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CONSTRAINTS ON CATACLYSMIC VARIABLE EVOLUTION FROM THE TRIPLE SYSTEM 4 DRACONIS

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ABSTRACT

We consider the evolution of the triple system 4 Dra, which now comprises a cataclysmic variable in a $P \approx 1700^d$ orbit with an M giant. We find that the requirement of dynamical stability imposes significant restrictions on the precursor system of the CV.

Subject headings: stars: binaries — stars: dwarf novae — stars: evolution — stars: individual (4 Dra)

Reimers, Griffin, and Brown (1988; hereafter RGB) have shown that 4 Dra (HR 4765; CQ Dra) is an extraordinary triple-star system consisting of an M giant in orbit with a cataclysmic variable (CV) of (probably) the AM Her subtype: ((WD + ?; 0.166^d) + M3 III; 1703^d, $e = 0.3$). It is perhaps not inconceivable that a merely binary model, in which the white dwarf accretes directly from the wind of the giant, could be constructed, in which case the 4 hr periodicity would represent a rotational period of the white dwarf. However, the interpretation put forward by RGB is sufficiently strong to impel us to explore some of the restrictions which the longer period orbit imposes on models for the formation of the CV. In particular, we find a substantial restriction on the initial size of the binary subsystem from which the present CV has presumably evolved. The restriction comes from the possibility of dynamical instability of the orbits in the precursor triple-star system and is made tighter by the probability that at least one component of the triple system must have lost substantial mass at some stage in the past. Thus the existence of the third star and the dynamics of the triple system provide constraints on the precursor of a C V, information which is otherwise difficult to obtain.

We shall for convenience identify the three components by a "suffix notation" that can generalize to hierarchical n-tuple systems (see e.g., Evans 1968). The long-period binary consists of two components, $*1$ (CV) and $*2$ (M3 III), and $*1$ itself consists of two subcomponents, $*11$ (WD) and $*12$ (presumably an M dwarf). Orbital quantities (e.g., period P or semimajor axis a), with a suffix 1 refer to the ($*11 + *12$) binary; those without a suffix refer to the ($*1 + *2$) binary. The mass of the CV is denoted $m_1 = m_{11} + m_{12}$.

Following Paczyński (1976), we suppose that the CV evolved from a moderately wide (late case B or late case C) binary. At the time mass transfer was initiated, $*11$ (the present white dwarf) would have been a red giant. The transfer of mass from $*11$ to $*12$ would have been unstable and would have resulted in a common envelope phase of evolution. The two cores inside the common envelope would subsequently have spiraled in to the present short period ($P_1 = 3.98$ hr). While this evolutionary scheme is widely believed, it has proven extremely difficult to test directly, largely because the complexity and short time scale of the spiral-in stage ($\lesssim 10^3$ yr; see Bodenheimer and Taam 1984) precludes identifying CVs with their precursor systems, either numerically or observationally. In the case of 4 Dra, however, the presence of $*2$ in its $P = 1703^d$ orbit restricts considerably the possible size of the precursor to $*1$. Bailyn (1984, 1987) showed that one needs $a/a_1 \gtrsim 3$–5 for dynamical stability of the orbits, with this critical ratio being somewhat dependent on the mass ratios m_1/m_2, m_{11}/m_{12}, and the inclination and eccentricity of the orbits.

The above consideration, described in more detail below, immediately restricts the initial period to $P_1 \lesssim 400^d$, supposing that P has not changed. However, P itself is unlikely to be constant, because the CV's mass is almost certainly less, and perhaps considerably less, than the initial value for its precursor. Furthermore, it is not unreasonable that m_2 (the mass of the observed giant) should also have decreased by stellar-wind mass loss. There is no unequivocal statement relating mass changes to period changes, because much depends on the mechanism by which mass is lost. But the simplest assumption, which we believe to be reasonable in the present case, is that the mass lost by $*1$ (and also any by $*2$) carries off the same angular momentum per unit mass as resides in the orbital motion of the star from which it was lost, and that the process was sufficiently slow (i.e., taking many times the 1703^d period) that the eccentricity e and reduced angular momentum h (total angular momentum divided by reduced mass of the system), being adiabatic invariants, are unchanged. These assumptions lead to

$$P \propto m^{-2}\,, \quad a \propto m^{-1}\,. \tag{1}$$

We show below that m can reasonably be expected to have nearly halved since the birth of the system, and thus the 1703^d period can be expected to have been smaller at birth, perhaps by as much as a factor of 4. This tightens considerably any restriction on the maximum radius that $*11$ would have been able to attain during a red giant phase. It turns out that this restriction is not especially sensitive to the (unknown) inclination i of the spectroscopic orbit of $*2$.

RGB found the mass function of the M3 III star to be $f(m_2) = m_1^3 \sin^3 i/m^2 = 0.0076\ M_\odot$. Since the masses of CVs are expected to be typically 0.7–2 $M_\odot$ (Robinson 1976), the implication is that $25\ M_\odot \gtrsim m_2 \gtrsim 4\ M_\odot$, but of course only if the inclination i is not far from its median value of 60°. Even with $i = 30°$ and $m_1 = 0.7\ M_\odot$, we still get $m_2 \gtrsim 2\ M_\odot$. Clearly the *initial* value of m_{11} must have been greater than m_2 in order for $*11$ to have evolved to a red giant before $*2$. Thus if $0.7\ M_\odot \lesssim m_1 \lesssim 2\ M_\odot$ now, at least one and more probably several solar masses must have been lost to the system from $*11$ if $i \gtrsim 30°$.

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We have computed quantitative limits on the size of the precursor to the white dwarf (*11) for a variety of assumptions about the current values of m_{11} and m_{12} and the inclination of the system. If one assumes values for these parameters, the mass function found by RGB sets m_2 (the mass of the M giant), and thus the total mass of the system. We then assume that the *initial* value of m_{11} was greater than m_2, so that *11 could have completed its red giant evolution before *2. Specifically, we take the initial value of $m_{11} = m_2 + 0.5\ M_\odot$. The minimum amount of mass that the system could have lost is then given by comparing the initial and final values of m_{11}. This, with Kepler's laws, in turn yield the maximum orbital period and separation for the (*1, *2) binary before mass loss.

Now triple systems in which $\frac{1}{2} < m_2/(m_{11} + m_{12}) < 1$, as is the case here, have been shown to be stable if $a(1 - e)/a_1 > 3.3$ (Harrington 1977; Bailyn 1984, 1987). If this criterion is not satisfied, the hierarchical nature of the system breaks down, generally leading to the ejection of the least massive body from the system within a few tens of binary orbits. Variations of this critical value with m_{11}/m_{12}, e, and small deviations from coplanar orbits ($i_1 \lesssim 40°$), do not result in changes in the stability limit for a_1 of more than 10%. Therefore, the initial value of a calculated above, together with RGB's value of $e = 0.3$, give a maximum value for a_1 prior to mass loss when the system must have been stable for many orbital periods. Eggleton's (1983) formula for relating effective Roche lobe size to separation and mass ratio then yields a maximum value for the radius of *11 immediately prior to Roche lobe overflow and the common envelope phase which presumably leads to mass loss and the formation of the current CV. Values for these limits are given for various values of the assumed parameters in Table 1.

We regard the values $m_{11} = 0.6\ M_\odot$, $m_{12} = 0.3\ M_\odot$ as the most probable current masses for the CV. The ≈ 4 hr period sets m_{12} (if *12 is a hydrogen burning star; see RGB), and, as we shall see, larger values of m_{11} are unlikely due to the relatively small core size attainable by *11's red giant progenitor. Using these guesses as input, the maximum size allowed for the progenitor to *11 ranges from 96 $R_\odot$ to 55 $R_\odot$, depending on the assumed inclination. While the maximum period of the (*11, *12) binary increases with decreasing inclination (due to the smaller total mass loss), the maximum size of the Roche lobe of *11 *decreases* with decreasing inclination, due to the less extreme mass ratio m_{11}/m_{12}. We note that inclinations less than 18° are unlikely, since in this case $m_2 < 0.8\ M_\odot$, so the star could not have evolved into a giant in less than a Hubble time. If mass loss from the giant is used to get around this limitation, the restrictions on size of the white dwarf progenitor become more severe, since the system will have lost even more mass than we assume. If one assumes somewhat larger current values for m_{11} and m_{12}, one can obtain larger initial values of m_{11}, and therefore somewhat looser limits on the size of the progenitor to *11. However, the limit on R_{11} *for a given original value of* m_{11} is almost independent of the current values of m_{11} and m_{12}. We note that the large value for m_2 (and hence for the original value of m_{11}) of $\approx 18\ M_\odot$ obtained with $m_{11} = 1.0\ M_\odot$ and $m_{12} = 0.4\ M_\odot$ and $i = 90°$ may be implausible given the current absolute luminosity for *2 of $M_V \approx -0.5$ (RGB).

We focus attention on models in Table 1 with inclinations in the range 15°–30°, because these give masses for the present red giant (*2) of $\sim$1–5 $M_\odot$. Such masses are low enough not to be particularly rare, while high enough for evolution to have taken place. They imply a maximum radius for the precursor red giant (*11) of 55–90 $R_\odot$. For a 3 $M_\odot$ theoretical model, we find that this is large enough to allow helium ignition before Roche lobe overflow (i.e., case C in the notation of Kippenhahn and Weigert 1967), but only by a factor of 2 or 3. When the 3 $M_\odot$ model reaches this size, its core mass is between 0.60 $M_\odot$ and 0.64 $M_\odot$. If common envelope evolution begins at this stage, we would therefore expect the white dwarf in the present CV to be much nearer 0.6 $M_\odot$ than 1.0 $M_\odot$.

TABLE 1

MAXIMUM RADIUS FOR WHITE DWARF PRECURSOR

m_{11}[a]	m_{12}[a]	i[b]	m_2[c]	$P_{t=0}$[d]	$P^{\max}_{1,t=0}$[e]	$R^{\max}_{11}$[f]
0.4.........	0.3	90	6.0	461	63	81
		60	4.7	477	63	74
		45	3.3	498	65	66
		30	1.7	558	71	54
0.6.........	0.3	90	8.9	473	64	96
		60	7.0	485	65	88
		30	2.6	582	75	66
		20	1.1	767	94	57
0.8.........	0.3	90	12.1	475	65	109
		60	9.6	488	66	100
		30	3.6	589	78	76
		20	1.5	787	99	66
1.0.........	0.4	90	17.6	472	64	123
		60	13.9	484	66	114
		30	5.3	578	77	86
		15	1.1	1107	135	71

NOTE:—Masses and radii are in solar units.
[a] Current masses of CV components.
[b] Inclination of systems.
[c] Current mass of red giant.
[d] Initial period of *outer* binary (days).
[e] Maximum period of CV precursor, *i.e., inner* binary (days).
[f] Maximum Roche lobe radius for white-dwarf precursor.

However, no known hierarchical triple with an outer period of $\sim$100 yr or less has a ratio of semimajor axes (or equivalently of periods) at all close to the instability limit. Fekel (1981) shows that period ratios are typically 100:1 rather than 5:1. The nearest known case to instability, λ Tau ((A4 IV + B3 V; 4^d.0) + ?; 33^d; Fekel and Tomkin 1982; Bailyn and Eggleton 1983) has a period ratio of 8:1. In this case the stability limit, expressed as a period ratio, is 3.3:1, which is considerably less than in the precursor to 4 Dra due to the lower mass of the third body and the circularity of the outer orbit in λ Tau. Although several triples are known whose longer period $\approx 10^3$ day, their shorter periods are always $\lesssim 10^d$. ξ Tau((B9 V + B9 V; 7^d.15) + B7 V; 145^d; Fekel 1981)) is moderately close to the kind of initial configuration we are driven to, but the shorter period is too short; HD 214608 ((G2 V + K2 V; 552^d) + F9 V; 30 yr; Duquennoy 1987) has a similar period ratio to ξ Tau, but the longer period is too long. If we suppose that a period ratio of ≈ 15 is more credible than an initial period of ratio ≈ 8 for the precursor to 4 Dra (which would be right at the boundary of stability; see Table 1) then the maximum radius of *11 is further reduced by a factor of 2. This makes it marginal whether *11 could evolve beyond He ignition or not, thus imposing a limit on the core mass and the current value of m_{11} of $\lesssim 0.5\ M_\odot$.

The often relatively high masses ($\gtrsim 1\ M_\odot$) of white dwarfs in CVs (Robinson 1976) suggests that their precursor red giants had evolved beyond He ignition. Indeed Livio and Soker (1984) have suggested that the precursors of all white dwarfs in CVs must have undergone He ignition. Systems that came into contact before this would form common envelopes with relatively high binding energy, and the subsequent spiral in would lead to a single coalesced object rather than a CV. The calculations described here, however, place an *upper* limit on the size of the precursor red giant of this CV. Comparison of this upper limit with improved stellar models based upon those described by Eggleton (1972) suggests that this CV has formed from a giant that may never have experienced He ignition. If we restrict ourselves to moderate masses for the precursor, we find that the CV is probably best modeled with a 3 $M_\odot$ precursor. Coincidentally this is about the mass at which He ignition occurs at the smallest radius, $\approx 26\ R_\odot$. After He ignition a 3 $M_\odot$ star will shrink back inside this radius for a while eventually achieving a core mass of $\sim 0.6\ M_\odot$ before becoming larger than 26 $R_\odot$ on its way up the asymptotic giant branch. At lower masses, interaction would have to take place before He ignition—a 2 $M_\odot$ star would not ignite He until its radius reaches 120 $R_\odot$ at which point it has a helium core of only 0.47 $M_\odot$. At slightly higher masses it is again unlikely that the precursor can ignite He before interacting—a 4 $M_\odot$ star reaches 40 $R_\odot$ and a core mass of 0.46 $M_\odot$ at He ignition, which while possible, it quite close to the limit. Such a star goes on to reach a core mass of 0.77 $M_\odot$ before again exceeding this radius. We cannot rule out the possibility of much higher mass stars leaving substantial cores ($\gtrsim 1\ M_\odot$) since such a core is already well-developed before the star reaches the giant branch, but such massive stars are of course much rarer than intermediate mass stars like those discussed above.

Thus there exist serious restrictions on the nature of the zero-age precursor to the CV in 4 Dra, a situation which may be unique. While this CV may not be "typical," it provides fairly firm evidence that a CV can be result from a precursor binary with $P \lesssim 100$ days. Although we have selected, as most likely, a model with a 3 $M_\odot$ precursor that leaves a 0.6 $M_\odot$ white dwarf in the CV, in line with observations (Robinson 1976) and theory (Livio and Soker 1984), it is a tight squeeze and a lower mass white dwarf remnant is perhaps more likely. In this case the presence of the third star may have tidally assisted in the removal of the, normally too tightly bound, common envelope.

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