# THE TIDES AND NEWTON

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## From the "Principia" to Laplace

Newton succeeded in doing for the tides what Galileo had done for the pendulum and the free fall of bodies: he isolated the effects of the main cause from those of the "incidental" causes; with a stroke of genius, perhaps among the most original to be found in the Principia, he isolated the latter, which often make the solution of problems difficult or even impossible, in order to explain what we may call the fundamental phenomenon, after many vain efforts and incredible raving of the philosophers and scientists who had preceded him. Newton enunciated the principle most concisively, almost with indifference, and to arrive at a clarification and a first theory, it was necessary, as it has been in many other cases, to wait for a change in generations.

The first theory based on Newton's enunciation was a static theory, obtained at the price of many simplifications, about which we shall speak later on. It was Laplace at the end of the 18th century who, not without other important abstractions, began to speak of a dynamic theory and lay the bases for it. And Laplace realized that the theory could determine the period of the phenomenon but not its phase (for example the instant of the maximum) and its entity (in the sense of the difference between maximum and minimum levels); these amplitudes, port by port, had to be inferred from a long series of observations, that is to say, empirically. But, to repeat, this depends on the large number of what we have called "incidental" causes: Newton had already singled out the principal cause completely and definitely, even though it had been expressed quite succintly.

## The Greeks and the Movements of the Sea

Of the attempts to find an explanation previous to Newton, a detailed review was made by Roberto Almagià even before he received his degree and went on to become one of our most important historians in the fields of geography and map-making. His paper was entitled La dottrina della marea nell'antichità classica e nel Medio Evo.

This paper of 140 pages was presented to the Reale Accademia dei Lincei in June 1905 by members Della Vedova[1] and Schiaparelli[2]. In this long report Almagià took into consideration ancient and medieval texts, citing the most significant passages and examining them closely up to Leonardo Da Vinci, then in a more general way up to Newton.

Almagià found among the Greeks scarce and belated knowledge of the tides and attributed this to two circumstances. Firstly, in the Mediterranean tides are slight, especially in the central part where the Greeks concentrated their activities. He gave some values of the maximum height of the tides deduced from the proceedings of the first Italian Geographical Congress which took place in 1892:

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| Corfu | 6 cm |
| Taranto | 4 " |
| Brindisi | 9 " |
| Reggio Calabria | 13 " |
| Naples | 22 " |

But on moving to the more distant coasts where special conditions are found, the values are higher:

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| Tripoli in Syria | 40 cm |
| Venice | 48 " |
| Trieste | 53 " |
| Malaga | 77 " |
| Sfax (Tunisia) | 125 " |
| Djerba (Tunisia) | 183 " |

The fact is that the first sure mention of tides that has come down to us, thanks to Herodotus, refers to the Red Sea (II,11,2):

ρηχιη [•••] καὶ αμπωτις ἀνὰ πασαν ἡμέρην γίνεται

which is to say: "every day there is ebb and flow".

Secondly, the Greeks were certainly put off the track by the observation of many sea currents with characteristics due only in part to tide; for example the strong, alternating currents in the Straits of Messina on the opposite shores (Scylla and Charybdis) and the much more frequent currents in the narrow channel separating Attica from Euboea in correspondence to the city of Khalkis, what the Greeks called Ευ ρ ι π ο and the Romans Eurīpus. This strait, which Livy defined as the Thermopylae of the sea, after being widened a century ago to allow the passage of steamships, is now thirty meters wide and eight meters deep and in reality is not particularly spectacular. But it is of interest because of its currents, which are quite frequent and have unequal periods. The ancients, without objective points of reference, claimed that it alternated seven times every day and seven times every night: one of the many cases in which, rightly or wrongly, the number seven is involved in rhythmic and serial phenomena.

Another anomaly in our seas is the "acqua alta" in the northern Adriatic which manifests itself especially in Venice, and which has become notorious for the consequences it has had on this historic city, whose condition has become more critical since in the last few decades, before the closing of the wells, it has subsided to an alarming degree. This phenomenon has been observed and studied with great care in relation to the period of oscillation of the Adriatic[3]. In fact, here we have a prevalent resonance phenomenon which can be enhanced by winds, atmospheric pressure and, in Venice, even by rainfall. In Venice, the tides as such contribute very little to the high water; in November 1966, the record and most disastrous high water level fortunately coincided with the lowest tide. Also to be mentioned is the Port of Leningrad where, although there is no tide to speak of, changes of up to two meters in level, caused by atmospheric phenomena, have taken place.

Tides and currents are more closely connected in the Straits of Messina where two fairly large seas, whose moments of high and low tide certainly do not coincide, come together. It was not easy to explain why the current alternates, passing from one side of the strait to the other. The rapid oscillations that take place in the two small basins between Euboea and Attica, separated by the extremely narrow and shallow Euripus, are also somehow connected with the tides. Because of their size, these basins must have their own very small, and quite different, period of <u>oscillation</u> which is, in any case, set off by the tidal wave arriving at the mouth, through which they both communicate with the open sea. Given the distance of the two mouths, the tidal wave reaches them at different times and the two basins, on beginning to oscillate, usually reach the highest levels at different times, thus causing the frequent and unequal pouring of the one into the other.

It is quite possible that to the ancients this must have appeared as inexplicable, and there is a legend to the effect that Aristotle, in exile at Khalkis and exasperated by his failure to find an explanation for the phenomenon, drowned himself in the Euripus.

## The Difficulty of Finding a Rational Solution

To the two reasons mentioned by Almagià, perhaps a third can be added, which may further explain the "scarce and belated" knowledge of the Greeks concerning tides.

Up to not so long ago tides were an enigma. In their simplest form, uncomplicated by concomitant causes of different origins, they follow the pattern of a high tide taking place shortly after the passage of the Moon at the meridian followed after about six hours by a low tide which is in turn followed by another high tide about twelve hours later. Now, after twelve hours, the Moon is on the opposite side, below the horizon to the north. It is quite reasonable to suppose that in the context of Greek rationalism it must have appeared impossible to attribute to the same cause, the Moon, two contrasting effects (today we would say of opposite sign). In any case, based on what has come done to us, no pre-Socratic sources speak of tides, although a lexicon for tides was already beginning to appear: πλημμρι or πλησμη as well as ρηχιη for flood tide; αμπωσι and αναπωσι from αναπινω = suck in or drink again) for ebb tide; παλιρροια = fluctuation, for tide; but these words were not used specifically and exclusively for the phenomenon under discussion.

As regards theory, the first to attempt an explanation appears to have been Plato, who hypothesized caves and undersea cavities from which these currents came and to which they returned. At least this is what later authors have stated.

In the time between Plato and Aristotle there was the diffusion of the observations of those who had participated in Alexander's expeditions to the Indian Ocean. At the same time, or some time later, Pytheas, who was believed to differing degrees, brought his observations from the Atlantic, explicitly attributing the phenomenon to the Moon. Aristotle held a different opinion which we can reconstruct above all from what Plutarch and the apologists of the Greek philosophers had to say. Tides, which our seas can reflect only to a limited degree, are typical of the oceans and are caused by winds driven by the Sun. Thus the Sun with its heat, and not the Moon, is to be considered responsible for the phenomenon. It appears to me that here we can recognize one of the characteristic features of the Classic Greek philosopher, who had always and invariably refused to acknowledge celestial "influences", including those of the Moon; while in the East these influences had been elaborately theorized several centuries before.

## The Astrologers' Explanation

Almagià went on with his review without taking into account the profound changes that had taken place in the Hellenistic age. Vitruvius tells us that Berosus, a famous historian and Chaldean priest, founded a school in Cos[(4)], probably towards the middle of the 3rd century, at the height of the Hellenistic period. From that renowned island Berosus began his teaching, which was not without religious overtones, explaining the mechanisms of celestial influences determined by the "aspects" of the stars and particular positions of the sky. To be brief, Greece too was conquered by the astrologers, a conquest favoured by the popularity enjoyed by Stoic philosophy, which was the first to introduce into Greek thought the fatalistic concept of Divine Providence; but we know very well that the Greek world made profound changes in Chaldean astrology and adapted this pseudo-science to its own way of thinking, which was more practical and individualistic.

From that moment on, the "influence" of the Moon was no longer surprising. It became detached from physical causes and disregarded models of a mechanistic type. If this mysterious influence agreed with observational data it was certainly accepted with few questions asked.

Research in the field of the tides thus became freer, and the two great founders of mathematical and physical geography, Eratosthenes in the third century and Posidonius between the second and the first centuries B.C., correctly described tides by connecting them with the phases of the Moon, that is to say, with the positions of the Moon relative to the Sun. It is to be remembered that Posidonius, in order to perfect his observations of the tides, resided at length in Cadiz, in the south of Spain, where the ocean opens out upon an unlimited horizon just outside the pillars of Hercules. Very few of the works of these two great philosophers have come down to us: there is, however, reason to believe that they produced a complete description of the tides and that if their work had been passed on to and continued by disciples capable of understanding them in their original form and handing them down in turn, they would probably have remained without equal up to the 18th century, although it is legitimate to suppose that these two did not really attempt a complete explanation of the mechanism at the base of this complex phenomenon. In any case, most of their conclusions, perhaps with some additions and misunderstandings, are to be found in Pliny, who repeats that the cause of tides is "in Sole Lunaque".

Approximately two centuries after Posidonius another step forward was represented by the works of Ptolemy.

Ptolemy did not speak of tides either in his astronomical or his geographical works, but in the Τετραβιβλοσ, the summa of knowledge concerning the celestial "aspects" and their influences, the work that was to become the "bible" of the astrologers. Ptolemy, unlike Eratosthenes and Posidonius, was not so much interested in describing the phenomenon in the greatest detail as he was in establishing the fact that tides were the physical and logical proof of the influence of the stars, and in particular that they were the consequence of the Moon's influence, to which the weaker influence of the Sun was to be added or subtracted.

From that time on it was generally admitted that the Moon, the ruler over humours and Dame of all kinds of different flows (including the menstrual flow), was the agent most responsible for tides. Every time opposition to the pseudo-science of astrology arose and attempts were made to deny its validity as being without foundation (for example by Pico della Mirandola or Leonardo Da Vinci) believers brought up the Aristotelian tradition which attributed the main cause to the Sun, whose heat caused the winds, which in turn raised up and pushed the waters of the ocean.

Substantially, the astrological explanation did not change even when the Arabs integrated it with effects of magnetic origin. And from the middle of the 13th century the physical phenomena proving the influence of the heavenly bodies became two: the tides and the magnetic needle, one end of which, in the words of Pietro Peregrino, was "influenced" by the north pole of the sky and, because it turned in that direction, was called the north pole of the magnet, which turned out to be incoherent but nevertheless has survived.

The astrological explanation is defended by Dante who has Cacciaguida say that the cause of the tides was "il volger del cielo della luna", which "Cuopre e discuopre i liti senza posa"(5), with an expression which is perhaps more cultured and up-to-date than it appeared to Almagià and most of the annotators of the "Commedia".

Astrology became bereft of the support of these two hypothetical proofs in less than a hundred years' time at the hands of Gilbert at the beginning of the 17th century and Newton towards the end of the same century.

## Newton and His Interpreters

We are indebted to Newton for the principle on which tides are based. As has been said, the first theory advanced was the static theory of tides attributed to Daniel Bernoulli, Leonhard Euler and Colin Maclaurin.

Daniel Bernoulli (1700-1782), the second son of the first Jean of this family and also his pupil, was the author of Hydrodynamica, completed in 1734, but published only in 1738.

Euler (1707-1783) was another of Jean Bernoulli's pupils, and for some time studied and carried on research together with Daniel, whom he succeeded in the chair of mathematics at St. Petersburg in 1733.

The Scotsman Maclaurin (1698-1746) was the first great mathematician in the British Isles to develop and divulge the works of Newton, whose pupil he was. In 1740, he published an essay entitled De causa physica fluxus et refluxus maris, with which he competed for a prize offered by the Royal Society. Daniel Bernoulli and Leonhard Euler also took part in this competition, upon the conclusion of which the jury decided to divide the prize into three equal parts, proclaiming all three the winners. In their studies of tides, all three had started from Proposition 24, Theorem 19 of the third volume of the Principia, which concerns the ebb and flow of the sea originating from the actions of the Sun and Moon. Despite the order in which these two celestial bodies are mentioned in the title, Newton points out at the beginning of the rather brief esposition that the influence of the Moon, as experience teaches us, is greater than that of the Sun, because their effects are inversely proportional to the cube of their apparent diameters. It is as if to say that they decrease with the square of the distance, but the form adopted allows for easy checking. In fact, when once a year the Sun, and once a month the Moon are at their perigees and therefore are at the maximum diameter of the cycle, they cause a more accentuated tide; furthermore, the increase in the tide when the two bodies are closest to the celestial equatoris described minutely. The more delicate question is however, almost neglected. Newton, on referring only to the gravitational effect of the Moon, shows with the aid of a figure that it deforms the mass of water into an ellipsoid, stating that the entire sea undergoes two hemispheric flows, one in the hemisphere to the north the other in the opposing hemisphere. And he underlines that these two flows are always opposite one to the other. Nothing more.

On concluding, Newton also contemplates the special case in which the tidal wave is propagated towards the same port through different straits. In such a case it may be that the two flows become equal, that is to say, arrive in phase opposition and therefore in such a port the water will cease to flow and remain quiet. Or there may be agreement of alternating phase and opposition phase, in which case over the twenty-four hour period there will be a high tide only. He then gives examples taken mostly from observations of the oceans and in different ports, from metropolitan ones to others on the other side of the globe. Newton wished to show that he had gathered his data carefully and that all the numerous anomalies were easily explanable and could be traced back to the basic phenomenon; but for the deformation of our planet into an ellipsoid, he substantially refers to Proposition 66 in the first volume (if three bodies, whose forces decrease in proportion to the square of the distances, they mutually attract one another, etc.), and in particular he cites Corollaries 19 and 20, where he posits a body T which, after being considered completely solid, is made to rotate while covered with water in the presence of the gravitational attraction of a body S which is at a fairly great distance. As the result of this gravity the water assumes a motion of ebb and flow, which may be more or less mitigated.

I believe that readers of that time, on finding extremely clear explanations, as are those concerning resonance and antiresonance phenomena that may occur in certain ports and those of the many causes that make normal tides differ widely one from another, side by side with others of far less clarity, must have found it rather difficult to follow the explanation, which is implicit and certainly fundamental for Newton himself, of that which had always been the most enigmatic, that is, the explanation of why, if we have a high tide in a point P of the globe we have it also at point p, diametrically opposite to P or, as was said then as well as today, at the antipodes of P.

The three mathematicians who won the prize in 1740 had the bad luck to receive only a third of it. However, if the solution had been presented by only one of them, I am not at all sure that the jury would have been persuaded to award the prize to him.

## A Key Solution

One is perplexed by Newton's undoubted wish to succeed in being comprehensible only to the few who could manage to overcome the objective difficulties. One could say that he wished to assure himself of priority without going into too many details or explanations.

The problem of the causes of semidiurnal tides has been of great importance in the history of science, and in particular in the economy of the *Principia* and in the affirmation of gravitation. The fable of the apple falling on Newton's head has enjoyed undeserved fame and has transformed the pieces of the tree from which it supposedly fell into sacred relics. It cannot be excluded that at the origin of the story was a bit of sarcasm aimed at the great non conformist scientist, just as there was undoubtedly much irony in the story of Archimedes' bath and Buridan's ass (which, among other things, was not even original). The falling apple certainly shows that heavy bodies are attracted by the Earth, but this had always been known. What it does not show is that the apple also attracts the Earth.

Thus, in the cosmic equilibrium prospected by Newton, the Sun's gravitational attraction on the Earth had a twofold demonstration: the Sun's gravity keeps the Earth in its planetary orbit and at the same time raises a weak tidal wave on it. But if we consider the Earth-Moon system, we finally have the first evident example of reciprocity, the reciprocity on which gravitation and the third law of mechanics, placed together with the other two at the beginning of the work preceding the first volume, among the axioms or laws of movement, are based. The third law states, as is known, that to every action corresponds an equal and opposite reaction. In the Earth-Moon system we necessarily have the effect of the Earth's gravity which forces the Moon, with a very strong action, to follow its satellite orbit; at the same time we have the effect, which for us is quite evident, of the Moon's gravitational pull on the Earth which deform it because of the systematic raising up of its waters, and this deformation follows, with a slight lag, the orbiting of the Moon, adding to, or subtracting from, the slighter deformation induced by the Sun.

It appears to me rather obvious that the Principia follow, in their exposition, a logical development that does not follow a chronological process. On the other hand Newton, contrary to Kepler, was reluctant to explain the genesis of his ideas and intuitions. As a young man, Newton was visibly attracted by complex problems connected with the real world. In the British Isles the tides were of the greatest importance because of the movements they caused in ports and the consequences of the brief but impetuous floods caused by rivers. In the first thirty years of the 17th century the problem of tides with a semidiurnal period had not been evaluated correctly either by Kepler or by Galileo, and in 1651 Riccioli, in Almagestum Novum, a work that Newton very probably had read, reviewed as many as seventeen different explanations of tides, discarding them all, but at the same time recognizing in almost all of them some measure of truth. And in conclusion Riccioli observed that the characteristics of this movement of the sea were so varied and surprising that it appeared that they would continue to stymie all attempts by the human mind to explain them.

This was a challenge that Newton accepted and won. And to me it does not appear impossible that in his approach to the law of gravity the solution to this problem was far more illuminating than his being hit on the head by a falling apple, if we are to believe that this ever happened.

Cagliari, 4th November 1988.

## N O T E S

(1) Giuseppe Dalla Vedova, who was born in Padua and who had been a professor in Rome since 1875, was at that time seventy-five years of age and was coming to the conclusion of a five-year term as president of the Società Geografica Italiana; he had earned his degree in Vienna and was remembered by his many students, and in particular by Almagià, as the person who gave new impetus to map-making and introduced scientific geography in Italy.

(2) The Piedmontese Giovanni Virginio Schiaparelli (he was from Savigliano) was a contemporary of Della Vedova's. He received his degree in hydraulics and architecture in Turin, but was also involved in astronomy even at that time, together with Plana. He immediately won a scholarship to study in Berlin, where he studied geography, astronomy, meteorology and ancient history with the greatest scholars of the time, before going on to Pulkov, the St Petersburg Observatory, together with the second exponent of the Struve dynasty. Following his return to Italy, in 1862 he became the director of the Brera Observatory in Milan, a position he held for forty years.

(3) S. Tibaldi, Dinamica delle onde lunghe mareali nell'Adriatico, Bologna, 1972.

(4) "Berosus was the first, he settled on the island and in the city of Cos, and there founded a school" (De Arch., IX, 7)

(5) Par. XVI, 82. Dante compares the alternating fortunes of Florence to the tides.