THE ASTROPHYSICAL JOURNAL, 348:L17–L20, 1990 January 1

CONVECTIVE INSTABILITIES IN SN 1987A

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Received 1989 June 23; accepted 1989 October 10

ABSTRACT

Following Bandiera, we show that the relevant criterion to determine the stability of a blast wave, propagating through the layers of a massive star in a supernova explosion, is the Schwarzschild (or Ledoux) criterion rather than the Rayleigh-Taylor criterion. Both criteria coincide only in the incompressible limit. Results of a linear stability analysis are presented for a one-dimensional (spherical) explosion in a realistic model for the progenitor of SN 1987A. When applying the Schwarzschild criterion, unstable regions get extended considerably. Convection is found to develop behind the shock, with a characteristic growth rate corresponding to a time scale much smaller than the shock traversal time. This ensures that efficient mixing will take place. Since the entire ejected mass is found to be convectively unstable, Ni can be transported outward, even into the hydrogen envelope, while hydrogen can be mixed deep into the helium core.

Subject headings: hydrodynamics — instabilities — stars: supernovae

I. INTRODUCTION

Many supernova remnants (such as, e.g., Cas A; Kirshner and Chevalier 1977) show mixing in the ejecta. While there existed some theoretical indications that this is due to instabilities of the propagating shock wave (e.g., Falk and Arnett 1973; Gull 1973; Chevalier and Klein 1978; Bandiera 1984), it has usually been related to the expansion into an inhomogeneous medium. SN 1987A now showed clearly that mixing is part of the supernova explosion itself. Several independent reasons lead to such a conclusion. The supernova produces large amounts of unstable long-lived nuclei; the dominant abundance is found in the doubly magic nucleus ^{56}Ni, which is produced in the innermost part of the ejecta; and ^{56}Ni decays with half-lives of 6.1 days to ^{56}Co and 77.8 days to ^{56}Fe. After the beta-transition, deexcitations to the ground state of the daughter nucleus lead to the emission of high-energy gamma rays. While these gamma rays would escape freely at low densities, Compton scattering will reduce their energies into the X-ray and even thermal regime at higher densities. With decreasing densities during the expansion, initially only thermalized photons will escape, then X-rays and finally gamma rays. X-ray observations (Dotani *et al.* 1987; Sunyaev *et al.* 1987; Wilson *et al.* 1988) and gamma-ray observations with the *SMM* (Matz *et al.* 1988) and balloons (Mahoney *et al.* 1988; Sandie *et al.* 1988; Cook *et al.* 1988; Gehrels, Leventhal, MacCallum 1988; Teegarden *et al.* 1989) actually agreed with this general behavior. The main problem was that the predicted time scales (Itoh *et al.* 1987; McCray, Shull, and Sutherland 1987; Gehrels, Leventhal, and MacCallum 1988; Xu *et al.* 1988; Ebisusaki and Shibazaki 1988*b*; Pinto and Woosley 1988*a*) did not agree with the observations, where X-rays and gamma rays appeared earlier than predicted. An agreement could be obtained only when part of the ^{56}Ni, being produced initially in the inner parts of the ejecta, was mixed out to larger distances (Ebisusaki and Shibazaki 1988*a*; Pinto and Woosley 1988*b*; Kumagai *et al.* 1988, 1989; Woosley, Pinto, and Ensman 1988; Grebenev and Sunayev 1988; Leising 1988; Arnett and Fu 1989; Fu and Arnett 1989; Yamada, Kasahara, and Nakamura 1989; Bussard, Borrows, and The 1989). Mixing is also required to explain the spread of expansion velocities seen in line widths of infrared observations for various elements (Erickson *et al.* 1988; Witteborn *et al.* 1989) and in the gamma-ray lines of ^{56}Co (Barthelmy *et al.* 1989). The inferred velocities differ strongly from the much smaller ones, expected from an expanding remnant, which maintains the stratified composition from explosive and hydrostatic nuclear burning.

Other indications come from the modeling of the optical light curve (Woosley 1988; Shigeyama, Nomoto, and Hashimoto 1988; Arnett and Fu 1989). In order to obtain the correct light curve behavior as a function of time, i.e., the rise after the initial adiabatic decline and a flattened rather than a sharp maximum before following the exponential decline of ^{56}Co, a mixed composition is necessary. Two effects are of importance here, the location of ^{56}Ni(^{56}Co) and of H in the expanding remnant. The location of ^{56}Ni controls the position where decay heat is released. The photosphere is mainly defined by the position of the hydrogen recombination front which causes a strong decrease in opacity and moves inward with respect to the Lagrangian mass coordinate. In order to obtain a flattened luminosity peak it seemed necessary to have hydrogen mixed deeply into the innermost mass zones (Woosley 1988; Nomoto *et al.* 1988; Woosley, Pinto, and Weaver 1988). While these calculations are still somewhat preliminary and the conclusions not necessarily stringent (P. A. Pinto, private communication), the best agreement between calculated and observed light curves were obtained for a composition which mixed a small fraction of Ni all the way into the 10 $M_\odot$ hydrogen envelope and hydrogen into the deeper layers, containing mostly heavy elements.

The existence of this extensive mixing can have several causes. Two major mechanisms were suggested: (1) instability of the nickel-bubble where the energy release from radioactive decay causes a pressure wave and leads to a density inversion which might grow into a Rayleigh-Taylor instability (Woosley 1988; Arnett 1988); and (2) otherwise the explosion shock wave itself can already become unstable, dependent on the density gradient (Chevalier 1976) or due to the reverse shock originating at the inner edge of the hydrogen envelope (Shigeyama, Nomoto, and Hashimoto 1988). Preliminary studies of the Ni bubble with a three-dimensional smooth particle hydro code (SPH), including a nuclear network for the Ni decay chain and starting from an unperturbed expansion, showed only small density fluctuations (clumping) of less than a factor of 2 (Benz and Thielemann 1989). But this picture might change when the initial blast wave is already unstable, leaving an inhomogeneous abundance and density distribution before Ni decay. Nagasawa, Nakamura, and Miyama (1988) followed the propagation of the supernova blast wave by depositing the supernova energy into an $n = 3$ polytrope of 10 $M_{\odot}$. Their calculations, done with a three-dimensional SPH code, showed that severe clumping would result in such a circumstance.

To examine the question of instabilities occurring during the explosion itself, we have run the same polytrope explosion simulation as Nagasawa, Nakamura, and Miyama (1988), using from 10,000 particles for the whole star up to 40,000 particles in one octant only. Preliminary results of these simulations have been reported in Benz and Thielemann (1989). These simulations did not show the extensive clumping and mixing claimed by Nagasawa, Nakamura, and Miyama (1988). Two percent density perturbations that were introduced in the initial configuration resulted in 15%–20% density fluctuations at the end of our simulation, compared to the 400% obtained by Nagasawa, Nakamura, and Miyama (1988). The relative stability of the exploding polytrope was also found by Müller *et al.* (1989) and by S. Clancy and R. Bowers (private communication), who both used classical finite difference codes. Extensive clumping can, however, be obtained by using highly perturbed initial conditions or an unrealistic explosion energy deposition scheme.

The search for instabilities in SN 1987A has to be performed with a realistic supernova model, rather than a polytrope. Arnett, Fryxell, and Müller (1989) could show the growth of convective fingers, but a clear analysis and understanding of the effect has still to be established. Ebisuzaki, Shigeyama, and Nomoto (1989) performed a linear stability analysis of a spherical explosion using the Rayleigh-Taylor criterion (derived in the incompressible limit) and could show that essentially the mass zones between the metal-He and He-H interfaces are unstable. In the present *Letter* we want to emphasize, following Bandiera (1984), that it is important to use the appropriate stability criterion, namely the Schwarzschild or the Ledoux criterion, when searching for instabilities in supernovae explosions. Convective motions will appear in compressible fluids over a wider range than predicted by the incompressible Rayleigh-Taylor criterion. Both coincide only in the incompressible limit.

II. STABILITY ANALYSIS

Chandrasekhar (1961) discusses the stability of an initially static, incompressible fluid in a gravitational field. In the case of supernova explosions, gravity is negligible and pressure gradients provide the required relative acceleration. The criterion for a Rayleigh-Taylor instability to develop in this case becomes

$$\left(\frac{\partial P}{\partial r}\right)\left(\frac{\partial \rho}{\partial r}\right) < 0 . \tag{1}$$

This criterion has been used by Chevalier (1976) to investigate the stability of blast waves propagating in power-law density distributions. He found that for a density distribution of the type $\rho \propto r^{-\omega}$ the Sedov solution was unstable in the sense of equation (1) for $2.53 \leq \omega \leq 3.0$. Recently Ebisuzaki, Shigeyama, and Nomoto (1989), using equation (1), performed a stability analysis of a realistic model of SN 1987A. They found that strong Rayleigh-Taylor instabilities were developing in the region between the H-He and He-metal interfaces. The growth rate was found to be large enough for significant mixing to take place in these regions during a dynamical time scale. However, as pointed out by the authors, such an extend of mixing would not be large enough to account for the emission line widths (3000 km s^{-1}) observed in the infrared and gamma-ray band. To explain these observations, mixing would have to extend over a much wider region.

Ebisuzaki, Shigeyama, and Nomoto (1989) used the stability criterion relevant for instabilities in incompressible fluids. For compressible fluids, however, the results are different. As shown by Bandiera (1984), who performed a local linear stability analysis in a plane-parallel geometry, instabilities will develop for a significantly wider range of density and pressure gradients than required by equation (1). In such a linear stability analysis, the growth rate for the unstable modes along the z-direction is found to be given by

$$\sigma_{\pm} = \pm \frac{c}{\gamma} \sqrt{\mathscr{P}^2 - \gamma \mathscr{P} \mathscr{R}} , \tag{2}$$

where $\mathscr{P} = (1/P)(\partial P/\partial z)$ and $\mathscr{R} = (1/\rho)/(\partial \rho/\partial z)$ are, respectively, the reciprocals of the pressure and density scale heights; $c = (\gamma P/\rho)^{1/2}$ is the sound speed; and γ is the adiabatic index of the gas. The above formula is valid in the regime were the wavenumber k satisfies $k \gg \mathscr{P}, \mathscr{R}$ (or the wavelength is small in comparison to the pressure and density scale heights). Notice that in this limit, the growth rate is found to be independent of the wavenumber. For instabilities to grow exponentially, the necessary condition is

$$\frac{\mathscr{R}}{\mathscr{P}} < \frac{1}{\gamma} , \tag{3}$$

which can be recognized as the well-known Schwarzschild criterion for convective instabilities. To allow an easy interpretation notice that the Rayleigh-Taylor criterion (1) can also be written as

$$\frac{\mathscr{R}}{\mathscr{P}} < 0 \tag{4}$$

with the corresponding growth rate given by

$$\sigma_{\pm} = \pm \frac{c}{\gamma} \sqrt{-\gamma \mathscr{P} \mathscr{R}} = \pm \sqrt{-\frac{P}{\rho} \mathscr{P} \mathscr{R}} . \tag{5}$$

When comparing equations (4) and (3), it appears that the Rayleigh-Taylor criterion is much more stringent than the actual stability criterion for a compressible fluid. Consequently for a compressible fluid, instabilities, i.e., convective motions, will set in even if condition (1) is not satisfied. Taking the limit $\gamma \to \infty$ in equation (3), that is assuming incompressibility, we indeed recover the classical criterion (4), and expression (2) changes to (5). We conclude that applying the Rayleigh-Taylor criterion (1) or (4) to determine the stability of compressible fluid flows can be misleading since instabilities will occur for situations predicted to be stable. Stability of compressible fluid flows should be checked using equation (3). It was pointed out by many authors that taking into account compressibility does not change significantly the growth rate of the instability. This is quite true; the effect of compressibility is not to change the growth rate drastically, but to *extend* considerably the region where the flow is subject to instabilities. Furthermore, the growth rate of the instabilities in these extended regions is comparable, over a large range, to the growth rate obtained with equation (5).

For these reasons, the existence and the extent of hydrodynamical instabilities in supernovae explosion in general, and SN 1987A in particular, should be studied using criterion (3) rather than (1). The same criterion should be applied when comparing results from two- or three-dimensional hydrodynamical simulations to a stability analysis done for one-dimensional explosions. Whether these instabilities are called Rayleigh-Taylor or convection is only of semantic interest, but to keep with astronomical traditions, we probably should call them convective instabilities. Notice that in the case where strong mean molecular weight (μ) gradients exist, the appropriate stability criterion would be the Ledoux criterion. In our simple approach, we neglected the effects of such μ gradients.

As a word of caution, we would like to point out that a dynamically driven instability as considered in this *Letter* is generally a " global " instability and should, in fact, be analyzed as such including the overall dynamics of the blast wave and a proper treatment of the boundary conditions. Nevertheless, the local satisfaction of the convective instability criterion usually indicates the existence of a global instability and therefore warrants its use in this context. The growth factor (see next section) derived from our local stability analysis may, however, not correspond to the true amount of the growth of the perturbations. First, the small wavelengths considered here may turn out not to be the dominant modes and second, once the instability has reached the nonlinear regime the linear analysis loses any meaning. The first point can possibly be addressed by a more general stability analysis whereas the second point will require fully three-dimensional numerical simulations. The growth rate derived in the next section should therefore be regarded only as a qualitative indicator of the relative strength of the instability throughout the star.

III. CONVECTION IN SN 1987A

We have performed explosion simulations with a one-dimensional Lagrangian hydrodynamics code, using the stellar model for the progenitor of SN 1987A with a 10 $M_\odot$ mass H-envelope by Nomoto *et al.* (1988). We deposited 10^{51} ergs in the center and followed the blast wave propagation through the entire star. For simplicity we adopted a polytropic equation of state with $\gamma = 4/3$. This should be a reasonable approximation since the shocked material is so hot as to be radiation-dominated (Weaver and Woosley 1980). We also neglected radiation transport because the envelope is optically thick due to Thompson scattering, and the resulting diffusion time scale is much longer than the dynamical time scale considered here. The code itself used a staggered mesh resulting in second-order accuracy in both time and space. A total of 700 zones were used.

When following the explosion we obtained pressure and density profiles identical to Figures 1 and 2 in Ebisuzaki, Shigeyama, and Nomoto (1989). We checked each zone for potential instabilities according to criteria (3) and (4). If a zone was found to be unstable, the corresponding growth rate was computed according to expressions (2) or (5). The growth of any perturbation ξ_0 over a given time t can be *estimated* (see discussion above—Ebisuzaki, Shigeyama, and Nomoto 1989) by

$$\frac{\xi}{\xi_0} = \exp\left(\int_0^t \sigma_+ \, dt'\right) . \qquad (6)$$

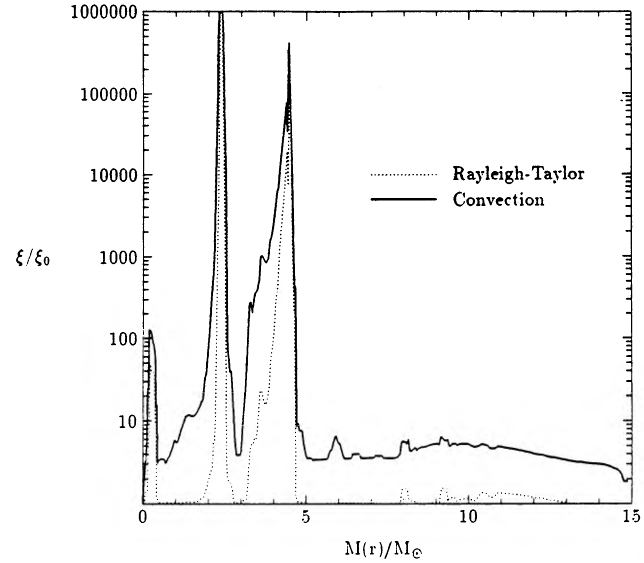

FIG. 1.—Growth of initial perturbations ξ_0 according to linear stability analysis (eq. [6]) 1 hr after the explosion when the shock has almost reached the surface. The solid line refers to convective instabilities (Schwarzschild criterion, eq. [2]) and the dotted line to Rayleigh-Taylor instabilities (eq. [5]).

The growth of initial perturbations according to equation (6) is displayed in Figure 1 against the Lagrangian mass coordinate of ejected mass (excluding the 1.6 $M_\odot$ neutron star—Thielemann, Hashimoto, and Nomoto 1990) for a time corresponding to 1 hr after the explosion. This coincides with the time when the shock front hits the surface of the star. The dotted curve gives to the integrated Rayleigh-Taylor growth rate (5), whereas the solid line corresponds to the integrated convective instability growth rate (2).

As expected, we recover a perturbation growth pattern very similar to the one obtained by Ebisuzaki, Shigeyama, and Nomoto (1989) for the Rayleigh-Taylor instability with the characteristic large spikes at the metal-He and He-H interfaces and a mostly stable H-envelope. Notice in this case the small growth of perturbations in the center and the existence of a sharp barrier just before 3 $M_\odot$ of ejected mass. Therefore, it is not quite clear whether the Ni formed in the center will be mixed to appreciable distances.

When the proper instability criterion is used, these problems disappear. The growth of the perturbations in the center is substantially larger although still not as large as at the metal-He interface, and the barrier at 3 $M_\odot$ has been partially removed. We expect therefore that convective motions will lead to substantial mixing inside the inner core. Furthermore, the hydrogen envelope is also found to be convectively unstable but with a significantly smaller growth rate. This makes it possible for some material (most probably only a small fraction) originating from the inner core to be convected into the envelope and conversely, for some hydrogen from the envelope to be transported into the core. The quantitative extend of the mixing and the nonlinear behavior of the instabilities can, of course, not be addressed by this linear stability analysis. Multidimensional, high-resolution numerical simulations of realistic stellar explosions similar to the one by Müller *et al.* (1989) and Arnett, Fryxell, and Müller (1989) have to be performed to answer these questions. We are currently investigating these questions with a three-dimensional Smooth Particle (SPH) code. Calculations of blast waves, propagating in power-law density distributions, have indeed shown the existence of convective instabilities behind the shock. These results, which will be published in a forthcoming paper, also indicate that significant mixing is obtained.

We would like to thank K. Nomoto for providing us with a progenitor model of SN 1987A, M. Birkinshaw for very helpful discussions, and E. Müller for providing us with unpublished results. This research was supported in part by NASA grant NGR 22-007-272 and computer time at the National Center for Supercomputer applications at the University of Illinois (AST 89-0009N). One of us (W. B.) also acknowledges partial support from the Swiss National Science Foundation and the William F. Milton Fund.

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