Bending instability in collisionless elliptical disks. Self-gravitating disks

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We investigate large-scale bending instability, representing the bars in SB galaxies, of Freeman models of elliptical stellar disks. The condition of stability against this type of perturbation, as well as against bar-like perturbations, constrains the global parameters (shape and global dynamical characteristics) of highly flattened, nonaxisymmetric stellar systems. We give the intersection of these domains of stability against bending perturbations with the well-known domain of stability against bar-like perturbations.

1. INTRODUCTION

The construction and study of nonaxisymmetric disk models of stellar systems is associated primarily with the explanation of the dynamical properties of the bars of SB galaxies – triaxial ellipsoidal configurations that are highly flattened along the rotational axis (if we adhere to the viewpoint that the bar is a material formation rather than a density wave [1)] – as well as galaxies that are in a tidal force field. Here certain information can be obtained by studying fairly simple self-consistent models with a quadratic potential.[2–6] These models cannot pretend to explain the fine structure and detailed dynamics of their actual prototypes, of course. Processes with a characteristic size much smaller than the size of the system, when boundary effects are not important, can be studied with models that have a boundary with a simple geometry but are relatively complex from the aspect of fine structure. As for global processes that involve the entire system (this is expressed mathematically by the fact that only second-order moments of the distribution function are involved in the description), one may hope that models which are idealized as to potential (density distribution) and thickness (in the case of disks) will be faithful to their actual prototypes. The results of an investigation of the stability of model stellar systems against global perturbations in the rotational plane – bar-like perturbations – are well known.[7] For models with round disks, they confirm the Ostriker–Peebles hypothesis that a gravitating system becomes unstable against bar-like perturbations if the ratio of the rotational to gravitational energy is $t_{OP} = \frac{E_{rot}}{|W|} \geqslant t_{OP}^{(b)} \approx 0.14$, with the critical value $t_{OP}^{(b)}$ depending little on the internal structure of the system.

The bending instability which results from global noncoplanar perturbations complements the bar-like instability in the sense that rotation, which destabilizes the bar mode, stabilizes a bend. Round disks having a quadratic potential become unstable against a large-scale bend (the plane changes into a second-order surface) when $t_{OP} \lesssim t_{OP}^{(B)} = 0.125$. This critical value changes little in models with different surface density distributions.[8]

The purpose of the present work is to study the stability of elliptical Freeman disks[2] against large-scale bending perturbations and to compare the stability domain with that for bar-like perturbations found in Ref. 9, in order to determine the disk models with different degrees of elongation which are stable against global perturbations.

2. EQUILIBRIUM MODEL

The equation for the boundary, the surface density, and the internal potential of the model in a rotating frame of reference associated with the principal axes of the disk have the form[2]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{1}$$

$$\sigma(x, y) = \sigma(0)\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}, \tag{2}$$

$$\Phi(x, y) = \frac{A_1}{2}x^2 + \frac{A_2}{2}y^2 + \text{const}, \tag{3}$$

where

$$A_1 = \frac{2\pi G\sigma(0)}{a^2}\frac{b}{\tilde{k}^2}[K(\tilde{k}) - E(\tilde{k})], \tag{4}$$

$$A_2 = \frac{2\pi G\sigma(0)}{b\tilde{k}^2}\left[E(\tilde{k}) - \frac{b^2}{a^2}K(\tilde{k})\right], \tag{5}$$

$\tilde{k}^2 = 1 - b^2/a^2$, and $K(\tilde{k})$ and $E(\tilde{k})$ are complete elliptic integrals of the first and second kinds. Later we shall need not the total distribution function of the two-parameter Freeman model with independent parameters $0 \leqslant e = \frac{b}{a} \leqslant 1$ and $0 \lesssim \eta = \Omega^2/A_1 \lesssim 1$, but only its first and second hydrodynamic moments. We give them here in the form found in Ref. 10. The velocities of the centroids and the anisotropic "pressure" have the following structure:

$$\bar{v}_x = \alpha_1 y, \quad \bar{v}_y = \alpha_2 x, \quad \overline{\sigma v_x'^2} = p_1\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{3/2},$$

$$\overline{\sigma v_y'^2} = p_2\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{3/2},$$

where $\alpha_1 = \frac{a}{b}\Lambda, \quad \alpha_2 = -\frac{b}{a}\Lambda,$

$$\Lambda = 2\sqrt{\eta A_1}\,\frac{e^2A_2 + [(1-e^2)\eta - 1]A_1}{e(A_2 - A_1)}, \tag{6}$$

$$p_1 = \frac{1}{3}\sigma(0)a^2A_1(1-\eta)\left\{1 + 4A_1\eta(1-e^2)\frac{e^2A_2 + [(1-e^2)\eta - 1]A_1}{e^2(A_2 - A_1)^2}\right\} \tag{7}$$

$$p_2 = \frac{1}{3}\sigma(0)b^2A_2\left(1 - \frac{A_1}{A_2}\eta\right)\left\{1 + 4A_1(1-e^2)\frac{e^2A_2 + [(1-e^2)\eta - 1]A_1}{e^2(A_2 - A_1)^2}\right\}, \tag{8}$$

The quantity Λ characterizes the magnitude and direction of internal flows and is related to the vorticity by the relation $\zeta = \mathrm{curl}_z\, \mathbf{v} = -\frac{a^2+b^2}{ab}\Lambda$. We note that there is a misprint in Eq. (15) in Ref. 10; Eq. (6) presented here should be taken as the correct expression.

3. DERIVATION OF THE DISPERSION RELATION

An equation for linear bending oscillations of a disk was derived in Ref. 8. We write it in the form

$$\left(\frac{\partial}{\partial t}+\bar{v}_x\frac{\partial}{\partial x}+\bar{v}_y\frac{\partial}{\partial y}\right)^2 h=F_\perp(h)-\frac{1}{\sigma}\left(\frac{\partial h}{\partial x}\frac{\partial}{\partial x}\sigma\overline{v_x'^2}+\frac{\partial h}{\partial y}\frac{\partial}{\partial y}\sigma\overline{v_y'^2}+\sigma\overline{v_x'^2}\frac{\partial^2 h}{\partial x^2}+\sigma\overline{v_y'^2}\frac{\partial^2 h}{\partial y^2}\right), \tag{9}$$

whre $h = h(x, y, t)$ is the vertical displacement of a disk element, $F_\perp(h)$ is the vertical component of the gravitational force exerted on a disk element by the entire bent disk, and $\bar{v}_x$, $\bar{v}_y$, and $\overline{v_y'^2}$ are the velocities of the centroids and the velocity dispersions of the equilibrium disk. Equation (9) has a fairly transparent physical meaning, and its structure obviously does not change when it is written in a rotating coordinate system.

To fine the force $F_\perp$ we use the approach taken by Hunter and Toomre,[11] who studied the bending instability of cold round disks. A disk element with coordinates x', y' exerts a force $G\,\sigma(x', y')/|\mathbf{r}-\mathbf{r}'|^2$ (neglecting terms $\sim h^2$) on a test particle with coordinates x, y. The vertical component of this force is $f_\perp = G\,\sigma(x', y')[h(x', y', t) - h(x, y, t)]/|\mathbf{r}-\mathbf{r}'|^3$. On the basis of this form of the force $f_\perp$, we can interpret $F_\perp$ to be the force exerted by a double layer in which the density of dipoles distributed over the unperturbed disk is

$$\tilde{\sigma}=\sigma(x', y')\,[h(x', y', t)-h(x, y, t)]. \tag{10}$$

Below we shall use the representation $F_\perp = F_1 + F_2$, where F_1 is the force exerted by a double layer with a density $\tilde{\sigma}_1 = \sigma(x', y')h(x', y', t)$, and F_2 is due to a layer with $\tilde{\sigma}_2 = -\sigma(x', y')h(x, y, t)$. The potential of each of the forces F_1 and F_2 undergoes a discontinuity at the disk plane, but the total potential is continuous, since the equivalent dipole density $\tilde{\sigma}$, which depends on the point x, y, vanishes at this point ($F_\perp$ is continuous, being the force exerted by a double layer).

In this paper we consider large-scale, "plane–second-order surface" bending deformations in which the direction of the rotation axis is constant, i.e., we take

$$h(x, y, t)=(\alpha x^2+i\beta xy+\gamma y^2+\delta)\,e^{-i\omega t}. \tag{11}$$

Let us find the force F_1. Since we are dealing with an elliptical disk, to find the external potential it is convenient to transform to ellipsoidal coordinates λ, μ, ν, taking the disk (1) for the primary ellipsoid. Then the relation to Cartesian coordinates has the form (see, e.g., Ref. 12, as well as Ref. 9)

$$x^2=\frac{(a^2+\mu)(a^2+\nu)(a^2+\lambda)}{(a^2-b^2)a^2},$$
$$y^2=-\frac{(b^2+\mu)(b^2+\nu)(b^2+\lambda)}{(a^2-b^2)b^2},$$
$$z^2=\frac{\lambda\mu\nu}{a^2b^2}, \tag{12}$$

where $a^2 \le \nu \le -b^2 \le \mu\ 0 \le \lambda$. As is well known, the Laplace equation can be separated in ellipsoidal coordinates and its solutions can be expanded in basis functions of the form

$$\Phi_n{}^m=F_n{}^m(\lambda)E_n{}^m(\mu)E_n{}^m(\nu),\quad n=0,1,\ldots,\ m=1,2,\ldots,2n+1, \tag{13}$$

where E_n^m are ellipsoidal Lamé functions of the first kind and F_n^m are Lamé functions of the second kind, which may be expressed in terms of E_n^m as follows[12]:

$$F_n{}^m(\lambda)=(2n+1)E_n{}^m(\lambda)\int\limits_\lambda^\infty\frac{ds}{[E_n{}^m(s)]^2\sqrt{(a^2+s)(b^2+s)s}}. \tag{14}$$

Since the potential being sought is that of a double layer, which undergoes a discontinuity in the plane $z = 0$, its expansion involves functions that are antisymmetric in z. We seek the potential Φ_1 corresponding to the density $\tilde{\sigma}_1$ in the form

$$\Phi_1=\left[\sum_{m=5}^{7}D_{m-4}F_3{}^m(\lambda)E_3{}^m(\mu)E_3{}^m(\nu)+D_4F_1{}^3(\lambda)E_1{}^3(\mu)E_1{}^3(\nu)\right]e^{-i\omega t}, \tag{15}$$

where

$$E_3{}^5(\lambda)=(\lambda+k_1)\sqrt{\lambda},$$
$$E_3{}^6(\lambda)=(\lambda+k_2)\sqrt{\lambda},$$
$$E_3{}^7(\lambda)=\sqrt{(a^2+\lambda)(b^2+\lambda)\lambda},$$
$$E_1{}^3(\lambda)=\sqrt{\lambda}, \tag{16}$$

and $k_{1,2}=\frac{2}{5}(a^2+b^2)\pm\frac{1}{5}\sqrt{4(a^4+b^4)-7a^2b^2}$. The Lamé derivatives in the plane $z = 0$ may be expressed in terms of the Cartesian coordinates as follows:

$$E_3{}^5(\mu)E_3{}^5(\nu)\,|_{\lambda=0}=\pm abG_3{}^5(x, y)\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}},$$
$$E_3{}^6(\mu)E_3{}^6(\nu)\,|_{\lambda=0}=\pm abG_3{}^6(x, y)\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}},$$
$$E_3{}^7(\mu)E_3{}^7(\nu)\,|_{\lambda=0}=\pm abG_3{}^7(x, y)\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}},$$
$$E_1{}^3(\mu)E_1{}^3(\nu)\,|_{\lambda=0}=\pm ab\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}, \tag{17}$$

where

$$G_3{}^5(x, y)=k_{11}x^2+k_{12}y^2+k_{11}k_{12},$$
$$G_3{}^6(x, y)=k_{21}x^2+k_{22}y^2+k_{21}k_{22},$$
$$G_3{}^7(x, y)=ik_{33}xy, \tag{18}$$

$$k_{11}=k_1-b^2,\ k_{12}=k_1-a^2,\ k_{21}=k_2-b^2,\ k_{22}=k_2-a^2,\ k_{33}=a^2-b^2.$$

The plus sign in (17) corresponds to the limit $z \to +0$ and the minus sign to $z \to -0$. We expand $h(x, y, t)$ in the polynomials (18),

$$h(x, y, t)=\left[\sum_{m=5}^{7}d_{m-4}G_3{}^m(x, y)+d_4\right]e^{-i\omega t}. \tag{19}$$

Using the property of the potential of a double layer[12] that $\Phi_{1(+0)} - \Phi_{1(-0)} = -4\pi G\,\tilde{\sigma}_1$, we find the coefficients D_j in the expansion (15): $D_{m-4} = -\frac{2\pi G\sigma(0)d_{m-4}}{abF_3{}^m(0)}$, $m = 5, 6, 7$, $D_4=-\frac{2\pi G\sigma(0)d_4}{abF_1{}^3(0)}$. Substituting these coefficients into Eq. (15) and differentiating with respect to z, we find

$$F_1=-\left.\frac{\partial\Phi_1}{\partial z}\right|_{z=0}=-\left[\frac{1}{h_\lambda}\frac{\partial\Phi_1}{\partial\lambda}\right]_{\lambda=0}=2\pi G\sigma(0)\times\left[\sum_{m=5}^{7}d_{m-4}\Delta_3{}^mG_3{}^m(x, y)+d_4\Delta_1{}^3\right]e^{-i\omega t}, \tag{20}$$

where

$$\Delta_n{}^m=\left[\frac{\sqrt{\mu\nu}}{abh_\lambda}\frac{dF_n{}^m(\lambda)}{d\lambda}\right]_{\lambda=0}\Big/\,F_n{}^m(0) \tag{21}$$

and $h_\lambda{}^2=\dfrac{(\lambda-\mu)(\lambda-\nu)}{4(a^2+\lambda)(b^2+\lambda)\lambda}$ is the coefficient of $(d\lambda)^2$ when the metric is written in ellipsoidal coordinates. Calculating Δ_n^m directly in (21) using Eqs. (14) and (16), we find

$$\Delta_3{}^5=K(\tilde{k})\frac{b}{a^2}\left[1-\frac{k_1}{k_{12}}-\frac{a^2}{2k_{12}}\left(1+\frac{k_1}{k_{12}}\right)\right]-E(\tilde{k})\frac{b}{a^2}\left(\frac{a^4}{2k_{11}k_{12}}+\frac{a^2}{b^2}\right)$$
$$+\Pi\left(1-\frac{k_1}{a^2},\tilde{k}\right)\frac{b}{2a^2}\left(\frac{3a^2}{k_{12}}+\frac{a^2k_1}{k_{11}k_{12}}+\frac{a^2k_1}{k_{12}{}^2}\right),$$
$$\Delta_3{}^6=K(\tilde{k})\frac{b}{a^2}\left[1-\frac{k_2}{k_{22}}-\frac{a^2}{2k_{22}}\left(1+\frac{k_2}{k_{22}}\right)\right]-E(\tilde{k})\frac{b}{a^2}\left(\frac{a^4}{2k_{21}k_{22}}+\frac{a^2}{b^2}\right)$$
$$+\Pi\left(1-\frac{k_2}{a^2},k\right)\frac{b}{2a^2}\left(\frac{3a^2}{k_{22}}+\frac{a^2k_2}{k_{21}k_{22}}+\frac{a^2k_2}{k_{22}{}^2}\right),$$
$$\Delta_3{}^7=K(\tilde{k})\frac{b}{a^2}\left[\frac{a^2(a^2+b^2)}{(a^2-b^2)^2}\right]-E(\tilde{k})\frac{b}{a^2}\left[\frac{a^2}{b^2}\frac{a^4+b^4}{(a^2-b^2)^2}+\frac{a^2}{b^2}\right]$$
$$\Delta_1{}^3=-\frac{E(\tilde{k})}{b}, \tag{22}$$

where $\tilde{k} = 1 - b^2/a^2$; $K(\tilde{k})$, $E(\tilde{k})$, and $\Pi(\ell, \tilde{k})$ are complete elliptic integrals of the first, second, and third kinds.

Let us find the force F_2. We seek the potential corresponding to the density $\tilde{\sigma}_2$ in the form

$$\Phi_2=AF_1{}^3(\lambda)E_1{}^3(\mu)E_1{}^3(\nu). \tag{23}$$

Using the property of the potential of a double layer that $\Phi_2(+0)-\Phi_2(-0) = -4\pi G\tilde{\sigma}_2$, we find $A = \dfrac{2\pi G\sigma(0)h(x,y,t)}{abF_1{}^3(0)}$. Hence we have

$$F_2=-\frac{\partial\Phi_2}{\partial z}\Big|_{z=0}=-\left[\frac{1}{h_\lambda}\frac{\partial\Phi_2}{\partial\lambda}\right]_{\lambda=0}$$
$$=-2\pi G\sigma(0)\Delta_1{}^3\left[\sum_{m=5}^{7}d_{m-4}G_3{}^m(x,y)+d_4\right]e^{-i\omega t}. \tag{24}$$

Combining F_1 and F_2, we obtain

$$F_\perp=2\pi G\sigma(0)\sum_{m=5}^{7}d_{m-4}\tilde{\Delta}_3{}^mG_3{}^m(x,y)e^{-i\omega t}, \tag{25}$$

where $\tilde{\Delta}_3{}^m = \Delta_3{}^m - \Delta_1{}^3$.

Substituting this expression for $F_\perp$, as well as those for $\bar{v}_x$, $\bar{v}_y$, $\overline{\sigma v_x'^2}$, $\overline{\sigma v_y'^2}$ and $h(x, y, t)$, into Eq. (9) and equating the coefficients to x^2, ixy, and y^2 to zero in the resulting polynomial, we obtain the following system of linear homogeneous equations for d_1, d_2, and d_3:

$$\sum_{j=1}^{3}a_{ij}d_j=0,\quad i=1,2,3, \tag{26}$$

where

$$a_{11}=(2\alpha_1\alpha_2-\omega^2)k_{11}+2\alpha_2{}^2k_{12}-2\pi G\sigma(0)\Delta_3{}^5k_{11}-\frac{2}{\sigma(0)a^2}(4p_1k_{11}+p_2k_2),$$
$$a_{12}=(2\alpha_1\alpha_2-\omega^2)k_{21}+2\alpha_2{}^2k_{22}-2\pi G\sigma(0)\Delta_3{}^6k_{21}-\frac{2}{\sigma(0)a^2}(4p_1k_{21}+p_2k_{22}),$$
$$a_{13}=2\omega\alpha_2k_{33},$$
$$a_{21}=(2\alpha_1\alpha_2-\omega^2)k_{12}+2\alpha_1{}^2k_{11}-2\pi G\sigma(0)\Delta_3{}^5k_{12}-\frac{2}{\sigma(0)b^2}(4p_2k_{12}+p_1k_{11}),$$
$$a_{22}=(2\alpha_1\alpha_2-\omega^2)k_{22}+2\alpha_1{}^2k_{21}-2\pi G\sigma(0)\Delta_3{}^6k_{22}-\frac{2}{\sigma(0)b^2}(4p_2k_{22}+p_1k_{21}),$$
$$a_{23}=2\omega\alpha_1k_{33},$$
$$a_{31}=4\omega(\alpha_1k_{11}+\alpha_2k_{12}),$$
$$a_{32}=4\omega(\alpha_1k_{21}+\alpha_2k_{22}),$$
$$a_{33}=\left[\omega^2-4\alpha_1\alpha_2+2\pi G\sigma(0)\Delta_3{}^7+\frac{3}{\sigma(0)}\left(\frac{p_1}{a^2}+\frac{p_2}{b^2}\right)\right]k_{33}, \tag{27}$$

(equating the free term in the polynomial to zero enables us to find d_4 and thus the form of the eigenfunctions of Eq. (9)). Consequently, the desired dispersion relation has the form

$$|(a_{ij})|=0. \tag{28}$$

4. RESULTS

Equation (28), reduced to a bicubic equation for $\omega/\sqrt{2\pi G\sigma(0)/a}$, was solved numerically. The domains of stability of elliptical Freeman disks against large-scale bending perturbations that were found using this solution are presented in tables I and II and in Figs. 1–3. There are two stability domains, shown in Fig. 1. Here we also give contours of equal values of the Ostriker–Peebles parameter $t_{OP} = E_{rot}/|W|$, taken from Ref. 3. In Figs. 2 and 3 the domains are shown on an enlarged scale, and the boundary of the domain of stability against bar-like perturbations found by Tremaine[9] is also given in Fig. 3. The parameters of the limiting models are given in Tables I and II (with a step 0.01 in b/a). The values of t_{OP} given in Tables I and II were calculated from the model parameters using the equation

$$t_{OP}=\frac{E_{rot}}{|W|}=\frac{(\Lambda^2+\Omega^2)(a^2+b^2)-4ab\Omega\Lambda}{2F(\tilde{k})}. \tag{29}$$

For b/a = 1 we solved Eq. (28) analytically after expanding all the parameters in $\tilde{k}^2 = 1 - b^2/a^2$ and proceeding to the limit $\tilde{k} \to 0$.

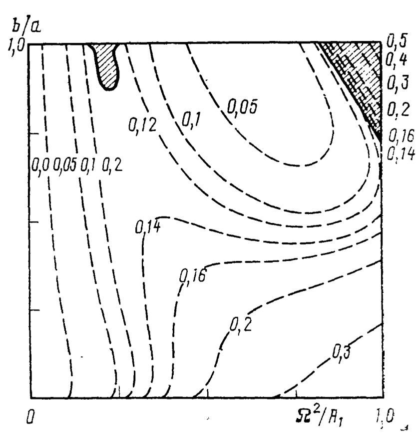

FIG. 1. Domains of stability (hatched) of elliptical Freeman disks against large-scale bending perturbations.

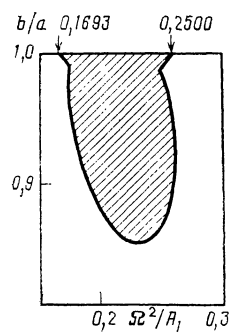

FIG. 2. Left-hand domain of stability of elliptical Freeman disks against large-scale bending perturbations.

TABLE I. Limiting Parameters for the Right-Hand Domain of Stability of Elliptical Disks Against Quadratic Bending Perturbations

b/a	η	t_{OP}	b/a	η	t_{OP}	b/a	η	t_{OP}
1	η_1 *	1/8	0,91	0,889	0,1285	0,82	0,950	0,1395
0,99	0,837	0,1252	0,90	0,896	0,1293	0,81	0,957	0,1414
0,98	0,843	0,1253	0,89	0,902	0,1302	0,80	0,964	0,1431
0,97	0,850	0,1254	0,88	0,909	0,1313	0,79	0,971	0,1450
0,96	0,856	0,1258	0,87	0,916	0,1325	0,78	0,978	0,1470
0,95	0,863	0,1261	0,86	0,923	0,1336	0,77	0,985	0,1491
0,94	0,869	0,1265	0,85	0,929	0,1350	0,76	0,992	0,1512
0,93	0,876	0,1270	0,84	0,936	0,1364	0,75	0,999	0,1534
0,92	0,882	0,1277	0,83	0,943	0,1380	0,7481	1,000	0,1539

* $\eta_1 = \frac{1}{2} + \frac{\sqrt{7}}{8}$.

TABLE II. Limiting Parameters for the Left-Hand Domain of Stability of Elliptical Disks Against Quadratic Bending Perturbations

b/a	η	t_{OP}	b/a	η	t_{OP}	b/a	η	t_{OP}
1	η_2 * 1/4	1/8 1/8	0,94	0,180 0,257	0,1256 0,1259	0,88	0,198 0,252	0,1270 0,1281
0,99	0,174 0,248	0,1251 0,1251	0,93	0,182 0,257	0,1257 0,1262	0,87	0,204 0,250	0,1276 0,1285
0,98	0,173 0,252	0,1251 0,1251	0,92	0,185 0,257	0,1260 0,1265	0,86	0,210 0,245	0,1281 0,1289
0,97	0,174 0,254	0,1251 0,1253	0,91	0,187 0,256	0,1261 0,1269	0,85	0,219 0,237	0,1284 0,1292
0,96	0,176 0,256	0,1254 0,1255	0,90	0,191 0,255	0,1265 0,1273	0,84	– –	– –
0,95	0,178 0,256	0,1254 0,1257	0,89	0,194 0,254	0,1267 0,1277			

* $\eta_2 = \frac{1}{2} - \frac{\sqrt{7}}{8}$.

As we see, the condition of stability against bending perturbations for round disks, $t_{OP} < 0.125$, is not satisfied for elongated disks. For a ratio of semiaxes $b/a \lesssim 0.85$ no stable models exist for any possible rotational energy. Models that are stable against bending perturbations (down to $b/a \approx 0.75$) are unstable against bar-like perturbations and vice versa. The limiting value $b/a \approx 0.85$ is taken for models from the left-hand domain in Fig. 1, which is considerably larger (measured in the plane of the parameters b/a and Ω^2/A_1) than the intersection of the right-hand domain with the domain of stability against bar-like perturbations. For models from this intersection, as seen in Fig. 3, b/a is even larger. Models from the left-hand and right-hand domains that have the same t_{OP}

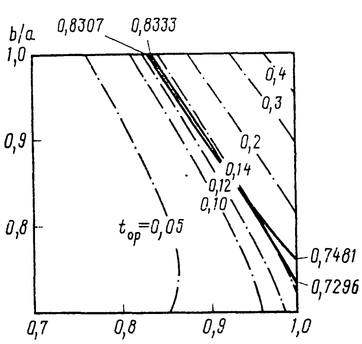

FIG. 3. Boundaries of the right-hand domain of stability against large-scale bending perturbations and boundary of the domain of stability against bar-like perturbations.

differ in the nature of the internal flows. In models from the left-hand domain the direction of internal flows coincides with the direction of rotation of the boundary; counterflows exist in models from the right-hand domain.

The instability of elliptical disks against small-scale bending perturbations follows from studies of the anisotropic instability of an infinite sheet (references and a presentation of the results can be found in Ref. 7). It is assumed that the development of such instability leads to heating of the disk in the direction perpendicular to the plane and its thickening. This thickness still remains small, however, so that the thin-disk model can be used to study processes in the plane of rotation. A large-scale bending instability becomes saturated, as one might suppose, upon reaching a thickness comparable with the "width" of the system. Thus, from the results obtained in the present work we may conclude that fairly elongated (with $b/a \lesssim 0.85$) self-gravitating systems should be represented by elongated ellipsoids of rotation rather than elliptical disks. In any case, a conclusion about the shape of fairly elongated systems should be based upon a calculation of the large-scale stability of triaxial ellipsoidal models.

Considering the location of the bar in SB galaxies and galaxies in compact groups, it would seem to be worthwhile to carry out a similar study of the stability of elliptical disks in a tidal force field, as well as disks embedded in systems representing the different dynamical components of galaxies. It is fairly trivial to generalize the dispersion relation (28) in these cases. We propose to publish the results of such a study in the future.

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