THE ASTROPHYSICAL JOURNAL, 347:112–117, 1989 December 1

NONLINEARITY OF THE TULLY-FISHER RELATION

JEREMY MOULD AND MINGSHENG HAN
Palomar Observatory, California Institute of Technology
AND
GREG BOTHUN
University of Michigan
Received 1989 February 13; accepted 1989 June 8

ABSTRACT

Because the fraction of dark matter in spiral galaxies varies with luminosity, or for a number of other possible reasons, the relation between infrared luminosity, and velocity width has previously been shown to be nonlinear. The relation between surface brightness and velocity width is now demonstrated to be nonlinear also. When these nonlinearities are taken into account, an apparent trend in surface brightness of cluster spirals with redshift is shown to be spurious. Apparent correlations of surface brightness and slope on cluster richness and cluster velocity dispersion also disappear. Neither this effect, nor an apparent trend in slope of the infrared Tully-Fisher relation with Hubble type, in themselves imply that the Tully-Fisher relation depends on the environment in which galaxies are located.

Subject headings: galaxies: photometry — galaxies: redshifts — radio sources: 21 cm radiation

I. INTRODUCTION

Aaronson, Huchra, and Mould (1979) pointed out that a set of rotationally supported galaxies of constant central surface brightness and constant mass-to-light ratio follow a Tully-Fisher (TF) relation of the form $L \propto V^4$. Since surface brightness and M/L underly the TF relation, it clearly makes sense to examine these properties of spiral galaxies more closely, in order to better understand the TF relation and the nature of its scatter. Kraan-Korteweg (1983) suggested that the loci of different clusters of galaxies in the surface brightness/velocity width plane might point to a variation of these distance-independent properties of galaxies with their environment. Aaronson *et al.* (1986) noted a disturbing tendency of the mean surface brightness of their cluster spirals to rise with increasing redshift. According to Silk (1983) and Blumenthal *et al.* (1984) the surface brightness/velocity width plane is directly related to the classic cooling diagram, in which the baryon density of a system is plotted against its virial temperature. A galaxy's location in this plane results from its past evolution and degree of dissipation.

We also have more knowledge now of the M/L systematics of disk galaxies. Tinsley (1981) showed that the relation of M/L with galaxy color could best be understood if late-type (dwarf) galaxies had a large fraction of their mass in dark form than their early-type (giant) counterparts. Aaronson *et al.* (1982*a*) found, when a large sample of galaxies was examined, that the infrared TF relation, (H, log ΔV), is not a linear relation: it is steeper for dwarf galaxies and shallower for giants. This is also understood if M/L_H varies systematically with velocity width, due to a variation of the dark matter fraction with galaxy mass (see also Salucci and Frenk 1988). Dependence of M/L_H on galaxy metallicity may also be a factor (Bothun *et al.* 1984).

In this paper we examine the nonlinearities of the surface brightness/velocity width relation (§ II), and the TF relation (§ III), and briefly discuss their origin and implication (§ IV).

II. THE SURFACE-BRIGHTNESS/VELOCITY-WIDTH RELATION

The H magnitudes of galaxies, their diameters, D, and velocity widths, ΔV, are the three crucial observables in studying the TF relation. While ΔV is distance independent, the other two are not, but they can together form another important distance-independent quantity, the mean surface-brightness, $\Sigma = H + 5 \log D$. It is known that Σ and ΔV are correlated. Aaronson *et al.* (1986) compared the $\Sigma/\Delta V$ correlations for the clusters of the Arecibo sample with the "fiducial relation" defined by the 308 member Local Supercluster galaxy sample of Aaronson *et al.* (1982*b*);

$$\Sigma_H = 17.72 - 6.89(\log \Delta V - 2.5) . \qquad (1)$$

Curiously, they found that the mean surface-brightness of Arecibo clusters systematically deviates from this fiducial relation with cluster redshift (see their Fig. 1), in the sense that distant clusters have lower surface-brightness than the prediction.

Arecibo cluster redshifts correlate with cluster properties (such as richness and velocity dispersion), so that more distant clusters appear to be denser or richer. This correlation might be a selection effect, or it might imply that the apparent systematic behavior of surface brightness with redshift reflects an environmental dependence of the $\Sigma/\Delta V$ relation (Kraan-Korteweg 1983). Indeed, the mean deviations of surface brightness from the fiducial line do show some systematic variation with cluster environmental parameters (richness class and velocity dispersion) as shown in Figure 1. The correlation is such that galaxies in denser or richer clusters tend to have lower surface brightness than the fiducial line would predict; in other words, denser or richer clusters tend to lie systematically above the fiducial line in the $\Sigma/\Delta V$ plane. What kind of environmental effects can possibly produce this behavior? In fact, one would expect to see an opposite trend according to

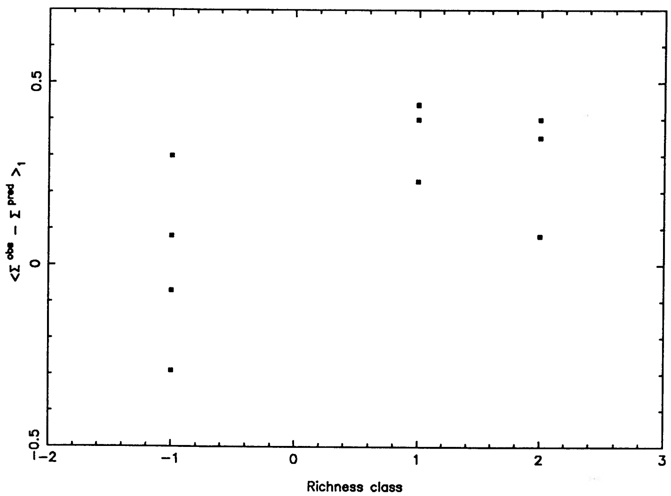

FIG. 1*a*

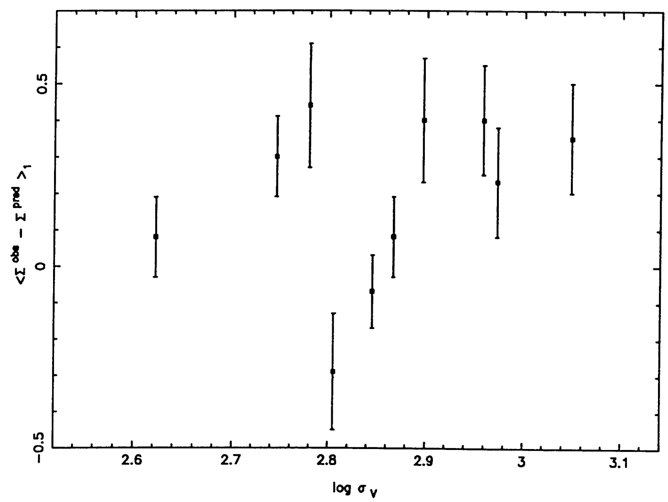

FIG. 1*b*

FIG. 1.—Mean deviation of surface brightness from eq. (1) for clusters in the Arecibo sample against (*a*) cluster richness class and (*b*) cluster velocity dispersion

the recent studies of Whitmore *et al.* (1988), and Rubin *et al.* (1988). These authors found that galaxies in denser environments tend to have falling rotation curves, implying that the velocity widths for galaxies of identical luminosities may be systematically lower in denser clusters (see, however, Guhathakurta *et al.* 1988). This would move the denser clusters away from the fiducial line to its left. Therefore, if the IR surface brightness is blind to any environmental effects, one would expect to see the denser clusters fall below the fiducial line. It seems difficult to explain the systematic surface brightness behavior in terms of environmental effects. Leaving aside the general question of whether or how the $\Sigma/\Delta V$ relation depends on environment, we show below that the observed systematic surface brightness behavior can be interpreted as being due to curvature of the $\Sigma/\Delta V$ relation.

In Figure 2, we plot Σ against $\log \Delta V$ for the 308 spiral galaxies in the sample of Aaronson *et al.* (1982*b*). A close inspection of Figure 2 shows that there is a subtle change in the relation at $\log \Delta V \sim 2.6$, beyond which the dependence of surface brightness on velocity width almost completely disappears. Barring systematic error in our measurements of the three observables, or some yet unknown selection effect in the sample we employed, we might suppose that this is a real effect relating to the structure of massive galaxies. This may merit further investigation. But for the purpose of this paper, this behavior just reminds us that better analytical representation

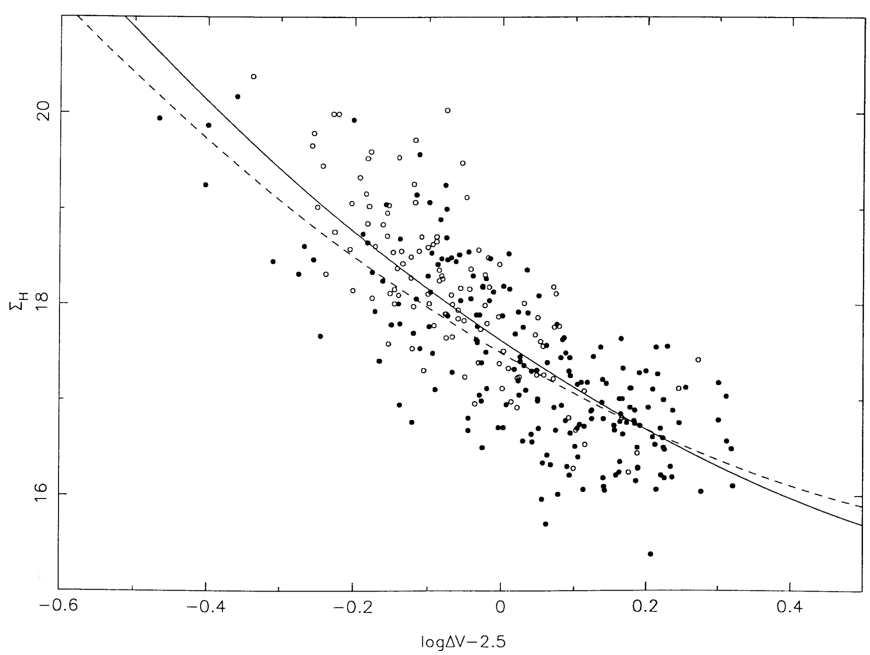

FIG. 2.—Surface-brightness/velocity-width relation for the 308 spiral galaxies in the sample of Aaronson *et al.* (1982*b*). Filled circles represent those galaxies in the "good sample" of Faber and Burstein (1988). The solid curve is the best quadratic fit to the full data set (eq. [2]); the dashed curve is the quadratic fit to the "good sample."

TABLE 1

ARECIBO CLUSTERS

Cluster Name	N[a]	V_0[b]	$\langle\log \Delta V\rangle$[c]	$\langle\Sigma^o - \Sigma^p\rangle_1$[d]	$\langle\Sigma^o - \Sigma^p\rangle_2$[e]	Δ Slope[f]
Pisces	20	5274	2.562	0.08 ± 0.11	0.02 ± 0.12	-0.58 ± 0.42
A400	7	7154	2.645	0.44 ± 0.17	0.22 ± 0.14	-0.42 ± 1.47
A539	9	8561	2.632	0.40 ± 0.17	0.22 ± 0.18	0.42 ± 1.47
Cancer	22	4790	2.531	-0.07 ± 0.10	-0.06 ± 0.11	-0.01 ± 0.98
A1367	20	6427	2.595	0.08 ± 0.11	-0.02 ± 0.10	-1.78 ± 1.11
Coma	13	6931	2.622	0.40 ± 0.15	0.24 ± 0.16	-0.51 ± 0.76
Z74-23	13	6025	2.504	0.30 ± 0.11	0.37 ± 0.14	-2.66 ± 1.17
Hercules	11	11077	2.627	0.35 ± 0.15	0.17 ± 0.13	1.43 ± 1.18
Pegasus	22	4078	2.440	-0.29 ± 0.16	-0.13 ± 0.15	2.09 ± 0.80
A263/66	11	8783	2.687	0.23 ± 0.15	-0.10 ± 0.10	0.64 ± 1.03

[a] Number of galaxies.
[b] Cluster mean velocity with respect to the LG.
[c] Mean of log ΔV for the N (col. [2]) cluster members.
[d] Mean SB deviation from eq. (1).
[e] Mean SB deviation from eq. (2).
[f] Difference between observed TF slope and that calculated from eq. (3).

of the global empirical $\Sigma/\log \Delta V$ relation is a curve rather than a straight line. We have performed a quadratic least-squares fit to this data set. The best fit, as shown in the diagram, is given by

$$\Sigma = 17.62(\pm 0.04) - 5.19(\pm 0.24)(\log \Delta V - 2.5) + 2.69(\pm 1.22)(\log \Delta V - 2.5)^2 . \quad (2)$$

This curvature is seen in both the full 308 galaxy sample, and the 211 galaxy subsample (plotted as filled circles in Fig. 2) preferred by Faber and Burstein (1988). This subsample is that subset of the data for which blue magnitudes are given in the Second Reference Catalog (de Vaucouleurs, de Vaucouleurs, and Corwin 1976). Equation (2) also fits the 56 galaxies in the Parkes sample of southern clusters studied by Aaronson *et al.* (1989), at least as well as equation (1), which appears in Figure 5 of that paper.

Now, we seek to find how the clusters of the Arecibo sample deviate from this quadratic relation. We calculate the mean deviation of surface brightness from equation (2) for each cluster. The results are listed in column (6) of Table 1, and are plotted against cluster redshift in Figure 3, and against cluster environmental parameters (richness class and velocity dispersion) in Figure 4. No systematic variations of $\Delta\Sigma$ with either redshift or environmental parameters are seen in Figures 3 and 4, indicating that the relation between surface brightness and velocity width is better represented in a quadratic form, and this relation does not show any apparent dependence on environment. It should be clear that our conclusion is very preliminary, because of the limited sample size and incomplete understanding of the sample selection effects.

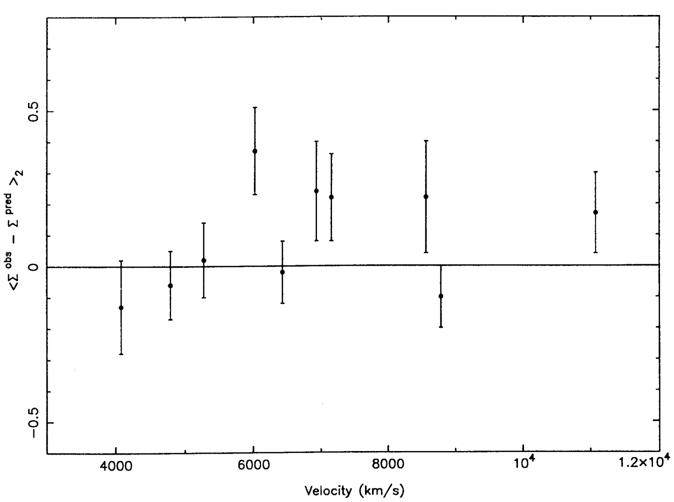

FIG. 3.—Mean surface brightness deviation from eq. (2) for the Arecibo cluster sample plotted against cluster redshift.

III. THE SLOPE OF THE TULLY-FISHER RELATION

Because the infrared TF relation is nonlinear, the slope $dH/d \log (\Delta V)$ is a function of velocity width. Since there is a correlation between morphological type and velocity width (Rubin, Ford, and Thonnard 1978), later type galaxies show a steeper slope. These correlations can be seen in Table 2, which is calculated from the sample of Aaronson *et al.* (1982*b*).

The slope of the infrared TF relation can also be calculated for any velocity width by differentiating equation (3) of Aaronson *et al.* (1986):

$$\frac{dH}{d \log (\Delta V)} = -11.18 + 15.0(\log \Delta V - 2.5) . \quad (3)$$

The difference between the observed slope (Aaronson and Mould 1983) and the prediction from this equation for $\langle\log \Delta V\rangle$ for each type is displayed in Figure 5. We conclude that there is no evidence for an *intrinsic* dependence of slope on morphological type, only a dependence of slope on velocity width, which is just the nonlinearity we have been discussing.

TABLE 2

MORPHOLOGICAL TYPE DEPENDENCE

Type	n	$\langle\log \Delta V\rangle$	— Slope[a]	Δ Slope[b]
Sa/Sab	10	2.58	9.91	0.14 ± 1.19
Sb	38	2.61	9.49	-0.41 ± 0.67
Sbc	42	2.59	9.83	-1.10 ± 0.71
Sc	88	2.52	10.88	-0.35 ± 0.53
Scd	40	2.45	11.89	-0.44 ± 0.72
Sd/Irr	46	2.38	13.04	-0.27 ± 1.39

[a] Calculated from eq. (3).
[b] Difference between observed and calculated slopes.

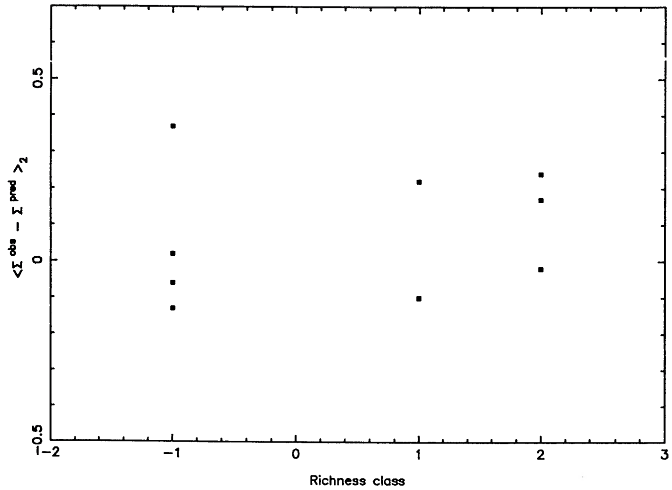

FIG. 4*a*

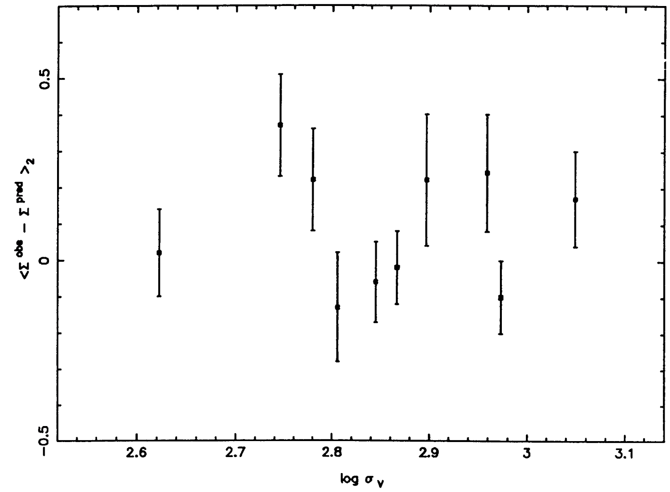

FIG. 4*b*

FIG. 4.—Mean surface brightness deviation from eq. (2) for the Arecibo cluster sample plotted against (*a*) richness class and (*b*) velocity dispersion

Aaronson *et al.* (1986) noticed that the slopes of TF relations for the clusters of their Arecibo sample systematically decreased with cluster redshift. Without carrying out formal tests, they believed this was completely due to the curvature of the TF relation. Therefore they maintained that no bias was introduced into distances of the Arecibo clusters estimated by applying a quadratic TF relation drawn from the Local Supercluster sample of Aaronson *et al.* (1982*b*). Recently, Djorgovski, Carvalho, and Han (1988) discussed the origin of distance-indicator relations (e.g., TF and Faber-Jackson relations), and their dependence on environment. They also studied the Arecibo data set, and concluded that there might be some environmental, or some as yet unknown sample biasing effects, involved in the observed trend of TF slope with redshift. Here we can perform another simple test on this question with equation (3). We calculate the predicted TF slope for each of the Arecibo clusters by simply substituting the cluster's mean velocity-width (col. [4] of Table 1) into equation (3). The difference between observed and predicted slopes are listed in column (7) of Table 1, and plotted in Figures 6*a*–6*d* against redshift, mean velocity-width, and the environmental parameters, respectively. None of these plots shows any correlation. So our simple test detects no environmental dependence of the slope (or more precisely, the curvature) of TF relation, suggesting that the TF relation has universal shape, approximated by equation (3).

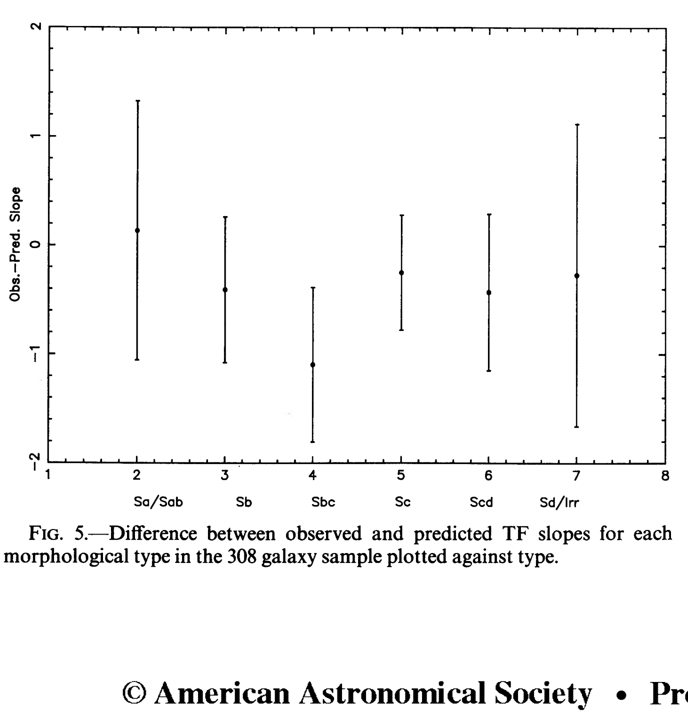

FIG. 5.—Difference between observed and predicted TF slopes for each morphological type in the 308 galaxy sample plotted against type.

IV. DISCUSSION

We have examined in §§ II and III the surface-brightness/velocity-width relation and the TF relation. Our preliminary studies seem to suggest that these two relations are both nonlinear, and their shapes are independent of cluster richness and velocity dispersion. To understand the implication of any observed behavior of the TF relation, we may need to examine its origin.

The underlying physics of the TF relation is generally believed to be very simple: it is fundamentally Newtonian gravity. The emergence of a TF relation from this simple physics is, however, highly nontrivial, since it involves practically all the puzzles of galaxy formation and evolution (Djorgovski, De Carvalho, and Han 1988). Based on the energy equation for a gravity bound system, these authors derived, what they call, "generalized distance indicator relations". The following equation is a copy of their equation (6), which corresponds to the TF relation

$$L = K_{SL} I^{-1} (M/L)^{-2} V^{-4} , \tag{4}$$

where V, I, and L are, respectively, the operationally defined velocity scale (e.g., the velocity width at 20% level for spiral galaxies), the central surface brightness, and the total luminosity within some radius. The structural parameter K_{SL} depends on the operational definition of the observables, as well as on the kinematical, density, and luminosity structures of galaxies, and therefore on the formative and evolutionary histories of galaxies, which again may be a function of environment. The mass to light ratio M/L may vary in a similar way.

We see that in order to have a good relation between the observed L, and V for a sample of galaxies, nature must produce and arrange the mass, light, and kinetic energy within each of the sample galaxies in a homogeneous way, so that when applying our adopted schemes for the definition of the observables, the resultant product $K_{SL} I^{-1} (M/L)^{-2} \equiv \gamma$ is

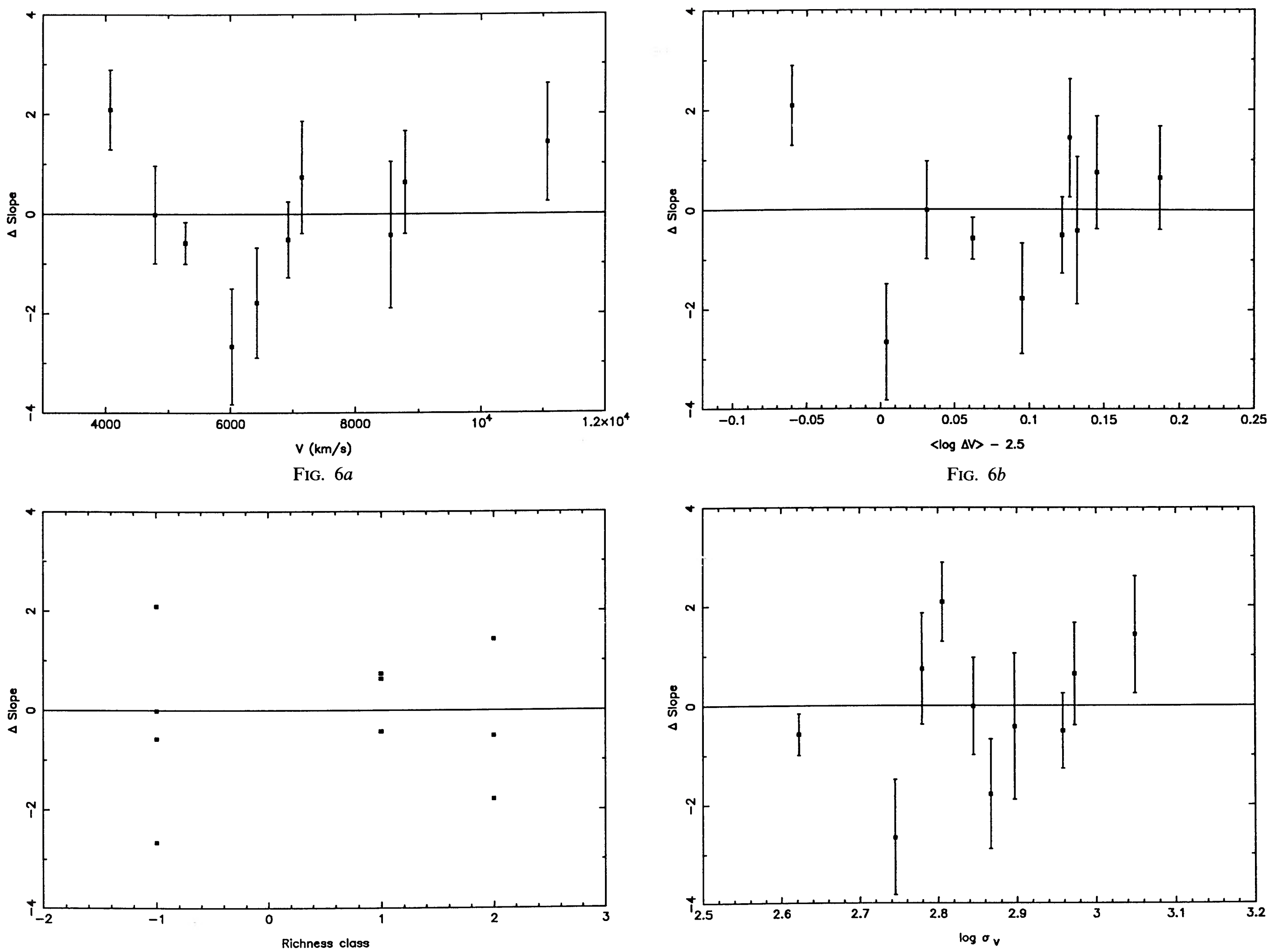

FIG. 6*a* FIG. 6*b* FIG. 6*c* FIG. 6*d*

FIG. 6.—Difference between observed and predicted TF slopes for each of the Arecibo clusters plotted against (*a*) cluster redshift, (*b*) mean velocity-width, (*c*) richness class of the cluster, and (*d*) velocity dispersion of the cluster.

nearly a constant or a function of V only. Noise in this product or dependence on a third parameter would directly introduce scatter in the observed TF relation. Therefore, improving the TF relation is basically an effort to make the parameter γ more constant, or a well-behaved function of V. Considering the fact that γ depends on both the internal structure of galaxies and the operational definition of the observables, there are two obvious ways to adjust γ, (1) choose a "homogeneous" sample by rejecting some *peculiar* galaxies or (2) optimize our observable definition scheme so that some *abnormal* galaxies become *normal* with the new definition of the observables, which somehow absorb the internal peculiarities of the abnormal galaxies. A real attempt to apply these two methods practically, and thus improve the TF relation, has been made by Bothun and Mould (1987). They showed that the scatter in the I band TF relation of two cluster samples was greatly reduced by (1) excluding those galaxies that have abnormal luminosity structure and (2) employing different definitions of isophotal magnitudes for large and small galaxies.

Returning to the matter of nonlinearity, we see from equation (4) that the TF relation is nonlinear, unless the parameter γ is a pure power-law function of V. Therefore the observed nonlinear TF relation implies that γ is a function, but not a power-law function, of V. This may reflect some internal regulative variation in the structure of galaxies, or equivalently, from our point of view, the definition of the observables we employ may not properly reflect what is physically varying in the sequence of galaxies in the sample.

For example, as Bottinelli *et al.* (1983) pointed out, the curvature of the TF relation may be caused by noncircular motions in spiral disks, indicating that the conventional definition of velocity width at the 20% level needs a correction for this effect in order to better represent the overall rotation of galaxies with different local kinematics. Unfortunately, our best estimate of this correction seems unable to straighten out the TF relation fully (Aaronson *et al.* 1986 and references therein), although this effect no doubt contributes to the observed curvatures of the TF and Σ/log ΔV relations. Artificial curvature may also result from indeterminacy of the inclination angles of irregular galaxies (van den Bergh 1989), although the selection criteria of Aaronson, Mould, and Huchra (1980) should act to exclude such galaxies from their samples. A plot of surface brightness residual versus inclination in the samples discussed here shows no correlation.

Another possible contribution to the observed curvature of the TF relation might have to do with the aperture definition [i.e., $\log (A/D_0) = -0.5$]. In order to sample a similar physical size in terms of disk scale length, this definition of aperture requires the central disk surface brightness to be constant (Romanishin *et al.* 1982). Systematic variation of central surface brightness with luminosity may result in a curved TF relation. In fact, such a variation is expected in some galaxy formation models (e.g., Silk and Norman 1981; Faber 1982). It is interesting to note that if we correct the aperture effect to straighten out the TF relation, the $\Sigma/\log \Delta V$ relation will become more curved. In other words, we cannot straighten out both the TF relation and the $\Sigma/\log \Delta V$ relation by redefining the aperture.

Recently, Persic and Salucci (1988) and Salucci and Frenk (1988) investigated the effect of dark matter on the TF relation, where it is shown that the dark matter fraction systematically varies with luminosity. The TF relation is found to become linear and much less noisy, after the velocity width is corrected for this effect by an expression of the form $\log \Delta V_* = \log \Delta V + \alpha M + \beta$, where M is absolute magnitude of galaxy (or alternatively the mean rotation-curve slope). It is obvious mathematically, that as long as $\alpha < 0$, this kind of correction will always reduce the scatter in the original $M/\log \Delta V$ relation, and tend to straighten it out, even if there is no physical basis for it at all. One should therefore be very careful in applying and interpreting this sort of correction.

In any case, it seems that the curvature of the TF relation implies some intrinsic systematic variation in the structure of spiral galaxies; it may also provide a tool for the investigation of such systematics. Our study in this paper shows that the curvature of the TF relation and that of the $\Sigma/\Delta V$ relation have no significant dependence on environmental parameters. This might suggest that the processes (formative or evolutionary) responsible for the relevant systematics in spiral galaxies must be independent of galaxies' environment. Further investigation of these problems is desirable and will provide us important information about galaxy formation and evolution, no matter what causes the curvature of the TF relation and whether or not it is universal. It is especially important to understand the degree of universality of the TF relation, when applying it as a distance indicator for the mapping of the large-scale velocity field.

We thank S. G. Djorgovski for helpful discussions. Partial support for this project was provided by NSF grants AST 85-02518 and 87-21705 to Caltech.

REFERENCES

Aaronson, M., Bothun, G., Mould, J., Huchra, J., Schommer, R., and Cornell, M. 1986, *Ap. J.*, **302**, 536.
Aaronson, M., Huchra, J., and Mould, J. 1979, *Ap. J.*, **229**, 1.
Aaronson, M., Huchra, J., Mould, J., Schechter, P. L., and Tully, R. B. 1982*a*, *Ap. J.*, **258**, 64.
Aaronson, M., and Mould, J. 1983, *Ap. J.*, **265**, 1.
Aaronson, M., Mould, J., and Huchra, J. 1980, *Ap. J.*, **237**, 655.
Aaronson, M., *et al.* 1982*b*, *Ap. J. Suppl.*, **50**, 241.
Aaronson, M., *et al.* 1989, *Ap. J.*, **338**, 654.
Blumenthal, G., Faber, S., Primack, J., and Rees, M. 1984, *Nature*, **311**, 517.
Bothun, G., and Mould, J. 1987, *Ap. J.*, **313**, 629.
Bottinelli, L., Gougenheim, L., Paturel, G., and de Vaucouleurs, G. 1983, *Astr. Ap.*, **118**, 4.
de Vaucouleurs, G., de Vaucouleurs, A., and Corwin, H. G. 1976, *Second Reference Catalogue of Bright Galaxies* (Austin: University of Texas Press).
Djorgovski, S., De Carvalho, R., and Han, M. 1989, in *The Extragalactic Distance Scale*, ed. S. van den Bergh and C. Pritchet, *A.S.P. Conf. Series*, **4**, 329.
Faber, S. 1982, in *Astrophysical Cosmology*, ed. H. Bruck, G. Coyne, and M. Longair (Vatican: Pontifica Academia Scientarum), p. 191.
Faber, S., and Burstein, D. 1988, in *Large-Scale Motions in The Universe*, ed. G. Coyne and V. Rubin (Princeton: Princeton University Press), p. 116.
Guhathakurta, P., *et al.* 1988, *A.J.*, **96**, 851.
Kraan-Korteweg, R. 1983, *Astr. Ap.*, **125**, 109.
Persic, M., and Salucci, P. 1988, *M.N.R.A.S.*, **234**, 131.
Romanishin, W., Krumm, N., Salpeter, E., Knapp, G., Strom, K., and Strom, S. 1982, *Ap. J.*, **263**, 94.
Rubin, V., Ford, W., and Thonnard, N. 1978, *Ap. J.* (*Letters*), **225**, L107.
Rubin, V., Whitmore, B., and Ford, W. K. 1988, *Ap. J.*, **333**, 522.
Salucci, P., and Frenk, C. 1988, preprint.
Silk, J. 1983, *Nature*, **301**, 574.
Silk, J., and Norman, C. 1981, *Ap. J.*, **247**, 59.
Tinsley, B. 1981, *M.N.R.A.S.*, **194**, 63.
van den Bergh, S. 1989, *Pub. A.S.P.*, **100**, 344.
Whitmore, B., Forbes, D., and Rubin, V. 1988, *Ap. J.*, **333**, 542.

GREG BOTHUN: Astronomy Department, University of Michigan, Ann Arbor, MI 48109

MINGSHENG HAN and JEREMY MOULD: Caltech 105-24, Pasadena CA 91125