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THE RADIUS-MASS RELATION FOR CLUSTERS OF GALAXIES: COSMOLOGICAL SCENARIOS VERSUS OBSERVATIONS

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ABSTRACT

The relationship between the characteristic radii and masses (luminosities) of rich clusters of galaxies is studied using N-body simulations in comparison with observations. Various scenarios for the formation of the large-scale structure of the universe within the gravitational instability picture are considered, including hierarchical clustering, pancake, and hybrid scenarios. A well-defined radius-mass relation is found for the simulated clusters in each model, the slope of which depends on the form of the initial density fluctuation spectrum. The radius-luminosity relation is also studied for a sample of 29 Abell clusters, and a well-defined correlation is found to exist, $R \propto L^{0.51 \pm 0.07}$. If M/L is roughly the same among clusters, and if the distributions of light and mass are similar, then best agreement with the observations is found for an initial spectrum with an effective slope $-1 \leq n \leq 0$ on the scale of protoclusters, as is expected in such scenarios as cold dark matter, isocurvature baryonic models, and certain hybrid scenarios. The pancake scenario, on the other hand, may have difficulty reproducing the observed radius-luminosity relation.

Subject headings: cosmology — dark matter — galaxies: clustering — galaxies: formation

I. INTRODUCTION

If structure in the universe has formed as a result of gravitational growth of primordial fluctuations, it should, in principle, be possible to relate certain properties of the presently observed structure to the cosmological initial conditions. Rich clusters of galaxies, being a well-observed class of objects, may provide interesting cosmological information despite the fact that they are nonlinear systems at present. In a series of papers (West, Dekel, and Oemler 1987, hereafter Paper I; West, Oemler, and Dekel 1988, hereafter Paper II; West, Dekel, and Oemler 1989, hereafter Paper III), various properties of rich clusters of galaxies have been studied, including density and velocity dispersion profiles, subclustering, ellipticities, and alignments of clusters with their surroundings. The results to date suggest that the relaxation process associated with cluster collapse quite efficiently and rapidly erases any memory of the cosmological initial conditions from the inner regions of clusters, but does leave sensitive tracers in the outer parts. It might be expected that the global properties of clusters, such as their masses and radii, have not changed drastically since the time of their formation, as postcollapse dynamical evolution should have little effect on these properties, and secondly infall has not yet had sufficient time to affect the structure of clusters significantly (Paper I). Thus, the radius-mass relation of rich clusters might be expected to still reflect the cosmological conditions at the time of their formation. That possibility is the focus of this paper.

The theoretical uncertainty regarding the type of primordial fluctuations and the nature of the dominant dark matter which affects their evolution allows several different scenarios to be considered within the general gravitational instability framework, each predicting a different sequence of formation of structure. Assuming a Gaussian density fluctuation field, if the universe is baryon-dominated and the fluctuations were isothermal or isocurvature, then gravitational clustering would result in a hierarchical formation of structure from small to large scales (e.g., Peebles and Dicke 1968; Peebles 1980; Peebles 1987). A similar hierarchical scenario would occur if the universe is dominated by cold, weakly interacting particles (cold dark matter), although in this case the rather flat fluctuation spectrum which is predicted would result in the nearly coeval collapse of structure on galactic scales (Peebles 1982; Blumenthal and Primack 1983; Blumenthal *et al.* 1984). If, on the other hand, the universe is baryon-dominated and the fluctuations were adiabatic, then photon diffusion would have erased small-scale fluctuations prior to recombination (Silk 1968), resulting in the collapse of supercluster-sized "pancakes" first, followed by fragmentation to galaxies and clusters of galaxies (Zel'dovich 1970; Doroshkevich *et al.* 1981). A similar pancake scenario would arise if the universe is dominated by massive neutrinos (e.g., Bond, Efstahiou, and Silk 1980). Hybrid scenarios are also possible, in which the initial density fluctuation spectrum might have possessed a coherence length as in the pancake scenario, but without all small-scale fluctuations having been completely damped out. Such a hybrid scenario could result from the presence of different types of dark matter or perturbations (Dekel 1983; Dekel and Aarseth 1984; Blumenthal, Dekel, and Primack 1988), or if the universe underwent more than one inflationary phase (Silk and Turner 1987; Turner *et al.* 1987).

When considering the growth of structure in the universe, one might expect that correlations would exist between certain characteristic properties of bound objects, and that these might depend on the cosmological initial conditions. For example, correlations among such parameters as mass (or luminosity), internal velocities, and radius would seem natural. Observationally, strong correlations are known to exist between various properties of elliptical galaxies (e.g., Faber and Jackson 1976; Tonry and Davis 1981; Terlevich *et al.* 1981; Faber 1982; Davies *et al.* 1983; Djorgovski and Davis 1987; Dressler *et al.* 1987; and others). Unfortunately, because dissipation is very likely to have influenced the formation of galaxies and their properties, the connection between the observed relations and the initial conditions is not necessarily straightforward. Clusters of galaxies, on the other hand, which are believed to have formed mainly via dissipationless collapse, are more likely to reflect some traces of the conditions at the time of their formation. Correlations have been found between certain properties of groups and clusters of galaxies. For example, Carpenter (1938) long ago noted a relationship between the number of member galaxies in a cluster and its size. The existence of a density-radius relation extending from individual galaxies through groups and clusters of galaxies has been suggested by de Vaucouleurs (1960, 1961, 1971). Bahcall (1981) also found a correlation between the average galaxy density within $0.25h^{-1}$ Mpc of the center of a rich cluster and its global line-of-sight velocity dispersion. Kashlinsky (1983) studied the relationship between the gravitational radius and velocity dispersion of clusters of galaxies, for which he found $R_{\rm grav} \propto \sigma^{1.55\pm0.45}$, which is consistent with the clusters having either constant surface density or constant space density, although the observational uncertainties here are quite large. Other correlations have also been noted (e.g., Kaastra and van Bueren 1981; Quintana and Melnick 1982).

One can make some approximate predictions as to the sort of radius-mass relation expected in different cosmological scenarios. In Paper I, it was shown that observed clusters exhibit a fairly universal surface brightness profile, and that the simulated clusters in an $\Omega = 1$ universe show a similar mass density profile that is not very sensitive to the actual formation scenario. This profile can be approximated by the de Vaucouleurs $r^{1/4}$ law,

$$S(r) = S_e \exp\left[-7.67(r/R_e^{1/4}) - 1\right], \quad (1)$$

where R_e is the effective radius, i.e., that radius within which half of the total mass (light) is contained, and S_e is the surface density at R_e. Integrating equation (1) over all radii then yields the total cluster mass

$$M_{\rm tot} = 7.22\pi R_e^2 S_e, \quad (2)$$

which is a simple relation between mass, radius, and surface density. It is important to note that even though the density profile of equation (1) does not vanish at any finite radius, the total mass, $M_{\rm tot}$, and therefore half-mass radius, R_e, are well-defined finite quantities. The values of $M_{\rm tot}$ and R_e are not very sensitive to the actual density profile at large radii, where it is not known very well, as long as it converges fairly rapidly, like the de Vaucouleurs profile.

Given equation (2), how might S_e vary from one cosmological scenario to another? Consider fluctuations at a given time in the linear regime with a power spectrum of the form

$$\langle|\delta_k|^2\rangle \propto k^n, \quad -3 \le n \le 4. \quad (3)$$

This corresponds to an rms mass fluctuation in spheres of average mass M, $\delta \propto M^{-(3+n)/6}$. Assuming that a given protocluster evolved according to a spherical "top hat" model until it turned around at maximum expansion, and that after collapse it relaxed to a final radius that is a fixed fraction of the radius at turnaround, then the final mean density inside R_e, ρ_e, is proportional to the cosmological density at the time when it collapsed, $\bar{\rho}(t_e)$. In an Einstein–de Sitter universe $\bar{\rho} \propto t^{-2}$, and according to linear perturbation theory $t_e \propto \delta^{-3/2}$, so $\rho_e \propto M^{-(3+n)/2}$. Assuming $\rho_e \propto S_e/R_e$, equation (2) then yields the predicted radius-mass relation

$$R_e \propto M_{\rm tot}^{\beta}, \quad \beta = \left(\frac{5+n}{9+n}\right). \quad (4)$$

For a fluctuation spectrum of the form in equation (3), this would predict $\beta = 0.56$, 0.50, 0.43, and 0.33 for $n = 0, -1, -2$, and -3, respectively.

The arguments leading to equation (4) are, however, likely to be an oversimplification of the problem, since they do not include such effects as subsequent mergers of smaller lumps, dynamical friction, secondary infall of outlying material, and the fact that there is inevitably some dispersion in the masses of objects entering the nonlinear regime at any given time. Nevertheless, equation (4) does suggest that a well-defined radius-mass relation should exist for clusters of galaxies formed via hierarchical clustering, and that it could be related to the form of the initial density fluctuation spectrum, even if the density profile is universal.

Similarly, a correlation between radius and mass would also seem likely for clusters formed in the pancake scenario, depending on the geometry of the superclusters in which these objects were born (e.g., two-dimensional sheets or one-dimensional filaments). For example, if clusters fragmented from pancakes of different thickness but of the same mean density, then one would expect a relation of the sort $R_e \propto M_{\rm tot}^{1/3}$. Of course, cluster formation in the pancake scenario is likely to be a more complicated process than envisioned by this simple argument, as they would tend to form at the nodes where sheets and filaments intersect.

A correlation between radius and mass might also be expected for clusters formed in models which are not based purely on gravitational instability, such as the explosion scenario (Ostriker and Cowie 1981; Ikeuchi 1981). Weinberg, Ostriker, and Dekel (1989) have shown that the most likely sites for the formation of rich clusters are the points where three expanding shells intersect, and have suggested a simple relationship between the cluster mass, $M_{\rm tot}$, and the geometrical average of the shell sizes, R_s, $M_{\rm tot} \propto R_s^3$. This would then lead to a radius-mass relation similar to that in the dissipative pancake scenario, although again this prediction is very rough; for more quantitative predictions one should appeal to N-body simulations (e.g., West, Weinberg, and Dekel 1989).

Given the range of values for β predicted for the different cosmogonic scenarios, it would seem that the radius-mass relation might provide a useful test of different theories for the formation of structure in the universe. In the present paper, the radius-mass relation for rich clusters of galaxies is studied using N-body simulations and then compared with observations. The N-body cluster simulations used in this study are described briefly in § II. The radius-mass relations for clusters formed in the different theoretical scenarios are then studied in § III. New data on the observed radius-luminosity relation for a sample of 29 Abell clusters are presented in § IV, and are compared to the theoretical radius-mass relations in § V, which also contains a discussion of these and previous results.

II. INITIAL CONDITIONS FOR N-BODY SIMULATIONS

The N-body simulations used in this study have been described in detail in Paper I and therefore are reviewed here only briefly. A two-step approach was used in order to generate high-resolution cluster simulations beginning from a wide range of cosmological initial conditions. Low-resolution, large-scale cosmological simulations of the different theoretical scenarios were first performed to find the locations where protoclusters formed for a given set of initial conditions, and these results then provided the initial conditions for the second major step, in which high-resolution simulations of individual clusters were performed. Using such a two-step approach makes it possible to study the detailed properties of rich clusters formed in a wide range of cosmogonic scenarios with sufficient resolution so that any systematic differences which may exist between clusters are not masked by the poor resolution on small scales that is inherent in most large-scale cosmological N-body simulations.

The desired initial fluctuation spectra for the different cosmogonic scenarios were generated using a method based on the approximation of Zel'dovich (1970) for describing the evolution of density fluctuations in the linear regime. The spectra considered here have the general power-law form of equation (3). Simulations were performed for the following five gravitational instability scenarios: (a) a pancake scenario originating from an initial fluctuation spectrum with $n = 0$ on large scales and truncated below a critical wavelength, (*b*), (*c*), and (*d*) three hierarchical clustering scenarios with power spectrum indicates $n = 0$, -1, and -2, and (*e*) a hybrid of these scenarios originating from an initial $n = 0$ perturbation spectrum possessing a coherence length as in the pancake scenario, with an additional low-amplitude small-scale component. All of these simulations assumed an Einstein–de Sitter universe ($\Omega = 1$). In addition, the $n = 0$ hierarchical clustering simulations were repeated for the case of an open universe, $\Omega_0 = 0.15$ at present. And although not explicitly simulated here, the initial fluctuation spectrum in the cold dark matter scenario can be approximated near the relevant scales for clusters by either the $n = 0$ or $n = -1$ hierarchical clustering cases, while $n = -2$ is more appropriate for cold dark matter on the scale of galaxies. The procedure used to generate the initial conditions was thoroughly checked by a variety of means, all of which confirmed that it is indeed capable of reproducing the desired initial density fluctuation spectra (Paper I; Braun and Dekel 1988).

The large-scale simulations used in the first step were performed with ~ 4000 equalmass particles using a comoving version of a direct N-body code (Aarseth 1985). Four random realizations were performed for each of the different theoretical scenarios. The stages of the simulations that correspond to the present epoch were determined by matching the slope of the two-point correlation function with that observed for galaxies. Equating the correlation length, r_0, of the simulations with the claimed value for galaxies, $r_0 = 5h^{-1}$ Mpc (Davis and Peebles 1983; although see Kirshner *et al.* 1989), then sets the scaling from simulation to physical units. With this scaling, the diameter of the simulated volumes corresponds to $\sim 100h^{-1}$ Mpc, and the coherence length in the pancake and hybrid scenarios results in superclusters of $\sim 30h^{-1}$ Mpc in diameter. Rich clusters were then identified in these large-scale simulations using a simple group finding algorithm described in Paper I. The clusters found by this procedure should correspond roughly to Abell clusters of richness class $R \geq 1$.

Once the rich clusters were identified, new simulations were then performed using these same initial conditions, but now modeling smaller volumes centered on the locations of each of the five richest clusters found in each of the large-scale simulations (thus, a total of 20 clusters per cosmogonic scenario). The initial radius of these individual cluster simulations was 45% that of the large-scale simulations (simulations using a larger volume give similar results; see Paper I). These high-resolution simulations of individual clusters were run using a noncomoving version of the Aarseth code, with ~ 1000 equal-mass particles. With the adopted scaling from simulation to physical units, each particle in these simulations should correspond roughly to an L^* galaxy. With such a two-step procedure, the resolution of the cluster simulations can be greatly improved, since by concentrating on the relatively smaller volume around each cluster, the mass of each individual particle in the new simulations can be smaller, and more of the particles eventually end up in the cluster itself rather than in other surrounding structures. Rich clusters formed in these high-resolution simulations contained typically 100–200 galaxies. Several representative clusters formed in the different cosmological scenarios are shown in Figure 1. Again, for further details on the initial conditions used in these N-body simulations, the reader is referred to Paper I.

III. RADIUS-MASS RELATION FOR SIMULATED CLUSTERS

Since the ultimate goal is to compare the results from the simulations with observations of real clusters, the simulated clusters have been analyzed in a manner similar to that which observers must use. Three orthogonal projected views of each of the simulated clusters were examined. The projected total cluster radius, R_{100}, was then determined from the cluster density profile, by defining R_{100} to be that radius at which the profile first falls to the mean background density of the simulated volume (see Paper I). The total projected cluster mass, $M_{\rm tot}$, was then found by counting all particles within this radius. The cluster half-mass radius, R_{50}, was taken as the projected radius encompassing half this total mass, and thus corresponds to R_e of equation (1). The values of R_{50} and $M_{\rm tot}$ determined in this manner were found to be quite robust, not depending sensitively on either the assumed background level or the procedure used to determine the cluster density profile.

The radius-mass relations for the simulated clusters in the different cosmogonic scenarios were determined by comparing $M_{\rm tot}$ with R_{50}. To make doubly certain that the measured values of the cluster half-mass radii and total masses were not in error as a result of contamination by other nearby groups or clusters, only reasonably well-isolated clusters were used, as determined from visual inspection. However, in only a few instances was it necessary to reject a projected view of a given cluster for this reason.

The results for each of the different scenarios are shown in Figure 2. A well-defined correlation between R_{50} and $M_{\rm tot}$ can be seen in most cases. Logarithmic slopes were determined for each of these distributions by a least-squares method, and are listed in Table 1, along with the corresponding correlation coefficients, $r_{\rm corr}$, for the data. Since the probabilities of having obtained these values of $r_{\rm corr}$ in the absence of any true correlations between R_{50} and $M_{\rm tot}$ are all $\ll 1\%$, the radius-mass relations found here can be considered highly statistically significant for all scenarios.

Several interesting conclusions can be drawn from the results of Figure 2 and Table 1. It is apparent that in general

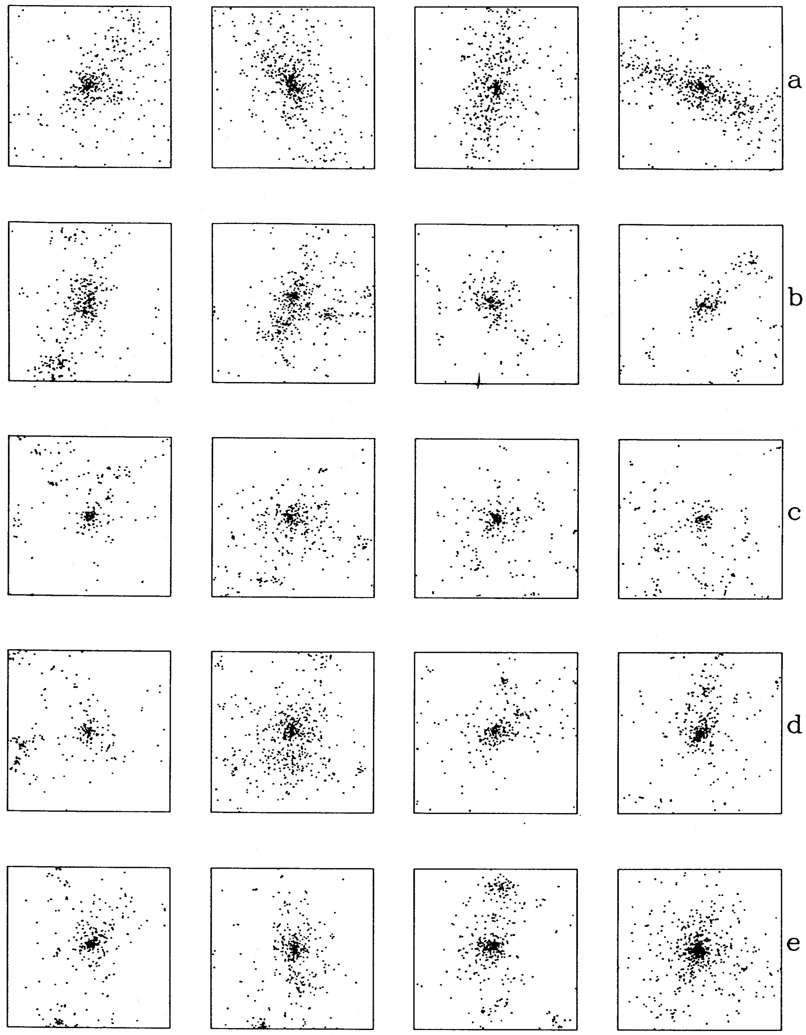

FIG. 1.—Typical clusters formed in the different cosmological simulations. Labels denote (*a*) pancake scenario; (*b*) hybrid scenario; (*c*), (*d*), and (*e*) hierarchical clustering scenarios with $n = 0, -1$, and -2, respectively. Each box is $10h^{-1}$ Mpc on a side. All simulations shown are for $\Omega = 1$.

the slope of the radius-mass relation found for all the simulated clusters depends quite sensitively on the cosmological initial conditions, with slopes ranging from 0.24 to 0.49. For example, the radius-mass relation for clusters formed in the $n = 0$ hierarchical clustering scenario shows the steepest slope, while clusters formed in the pancake scenario exhibit the shallowest slope, in qualitative agreement with the theoretical predictions. Furthermore, the actual slopes found for the simulated clusters agree fairly well with the predicted values in § I. Interesting too is the fact that the radius-mass relation for the $n = 0$ hierarchical clustering scenario seems to show little dependence on Ω, although the spread in this relation is larger for clusters formed in an open universe.

TABLE 1

RADIUS-MASS RELATION: $R \propto M^{\beta}$

Scenario	β	r_{corr}
Pancake	0.24 ± 0.05	0.55
Hybrid	0.48 ± 0.06	0.77
Hierarchical ($n = 0$)	0.49 ± 0.07	0.70
Hierarchical ($n = -1$)	0.42 ± 0.05	0.74
Hierarchical ($n = -2$)	0.34 ± 0.05	0.70
Hierarchical ($n = 0, \Omega = 0.15$)	0.47 ± 0.10	0.55

In conclusion, the fairly wide range of slopes found here for the radius-mass relation in different cosmogonies suggests that comparison with observations might allow one or more of the different scenarios to be ruled out. This will be discussed further in § V.

IV. RADIUS-LUMINOSITY RELATION FOR ABELL CLUSTERS

To compare the above results with observations, data have been collected on the properties of 29 clusters of galaxies. With a few exceptions, these represent all clusters with reliable determinations of both the cluster profile and luminosity function. The exceptions are those few clusters, discussed briefly in Paper I, which have very flat, low central concentration profiles. These clusters, which also have distinctive galaxy populations (see Butcher and Oemler 1984), appear to represent a separate class of objects, whose relation to the class of normal clusters is unclear. We have included among the group of excluded objects the Virgo Cluster, whose structure is intermediate between those of low- and high-concentration clusters.

In Paper I we gathered all of the extant high-quality data on

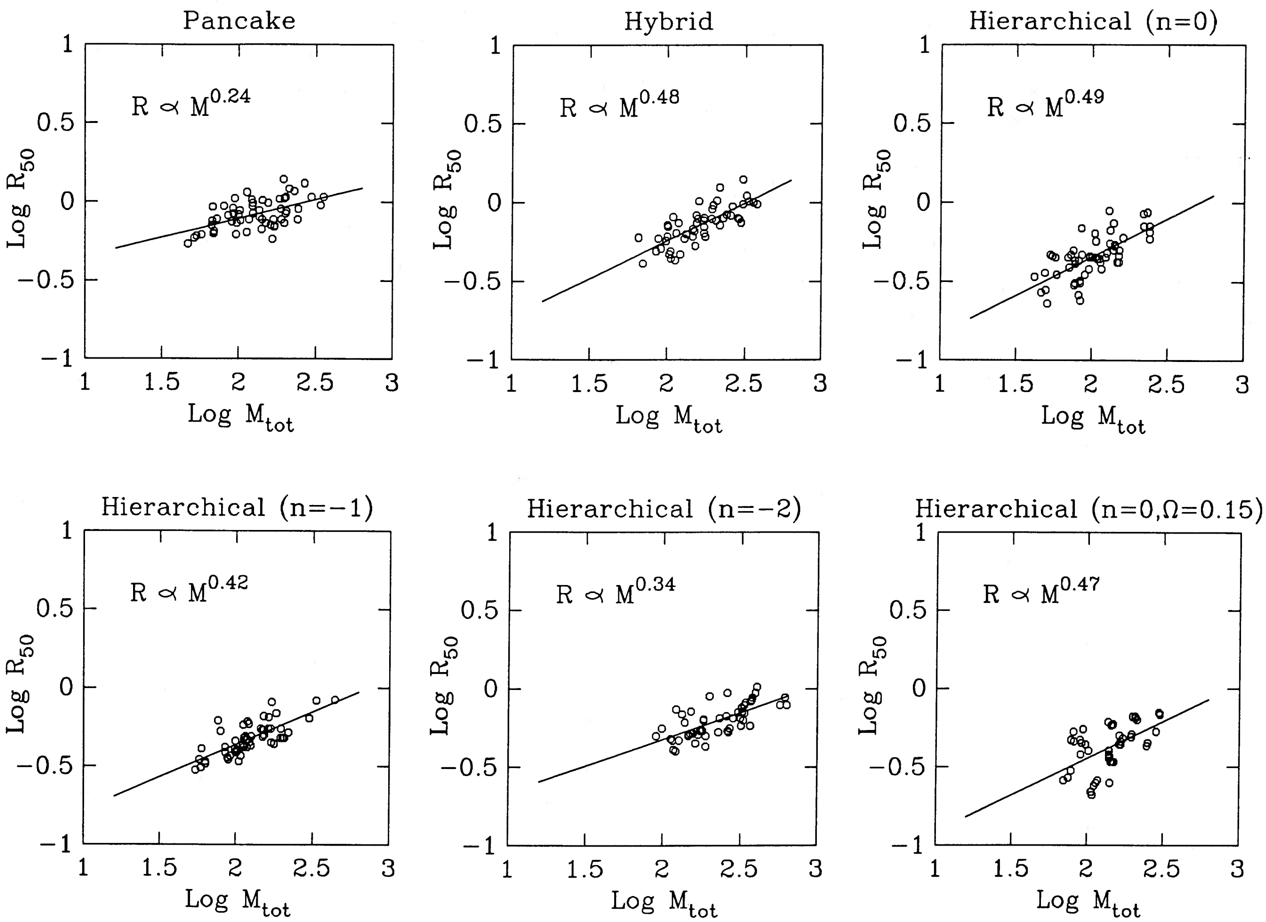

FIG. 2.—Radius-mass relation for simulated clusters in the different cosmological scenarios. R_{50} is the projected half-mass radius in h^{-1} Mpc, and M_{tot} is the total cluster mass in units of M^* galaxies.

cluster profiles. (The reader is referred to that paper for a discussion of the uncertainties in determining the cluster galaxy distributions.) Reliable photometry is also available for all but three of these clusters. After excluding five low-concentration clusters, 19 remain. To these we add 10 clusters which were studied by Butcher, Oemler, and Wells (1983). Although only one determination of the cluster profile is available for each of these, we think that they are reliable enough to use. In Table 2 we present the structural and luminosity data on the 29 clusters. Columns (1)–(6) contain the cluster name, redshift, R_{50} in arcmin, the source of that number, the total cluster luminosity, L, in units of L^*, and the source or sources of the data used to determine L.

The determination of cluster luminosities is fraught with even more uncertainties than that of cluster profiles. Relatively minor uncertainties include correction for Galactic extinction and background galaxy contamination, calibration of the photometry, and correction of (the usual) isophotal galaxy magnitudes to total magnitudes. The major problems are due to the necessity of extrapolating the sum of the measured luminosities of the cluster members to include galaxies below the completeness limit of the photometry, and to include the outer parts of the cluster beyond the region studied. These extrapolations can be large and depend sensitively on the form of the galaxy luminosity function and cluster profile. For five clusters in common between the study of Dressler (1978) and those of Oemler (1974), Butcher, Oemler, and Wells (1983), and Butcher and Oemler (1985), Dressler's total cluster luminosities are larger than those of the other studies by a mean factor of 1.7, due, almost entirely, to differences in these extrapolations.

To produce a homogeneous data set, we have re-reduced all of the cluster photometry cited in Table 2 in a more uniform manner. The corrections for Galactic extinction, isophotal magnitudes, and background contamination used by the original workers have been retained. However, we have extrapolated to infinite radius using our own determinations of the cluster profiles, as presented in Paper I, or as taken from Butcher, Oemler, and Wells (1983). In all but one case, our extrapolation to fainter galaxies has been made using a luminosity function of the Schechter (1976) form, with parameters taken from Kirshner *et al.* (1983). The one exception is Abell 2670. Its luminosity function fainter than L^* is unusually flat, and we have used a smaller than normal extrapolation to account for that. With these extrapolations, the discrepancy between the Dressler photometry and that of Oemler and collaborators is reduced to 15%, and that is due almost entirely to one cluster, A401, for which the Dressler photometry produces a cluster luminosity 50% greater than do the Butcher and Oemler data, for reasons we do not entirely understand. From the scatter of individual determinations, we estimate that the contribution of random errors to uncertainty in the cluster luminosity is ~15%. Systematic errors may be much larger, if

TABLE 2

ABELL CLUSTER PARAMETERS

Cluster	z	R'_{50}	References	L/L^*	References
Coma	0.0235	51	1	190	3, 4
Corona Borealis	0.0721	14	1	250	5
Fornax	0.0044	75	1	27	6
Perseus	0.0183	59	1	230	7
A154	0.0658	31	1	240	8
A168	0.0452	42	1	150	8
A194	0.0186	28	1	35	4
A400	0.0232	29	1	65	3, 4
A401	0.0748	21	1	300	3, 8
A520	0.0203	6.6	2	270	2
A539	0.0267	14	1	45	4
A665	0.1816	7.5	1	270	4
A777	0.226	2.1	2	32	2
A963	0.206	6.5	2	190	2
A1314	0.0341	15	1	55	4
A1413	0.1427	7.1	1	270	4
A1758	0.280	4	2	200	2
A1904	0.0714	18	1	150	3, 4
A1942	0.224	6.8	2	120	2
A1961	0.232	5.6	2	190	2
A1963	0.23	2.6	2	80	2
A2029	0.0767	25	1	330	8
A2111	0.229	5	2	250	2
A2125	0.247	3.9	2	130	2
A2199	0.0305	32	1	120	3, 4
A2218	0.171	11.5	1	430	2, 8
A2256	0.0601	19	1	350	8
A2397	0.224	4.2	2	50	2
A2670	0.0749	8.4	1	100	2, 4, 8

REFERENCES.—(1) West, Dekel, and Oemler (1987, Paper I); (2) Butcher, Oemler, and Wells (1983); (3) Butcher and Oemler (1984); (4) Oemler (1974); (5) Hoffman and Crane (1977); (6) Duus (1977); (7) Bucknell, Goodwin, and Peach (1979); (8) Dressler (1978).

our extrapolations are far wrong, but such an error should not affect the slope of the radius-luminosity relation which is our goal.

The data from Table 2 are plotted in Figure 3, where we have converted from angular to linear sizes assuming $H_0 = 100$ km s^{-1} Mpc^{-1} and $q_0 = 0.5$. A well-defined correlation between radius and luminosity is found to exist, with

$$R_{59} \propto L_{\rm tot}^{0.51 \pm 0.07} ,$$

and correlation coefficient $r_{\rm corr} = 0.81$.

One issue of concern here is possible selection effects on the observed correlation. For example, a worry is that the Abell classification scheme might itself introduce a lower cutoff in surface brightness. Abell (1958) selected and classified clusters into richness groups according to their galaxy count within a fixed Abell radius, $R_A = 1.5h^{-1}$ Mpc. The bottom of the $R = 0$ richness class, which defines the criterion for inclusion in the catalog, corresponds to a lower limit of the surface brightness within a fixed radius. Since the Abell radius is typically 1.5–2 times larger than the half-light radius, one can assume that the luminosity contained within the Abell radius is a good approximation (better than a factor of 2) for the total cluster luminosity. So the limit imposed by the Abell selection procedure is effectively rather a lower limit on $L_{\rm tot}$, not on $S_e \propto L_{\rm tot}/R_{50}$, where R_{50} varies from cluster to cluster. Such a lower limit on $L_{\rm tot}$ in the radius-luminosity distribution is not likely to generate a false correlation of the sort $R_{50} \propto L_{\rm tot}^{1/2}$.

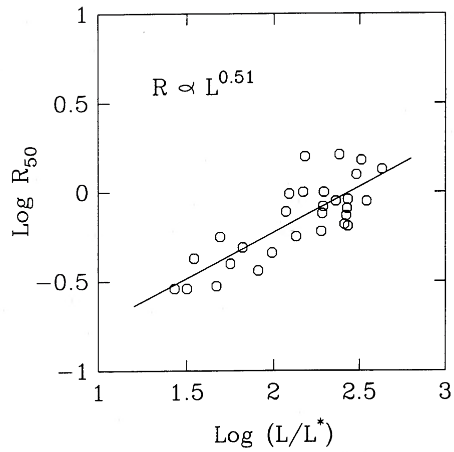

FIG. 3.—Observed radius-luminosity relation for the sample of 29 Abell clusters. R_{50} is the projected half-light radius in h^{-1} Mpc, and $L_{\rm tot}$ is the total cluster luminosity in units of L^*.

V. DISCUSSION

To compare the observed radius-luminosity relation with the radius-mass relation of the simulated clusters, one must make some assumptions about how the mass-to-light ratio (M/L) varies both within a given cluster, and among different clusters. If one adopts the simplest assumptions, namely, that the dark and luminous matter are similarly distributed within clusters (e.g., Paper I; Kent and Gunn 1982; Merritt 1987; although see West and Richstone 1988) and that their ratio remains fairly constant among rich clusters (e.g., Dressler 1978; Blumenthal *et al.* 1984), then a direct comparison between the theoretical radius-mass relations and the observed radius-luminosity relation is possible. In that case, there are several scenarios which would seem to be in agreement with the observed relationship, namely, the hybrid and the $n = 0$ and $n = -1$ hierarchical clustering scenarios (for both $\Omega_0 = 1$ and $\Omega_0 = 0.15$). This range of n is also consistent with what one would expect for a cold dark matter initial fluctuation spectrum, or for the baryonic-isocurvature model, near the relevant scales for rich clusters. The radius-mass relation for clusters formed in the pancake scenario, on the other hand, is markedly inconsistent with the radius-luminosity relation for the observed clusters, *unless* M/L has a strong radial dependence within clusters, *or* the global M/L *decreases* with increasing system size (neither of which possibilities has much observational support at present). Thus, *if* the aforementioned assumptions regarding M/L are valid, then the observed radius-luminosity relation places constraints on the sequence of cosmogony, requiring that clusters have formed as the result of hierarchical clustering originating from an initial fluctuation spectrum with some power on small scales. However, this conclusion must be tempered somewhat until present uncertainties regarding M/L variations can be resolved. Further studies of the radius-luminosity relation for rich clusters may help to narrow the range of viable cosmogonic scenarios, although converting from the observed radius-luminosity relation to a radius-mass relation will remain problematic until it can be shown convincingly that M/L either remains constant or varies in some systematic way.

Finally, it is worth summarizing the basic conclusions from this series of papers (West, Dekel, and Oemler 1987, 1989;

West, Oemler, and Dekel 1988; this paper) in which the systematic properties of clusters of galaxies formed in different cosmological scenarios have been studied by using N-body simulations in comparison with observations. The facts would seem to suggest that, although able to reproduce many of the observed properties of rich clusters fairly well (e.g., density profiles and velocity dispersion profiles), none of the simple gravitational instability scenarios which have been studied is capable of reproducing *all* of them simultaneously. For example, simulations of dissipationless hierarchical clustering originating from initial fluctuation spectra with $n \simeq 0$ on cluster scales (such as the cold dark matter scenario, or Poisson initial conditions) simply cannot reproduce the observed tendency for clusters to be aligned with one another and with their surroundings (Dekel, West, and Aarseth 1984; Paper III), yet it is these scenarios which were found here to produce the best agreement with the observed radius-luminosity relation. The pancake scenario, on the other hand, although nicely reproducing the cluster alignments, has been found in the present paper to suffer from a potential flaw, namely, that clusters formed within pancakes have a radius-mass relation which seems to be quite inconsistent with the observed radius-luminosity relation for clusters. The gravitational instability scenario which seems to come closest to reproducing most of the observed cluster properties is the hybrid scenario. Although such an initial fluctuation spectrum is admittedly somewhat ad hoc, it does seem to combine the desirable features of both pancake and hierarchical clustering scenarios (namely a coherence length coupled with small-scale power), while also avoiding their pitfalls. The isocurvature-baryonic model of Peebles (1987) might also be expected to produce similar results. The conclusions concerning cluster formation in open and closed cosmological models are also ambiguous. For example, clusters formed via hierarchical clustering in an open universe were found in Paper I to have density profiles which were a poor match to those observed, while identical simulations run for an $\Omega = 1$ universe produced good agreement. Yet no dependence of the radius-mass relation on Ω was found in the present paper, with clusters formed via hierarchical clustering in both open and Einstein–de Sitter universes agreeing well with the observed relation for Abell clusters. Given then the seemingly paradoxical situation which arises when most simple gravitational instability models are confronted with observations of rich clusters, one might be encouraged to explore other possibilities for the origin of the large-scale structure of the universe.

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