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VORTEX CREEP AND THE INTERNAL TEMPERATURE OF NEUTRON STARS: LINEAR AND NONLINEAR RESPONSE TO A GLITCH

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ABSTRACT

The dynamics of pinned superfluid in neutron stars is determined by the thermal "creep" of vortices. Vortex creep can respond to changes in the rotation rate of the neutron star crust and provide the observed types of dynamical relaxation following pulsar glitches. It also gives rise to energy dissipation, which determines the thermal evolution of pulsars once the initial heat content has been radiated away. We explore the different possible regimes of vortex creep and show that the nature of the dynamical response of the pinned superfluid evolves with a pulsar's age. Younger pulsars display a linear regime, where the response is linear in the initial perturbation and is a simple exponential relaxation as a function of time. A nonlinear response, with a characteristic nonlinear dependence on the initial perturbation, is responsible for energy dissipation and becomes the predominant mode of response as the pulsar ages. The transition from the linear to the nonlinear regime depends sensitively on the temperature of the neutron star interior. We give a preliminary review of existing postglitch observations within this general evolutionary framework.

Subject headings: pulsars — stars: neutron

I. INTRODUCTION

A dynamical model for the superfluid parts of the neutron star crust, the vortex creep model, was developed in recent years (Pines and Alpar 1985 and references therein) and applied to the observed relaxation of pulsar rotational behavior following each of a number of glitches (sudden discontinuities in rotation and spindown rates) exhibited by the Vela and Crab pulsars, and by PSR 0525−21. The evaluation of postglitch relaxation following the recent large glitch from PSR 0355+54 (Alpar *et al.* 1988) has involved a response of the crust superfluid in a linear regime of the model different from the regime evoked in previous evaluations of postglitch data. Further, the four most recent glitches from the Vela pulsar (McCulloch *et al.* 1983; Hamilton, McCulloch, and Royle 1982; Klekociuk, McCulloch, and Hamilton 1985; Flanagan 1989; Hamilton *et al.* 1989) and the latest Crab pulsar glitch (Lyne and Pritchard 1987), unlike previous glitches where the time of the event was uncertain by several weeks, were observed at times quite close (~hr to ~1 day) to the glitch event itself. These observations of the earliest, prompt response of the crustal neutron superfluid to a glitch raise the possibility that prompt response in these pulsars may also involve the linear regime of vortex creep. Finally, recent calculations of the superfluid energy gap in neutron matter (Chen *et al.* 1986; Ainsworth, Pines, and Wambach 1989) have indicated values somewhat less than the results of earlier gap calculations. As a consequence, pinning energies may be reduced compared to our earlier estimates. For all of these reasons, it seems appropriate to reexamine the vortex creep model and the conclusions it enables one to draw from postglitch relaxation data, including estimates of the internal temperatures of neutron stars.

II. DIFFERENT REGIMES OF VORTEX CREEP

The rotational dynamics of a superfluid is determined by the distribution and motion of its quantized vortex lines. The vortices in the neutron star crust superfluid coexist with a lattice of nuclei, which have dimensions and spacings comparable to the size of the vortex and which can therefore act as pinning centers. As a result, the vortices move in a very inhomogeneous medium of pinning nuclei. Vortex creep theory models this medium as a random pinning potential. As the crust of the star spins down under external torques, the rotation rate of the pinned superfluid will lag behind that of the crust and the random potential will be biased in favor of motions of vortex lines away (i.e., radially outward) from the rotation axis, which allows the superfluid to spin down. Vortex creep theory models the spin-down of the pinned superfluid in terms of the thermal motion of vortex lines in this biased pinning potential. An analogous treatment is possible for other systems whose macroscopic dynamics are governed by the motion of a distribution of singular structures in some random potential (e.g., dislocations in crystals and flux lines in type II superconductors), so that a discussion of the different regimes of vortex creep may be of some relevance for such systems.

The equation of motion for a rotating superfluid is

$$\dot{\Omega} = -\frac{(2\Omega)}{r} v_r , \quad (1)$$

where $|\dot{\Omega}|$ is the rotation rate, $2\Omega = n\kappa$ the vorticity, n is the area density of vortex lines and κ the vortex quantum; v_r is the average radial velocity of the vortex lines and r the distance from the rotation axis. For our purposes, changes in Ω are small compared to Ω, and the extension of the superfluid in r is small compared to r, so that we may treat $2\Omega/r \equiv 2\Omega_0/r$ as a constant. The velocity of an individual vortex line, with respect to the superfluid, is determined by the forces f per unit length acting on it, through the Magnus equation of motion:

$$f = \rho \boldsymbol{\kappa} \times (\boldsymbol{v}_L - \boldsymbol{v}_s) , \tag{2}$$

where ρ is the superfluid density, $\boldsymbol{\kappa}$ the vorticity vector directed along the line, with magnitude κ, $\boldsymbol{v}_L$ and $\boldsymbol{v}_s$ are the line and superfluid velocities, respectively. A vortex pinned to the crust will move with the crust, at a speed $v_L = r\Omega_c \equiv r(\Omega_s - \omega)$, lagging the superfluid by $r\omega$. Because of pinning, the superfluid velocity tends to remain somewhat larger, by the lag ω, than the velocity of the crust as the crust spins down. To sustain this relative motion between the superfluid and the pinned vortex lines moving with the crust, a force $f = \rho\kappa r\omega$ must be exerted per unit length of the line. The largest force f_p available from the pinning potential defines a critical value of ω, through $f_p = \rho\kappa r\omega_{\rm cr}$. If the velocity difference between the crust lattice (the pinning sites) and the superfluid exceeds $r\omega_{\rm cr}$, the lines cannot remain pinned. Based on this microscopic picture, the thermal creep model gives a mean value v_r of the radial velocity of vortex lines, for lines moving through the random pinning potential, at a temperature T (Alpar *et al.* 1984*a*) as

$$v_r = v_0 e^{-E_p/kT} 2 \sinh \frac{E_p}{kT} \frac{\omega}{\omega_{\rm cr}} \equiv 2 v_0 e^{-E_p/kT} \sinh \frac{\omega}{\varpi} , \tag{3}$$

where v_0 is a microscopic velocity, E_p the pinning energy which characterizes the random pinning potential. We have introduced the notation $\varpi \equiv \omega_{\rm cr} kT/E_p$. If one wishes to consider cases in which $\omega > \omega_{\rm cr}$ where the partition function is different from 1, the corresponding expression is

$$v_r = v_0 \frac{2e^{-E_p/kT} \sinh \omega/\varpi}{1 + 2e^{-E_p/kT} \cosh \omega/\varpi} . \tag{4}$$

While the discussion that follows can be carried through using this general expression, the simpler form in equation (3) is a very good approximation for the purposes of most applications, and we shall use it in our analysis here. The critical lag, $\omega_{\rm cr}$, is related to E_p by

$$\omega_{\rm cr} = \frac{E_p}{\rho\kappa r b \xi} , \tag{5}$$

where ρ is the density of the superfluid, ξ is the superfluid coherence length (the radius of a vortex core), and b is the distance between successive pinning sites along a vortex line. The superfluid coherence length is given by $\xi = \hbar^2 k_F/(\pi m \Delta)$ where k_F is the neutron Fermi wavenumber, m the neutron mass, and Δ the superfluid gap.

For a complete dynamical model, equations (1) and (3) are supplemented by the equation of motion for the crust rotation rate Ω_c:

$$I_c \dot{\Omega}_c = N_{\rm ext} - I_p \dot{\Omega} . \tag{6}$$

Here I_c is the total effective moment of inertia of the *observed* crust; it includes the actual crust plus the core superfluid which is coupled rigidly to the crust on the time scales of interest (Alpar, Langer, and Sauls 1984; Alpar and Sauls 1988). I_c therefore is the inertial moment of the entire star, minus that of the pinned superfluid, I_p. Thus one has $I_c \equiv I - I_p$, while $N_{\rm ext}$ is the external (magnetospheric) torque on the crust. From equations (1), (3), and (6) we obtain an equation for the lag $\omega = \Omega - \Omega_c$,

$$\dot{\omega} = -\frac{N_{\rm ext}}{I_c} - \frac{I}{I_c} \frac{4\Omega_0}{r} v_0 e^{-E_p/kT} \sinh \frac{E_p}{kT} \frac{\omega}{\omega_{\rm cr}} \tag{7}$$

$$\cong |\dot{\Omega}|_\infty \{1 - [\sinh(\omega/\bar{\omega})]/\eta\} , \tag{8}$$

where $|\dot{\Omega}|_\infty \equiv N_{\rm ext}/I$ is the steady-state value of $|\dot{\Omega}|$ and $|\dot{\Omega}_c|$ when the superfluid and crust share the total external torque, and we take $I_p \ll I_c$ so that $I \equiv I_p + I_c \cong I_c$. In equation (8) the response parameter η is given by

$$\eta \equiv \frac{|\dot{\Omega}_\infty| r}{4\Omega_0 v_0} \exp \frac{E_p}{kT} \equiv \frac{t_{\rm tr}}{8t_s} \tag{9}$$

and is the ratio of a characteristic transit time

$$t_{tr} \equiv r/[v_0 \exp(-E_p/kT)] \tag{10}$$

to the spin-down time of the external torque,

$$t_s = \Omega_0/2|\dot{\Omega}|_\infty . \tag{11}$$

Note that the observed perturbations $\Delta\dot{\Omega}_c$ to $\dot{\Omega}_c$ in glitches and postglitch relaxation do not exceed $\Delta\dot{\Omega}_c/\dot{\Omega}_c \sim 0(10^{-2})$ so that any observed value of $|\dot{\Omega}_c|$ can be used to represent $|\dot{\Omega}|_\infty$. Since t_s is much longer than the relaxation time scales of the vortex creep process, Ω_0 and $|\dot{\Omega}_c|$ at a given epoch can be taken as constants that specify the parameter η.

Equation (8) serves as the starting point of our discussion of the different regions of vortex creep. One can integrate it to obtain the evolution of ω and hence $\Omega(t)$ and $\Omega_c(t)$, for a given set of initial conditions. The solution is

$$\frac{\eta\varpi}{(1+\eta^2)^{1/2}} \times \left\{ \ln\left[\frac{\exp(\omega/\varpi) - \eta + (1+\eta^2)^{1/2}}{\exp(\omega/\varpi) - \eta - (1+\eta^2)^{1/2}}\right] - \ln\left[\frac{\exp(\omega(0)/\varpi) - \eta + (1+\eta^2)^{1/2}}{\exp(\omega(0)/\varpi) - \eta - (1+\eta^2)^{1/2}}\right] \right\} = |\dot{\Omega}|_\infty t . \tag{12}$$

Inspection of equation (8) shows that it possesses a steady-state solution, characterized by a lag ω_∞, such that $\dot{\omega} = 0$ (i.e., $\dot{\Omega} = \dot{\Omega}_c$)

$$\omega_\infty = \varpi \sinh^{-1} \eta . \tag{13}$$

It is evident from equation (12) that $\omega \to \omega_\infty$ asympotically as $t \to \infty$. From equations (8) and (13) it follows that

$$\operatorname{sgn} \dot{\omega}(t) = \operatorname{sgn} [\omega_\infty - \omega(t)] , \tag{14}$$

i.e., at all times the system evolves monotonically toward its steady state.

The creep response parameter η characterizes both the steady-state behavior and the evolution of the system. If conditions are such that $\eta \ll 1$, we see from equation (13) that $\omega_\infty \ll \varpi \equiv \omega_{\rm cr} kT/E_p$, and, since $kT \ll E_p$, we have $\omega_\infty \ll \omega_{\rm cr}$. Under these circumstances, the steady-state angular velocity difference is such that in the pinning region in question, one is never close to the critical angular velocity difference for unpinning, $\omega_{\rm cr}$. *The approach to steady state then depends linearly on the initial conditions.* Equation (8) becomes

$$\dot{\omega} = |\dot{\Omega}|_\infty [1 - \omega/(\eta\varpi)] = |\dot{\Omega}|_\infty - \omega/\tau_l , \tag{15}$$

where

$$\tau_l = \eta \frac{\varpi}{|\dot{\Omega}|_\infty} = \left(\frac{kT}{E_p}\right) \frac{\omega_{\rm cr} r}{4\Omega_0 v_0} \exp \frac{E_p}{kT} \tag{16}$$

is a relaxation time which depends on the transit time, the pinning parameters, pinning energy, the temperature of the inner crust, and the pulsar period. At steady state, we see from equation (15) that

$$[\omega_\infty]_l = |\dot{\Omega}|_\infty \tau_l , \tag{17}$$

while the explicit solution for $\omega(t)$ is

$$\omega(t) = \omega_\infty - \delta\omega(0)e^{-t/\tau_l} , \tag{18}$$

where

$$\delta\omega(0) = \omega_\infty - \omega(0) , \tag{19}$$

and one sees explicitly the linear dependence of $\omega(t)$ on $\delta\omega(0)$. The observed behavior of the crust is

$$\dot{\Omega}_c = \dot{\Omega}_\infty - (I_p/I)[\delta\omega(0)/\tau_l]e^{-t_l} . \tag{20}$$

Under these circumstances, vortex creep gives rise to energy dissipation at a rate

$$\dot{E}_{\rm diss} = I_p \omega_\infty |\dot{\Omega}_\infty| = I_p(\dot{\Omega}_\infty)^2 \tau_l . \tag{21}$$

If, on the other hand, conditions are such that $\eta \gg 1$, we see from equation (13) that

$$\eta = \sinh [\omega_\infty/\varpi] \cong (\tfrac{1}{2}) \exp [\omega_\infty/\varpi] \tag{22}$$

and

$$[\omega_\infty]_{nl} = \varpi \ln (2\eta) = \omega_{\rm cr}[1 - (kT/E_p) \ln (4v_0 t_s/r)] . \tag{23}$$

On making use of the definition of η, equation (9), one finds that the condition $\eta \gg 1$ implies that $(kT/E_p) \ln [4v_0 t_s/r] \ll 1$ so that $\omega_\infty \lesssim \omega_{\rm cr}$. Thus, for $\eta \gg 1$, the approach to steady state depends nonlinearly on the initial conditions. Equation (8) becomes

$$\dot{\omega} = |\dot{\Omega}|_\infty - \frac{1}{2\eta} \exp \left(\frac{\omega}{\varpi}\right) \tag{24}$$

which possesses the solution

$$\exp \{[(\omega_\infty)_{nl} - \omega(t)]/\varpi\}$$
$$= 1 + [\exp \{[(\omega_\infty)_{nl} - \omega(0)]/\varpi\} - 1] \exp (-t/\tau_{nl}) , \tag{25}$$

where

$$\tau_{nl} = \frac{\varpi}{|\dot{\Omega}|_\infty} = \frac{kT}{E_p} \frac{\omega_{\rm cr}}{|\dot{\Omega}|_\infty} . \tag{26}$$

The observed behavior of the crust is given by

$$\dot{\Omega}_c = \frac{I}{I_c} \dot{\Omega}_\infty - \frac{I_p}{I_c} \dot{\Omega}_\infty \frac{1}{1 + [\exp (t_0/\tau_{nl}) - 1] \exp (-t/\tau_{nl})} , \tag{27}$$

where t_0, the "offset time," reflects the nonlinear dependence on initial conditions, and is given by

$$t_0 \equiv \frac{\delta\omega(0)}{|\dot{\Omega}|_\infty} \equiv \frac{(\omega_\infty)_{nl} - \omega(0)}{|\dot{\Omega}|_\infty} . \tag{28}$$

The energy dissipation rate in steady state is

$$\dot{E}_{\rm diss} = I_p(\omega_\infty)_{nl} |\dot{\Omega}|_\infty \cong I_p \omega_{\rm cr} |\dot{\Omega}|_\infty \tag{29}$$

for pinned superfluid regions in this nonlinear creep regime.

A comparison of the linear regime, equations (15)–(21), and the nonlinear regime, equations (22)–(29), shows that the observed behavior is quite distinct in the two regimes on several counts. In the linear regime the observed relaxation of the crust in response to a perturbation from steady state is *always* a simple exponential decay in time, formally identical to the old two-component model (Baym *et al.* 1969), though in a different context. This response is linear in the initial perturbation, which appears as the amplitude of the exponentially decaying term in $\dot{\Omega}_c(t)$. By contrast, in the nonlinear creep regime, the response of the crust can be either linear or nonlinear depending on the size of the glitch. For perturbations such that $t_0/\tau_{nl} \gtrsim 1$, in fact, the response has an exponential dependence on the initial perturbation $\delta\omega(0)$, while the subsequent evolution in time of $\dot{\Omega}_c(t)$ is itself a more complicated function than simple exponential decay. The most characteristic feature of this response is its Fermi function behavior (Alpar *et al.* 1984*a*): $\dot{\Omega}_c(t)$ is constant initially, showing no recovery from its initial postglitch value $\dot{\Omega}_c(0^+) = \dot{\Omega}_\infty I/I_c$, until $t \sim t_0$; within a time interval $\sim\tau_{nl}$ around $t = t_0$, $\dot{\Omega}_c(t)$ relaxes back to its steady-state value $\dot{\Omega}_\infty$. Observation of such constant offsets in $\dot{\Omega}_c$ (until some offset time t_0) constitutes an indication of the presence of nonlinear creep regions. Thus, the observed "persistent shifts" in $\dot{\Omega}_c$ are likely indicators of this regime, while the observation of a sudden increase in $\dot{\Omega}_c$ some time after a glitch would provide conclusive evidence of creep in the nonlinear regime.

On the other hand, in the case of a weak perturbation, $t_0 \ll \tau_{nl}$, the response in the nonlinear regime also becomes a simple exponential in time, with linear dependence on the initial perturbation, $\delta\omega(0)$. Thus, a pinned superfluid region in the nonlinear regime will also respond linearly to small perturbations from *its steady state*, when $\delta\omega(0) = \omega_\infty - \omega(0) \ll (kT/E_p)\, \omega_{\rm cr}$. By contrast, nonlinear creep will respond very nonlinearly when $\delta\omega(0) \cong \omega_\infty$, i.e. $\omega(0) \ll \varpi$, which is a perturbation that takes it far from its steady state. Similarly, in the linear regime $\eta \ll 1$, if the initial conditions are such that $\sinh [\omega(0)/\varpi] > 1$, the initial evolution will rapidly converge to the appropriate time dependence of the linear regime given in equation (18), so that after some short time t_1, equation (18) will be followed with $\delta\omega(t_1) \ll \delta\omega(0)$ replacing $\delta\omega(0)$ and $\omega(t_1) < \varpi$. Thus, in either of the regimes specified by η, the initial conditions may be close to the steady state of the other regime, but the system will not remain at such "off" conditions. Rather, a very rapid initial evolution will ensue, converging to the general solution of the appropriate regime as specified by η.

The steady state in the linear regime, $\omega_\infty = |\dot{\Omega}|_\infty \tau_l$, is determined by the relaxation time, as is characteristic of linear systems operating at a small bias ($|\dot{\Omega}|_\infty \tau \ll \omega_{\rm cr}$). The relaxation time, again typical of such systems, is determined by *thermal* factors; i.e., $\tau \propto \exp E_p/kT$. (Early work on relaxation times [Feibelman 1971; Greenstein 1979] in the old two-component model also had this Boltzmann exponential dependence on T^{-1}, but with a different physical context, and a different energy spectrum.) By contrast, the steady state in the nonlinear regime, $\omega_\infty \lesssim \omega_{\rm cr}$, is set by the strength of pinning. The system is now operating close to unpinning conditions. The relaxation time toward the steady state is proportional to temperature (eq. [26]). This result, which seems strange on the basis of an intuition for almost "free" systems, whose equilibrium properties are dominated by *thermal* processes, is typical of highly driven systems, where the enthalpy plays a dominant role. Thus the colder the system is, the larger the bias ω_∞ must be to achieve the steady-state creep rate for the $|\dot{\Omega}|_\infty$ required by the external torque, and the more rapid the relaxation processes near the highly biased steady state. In this connection, we note that the linearity parameter η reflects, in fact, a comparison of the rate $|\dot{\Omega}|_\infty$ imposed by the external torque with the thermal rate $\propto \exp(-E_p/kT)$.

The rate of energy dissipation in the linear regime is smaller by $\sim |\dot{\Omega}|_\infty \tau_l/\omega_{cr} \ll 1$ than the rate in the nonlinear regime. If and when the entire pinned superfluid is in the linear regime, energy dissipation from vortex creep can be shown to be totally insignificant in the star's thermal evolution. On the other hand, regions in the nonlinear creep regime, as indicated by the observation of persistent shifts in $\dot{\Omega}_c$, provide a significant source of energy dissipation and determine the neutron star's temperature once the initial heat content of the star has been radiated away (Alpar, Nandkumar, and Pines 1985; Shibazaki and Lamb 1989).

III. THE TRANSITION

We now discuss the criterion for distinguishing the linear and nonlinear creep regimes, as a function of the star's age (Ω and $\dot{\Omega}_c$) and temperature, and of the pinning energy E_p (hence the superfluid gap Δ) in a given part of the pinned superfluid. From equation (9), a transition value of E_p/kT is obtained by setting $\eta = 1$, so that larger values of E_p/kT will imply the nonlinear creep regime and smaller values will imply linear creep:

$$\left(\frac{E_p}{kT}\right)_{\rm tr} = \ln\left(8t_s \frac{v_0}{r}\right) = 35.46 + \ln t_{s,6} + \ln\left(\frac{v_{0,7}}{r_6}\right), \quad (30)$$

where $t_{s,6}$ is the spin-down time $t_s \equiv \Omega/2|\dot{\Omega}|$, in units of 10^6 yr, and we have taken typical values $v_0 = 10^7$ cm s^{-1} and $r = 10^6$ cm of the microscopic velocity of vortex lines and the radius of the neutron star, respectively. It is desirable to have a framework for deciding whether a particular observed relaxation time τ reflects a nonlinear or linear response of vortex creep to the glitch. To this end we note that the criterion $\eta = 1$ can be expressed as

$$\eta = \frac{\tau_l}{\tau_{nl}} = 1 \quad (31)$$

using equations (9), (16), and (26). Thus, if $\tau_l < (>) \tau_{nl}$ creep will be in the linear (nonlinear) regime: the system is always in that regime for which the relaxation time to the appropriate steady state is shortest. The relation between τ_l and τ_{nl} can be most simply expressed as

$$\ln \tau_l = \ln \tau_{nl} + \frac{E_p}{kT} - \left(\frac{E_p}{kT}\right)_{\rm tr}. \quad (32)$$

This expression shows clearly that the transition between linear and nonlinear creep is determined essentially by the exponential dependence on E_p/kT. In principle, an understanding of the transition can be used to determine the range of pinning energies for which creep will be (non)-linear if the temperature of the star is known or, on the basis of a calculated range of pinning energies, to estimate a temperature below which vortex creep throughout the pinned superfluid parts of the star will be predominantly in the nonlinear regime. There are uncertainties in theoretical calculations of both the pinning energies E_p and of the cooling history of neutron stars. Though subject to these uncertainties, pinning energies are a structural property of neutron stars; we expect similar ranges of E_p to obtain in neutron stars of different ages. The transition in dynamical relaxation, from the linear to the nonlinear regime, is then an evolutionary property, linked sensitively to the temperature. We now address the issues of pinning and cooling in turn.

As discussed in earlier work, the physical conditions of pinning can be classified into various possibilities—strong, weak, and superweak pinning—depending on the relation of E_p to a lattice binding energy E_L, and the relation of the vortex core radius ξ to the lattice spacing b_z (Anderson *et al.* 1982; Alpar *et al.* 1984*a*, *b*). Observational upper limits (Alpar *et al.* 1987), as well as recent calculations yielding lower values of the superfluid gap Δ and the pinning energy E_p, indicate that strong pinning is unlikely to obtain. The transition from weak to superweak pinning probably involves a decrease in E_p, with a large reduction in ω_{cr} and, as may be seen in equation (5), a correspondingly large increase in b, the distance between successive pinning centers along a vortex line. Although we can estimate the various pinning parameters for weak pinning theoretically (Alpar *et al.* 1984*b*), such estimates are not at present possible for superweak pinning. We shall illustrate the transition by using a model calculation with weak pinning everywhere. The criterion for superweak pinning, that the vortex core radius ξ is greater than $b_z/2$ (Alpar *et al.* 1984*b*) implies, with the gap values we employ, that superweak pinning would obtain at densities greater than 9×10^{13} g cm^{-3}. The effect of superweak pinning is indicated qualitatively in the discussion below. The pinning parameters depend on the superfluid gap Δ and the temperature in the following way:

$$E_p({\rm MeV}) = 0.87\Delta^2({\rm MeV})k_{\rm F}({\rm fm}^{-1})\gamma, \quad (33)$$

where γ is a dimensionless factor of order 1. This formula is based on the following estimate of the pinning energy that binds a vortex line to a nucleus in the crust superfluid:

$$E_p = \frac{1}{8\pi^2}\left[\left(\frac{\Delta^2 k_{\rm F}{}^3}{E_{\rm F}}\right)_{\rm out} - \left(\frac{\Delta^2 k_{\rm F}{}^3}{E_{\rm F}}\right)_{\rm in}\right]V, \quad (34)$$

where "out" and "in" refer to the values of superfluid neutron density, and gaps, inside and outside the nuclei, respectively, and V is the volume of the nucleus. In equation (33) we have chosen to scale E_p with the gap and k_F values corresponding to the total density in a given pinning region, which are quite close to the "out" values, in view of the uncertainties in the rough estimate of pinning given in equation (34). A model calculation employing densities inside and outside nuclei used in earlier work (Alpar *et al.* 1984*a*, *b*) and the recent gap values of Ainsworth, Pines, and Wambach (1989) gives values of γ between 6×10^{-2} and 1 throughout the pinned superfluid:

$$b^{\rm weak}({\rm fm}) \cong 230\, \frac{(b_z/50\ {\rm fm})^3\, \Delta^2({\rm MeV})}{k_{\rm F}^2({\rm fm}^{-1})} \quad (35)$$

$$\omega_{\rm cr}{}^{\rm weak}({\rm rad\ s}^{-1}) \cong 0.4\gamma\, \frac{\Delta({\rm MeV})}{r_6\, k_{\rm F}({\rm fm}^{-1})(b_z/50\ {\rm fm})^3}. \quad (36)$$

For the relaxation time in the nonlinear creep regime with weak pinning, we have

$$\tau_{nl} = \frac{kT}{E_p}\frac{\omega_{\rm cr}}{|\dot{\Omega}|_\infty}$$

$$= 55\ {\rm days}\, \frac{kT({\rm keV})}{\Delta({\rm MeV})}\, \frac{1}{|\dot{\Omega}|_{-10}}\, \frac{1}{r_6(b_z/50\ {\rm fm})^3[k_{\rm F}({\rm fm}^{-1})]^2} \quad (37)$$

using equations (5) and (33–36). Here we have normalized the lattice spacing b_z to a typical value, 50 fm, and $|\dot{\Omega}|$ to the value

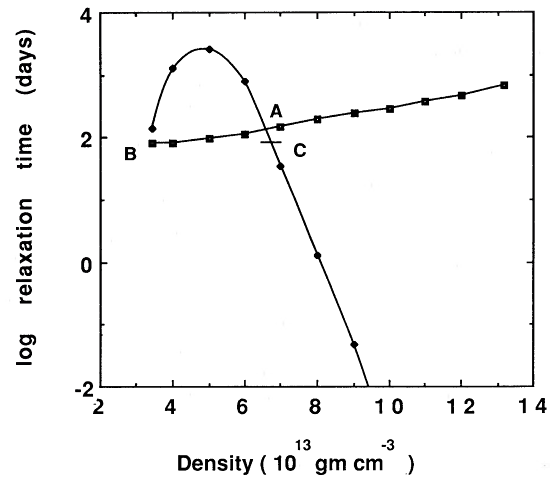

FIG. 1*a*

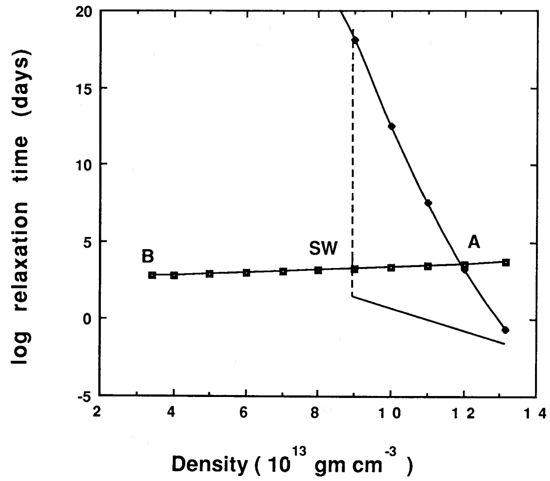

FIG. 1*b*

FIG. 1.—(*a*) Relaxation times, in days, from a model calculation for the Crab pulsar. The interior temperature is taken to be 35 keV. Values of the superfluid gap are those shown in Fig. 2, taken from Ainsworth, Pines, and Wambach (1989). γ is taken to be 0.7. The vertical axis is the logarithm (base 10) of the relaxation time in days, to accommodate the rapid variation of τ_l (*diamonds*). The more slowly varying τ_{nl} (*open squares*) has values of several hundred days. Regions between points A and B are in the nonlinear regime. The observed prompt relaxation times of a few days must represent higher density regions to the right of the transition point A, responding in the linear regime, according to this particular model calculation. (*b*) Relaxation times, in days, from a similar model calculation for the Vela pulsar. The interior temperature is taken to be 11 keV. Values of the superfluid gap are again those shown in Fig. 2, and $\gamma = 0.7$. The upper parts of the τ_l curve are not shown. The nonlinear relaxation times are several thousand days. The observed relaxation times of a few days and a few months indicate a possible extension of the linear creep regions by a transition to superweak pinning at point SW. The location of this transition is taken from the inequality (56), shown in Fig. 2. Regions between points B and A, or if there is superweak pinning, regions between points B and SW, are in the nonlinear regime. The dashed lines indicate superweak relaxation times qualitatively and do not reflect a model calculation for superweak pinning.

10^{-10} rad s^{-2}, the spindown rate of the Vela pulsar. (Values of the lattice spacing b_z as a function of density in the neutron star crust can be found in Negele and Vautherin 1975). Then, at a given temperature, $\tau_{nl} \propto 1/\Delta$, while for an observed τ_{nl}, the inferred value of the temperature scales with Δ. In the linear regime, the relaxation time

$$\tau_l = \frac{kT}{E_p}\,\omega_{\rm cr}\,\frac{r}{4\Omega_0 v_0}\,\exp\frac{E_p}{kT} \propto \exp a\,\frac{\Delta^2}{T}\,, \qquad (38)$$

where a is a constant. Thus, at given T, $\tau_l \propto (1/\Delta)\exp a'\Delta^2$, $a' = a/T$, and for an observed value of τ_l, the inferred temperature would be

$$T \propto \Delta^2/\ln(\Delta\tau_l/T) \propto \Delta^2\,. \qquad (39)$$

For a pulsar with internal temperature T, to the extent that one knows both $\Delta(\rho)$, the superfluid gap as a function of density, and $E_p(\rho)$, one can now plot τ_l and τ_{nl} as functions of density through the pinned superfluid. We emphasize again that this is subject to much uncertainty stemming from uncertainties in the calculations of $E_p(\rho)$ and $\Delta(\rho)$. The run of the two relaxation times is that shown in Figure 1. The nonlinear regime obtains at lower densities, to the left of the transition points marked A where $\tau_{nl} = \tau_l$. The $\Delta(\rho)$ curve we used is taken from the recent results of Ainsworth, Pines, and Wambach (1989) and is shown in Figure 2. A range of $\Delta(\rho)$ curves are allowed according to these recent calculations, but they are all typically less than the early gap results (Hoffberg *et al.* 1970); Takatsuka 1972) by a factor of 2. For illustrative purposes, in Figure 1*a*, we have used the Crab pulsar's parameters, with an interior temperature of 35 keV, consistent with observational upper limits and with standard cooling calculations. Figure 1*b* employs the Vela pulsar, with an interior temperature of 11 keV, consistent with the recent observational results of Ögelman and Zimmermann (1988). In calculating E_p, we set $\gamma = 0.7$ in equation (33), to illustrate some of the features of the transition in the figures. This value of γ is consistent with estimates of E_p using equation (34). Both relaxation times were evaluated for the weak pinning regime, using the expressions above. The effect of superweak pinning has been shown qualitatively in Figure 1*b*, with the alternative transition point SW, and will be discussed in the next section in connection with the Vela pulsar.

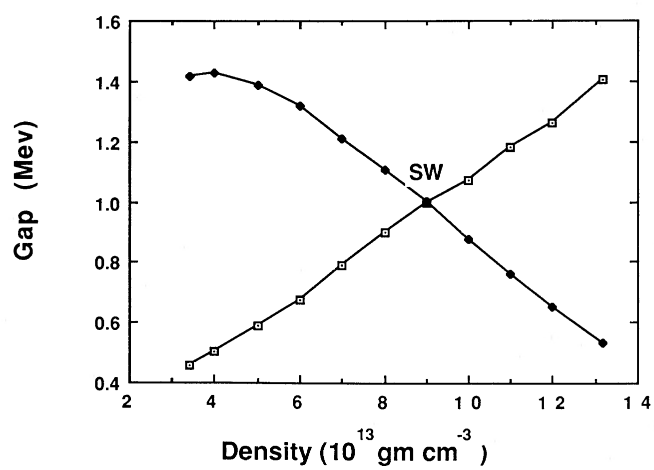

FIG. 2.—The superfluid gap in MeV as a function of neutron density. This is representative of a family of gap curves calculated recently by Ainsworth, Pines, and Wambach (1989). When the gap value is less than the threshold curve given in inequality (56) and represented by the open squares here, pinning is expected to be in the superweak regime. This is expected at densities above about 9×10^{13} g cm^{-3}.

Let us now discuss the effect of cooling on the transition. As the neutron star becomes older, the curve for τ_l will move upward, in proportion to the increasing period (decreasing rotation rate Ω) of the star. The τ_{nl} curve will also move upward, in inverse proportion to the decreasing $|\dot{\Omega}|$ of the star. As $|\dot{\Omega}| \propto \Omega^n$ with $n \sim 3$ for most pulsars, the τ_{nl} curve will move faster, resulting in a tendency against nonlinear creep. However, this effect of the dynamical evolution of the pulsar is completely overcome and reversed by the effect of its cooling. As the pulsar ages and its internal temperature T decreases, τ_{nl} will come down linearly in T, while τ_l will move up, exponentially in T^{-1}, resulting in a rather rapid extension of the nonlinear creep regime to encompass larger and larger portions of the pinned superfluid and rapidly prevailing over the entire pinned superfluid in older pulsars. A comparison of Figure 1*a* with Figure 1*b* shows this evolutionary effect clearly.

For pulsars much older than the Vela pulsar, at ages greater than a few 10^5 yr, the initial heat content will have been radiated away. The *surface* temperature of such older pulsars is determined by balancing their blackbody luminosity with the energy dissipation rate in nonlinear creep (eq. [29]) (Alpar, Nandkumar, and Pines 1985),

$$L_s \equiv 4\pi R^2 \sigma T_s^4 = \dot{E}_{\rm diss} = I_p \omega_{\rm cr} |\dot{\Omega}| . \qquad (40)$$

For neutron stars with $T_{s,6}{}^4/g_{14} \gtrsim 10^{-2}$, where g_{14} is the gravitational acceleration at the surface in units of 10^{14} cm s^{-2}, Gudmundsson, Pethick, and Epstein (1982) have shown that the interior (including the pinned superfluid regions) has a temperature $T \propto (T_s^4)^{0.455}$ giving

$$T \propto |\dot{\Omega}|^{0.455}, \quad \tau_{nl} \propto \frac{T}{|\dot{\Omega}|} \propto |\dot{\Omega}|^{-0.545} , \qquad (41)$$

using equation (26). For still older pulsars, the surface temperature and flux are so low that a very small temperature gradient from the interior to the surface suffices to drive the thermal flux. If we assume $T_s \lesssim T$ for such pulsars, we obtain

$$T \propto |\dot{\Omega}|^{1/4}, \quad \tau_{nl} \propto |\dot{\Omega}|^{-0.75} . \qquad (42)$$

Equations (41) and (42) give the scaling of the temperature in the interior and of postglitch relaxation times with $|\dot{\Omega}|$. The condition $T_{s,6}{}^4/g_{14} \gtrsim 10^{-2}$ for the validity of the Gudmundsson, Pethick, and Epstein (1982) results translates to $|\dot{\Omega}|$ (rad s^{-2}) $\gtrsim 10^{-12}(M/M_\odot)/(I_{p,43}\,\omega_{\rm cr})$.

If there is no significant nonlinear creep initially, the energy dissipation will be insignificant (cf. eq. [21] and eq. [29]), and cooling will be more rapid. Once the initial heat content of the star has been radiated away, nonlinear creep and energy dissipation will rapidly increase their role. The nonlinear creep, dominated by the external torque, becomes prevalent as the temperature drops and thermal rates become less important. The energy dissipation rate increases to a level where the further drop of temperature is regulated by a rate determined by the external torque as in equations (41) and (42). The time scale over which nonlinear creep becomes dominant depends sensitively (exponentially) on the barrier heights E_p for thermal creep, therefore on Δ^2. At a given epoch a pulsar, at some uniform internal temperature, will possess a spectrum of relaxation times reflecting the nature and strength of pinning, i.e., the variation of E_p, $\omega_{\rm cr}$, and b throughout the pinned inner crust superfluid. From this spectrum of possible relaxation times those which characterize substantial portions of the pinned superfluid will be *observable* in the pulsar's postglitch behavior. For the earlier epochs when both the linear and the nonlinear regimes of creep prevail in suitable portions of the superfluid, the shortest of the relaxation times will reflect creep in the linear regime. These may, in some pulsars, be too short to be observable. As the pulsar evolves, the nonlinear creep regime will take over in most of the pinned superfluid. It will still be true that the shortest relaxation times will come from the surviving regions of linear creep. But such regions of linear creep become progressively negligible in terms of the moment of inertia they represent, so that eventually the linear creep regime, with the shortest underlying relaxation times, will drop out of the hierarchy of observed relaxation times. Thus neutron stars, after a certain age, will display a set of pinning regions all responding in the nonlinear creep regime. In either creep regime, the shortest possible (or observed) relaxation times reflect the weakest (superweak) pinning regions, as both τ_{nl} and τ_l depend inversely on the distance b between successive pinnings along a vortex line, and b increases rapidly, in a way difficult to estimate, in the superweak pinning region. Whether a superweak pinning region is in the linear or the nonlinear regime depends most sensitively on the value of E_p/kT. E_p is also difficult to estimate in the superweak region. At present there is no calculation of the superweak pinning properties and relaxation times.

As can be seen in Figure 1, the longest relaxation times of a given pulsar are associated with the nonlinear regime. Some values of τ can obtain for one part of the pinned superfluid in the linear creep regime and another part, with stronger pinning, in the nonlinear creep regime (e.g., in Fig. 1*a*, τ_l at point C and τ_{nl} at point B have the same value). Thus an observed relaxation time might be consistent with either linear or nonlinear creep. The ambiguity is in principle resolved since the time-dependence of creep relaxation is different in the two regimes. In practice resolving this ambiguity may prove difficult: (*a*) because for small enough perturbations the response in the nonlinear creep regime becomes linear, and the time-dependence is a simple exponential decay, as for the linear creep regime, and (*b*) because the fits to the observed relaxation typically consist of several components with different relaxation times, and it can be difficult to distinguish significantly between different combinations of linear or nonlinear creep response.

Such ambiguity will mainly arise in relatively young pulsars, in which the linear *and* nonlinear creep regimes coexist. For an older pulsar in which creep is nonlinear, and any remaining linear creep becomes observationally insignificant, the shortest relaxation time from the *weak pinning regions* would correspond to the maximum gap Δ (cf. eq. [37]). However, the global minimum in the relaxation time will probably come from the superweak pinning regions, since

$$\tau^{sw} = \frac{b^w}{b^{sw}} \tau^w \ll \tau^w .$$

IV. DISCUSSION OF POSTGLITCH OBSERVATIONS

In this section we shall review the evaluation of postglitch behavior in terms of vortex creep theory, for each of the four pulsars whose postglitch relaxation has been observed, the Crab and Vela pulsars, PSR 0355+54 and PSR 0525+21. In our earlier work on postglitch relaxation, the data following the first two observed glitches of the Crab pulsar, the first four of the Vela pulsar and the one glitch observed from PSR 0525+21 were fitted in terms of nonlinear creep response alone (Alpar *et al.* 1984*b*; Alpar, Nandkumar, and Pines 1985). This was in part because nonlinear creep was required by shifts in $\dot{\Omega}_c$ that persisted for years after a glitch, and for simplicity, the same regime was employed, successfully, to evaluate the shorter time scale components of the relaxation. However, in all of these cases, the actual date of the glitch was not known, the uncertainty being at least a week. This may have masked the response of creep in the linear regime in the form of initial exponential relaxation, with short relaxation times. Postglitch observations following the recent large glitch of PSR 0355+54, when evaluated within the vortex creep model, require the presence of both linear and nonlinear creep regimes (Alpar *et al.* 1988). The more recent fifth (Hamilton, McCulloch, and Royle 1982), sixth (McCulloch *et al.* 1983), seventh (Klekociuk, McCulloch, and Hamilton 1985), and eighth (Flanagan 1989; Hamilton *et al.* 1989) glitches of the Vela pulsar, and the third Crab pulsar glitch (Lyne and Pritchard 1987) have all been caught within a day; fits to the postglitch data can be made with combinations of simple exponential decays and persistent shifts. In particular, Cordes, Downs, and Krause-Polstorff (1988) have fitted the relaxation following each of the first six Vela pulsar glitches with a "short" ($\tau = 4$–10 day), and an "intermediate" ($\tau = 14$–120 day) exponential decay followed by a long-term relaxation in which $\dot{\Omega}_c(t)$ is linear in time. It would seem timely to carry out a comprehensive evaluation of the postglitch fits, with particular emphasis on whether fits to the observed time-dependence of $\dot{\Omega}_c$ following the glitch can distinguish significantly between the alternatives of linear and nonlinear creep. Such a reexamination is being carried out (Nandkumar *et al.* 1989, Alpar *et al.* 1989). Here we present a preliminary analysis, based on consistency arguments, of the implications of observed relaxation times.

We can check for the consistency of interpreting a particular observed relaxation time as reflecting the linear creep regime, as follows:

(i) Note that a requirement on the linear regime is that (E_p/kT) be less than its transition value, given by equation (30).

(ii) Use this requirement, together with the expression, equation (16), which relates the linear creep relaxation time to the pinning energy, temperature, and ω_{cr}, to obtain a minimum value for ω_{cr}. This minimum is, from equation (16),

$$\omega_{cr} > \frac{40\Omega_0 v_{0,7} \tau_l}{r_6} \left(\frac{E_p}{kT} \exp\left(-\frac{E_p}{kT}\right)\right)_{tr} . \quad (43)$$

(iii) Check to see whether the minimum value so obtained can be accommodated by observational or theoretical upper bounds on ω_{cr}. The existence of *linear creep* will imply that

$$E_p < kT\left(\frac{E_p}{kT}\right)_{tr} \quad (44)$$

in the regions of the star where creep is linear. Similarly, evidence for *nonlinear creep* means there are regions in the pulsar where pinning energies are larger than a certain value,

$$E_p > kT\left(\frac{E_p}{kT}\right)_{tr} . \quad (45)$$

The bounds (37) and (45) could be used as bounds on the interior temperature of neutron stars if an accurate calculation of E_p were available. More realistically, with theoretical and observational information on the temperature, they can be used to yield the smallest and largest ranges in the spectrum of pinning energies. The bound (44) from the existence of linear creep is most restrictive using the temperature of the oldest and coldest pulsar that displays linear creep (PSR 0355+54). Similarly, the existence of nonlinear creep in the youngest and hottest, the Crab pulsar, supplies the most restrictive use of the bound (45) on the range of strongest pinning energies.

a) The Crab Pulsar

The most recent, third, observed glitch of the Crab pulsar (Lyne and Pritchard 1987) was recorded with an uncertainty of only one hour in its time of occurrence. The initial postglitch data can be fitted with an exponentially decaying phase residual [which is the second integral of $\dot{\Omega}_c(t)$], with a time constant $\tau = 2.5$–5.5 days, depending on the length of data employed, followed by a component that has a longer relaxation time. Exponential decays with various combinations of two relaxation times provide an equally good fit to the data. Our earlier work yielded 3 day and 60 day relaxation times for the nonlinear creep fits to the postglitch relaxation following the first two glitches. Work in progress (Nandkumar *et al.* 1989) will compare linear and nonlinear creep fits to the Crab postglitch relaxation data. A model calculation for Crab pulsar relaxation times is shown in Figure 1*a*. Using $T = 35$ keV, and the recent gap values of Ainsworth, Pines, and Wambach (1989) shown in Figure 2, nonlinear relaxation times of 80–120 day are indicated, along with a wide range of shorter relaxation times for the linear creep regions.

Using equation (30), we find $(E_p/kT)_{tr} \sim 29$. Applying the inequality, equation (43), to the Crab pulsar, we obtain $\omega_{cr} \gtrsim 1.5 \times 10^{-2}$ rad s^{-1} as the consistency condition for the appearance of a 3 day relaxation time which represents a linear creep region. Correspondingly, $\omega_{cr} \gtrsim 0.3$ rad s^{-1} for a 60 day relaxation time to represent linear creep. The latter limit is close to the observational upper limit (Alpar *et al.* 1987), $\omega_{cr} \sim 0.7$ rad s^{-1}, on the average value of ω_{cr} of the pinned crustal superfluid.

What are the consistency conditions imposed by the assumption that one or both relaxation times represent nonlinear creep? As we have noted, the presence of a persistent shift in $\dot{\Omega}_c$ is a sign of nonlinear creep. Since such a shift has been observed following the 1975 glitch, there must exist a region of nonlinear creep in the Crab pulsar. It follows that the pinning energy in the nonlinear creep regions, E_p^{nl}, must be greater than the transition value given in the inequality (45),

$$E_p^{nl} > 29kT_{\rm Crab} . \quad (46)$$

From standard cooling theory (Tsuruta 1986), $kT_{\rm Crab} \sim 35$ keV, so that there must be present, in the Crab pulsar, substantial ($I_p/I \sim 10^{-3}$) regions in which the pinning energy exceeds 1 MeV. Such regions may be expected to correspond to weak pinning (Alpar *et al.* 1984*b*). On making use of the microscopic expressions for the pinning energy in the weak pinning region (eq. [33]), this limit in turn implies a limit on the superfluid energy gap,

$$\Delta({\rm MeV}) \gtrsim 1.1[(kT)_{\rm Crab}/35 \text{ keV}]^{1/2}[\gamma k_F({\rm fm}^{-1})]^{-1/2} . \quad (47)$$

This limit does not depend on any observed relaxation times, but only on the existence of nonlinear creep as evidenced by the persistent shifts. It does, however, use the standard cooling theory results for the temperature.

One can obtain a particularly important limit on ω_{cr} from (36) and (47),

$$\omega_{cr} \gtrsim 0.45\gamma^{1/2} \frac{[kT({\rm Crab})/35 \text{ keV}]^{1/2}}{r_6(b_z/50 \text{ fm})^3 k_F^{3/2}({\rm fm}^{-1})} . \quad (48)$$

Now, the persistent shift in $\dot{\Omega}_c$ in the Crab postglitch data involves a region with $I_p/I \sim 10^{-3}$, so that $I_p \sim 10^{42}$ g cm^2 for a typical neutron star with total moment of inertia 10^{45} g cm^2. Thus the Crab data for nonlinear relaxation implies

$$I_{p,43}\varpi_{\rm cr} \gtrsim 4.5 \times 10^{-2}[kT(\text{Crab})/35 \text{ keV}]^{1/2}\alpha , \quad (49)$$

where $\alpha \sim O(1)$ is the average of the density dependent factors in the inequality (48) over the nonlinear creep regions. As older pulsars will have more nonlinear creep regions, and $\omega_{\rm cr}$ is a temperature independent pinning parameter, this gives a lower limit, applicable to all pulsars, on the coefficient of $|\dot{\Omega}|_c$ in the rate of energy dissipation, equation (29). This lower limit is consistent with the current observational upper limit (Alpar *et al.* 1987), $I_{p,43}\varpi_{\rm cr} \lesssim 0.7$ rad s^{-1}. For the Crab pulsar, with $|\dot{\Omega}|_c = 2.5 \times 10^{-9}$ rad s^{-2}, equation (37) reads

$$\Delta(\text{MeV}) \cong 2.2kT(\text{keV})[\tau_{nl}(d)]^{-1}(|\dot{\Omega}|/2.5 \times 10^{-9})^{-1} \times r_6^{-1}(b_z/50 \text{ fm})^{-3}[k_F(\text{fm}^{-1})]^{-2} . \quad (50)$$

With the current gap values of Ainsworth, Pines, and Wambach (1989), shown in Figure 2, and at a temperature of 35 keV, the nonlinear relaxation times shown in Figure 1*a*, $\tau_{nl} = 80$–120 days are obtained. In earlier work, Alpar, Nandkumar, and Pines (1985) used this relation, with a smaller nonlinear relaxation time of 60 days obtained from fits to postglitch relaxation data then available, and with larger gap values (Hoffberg *et al.* 1970). They *deduced* an internal temperature $T \cong 4 \times 10^8$ K, consistent with $T = 35$ keV obtained from standard cooling scenarios.

If an observed relaxation time, for example 60 days, reflects nonlinear creep and one does not assume any prior knowledge of the superfluid gap or the temperature, but retains the assumption of weak pinning, then an interesting lower bound on the superfluid gap Δ and on the temperature follows from equation (50) and the inequality (47):

$$\Delta(\text{MeV}) \gtrsim 0.9(\tau_{nl}/60 \text{ d})(|\dot{\Omega}|/|\dot{\Omega}|_{\rm Crab}) \times r_6(b_z/50 \text{ fm})^3 k_F(\text{fm}^{-1})\gamma^{-1} \quad (51)$$

and

$$kT(\text{keV}) \gtrsim 25(\tau_{nl}/60 \text{ d})^2(|\dot{\Omega}|/|\dot{\Omega}|_{\rm Crab})^2 \times r_6^{\,2}(b_z/50 \text{ fm})^6 k_F^{\,3}(\text{fm}^{-1})\gamma^{-1} \quad (52)$$

These lower bounds are proportional to $|\dot{\Omega}|\tau_{nl}$ and $(|\dot{\Omega}|\tau_{nl})^2$ respectively, so the Crab pulsar, having the largest $|\dot{\Omega}|$, is the one glitching pulsar, at present, where these bounds are interesting. They apply if the relaxation time can be confirmed to reflect the nonlinear regime, but are independent of any knowledge of the temperature.

If at least one component of postglitch relaxation can be shown to be in the linear regime, then there must be pinning regions with $E_p^{\,l} < 29\, kT_{\rm Crab}$. It will be very interesting to see if the short time scale postglitch relaxation following the third glitch can be fitted better with linear or nonlinear creep (Nandkumar *et al.* 1989). In either case the long time scale persistent shift in $|\dot{\Omega}|_c$, which was clearly part of the postglitch behavior following the 1975 glitch and may have been present after the 1969 and 1986 glitches as well, indicates the unequivocal presence of the nonlinear creep regime.

b) The Vela Pulsar

The situation for the Vela pulsar is similar to that for the Crab. Our earlier fits to postglitch relaxation data involved short-term relaxation components, characterized by $\tau \sim 3$ day and $\tau \sim 60$ day, followed by a long time scale response, with $\dot{\Omega}_c$ healing gradually, and only linearly in time. (The observation of similar relaxation times for Crab is a numerical coincidence.) For these first few observed glitches, the glitch date was uncertain by 5 to 23 days, and our postglitch fits employed the nonlinear creep regime alone. By contrast, the fifth, sixth, and seventh glitches were each caught within a day, while the recent eighth glitch (Flanagan 1989; Hamilton *et al.* 1989) actually took place during an observation (Hamilton *et al.* 1989). The short-term relaxation has been fitted by a pair of simple exponentials, with relaxation times $\tau_1 = 1.6$–3.2 days and $\tau_2 = 60$–233 days (Klekociuk 1987) for the three sets of postglitch data following glitches 5, 6, and 7. An extensive study of JPL data for relaxation following glitches 1 through 6 (Cordes, Downs, and Krause-Polstorff 1988) has also shown that each postglitch data set can be fitted with a pair of exponentials with $\tau_1 = 4$–10 days and $\tau_2 = 14$–120 days. It will be very interesting to see if these data sets can be fitted equally well with combinations including nonlinear creep response, and whether the observed time dependence can distinguish significantly between the two possible regimes. Figure 1*b* gives relaxation times for the Vela pulsar, employing the gap values of Ainsworth, Pines, and Wambach (1989), and $kT = 11$ keV for the interior temperature. This value of the interior temperature is deduced from the recent observational limits on the Vela pulsar's surface temperature (Ögelman and Zimmermann 1989) and is somewhat lower than the predictions of most standard cooling theories.

For the Vela pulsar the transition value of E_p/kT is $(E_p/kT)_{\rm tr} \cong 31$, so that if an observed relaxation time reflects linear regime vortex creep, the bound on $\omega_{\rm cr}$ using equation (43) is seen to be $\omega_{\rm cr} > 6 \times 10^{-2}$ rad s^{-1} for the longest reported relaxation time, 233 days (Klekociuk 1987). Thus the observed relaxation times can be easily associated with the linear creep regime. However, as was the case with the Crab pulsar, the observation of a slow component of the relaxation (with $\Delta\dot{\Omega}_c$ linear in t in the case of the Vela pulsar) indicates the presence of nonlinear creep. The *simultaneous* presence of the two creep regimes would imply:

$$E_p^{\,l} < (E_p/kT)_{\rm tr}\, kT < E_p^{\,nl} \quad (53)$$

so that for the Vela pulsar one has

$$E_p^{\,l} < 31kT_{\rm Vela} < E_p^{\,nl} . \quad (54)$$

The internal temperature of $kT \cong 11$ keV implied by the observations yields $E_p^{\,nl} \gtrsim 0.34$ MeV. The internal temperature predicted by standard neutron star cooling theories, $kT \sim 16$ keV at the age of the Vela pulsar, gives $E_p^{\,nl} \gtrsim 0.5$ MeV. If all relaxation times, including the prompt relaxation, reflect the nonlinear creep regime only the upper bound remains in effect. The internal temperature can then be determined from τ_{nl}, using equation (37) as was done by Alpar *et al.* (1984*b*) for the postglitch relaxation times for the first four Vela glitches; they found a temperature $T \cong 1.5$ keV, in clear disagreement with the results of standard cooling theory, using earlier, large gap values. According to equation (37), the currently available, lower values of the superfluid gap will further decrease the temperature estimate. A demonstration that nonlinear creep response provides a significantly better fit to the recent data than the fits provided by linear creep will be necessary, if we are to confirm these low-temperature estimates and call for a nonstandard, rapid cooling scenario for the Vela pulsar. We note that the current observational upper limit to the Vela pulsar's surface temperature is also somewhat below the surface temperatures inferred from standard cooling theory, but the discrepancy there is relatively small and can perhaps be accounted for if the spectrum that would emerge from a pulsar atmosphere (Romani 1987) is used instead of a blackbody in deriving a temperature upper limit from the observed count rate data.

Let us assume that it proves possible to fit the comparatively prompt relaxation ($\tau \lesssim 60$ d) observed in all eight Vela glitches with linear response, and, also, that the standard cooling scenario is applicable. In this case we can use equation (37) to obtain an estimate of the value for the relaxation time associated with the persistent shift, which characterizes vortex creep in a nonlinear region. With the internal temperature $T = 11$ keV, the nonlinear relaxation times, shown in Figure 1*b* range from 640 to several thousand days. The appearance of such a relaxation process would serve to confirm both the above assignment of relaxation times and calculations based on the standard cooling scenario. Figure 1*b* also shows the effect of regions of superweak pinning on the spectrum of relaxation times. A transition to superweak pinning might occur when ξ vortex cores encompass several neighboring pinning sites. Using the criterion $\xi \gtrsim b_z/2$, we find that with the current gap values, this transition would take place at a density of 9×10^{13} g cm^{-3}. The resulting reduction in E_p and increase of b, the distance between effective pinning sites along a vortex line, change the relaxation times. Superweak pinning parameters are not available at present; the dashed lines in Figure 1*b* are meant as a qualitative sketch. Note that because of the sensitive dependence of r_l on E_p, even a modest reduction in E_p will extend the linear creep regions significantly, while the reduction of τ_{nl} due to the increase in b (eqs. [5] and [26]) will be less important. With superweak pinning the longest nonlinear relaxation times may be reduced; down to a value of $\tau_{nl} \sim 100$ d at $\rho = 9 \times 10^{13}$ g cm^{-3}, from about 2000 days in the model calculation displayed in Figure 1*b*. The moment of inertia in the linear creep regions will increase, and a prevalent range of linear creep relaxation times might be chosen, depending on the run of E_p in the superweak regime; the dashed line in Figure 1*b* was drawn to reflect prevailing linear creep relaxation times in the 10–100 d range.

In short, *if* the observed relaxation times less than ~ 100 d reflect the nonlinear regime *and* the Vela pulsar has an interior temperature like 11 keV, then there must be superweak pinning. Alternatively, these relaxation times could reflect the linear regime and weak pinning. In earlier work (Alpar *et al.* 1984*b*), associating weak pinning and nonlinear creep with a 60 d relaxation time had led to the low-temperature estimate $T \cong 1.4$ keV. An evaluation of the postglitch fits following the first seven Vela glitches is in progress (Alpar *et al.* 1989).

c) PSR 0355+54

Alpar *et al.* (1988) recently evaluated the postglitch relaxation data following the large glitch observed from this pulsar (Lyne 1987) in terms of an initial linear creep response, observed as an exponential relaxation with time constant 44 d, and a persistent shift in $\dot{\Omega}_c$ due to a nonlinear creep region. The transition value of E_p for this pulsar is $(E_p)_{\rm tr} \sim 35kT$ so that the relaxation time of 44 d, when interpreted as a linear creep relaxation time, requires $\omega_{\rm cr} \gtrsim 4 \times 10^{-5}$ which is compatible with the pinning conditions almost anywhere in the pinned superfluid. The existence of a persistent shift in $\dot{\Omega}_c$ indicates the nonlinear creep regime. The simultaneous presence of the two creep regions implies that

$$E_p{}^l \lesssim 35kT_{0355+54} \lesssim E_p{}^{nl} \qquad (55)$$

The internal temperature of PSR 0355+54 is ~ 1.5 keV if we assume that this pulsar is old enough to be heated primarily by vortex creep; a comparable value is obtained using some standard cooling scenarios. With this assumed low temperature, one has an interesting upper limit on the pinning energy which characterizes the linear response regime, $E_p{}^l \lesssim 50$ keV. Such small pinning energies indicate superweak pinning. Indeed, a model calculation based on weak pinning alone shows that the nonlinear creep regime would prevail everywhere in PSR 0355+54; the observation of the 44 d linear relaxation time requires that superweak pinning must be present. The condition for superweak pinning, $\xi \gtrsim b_z/2$, means Δ must be small,

$$\Delta < \frac{2\hbar^2 k_F}{\pi m b_z} \sim 1 \text{ MeV} . \qquad (56)$$

This bound is indicated in Figure 2; the observation of linear creep from PSR 0355+54 implies the presence of $\Delta \lesssim 1$ MeV over a region with $I_p/I \sim 10^{-3}$. Note that some standard cooling scenarios give temperatures as high as 9 keV. In this case we only have $E_p{}^l \lesssim 315$ keV and no strong argument for superweak pinning.

Thus, PSR 0355+54 is still in that early stage in a pulsar's life when the presence of linear creep is observable as the response of a substantial part of the pinned superfluid, but only because pinning is superweak there. As the pulsar cools further, its temperature will no longer satisfy the first inequality in (55), even in comparison to the minimum pinning energies available in the superweak regions, leading to the prevalence of nonlinear creep at later stages of the pulsar's life.

d) PSR 0525+21

In this case, two relaxation times, $\tau_1 = 150$ d and $\tau_2 = 3000$ d were extracted by Alpar, Nandkumar, and Pines (1985) from their postglitch fits with nonlinear creep, along with a long-term persistent shift. As we would expect, for this old pulsar with $t_s = 1.5 \times 10^6$ yr and $T = 1.3 \times 10^{-2}$ keV (inferred from the dissipation rate by vortex creep), the entire relaxation process observed reflects nonlinear creep. The requirement that $E_p{}^l$ be less than the transition value $(E_p)_{\rm tr} \sim 36kT$, leads to a maximum pinning energy, $(E_p{}^l)_{\rm max} \sim 0.5$ keV. It is not very plausible that such very weak pinning energies could lead to the pinning of vortices in a substantial portion ($I_p/I \sim 10^{-3}$) of the star, required to explain relaxation in this pulsar as a linear response. As noted by Alpar, Nandkumar, and Pines (1985), the relaxation time obtained from the explicit expression for τ_{nl}, using a temperature $\sim 1.3 \times 10^{-2}$ keV, is ~ 140 d, in good accord with the observed time ~ 150 d. This comparison depends on the relation between the surface temperature and the interior temperature, as well as on the superfluid gap and pinning energies. Within the uncertainties involved, the more recent calculations of the gap do not alter the agreement between the theoretical estimate of τ_{nl} and the observed relaxation time.

V. CONCLUSIONS

We have presented a general analysis of vortex creep relaxation phenomena, with special attention to the possible presence of regions in which creep is a *linear* response to a glitch.

We have also considered in some detail the consistency criteria which enable one to assign an association of observed τ's with linear or nonlinear creep regions within the star and have applied these conditions to the few pulsars for which postglitch observations have been made: the Crab and Vela pulsars, PSR 0355+54 and PSR 0525+21. We conclude that linear creep can be important in the early stages ($\lesssim 10^6$ yr) of the life of a pulsar, while nonlinear creep becomes the dominant response as a pulsar ages ($\gtrsim 10^6$ yr) and cools. Nonlinear creep is, in fact, present in all four pulsars, since each exhibits relaxation phenomena which fall outside the expected behavior for linear creep response.

In PSR 0355+54, the observation of prompt postglitch relaxation requires the presence of linear creep and of superweak pinning, while for the Crab and Vela pulsars, a detailed analysis of the postglitch behavior which takes into account the possible presence of linear creep is needed to see whether linear creep, nonlinear creep, or a combination of the two, provides a significantly better fit to the data.

The phenomenology presented here shows that pulsar relaxation evolves from linear response, limited to superweak pinning as the pulsar ages and cools, to nonlinear response only, for pulsars older than about 10^6 yr. If upheld by future observations and supported by accurate calculations of pinning, vortex creep theory can become an important diagnostic of thermal evolution of pulsars (Shibazaki and Lamb 1989). The observation of a persistent shift in $\dot{\Omega}$ healing in the manner characteristic of nonlinear creep would be the most direct and significant support of vortex creep theory. Future detections of old pulsars in soft X-rays, along the lines of the recent X-ray identification of PSR 0656+14 (Cordova *et al.* 1989) will also be interesting: the thermal luminosities should scale with $|\dot{\Omega}|$ (eq. [29]), with a coefficient $I_p\,\omega_{cr}$ that depends on neutron star structure and so should be roughly similar in all pulsars. Finally, according to our analysis, old pulsars are not expected to exhibit linear response.

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