

To determine which oscillation modes may be excited we assume that

$$H \approx l_M h_{pr} , \qquad (6)$$

where $h_{pr}$ is the pressure scale height and $l_M$ is the mixing length. Assuming the modes are excited near where they are observed and using convective velocities calculated from the brown dwarf atmosphere models (Table 2), we calculate $\tau_H$. For all three atmosphere models $(1/\tau_H) \sim 1.7$–$4.2 \times 10^{-2}$ s$^{-1}$. Thus, depending on the brown dwarf atmospheres considered, acoustic oscillation modes with periods longer than 2.5–6 minutes should be excited by interaction with turbulence.

To estimate the expected total velocity amplitude $V_p$ of oscillation modes in equilibrium with turbulence we follow a similar calculation by Deming *et al.* (1989). We integrate $E(\omega)dN$ over the frequency range of observed periodicities ($\omega_{min} = 2.59 \times 10^{-2}$ rad s$^{-1}$ to $\omega_{max} = 3.47 \times 10^{-2}$ rad s$^{-1}$) and use the Goldreich and Kumar (1988) expression for the number of modes per unit frequency, in volume $V$:

$$dN/d\omega \sim \omega^2 c^{-3} V . \qquad (7)$$

Setting the kinetic energy of the modes in the frequency range of interest equal to calculated energy in the modes,

$$\rho_{obs} V_p^2 \sim \int_{\omega_{min}}^{\omega_{max}} E(\omega)\omega^2 c^{-3} d\omega , \qquad (8)$$

we derive

$$V_p^2 \sim \frac{\rho_0}{\rho_{obs}} \frac{H^3}{3c} (\omega_{max}^3 - \omega_{min}^3) , \qquad (9)$$

provided $\omega_{max}\tau_H \lesssim 1$. The ratio $\rho_0/\rho_{obs}$ accounts for the modes being excited deeper in the atmosphere than where they are observed. The calculated values of $V_p$ for each atmosphere model are shown in Table 2. Depending on the mass of the putative brown dwarf, this total velocity amplitude will be distributed among the 40–80 $p$-modes with frequencies in the frequency range.

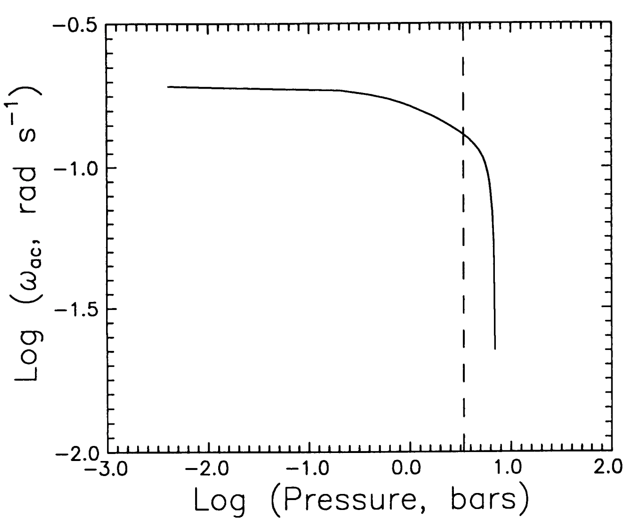


FIG. 1.—Acoustic cutoff frequency vs. prsssure in the atmosphere of brown dwarf case B, with effective temperature 1400 K. Values are inexact in the region close to 10 bar, where the cutoff frequency drops rapidly to zero, because of the approximate nature of the calculation as discussed in the text. The dashed line represents the pressure level of optical depth $\frac{2}{3}$.

## V. DISCUSSION

We have demonstrated that the low-degree $p$-mode oscillation frequencies of brown dwarfs are similar to the frequencies observed in the infrared excess of G29–38. The modes have large amplitudes in the atmosphere within a scale height of optical depth $\frac{2}{3}$ and should be observable if they are excited to sufficient amplitudes inside the planet. From considerations of turbulent forcing, brown dwarf $p$-modes with periods longer than ~3–6 minutes may be excited by interaction with turbulence. The observed periods of approximately 3 and 4 minutes are consistent with this calculation. Finally, the predicted velocity amplitude for modes in this frequency range is sensitive to the mixing length used in the calculation. Models A ($l_M = 0.1$) predict individual mode amplitudes of $v_p \sim 3$ cm s$^{-1}$, while models B and C ($l_M = 1.6$) predict $v_p \sim 50$–500 cm s$^{-1}$. These latter values are comparable to the required velocities of $\sim 3 \times 10^2$ cm s$^{-1}$ for each of the two modes necessary to produce the observed infrared variability. Since observable properties of brown dwarfs in this temperature range are insensitive to mixing length (Burrows *et al.* 1989), requiring the larger value of $l_M$ is not a significant constraint.

Only a limited number of modes should both be excited and detectable. Short-period modes will not be excited by interaction with turbulence and the longer period fundamental modes may reflect too deeply inside the planet to be observed. Also oscillation modes with $l > 3$ are not detectable. Hence it is not unlikely that only two oscillation modes would be detected. Depending on $M$ and $R$ of the brown dwarf, there may only be several additional detectable modes.

Nonradial oscillation modes of the hypothesized brown dwarf companion to Giclas 29–38 are consistent with the observed periodicities in the infrared excess. Furthermore, this interpretation places no particular constraints on required observing geometry or time, unlike the dust ring model. The oscillation hypothesis can be tested by searching for additional periodicities. If additional periodicities are found, the brown dwarf interpretation would be considerably strengthened as the $p$-mode model naturally predicts the presence of a limited number of additional modes.

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*Note added in proof.*—The interpretation presented in this *Letter*, that the optical and infrared periodicities of G29–38 are produced by two separate objects—a white dwarf and a brown dwarf, rests on the observation that the two infrared periodicities were not detected at shorter wavelengths. We have now seen data from simultaneous optical and infrared observations by J. McGraw, R. Cutri, and D. McCarthy of Steward Observatory, which clearly demonstrate the presence of the higher frequency variation at optical wavelengths. Since half the 2 $\mu$m flux is attributable to the white dwarf, similar infrared and optical signatures of white dwarf oscillations are expected. The lower frequency variation, however, is still not detected at shorter wavelengths. Thus while less compelling, an oscillating brown dwarf may still be required to explain this periodicity.

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