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GROUPS OF GALAXIES IN THE CENTER FOR ASTROPHYSICS REDSHIFT SURVEY¹

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ABSTRACT

We apply an objective group identification algorithm described by Huchra and Geller to the Center for Astrophysics redshift survey complete to $m_{B(0)} = 15.5$ over the right ascension range $8^{h} \le \alpha \le 17^{h}$ and declination range $26.5 \le \delta < 38.5$. We extract a catalog of 128 groups with three or more members; 92 of these groups constitute our "statistical sample." Simulations of the geometry of large-scale structure indicate that $\gtrsim 30\%$ of the groups with three or four members are probably an artifact of the geometry.

The median velocity dispersion for the 36 groups in the "statistical sample" with five or more members is $\sigma_v = 228 \text{ km s}^{-1}$, and the median $M/L_{B(0)}$ is 178 h M_{\odot}/L_{\odot} , where the Hubble constant H_0 is 100 h km s⁻¹ Mpc^{-1} (we take h = 1 unless otherwise indicated). The median parameters for the 92 group sample are similar. The sample contains seven Abell clusters; the physical properties of these clusters overlap substantially with those of groups. In fact, the distinction between groups and clusters is not generally apparent on the basis of selection of systems in redshift space.

Comparison of the distribution of group centers with the distribution of all of the galaxies in the survey shows qualitatively that groups trace the large-scale structure in the region. The physical properties of groups may be related to the details of the large-scale structure. Groups reextracted from the earlier CfA survey complete to $m_{B(0)} = 14.5$ have a significantly lower median velocity dispersion, $\sigma_v = 131$ km s⁻¹, than the groups in the 15.5 survey.

About 58% of the groups in the $m_{B(0)} \le 15.5$ survey contains three or more galaxies brighter than L*, the characteristic luminosity in the Schechter form of the luminosity function; in contrast, 77% of the groups in the 14.5 survey have fewer than three members brighter than L*. The difference in the group catalogs is probably largely a result of the properties of large-scale structures and their location relative to the survey limits. Subject headings: galaxies: clustering — galaxies: redshifts

I. INTRODUCTION

Studies of the dynamics of systems of galaxies-from small groups to rich clusters-have long provided evidence for dark matter in the universe (Zwicky 1933; Rood, Rothman, and Turnrose 1970; Faber and Gallagher 1979). For groups of galaxies, there are substantial uncertainties in the determination of mass-to-light ratios. Some problems arise in the identification and statistical analysis of groups; others reflect deeper issues in understanding the relative distribution of dark and light-emitting matter in the universe.

Approaches to the construction of catalogs of groups of galaxies have become increasingly objective and more closely related to the physics of systems of galaxies. In early catalogs (e.g., Holmberg 1969; de Vaucouleurs 1975) groups were selected primarily by visual inspection of plates. In the absence of complete redshift surveys, Turner and Gott (1976) first used an objective algorithm to extract groups from a catalog of the positions of galaxies in the sky. With the increasing availability of extensive redshift surveys, the focus has now shifted to the objective identification of groups of galaxies from these threedimensional data (Materne 1978, 1979; Tully 1980; Huchra

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and Geller 1982, hereafter HG82; Press and Davis 1982; Geller and Huchra 1983, hereafter GH83; Vennik 1984; Tully 1987; Nolthenius and White 1987, hereafter NW). Even in objectively constructed group catalogs, group mass-to-light ratios range over 3 orders of magnitude. Perhaps more sobering, the median mass-to-light ratios varies by more than a factor of 2 from one catalog to another. Heisler, Tremaine, and Bahcall (1985) and NW conclude that the large range and variation in median mass-to-light ratio are probably unavoidable statistical problems which result from studying systems with small numbers of members.

The detailed properties of the groups in any catalog are a function of (1) the galaxy survey, (2) the general characteristics of the larger scale structure in which groups of galaxies are embedded, and (3) the algorithm applied to select groups. Here we examine groups in two slices of the Center for Astrophysics redshift survey extension. The catalog of 1766 galaxies covers the right ascension range $8^{h} \le \alpha \le 17^{h}$ and the declination range $26^{\circ}.5 \le \delta < 38^{\circ}.5$ (Fig. 1). The large-scale structure in the region is characterized by voids surrounded (or nearly surrounded) by thin surfaces (FWHM $\simeq 500$ km s⁻¹). The set of systems of galaxies included in this survey differs from earlier surveys in the inclusion of more rich clusters; there are seven Abell clusters in the region. The extension of systems of galaxies along the line of sight ("fingers" in redshift space) produces distortion of a map in redshift space relative to its 58

counterpart in real three-dimensional space. These distortions are obvious for rich systems like the Coma Cluster which have a velocity dispersion large compared with the typical FWHM of the extended sheets in the survey (see Fig. 1). The distortions caused by small groups of galaxies with a typical line-of-sight velocity dispersion of $\lesssim 200 \text{ km s}^{-1}$ (Rood and Dickel 1978; HG82; Tully 1987) are more subtle.

We identify groups of galaxies in the survey by applying the percolation algorithm originally suggested by HG82. In order to examine the impact of large-scale structure on group identification, we also apply the algorithm to a geometric model for the structure in the region. Comparison of this model with the data suggests that at least a third of the sample of groups with three or four members are spurious-they are merely an "accident" of the geometry of large-scale structure. The "pseudogroups" have reasonable physical parameters because the FWHM of the sheets is about twice the velocity dispersion of a typical group. At the same time that the geometry causes contamination of a group catalog, it also enables identification of the system with little foreground/background contamination. The existence of voids with diameters large compared with the FWHM of the structures suppresses the number of velocity interlopers.

We review the group finding algorithm in § II. We discuss the survey in § III. Section IV is a discussion of the optimization of the selection parameters for the group finding algorithm, and § V describes the group catalog. We examine

the physical properties of groups in § VI. We compare the properties of groups with the properties of the Abell clusters in the survey with the properties of groups identified in the earlier CfA survey complete to
$$m_{B(0)} = 14.5$$
. We also qualitatively examine the role of groups as tracers of large-scale structure. Section VII summarizes the results and their implications.

II. THE ALGORITHM

We use the group finding algorithm described in HG82. Here we summarize the main features.

The cluster algorithm identifies isodensity contours of the galaxy distribution in a magnitude-limited sample. We start with a galaxy which has not yet been assigned to a group. We then search around it for companions with projected separation

$$D_{12} \le D_L(V_1, V_2, m_1, m_2) \tag{1}$$

and with line-of-sight velocity difference

$$V_{12} \le V_L(V_1, V_2, m_1, m_2), \qquad (2)$$

where V_1 and V_2 are the redshifts of the galaxy and its companion; m_1 and m_2 are their magnitudes. We add companions to the list of group members. Then we search the surroundings of each companion. We repeat the loop until we can identify no further members. This completely objective algorithm is commutative and produces a unique catalog.





To account for the variation in the sampling of the galaxy luminosity function, $\phi(M)$, with distance, we scale the parameter D_L according to

where

$$D_L = D_0 R , \qquad (3)$$

$$R = \left[\int_{-\infty}^{M_{12}} \Phi(M) dM \middle/ \int_{-\infty}^{M_{\rm lim}} \Phi(M) dM \right]^{-1/2}, \qquad (4)$$

$$M_{\rm lim} = m_{\rm lim} - 25 - 5 \log \left(V_f / H_0 \right) \,, \tag{5}$$

$$M_{12} = m_{\rm lim} - 25 - 5 \log \left[(V_1 + V_2)/2H_0 \right], \qquad (6)$$

and D_0 is the selection parameter at a fixed fiducial redshift V_f . We scale V_L in the same way as D_L ; thus the ratio D_L/V_L is constant.

The number density contour surrounding a group corresponds to a fixed enhancement above the global mean number density of

$$\frac{\delta\rho}{\rho} = \frac{3}{4\pi D_0^3} \left[\int_{-\infty}^{M_{\rm lim}} \Phi(M) dM \right]^{-1} - 1 .$$
 (7)

For each of the groups we derive the following set of physical characteristics. The virial radius is

$$R_{h} = \frac{\pi V}{H_{0}} \sin \left\{ \frac{1}{2} \left[\frac{N_{\text{mem}}(N_{\text{mem}} - 1)}{2} \left(\sum_{i} \sum_{j > i} \theta_{ij}^{-1} \right)^{-1} \right] \right\}, \quad (8)$$

where V is the mean velocity of the group, N_{mem} is the number of group members, and θ_{ij} is the angular separation of group members *i* and *j*. The mean projected separation is

$$R_p = \frac{8V}{\pi H_0} \sin\left[\frac{1}{N_{\text{mem}}(N_{\text{mem}}-1)} \sum_{i} \sum_{j>i} \theta_{ij}\right].$$
 (9)

The virial mass is

$$M_v = 6\sigma_v^2 R_h/G , \qquad (10)$$

and the virial crossing time (in units of the Hubble time) is

$$t_c = \frac{3}{5^{3/2}} \frac{R_h H_0}{\sigma_v} \,, \tag{11}$$

where σ_v is the line-of-sight velocity dispersion of the group. These definitions agree with those in GH83 and with NW [except that the crossing time in NW is $t_c = (2/\sqrt{3})R_hH_0/\sigma_v$].

III. THE REDSHIFT SURVEY AND LUMINOSITY FUNCTION

We apply the group finding algorithm to the first two complete strips of the Center for Astrophysics redshift survey extension (Huchra *et al.* 1989). The redshift survey contains 1766 galaxies with $m_{B(0)} \le 15.5$ in the Zwicky-Nilson merged catalog and within the limits $8^{h} \le \alpha \le 17^{h}$, $26^{\circ}.5 \le \delta < 38^{\circ}.5$. The sample covers 0.42 sr of the sky. The sample we analyze includes only the galaxies with $cz \le 15,000$ km s⁻¹. We apply the algorithm in the declination range $26^{\circ}.5 \le \delta < 32^{\circ}.5$ (6° *slice*) and to the full 12° *slice*. The velocities are *not* corrected for Virgocentric flow. At the depth of this slice, the correction has a completely negligible effect on the results.

The luminosity function we use to scale the selection parameters D_0 and V_0 is from de Lapparent, Geller, and Huchra (1988) and is consistent with the more detailed analysis by de Lapparent, Geller, and Huchra (1989; see Table 1 of their paper). Magnitudes are on the Zwicky B(0) system, and we make no absorption correction. We parameterize the luminosity function with the Schechter (1976) form with

$$\phi^* = 0.025$$
 galaxies mag⁻¹ Mpc⁻³;
 $M^*_{B(0)} = -19.15; \quad \alpha = -1.2.$ (12)

The corresponding luminosity density is $2.05 \times 10^8 L_{\odot}$ Mpc⁻³. Thus $\Omega = 1$ for a critical mass-to-light ratio $M/L_{B(0)} = 1360 M_{\odot}/L_{\odot}$.

IV. TUNING THE SEARCH ALGORITHM

Several requirements dictate the choice of the two selection parameters D_0 and V_0 . The choices should minimize the number of interlopers without biasing the velocity dispersion toward artificially low values. The choice of selection parameters should also maximize the identification of real physical systems relative to accidental superpositions. In other words, we seek to identify regions where the Hubble flow is distorted by condensations of galaxies. For virialized systems, the distortion has the characteristic form of an extension or "finger" along the line of sight. The distorted density enhancements in redshift space correspond to density enhancements in real three-dimensional space.

In the redshift survey slices, groups and clusters of galaxies are embedded in thin sheetlike structures (FWHM ≤ 500 km s⁻¹; de Lapparent 1986; Geller 1987). Even without gravitational distortion, the group finding algorithm will identify condensations in a geometry where galaxies are randomly distributed in thin sheets which surround (or nearly surround) voids. These "pseudogroups" will contaminate a catalog drawn from the survey.

To examine the degree of contamination and the properties of the "pseudogroups," we apply the selection algorithm to a simulation of the 6° slice. In this simulation (de Lapparent 1986) points are first randomly distributed. The simulation is magnitude-limited and has the luminosity function parameters appropriate for the data. Then we remove points in spherical voids with diameters comparable to those in the data until the fraction of the volume filled by the points is comparable to the volume filled by the galaxies in the survey (~20%). In this simulation there is no gravitational distortion of the distribution of points.

To explore the consequences of various choices of the parameters, we produce a set of catalogs for both sets of data and for the simulation. In all catalogs, the groups have mean velocities $cz \le 12,000 \text{ km s}^{-1}$; members may have velocities as large as 15,000 km s⁻¹.

a) The Parameter D_0 and the Density Enhancement

We produce group catalogs at density enhancements $\delta\rho/\rho$ of 20, 80, and 160 corresponding to the D_0 values 0.42, 0.27, and 0.21 Mpc, respectively. We set $V_0 = 350$ km s⁻¹. We analyze the effects of variation in V_0 separately.

Table 1 lists the number of groups N_{groups} , the total number of galaxies in groups N_{gal} , the median velocity dispersion $\sigma_{v,\text{med}}$, and the first and third quartiles of the σ_v distribution for the 6° and 12° slices as well as for the simulation.

The total number of groups with a particular number of members, N_{mem} , is more useful than the total number of groups in the catalog as an indicator of the best choice of the D_0 parameter. The decrease in the total number of real groups for very large values of D_0 —several groups merge into fewer low-density systems—is balanced by an increase in the number of

VARIA	TION OF D_0 (δ	(ho/ ho)	
Sample	N_{groups}	$N_{\rm gal}$	σ_{med} (km s ⁻¹)
	$\delta ho / ho = 20$		
Simulation	90	416	242372
6° slice	81	681	177_{113}^{377}
12°slice	139	1088	177^{365}_{109}
	$\delta ho/ ho=80$	- 4-	
Simulation	41	135	253 ³³⁴ ₁₁₈
6° slice	73	501	202_{116}^{271}
12° slice	128	778	196284
	$\delta ho / ho = 160$		
Simulation	17	51	169^{345}_{112}
6° slice	67	421	185 ²⁹⁵ 82
12° slice	117	634	171_{97}^{266}

TABLE 1

pseudogroups. For very small values of D_0 , i.e., high overdensities, the total number of real groups decreases because we exclude many low-density systems from the catalog. However, this decrease is compensated by an increase in the number of groups which are high-density subclumps within the richest systems. The optimal choice of D_0 is a compromise between the need to identify very low density associations of galaxies and the need to minimize the number of pseudogroups in the catalog.

Figures 2a, 2b, and 2c show the cumulative number of galaxies in groups, $N_{gal}(\leq N_{mem})$, with at most N_{mem} members as a function of N_{mem} . For $\delta\rho/\rho = 20$, 80, and 160, the plots compare the 6° slice with the simulation. At $\delta\rho/\rho = 20$ there are more galaxies in pseudogroups with $N_{mem} \leq 10$ than there are in the real groups. At $\delta\rho/\rho = 80$ most of the pseudogroups have $N_{mem} \leq 4$; many of the real groups still have $N_{mem} \geq 4$. At the highest density enhancement the difference between the histograms for the real groups and the pseudogroups is still larger. However, we still find pseudogroups with three members. The list of real groups consists primarily of the cores of the densest groups and of density peaks within the clusters in the survey.

At low $\delta\rho/\rho$, there are many groups in the simulation. Small random fluctuations in the space density with chance alignment nearly along the line of sight produces these pseudogroups. These pseudogroups have properties similar to those of physical groups in the survey. For these pseudogroups, the internal velocity dispersion is dictated by the typical thickness of the structures which surround the voids ($\sim 500 \text{ km s}^{-1}$). The properties of the pseudogroups are sensitive to the selection parameters.

In contrast, the groups extracted from the redshift survey are stable against variation of $\delta\rho/\rho$. Thirty-four of the groups are stable over the contrast range from $\delta\rho/\rho = 20$ to $\delta\rho/\rho = 80$. Sixty-five groups are stable over the range $\delta\rho/\rho = 20$ to $\delta\rho/\rho = 160$, and 22 over the range $\delta\rho/\rho = 20$ to $\delta\rho/\rho = 160$. In particular, derived parameters of the Coma Cluster are insensitive to the choice of the D_0 parameter. Table 2 is a summary of the properties of the Coma Cluster (velocity dispersion, mass-to-light ratio, and number of members) as a function of $\delta\rho/\rho$.

Following the criterion given at the beginning of this sub-

TABLE 2

	Тне Сома	CLUSTER	
Density Parameter $\delta \rho / \rho$	N _{mem}	σ_v (km s ⁻¹)	$M/L_{B(0)} \ (M_{\odot}/L_{\odot})$
20	182	842	377
80	139	868	357
160	119	900	400

section, the comparison of the data with the simulation indicates that $\delta\rho/\rho \simeq 80$ optimizes the identification of physical systems. Results are similar for the 12° slice. Given the coherence of large-scale structure in going from one 6° slice to the adjacent one, the similarity of group catalogs for the 6° and 12° slices is not surprising.

Although the simulated catalog mimics several of the salient features of the data, the group identification algorithm could be sensitive to some of the details of the model. We therefore construct a much simpler model based on a small portion of the data. The data contain a shell (S1) with $14^{h}30^{m} \le \alpha \le 17^{h}$, $26^{\circ}.5 \le \delta < 32^{\circ}.5$, and 9200 km s⁻¹ $\le cz \le 10,500$ km s⁻¹ (see Fig. 1). The sample S1 contains 111 galaxies. This simple structure extending perpendicular to the line of sight is representative of the environment of most galaxies in the survey. It is narrow enough in velocity space that the scaling of the luminosity function is irrelevant to group selection. This model yields a lower limit on the fraction of spurious groups as a function of N_{mem} .

Our simulation of S1 consists of 111 points distributed over the same solid angle and within the velocity range 9200–10,500 km s⁻¹. In an average over 10 simulations, each with 111 randomly distributed points, we find 4 groups with 3 members and 1 group with 4 members. In the data 6 groups have 3 members, 1 has 4, 1 has 5, 2 have 6, and 2 have 10 members. In the data the number of galaxies in rich groups $(N_{mem} \ge 5)$ is about a third of the total number of galaxies in groups, approximately the same as the fraction in rich systems over the entire survey.

Many of the groups with 5 or more members are probably real physical systems (see Fig. 2b); in other words, they are probably smaller in space than they are in redshift space. Our model of S1 may, therefore, be overly dense. To simulate the effect of this effectively lower background density on group selection, we use 79 randomly distributed galaxies in the simulation of S1. On average, we find 2 groups with 3 members, i.e., about one-third of the number of triples in the data. We find no groups with $N_{mem} > 3$. Taken together, the simulations of S1 suggest that at least a third of the triples in the real group catalog are spurious (although we cannot tell which ones) but that almost all of the $N_{mem} \ge 5$ groups are real.

b) The Selection Parameter V_0 and the Reference Velocity V_f

For each of the data sets in the previous subsection we construct one catalog with $V_0 = 350$ km s⁻¹ and one with $V_0 = 600$ km s⁻¹ (the first value is close to the one in HG82; the second is the one in GH83). All the catalogs have $D_0 = 0.27$ Mpc ($\delta\rho/\rho = 80$). For each catalog Table 3 lists the number of groups, N_{groups} , the total number of galaxies in groups N_{gal} , the median velocity dispersion $\sigma_{v,med}$, and the first and third quartiles of the σ_v distribution.

Just as for variation of D_0 , the catalogs obtained from the data by varying V_0 are much more stable than those obtained





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TABLE	3	

VAR	IATION OF V	ó	
Sample	N_{groups}	N_{gal}	$\sigma_{med} \ (km \ s^{-1})$
$V_0 =$	= 350 km s ⁻	1	
Simulation	41	135	253334
6° slice	73	501	202_{116}^{271}
12° slice	128	778	196284
	= 600 km s ⁻	• 1	
Simulation	57	213	329224
6° slice	-79	549	220 ⁴⁷⁵ ₁₂₄
12° slice	135	855	220_{122}^{461}

from the simulation. Contamination of the real catalog with pseudogroups is minimized for the smallest reasonable V_0 . The lower bound on V_0 is set by the requirement that the velocity dispersions of groups and clusters not be artificially biased toward low values. The scaling of V_0 yields $V_L = 1335$ km s⁻¹ at cz = 10,000 km s⁻¹ for $V_0 = 350$ km s⁻¹. This value of V_L is smaller than the typical radii of voids and larger than necessary to avoid biasing the velocity dispersion of the richest clusters. For $V_0 = 600$ km s⁻¹, $V_L = 2288$ km s⁻¹. We now demonstrate that this larger value leads to significant contamination of groups.

Figures 3a and 3b show the velocity dispersion σ_v as a func-



FIG. 3.—Line-of-sight velocity dispersion, σ_v , as a function of redshift for groups with $\delta\rho/\rho = 80$ and with (a) $V_0 = 600$ km s⁻¹ (135 groups) and (b) $V_0 = 350$ km s⁻¹ (128 groups).

tion of redshift, cz, for all groups in the 12° slice with $V_0 = 350$ km s⁻¹ and $V_0 = 600$ km s⁻¹, respectively. Among the systems with $\sigma_v \le 400$ km s⁻¹, 86/110 ($V_0 = 350$ km s⁻¹) or 86/94 ($V_0 = 600$ km s⁻¹) groups are unaffected by the variation in V_0 . Most of the differences between the catalogs are for $\sigma_v \ge 400$ km s⁻¹. In the $V_0 = 350$ km s⁻¹ catalog there are 18 groups with $\sigma_v \ge 400$ km s⁻¹; with $V_0 = 600$ km s⁻¹ there are 41 of these groups. About half of these groups have fewer than 5 members. Because many groups with a similar range of properties occur in the simulation, a substantial fraction of the groups in the redshift survey are likely to be spurious.

At $V_0 = 350 \text{ km s}^{-1}$ the algorithm identifies all seven Abell clusters in the survey. All have $M/L_{B(0)} \le 1000 M_{\odot}/L_{\odot}$, and the two richest have $\sigma_v \ge 400 \text{ km s}^{-1}$. The properties of the latter two systems are, in fact, insensitive to the choice of V_0 . For $V_0 = 350 \text{ km s}^{-1}$ Coma has 139 members with $\sigma_v = 868 \text{ km s}^{-1}$, and $M/L_{B(0)} = 357 M_{\odot}/L_{\odot}$; for $V_0 = 600 \text{ km s}^{-1}$, there are 153 members with $\sigma_v = 875 \text{ km s}^{-1}$ and $M/L_{B(0)} = 376 M_{\odot}/L_{\odot}$. The cluster A1185 does not vary at all. Among all of the groups with $\sigma_v \ge 400 \text{ km s}^{-1}$ in the $V_0 = 600 \text{ km s}^{-1}$ catalog, 27 have $M/L_{B(0)} \ge 1000 M_{\odot}/L_{\odot}$. In the $V_0 = 350 \text{ km}$ s⁻¹ catalog, only 10 groups have such suspiciously high massto-light ratios.

The selection of $V_0 = 350 \text{ km s}^{-1}$ diminishes the number of high M/L systems in the catalog and preserves most of the richer systems ($N_{\text{mem}} \ge 5$) with $\sigma_v \ge 400 \text{ km s}^{-1}$ and $M/L_{B(0)} \le 1000 M_{\odot}/L_{\odot}$. With $V_0 = 600 \text{ km s}^{-1}$, there are 8 of these systems; 7 of them also appear in the $V_0 = 350 \text{ km s}^{-1}$ catalog. Thus, the lower value of V_0 appears to minimize the contribution of contaminated or spurious groups to the set of rich, high velocity dispersion systems in the catalog.

We adopt $V_0 = 350$ km s⁻¹ for the final catalog. However, most groups are insensitive to this choice; they are generally more sensitive to the choice of D_0 . The insensitivity to the choice of V_0 is probably related to the geometry of large-scale structure. Most of the galaxies in this region are in structures perpendicular to the line of sight. These sheets are generally separated by large voids (with diameters generally larger than V_L). This structure severely restricts the number of interlopers with velocities larger than the ~500 km s⁻¹ FWHM of the sheets, provided that V_L remains small compared with the diameters of the voids. As an example of this effect, the shell S1 discussed in the previous subsection yields exactly the same groups for both values of V_0 . In this case the foreground void has a diameter of ~5000 km s⁻¹.

We have also examined the sensitivity of the catalog to the choice of the reference velocity, V_f . As in our previous papers (HG82; GH83), we adopt $V_f = 1000$ km s⁻¹. We find that catalogs with $\delta\rho/\rho = 80$ and $V_0 = 350$ km s⁻¹ for $V_f = 1000$ km s⁻¹ and for $V_f = 3000$ km s⁻¹ (note that $\delta\rho/\rho = 80$ for $D_0 = 0.39$ Mpc with $V_f = 3000$ km s⁻¹) have nearly indistinguishable properties.

V. THE GROUP CATALOG

The selection parameters for the group catalog in Table 4 are $D_0 = 0.27$ Mpc ($\delta \rho / \rho = 80$) and $V_0 = 350$ km s⁻¹. Each group has at least three members. The mean velocities of the groups are $\leq 12,000$ km s⁻¹, but members may have velocities are large as 15,000 km s⁻¹. In Table 4 each group member is listed with the name assigned in the Center for Astrophysics survey (Huchra *et al.* 1989). The names are related to the right ascension and declination in the usual way.

The catalog includes 128 groups with a total of 778

						GROUP (CATALO	5					
A			dnorg	members			e			group	members		
1		1044+2633 1045+2631	1044+2648	1045 + 2651	1046+2702	1047+2700			1207+3643	N4359	1219+3222	I3308	
3	ł	N3826 1140+2632	N3830A	N3830B	1142+2702	1142+2707	12	!	1125 + 2711 1123 + 2702	1121+2718	1125+2740	1126+2741	1125+2744
6		1413+9649	1416-19650	14205	1410 0706	11200	13	:	I3376	N4475	I3336		
4		N4849	1255+2707	1254+2710	1410+2100 I 837	14399 N4859	14	•	1426+2728 14452	N5635 1425+2710	1428+2727	1427 + 2745	1426 + 2729
		N4821 1966-19711	N4819 1950 9656	1256+2732	I 835 1956 - 9706	1255+2708 M 1767	15		1108 + 2722	N3570	N3563		
		N4827	12000 13900	123072122 [3913	1254+2744	1258+2740	1	+	1533 ± 9730	1531 ± 9730	1534-9790		
		N4842	N4839	N4853	N4854	1257 + 2758				10017	67 17 L LOOT		
		I3959 1253 $+2756$	N4789 1254+2801	1250+2740 N4816	N4798 1253+2805	N4807 I3949	17		N3414 N3451	N3418	1046 + 2811	N3400	N3380
		1258+2754 1250-12600	N4926A	1259+2803 NA011	N4919 N4679	I3976 N 467 4	18	*	1311 + 2801	N5032	1309 + 2735		
		N4871	1256+2817	N4911 N4883	N4812 N4860	N48/4 1255+2824	19		N6269	N6272	N6263	N6264	N6261
		I4026 1256+2820	N4926 13060	N4908 N4876	I4040 N4860	N4895 N4808W			1655 + 2816	1658 + 2805	N6265	1659 + 2830	1658 + 2740
		123072023 N4889	N4864	1957-19810	IADA5	1955 19297							
		1255+2828 N4788	N4848	N4881 N4027	1248+2739 14106F	1249+2751 NAMA	20	*	N4016 N4017	I2982	N4004A	N4004	N4008
		1259 + 2829	1258 + 2838	1259+2840	14100E N4841A	N4944 N4858	21	*	1447 + 2759	1447 + 2805	I4514		
		1259 + 2857	1255 + 2859	I4042	N4886	N4929	66		TOFOO	TOPOO	Toroo	1000 10001	
		I3946	N4850	N4721	1250 + 2721	1300 + 2850	77	*	15095	76651	13598	1236+2801	
		1257+2906	N4841B	1255+2913	1254 + 2912	1254+2918	23	*	I4572	I4568	I4570	I4569	1538 + 2831
		1224+2319 N4896	14133 14133	N4931 N4934	1209+2822 1301+2831	1256+2740			14580	14581	1541 + 2835	1541 + 2841	1539 + 2809
		N4840	N4873	N4828	1255 + 2749	1247 + 2710							
		N4692	1248 + 2806	1249 + 2718	I 842	I4088	24	*	1204 + 2825	N4104	1201 + 2826	1204 + 2813	
		I 843 1200 2620	N4922A N4057	N4922B	N4923	N4921	25	*	N4295	I3210	I3263	I3165	
		1246+2707	1245+2715	1245+2716	1230+2/33 N4745	1257+2808 N4715	26	i	N3026	N3032	0947 + 2815		
		14032 N4673	1259+2931 1944-9744	14051 N4865	13973	N4867	27	*	N4983	N4971	1306 + 2827	N4966	1305 + 2858
ъ		1146 + 2701	N3900	N3912	0007 - 1071				1303 + 2934	1303 + 2851	1302 + 2905	1303 + 2933	1304+2818
9		I3585	N4556	N4558	I3508	N4555	28	*	N3713	1128 + 2828	N3714	1126 + 2849	1129 + 2819
7		1240 + 2655	I3618	I3623	I3645		29	*	1525 + 2857	I4547	I4546	1525 + 2901	1524 + 2901
90	1	N4670	1242 + 2845	1244 + 2650					1525+2824				
6		1212 + 2710	1213+2717	1213 + 2743	1213 + 2656		30	*	1115 + 2833	1116 + 2848	1115 + 2830		
10		0916+2708	0916 + 2740	0914 + 2756	0917+2658		31	*	N3536 1104+2850	N3539 1108±2000	N3550 1107±2000	N3527 N3558	1104+2852 N3554
Π		1217+3010	N4136	N4308	N4150	N4395			1108+2858	N3561	1107 + 2835	OPPON	FOODL
		1202+2839 NA272	N4245 N4286	1202+3108 N4974	N4448	N4310	32	*	1548 + 2847	N6001	1548 + 2857	1547 + 2842	
		1226+3559 N4080	N4163 N4251	1212+3630 1212+3630 1914+9000	N4214 N4173	N4190	33	*	N4185 N4169	1209+2905 N4196	1210+2907	N4174	N4175
		N4314	1218+3104	N4020	1157 + 3129	N4062	34	*	1353 + 2847	1355 + 2902	1356 + 2906	1356 + 2906	1354 + 2845
		N4393	N4559	N4509	1230 + 3753	N4244	35		N3277	N3245A	N3265	N3254	N3245

TABLE 4

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							1							
E			group	members			B			group	members			11
36	*	1217+2907	1217 + 2909	1217 + 2900			67	*	N5025	1309+3146	1308+3143	1310+3131		
37	*	I4442	N5657	N5641			68	-	1140 + 3144	1140 + 3145	1139 + 3217	1143 + 3135		
38	:	N3486	1102 + 2925	N3510			69	*	0933 + 3202	0931 + 3217	N2918			
39	*	1306 + 2938	I4210	N5004	N5004A	N5000	70	:	0939 + 3205	N2968	N2970	N2964		
40	*	0847 + 2925	0846 + 2943	I2404	0848 + 2923		11	*	1158 + 3209	N4031	1157 + 3205			
41	*	N4132	N4131	N4134			72	*	1234 + 3221	1235 + 3222	1234 + 3223	1236 + 3216		
42	*	N6086	1610 + 2947	1612 + 2959	1612 + 3006	N6085	73	*	N3986	1154 + 3225	N3966	N3994	N3995	
43	*	1348 + 2937	I4334	1349 + 2935					N3991	1154 + 3253	I2978			
44	*	1159 + 2945	1159 + 3008	1159 + 3007	1200 + 2942		74	*	1006 + 3245	1003 + 3229	1007 + 3219			
45	*	1358 + 2948	1358 + 3019	1358 + 2946			75	*	N2944	0936 + 3236	0936 + 3236	0938 + 3223		
46	*	1103 + 3012	1102 + 3018	1101 + 2947			76		N4631	N4627	1242 + 3440	1244 + 3645	N4656	
47		N6282 1658+2948	1656 + 2959	N6274A	1659 + 3015	1659 + 3024	77	ł	0945+3307 N3067 N3011	0948+3310 I2524	N3003 0958+3322	N3021 1003+3311	0952+3330 N3118	
48	*	I2476	I2479	0926 + 3022	I2473	I2480	78	*	1624 + 3252	N6162	N6163	1623 ± 3235		
49	*	N2783	N2783B	0911 + 3020	I2444	N2789	79	*	I2439	0908 + 3302	0906+3243			
50	*	N5277 1340+3036	N5282 1338+3038	N5271 N5280	1340 + 3030	1342 + 3035	80	*	1131 + 3327	1130+3346	1131 + 3254			
51	*	N5642	1426 + 3018	1425 + 3010			81	*	I3003	N4122	I2993	1206 + 3256		
52	*	1650 + 3048	1650 + 3048	1651 + 3035			82	*	N3871 12953	N3881 N3847	N3880 1141+3256	N3878	I 729	
53	*	1426 + 3051 1424 + 3145	1429+3100 N5639A	I1012	I4447	N5653	83	3	N3424 1050+3417	N3430 1049+3445	N3395 N3381	N3396 I2604	N3442	
54	*	1519 + 3050	1519 + 3050	1518 + 3039			84	*	0918+3337	N2832	0920 + 3358	0919 + 3403	0917 + 3317	
55	*	12985	I2986	1154 + 3040			85	*	N5098A	N5098A	1318 + 3333	1318+3336	N5096	
56	*	1436 + 3101	1436 + 3110	1435 + 3042			86	*	1638 + 3346	1639 + 3352	1639 + 3327			
57	*	1314+3056 N5074	N5056 1312+3045	N5057	1315+3118	N5065	87	*	I4496	I4505	I4506			
58	*	1316 + 3102	1316+3107	1318 + 3051	1314 + 3047		88	*	N5321	N5318	N5312			
59	1	N5961	1532 + 3114	1534 + 3051			89	*	0930 + 3408	0931 + 3413	0932+3413	0930 + 3416	0930+3353	
60	*	I4256	N5187	1327 + 3139	1328 + 3153	1326 + 3105	60	*	1135 + 3408	1136 + 3413	1138 + 3357			
61	*	1317+3115	1319 + 3130	1319+3139	1319 + 3148	1317+3110	16	*	1119+3436 I2744	1120+3437 12738	1117 + 3422 1120 + 3406	1116 + 3424	I2735	
62	*	1615 + 3119	1613+3131	1613 + 3112			92	*	1344 + 3408	1345 + 3424	1344+3407			
63	*	1202 + 3126 1203 + 3120	1202 + 3127	I3007	1206 + 3151	1208+3156	93	*	1617 + 3440	1619 + 3453	1621 + 3507	1617 + 3427		
64	*	1520+3124	1517+3133	N5924		4	94	ł	1312+3509 N5002	I4213 N5005	1311 + 3629 1307 + 3427	N5014	N5033	
65	*	1319 + 3137 1320 + 3205	1318+3147 14225	1321 + 3136 14226	N5127 1318+3129	1321 + 3154	95	ł	N2780	N2778	0909 + 3544	N2793		
99	+	1337+3134	1336+3138	1336+3131			96	*	1211 + 3454	1210 + 3459	1212 + 3453			

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TABLE 4—Continued

			0942 ± 3645		1322 + 3651			12405				N5686			0906 + 3754	1620 + 3829		1357 + 3826			
			N2971		N5143			0845 + 3657		1114 + 3620	N5529	1433 + 3701			I 527	N6129	1621 + 3740	N5403	NE241	THONT	
	members	1136 + 3602	0942 + 3623		N5142		1256 + 3611	N2668		1112 + 3647	N5545	1433 + 3656	1109 + 3700	N3941	N2759 I2434	1621 + 3804	1618 + 3821	N5378	1351-13898	1102+3817	
	group	1137+3657	I2500		N5141		I4049	0847 + 3639		1114 + 3625	N5544	N5695	1107 + 3716	1155 + 3821	$0905 + 3742 \\0909 + 3752$	1620 + 3804	N6119	N5380	1340+3810	1101 + 3828	
		1137 + 3624	N2965	N2955	N5149	1322+3010	I4028	0847 + 3630	0846+3617	1117 + 3622	N5557	N5684	N3542	N3930	$0904 + 3742 \\ 0906 + 3814$	N6137	N6120	N5394 N5395	N5351	1102 + 3820	
pənı		*	*		*		*	*		*	*	*	*		ł			-			
Contin	A	114	115		116		117	118		119	120	121	122	123	124	125		126	127	128	
TABLE 4-			N5233	1402 + 3547			N3755						1124 + 3619		N6126	1406 + 3551	1646 + 3555			1635 + 3631	
			1334 + 3500	N5399			1129 + 3536	1143 + 3508		1150 + 3518		N5579	N3695		1617 + 3612	N5517	1644 + 3610			N6194	0909+3559
	members	0941 + 3455	N5223	N5444		N6107	1133 + 3536	1122 + 3554	0932 + 3513	N3897	1304 + 3522	1422 + 3529	1122 + 3547		1622 + 3620	1402 + 3602	1647 + 3618	N2719	1607 + 3555	N6197	0911+3618
	group	0944 + 3500	1333 + 3515	N5445	N5440	N6116	1135 + 3528	N3813	0934 + 3549	1148 + 3529	N4986	N5589	1123 + 3542		1619 + 3610	N5499	1646 + 3601	N2724	1609 + 3607	I4614 1633+3556	0911+3617
		0942 + 3455	N5228	1400 + 3506	1401 + 3559	N6109	1138 + 3529	N3694A	0934 + 3526	1151 + 3526	N4956	N5590	1123 + 3537	1124 + 3531	1621 + 3557 1620 + 3546	1404 + 3601	1646 + 3559	N2719A	1609 + 3605	N6199 1635+3633	0909+3608
		*	* * * *	*		* 0	1		2 *	3 *	4 *	5 *	6 *		7 *	8	9 *	* 0	1 *	2 *	*
	H	6	6	õ		Ц	10		10	10	10	10	10		10	10	10	11	11	11	=

NOTE.—An asterisk denotes a group belonging to the statistical sample of 92 groups used in the analysis of group properties.

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FIG. 4.—(a) Cone diagram for group members in the declination range $26^{\circ}.5 \le \delta < 38^{\circ}.5$. There are 778 galaxies. (b) Cone diagram for groups; each point represents one of the 128 groups in the catalog with mean redshift $cz \le 12,000 \text{ km s}^{-1}$. The crosses indicate the locations of Abell clusters. Compare Fig. 1.

members. The algorithm also identifies 142 binaries (not included in this analysis). The number of galaxies not assigned to a group or binary is 614. The sum of these numbers is not equal to the total number of galaxies in the survey (1766) because galaxies with velocities greater than 12,000 km s⁻¹ are considered only if they are contained in groups with mean velocity less than 12,000 km s⁻¹.

Figures 4a and 4b are cone diagrams which show the distribution of group members and group centers, respectively. Comparison of Figure 4b with Figure 1 shows that the group centers mark the large-scale structure in a nearly unbiased way. More detailed comparison of Figures 4a-4b with Figure 1 shows that projection effects in Figure 1 can hide "fingers" as well as produce misleadingly impressive ones. These impressions are a result of averaging over an angular scale which is large compared with a typical group diameter.

VI. GROUP PROPERTIES

Here we examine the typical physical properties of the groups in the catalog. We first trim the catalog by eliminating groups which may have properties affected by the proximity of the sample limits. We include only groups with velocities > 2000 km s⁻¹. At smaller distances the declination angle spanned by the survey is smaller than or at best comparable to the angular scale of a typical group. We also discard groups with baricenters closer to the edges of the survey than $2R_p$. This criterion deletes the Coma Cluster from our "statistical sample" of groups. Asterisks in Table 4 mark the groups in the statistical sample.

a) Physical Properties

Table 5 lists the properties of the 92 groups in the "statistical sample." For each group, we list the group number (col. [1]), the number of members (col. [2]), the right ascension (col. [3]), the declination (col. [4]), the mean velocity (col. [5]), the rms line-of-sight velocity dispersion σ_v (col. [6]), the apparent magnitude corresponding to the sum of the apparent luminosities of the observed group members (col. [7]), the mean pairwise separation R_p (col. [8]), the mean harmonic radius R_h (col. [9]),

the crossing time t_c in units of $t_0 = H_0^{-1}$ (col. [10]), the logarithm of the virial mass, log M (col. [11]), and the mass-tolight ratio, $M/L_{B(0)}$ (col. [12]). These quantities are also available for groups in the catalog of HG82. Table 6 gives medians for the distributions of σ_v , log M, M/L, T_c , R_h , and R_p . We also include the first and third quartiles for these distributions.

The quantities in Table 5 and 6 are number-weighted. Weighting with luminosity in the calculation of group parameters does not significantly affect the median quantities in Table 6 (see Heisler, Tremaine, and Bahcall 1985). The median mass-to-light ratio and crossing time do not change at all. Figure 5 shows the logarithm of the unweighted $M/L_{B(0)}$ as a function of the logarithm of the luminosity-weighted value. The scatter around the mean relation is remarkably small.

We next consider the properties of groups with $N_{\text{mem}} \ge 5$. These groups are statistically the most reliable; at least 33% of groups with $N_{\text{mem}} < 5$ are spurious (see § IV). Table 6 gives the median values and quartiles for the physical parameters of the 36 groups with $N_{\text{mem}} \ge 5$.

Figures 6a-6f are histograms of the distribution of σ_v , R_h , R_p , t_c , $\log (M/L_{B(0)})$, and $\log M$, respectively. The unfilled histograms refer to the entire sample of 92 groups; the hatched histograms refer to the 36 groups with $N_{\text{mem}} \ge 5$. There are no salient differences between the distributions for the groups with $N_{\text{mem}} \ge 5$ and the full sample. Because the properties of contaminated and/or spurious groups containing three or four members are statistically indistinguishable from those of real physical systems, we cannot remove pseudogroups from the sample. However, the median values obtained for the whole sample appear to be a fair representation of the properties of real groups.

The spread of mass-to-light ratios as measured by the quartile range is similar to the spread obtained for other catalogs (Rood and Dickel 1978; Tully 1987; Huchra and Geller 1982). We obtain a spread of 0.9 in the logarithm of $M/L_{B(0)}$ as measured by the range between the first and third quartiles. This spread is somewhat larger than the statistical spread (~0.7) obtained by Heisler, Tremaine, and Bahcall (1985) for systems with fixed mass-to-light ratio. The significance of the difference



FIG. 5.—Logarithm of the luminosity-weighted mass-to-light ratio as a function of the logarithm of the unweighted value. We include only the 92 groups in the statistical sample (Table 5).

No. 1, 1989

TABLE 5 Group Properties

$ L_B $																																						
M' (M'	2531. 2752	92.	183.	136.	47.	586.	404. 659.	371.	440.	635.	427.	446. 1452	5.	62.	304.	411.	1. 1149	18.	1838	676.	11.	31. 106	473.	38.	ы.	14.	453. 89	660.	167.	283.	1585	221.	137.	81.	101.	168.	1383	0.0
$\log_{M/M_{\odot}}$	14.39	14.04 12.78	13.26	13.91	12.62	13.58	13.89	13.77	12.78	14.83	12.87	13.07	12.47	12.71	13.26	13.83	12.06	11.41	13.81	13.37	12.31	12.81	13.83	12.86	11.72	12.99	13.89	14.22	13.33	14.04	13.84	13.56	13.53	13.53	13.60	13.94	14 60	10.27
t_c/t_0	0.03	0.08	0.14 0.34	0.03	0.36	0.13	0.19	0.09	0.07	0.04	0.02	0.03	0.00	0.10	0.14	0.52	0.00	0.24	0.11	0.09	0.13	0.55	0.10	0.07	0.20	0.03	0.16	0.11	0.11	0.13	0.18	0.15	0.55	0.51	0.06	0.08	00.0	-00 0
$\frac{R_h}{(Mpc)}$	0.38	0.22	0.46 0.83	0.26	0.51	0.54	0.90	0.49	0.19	0.98	0.09	01.10	0.01	0.23	0.44	1.65	0.00	0.16	0.59	0.38	0.21	0.78	0.55	0.20	0.17	0.14	0.81	0.79	0.40	0.76	0.82	0.58	1.37	1.31	0.32	0.53	0.70	0.00
$\frac{R_p}{(Mpc)}$	0.66	0.53	0.65	0.40	0.63	0.67	0.91	0.51	0.16	0.97	0.20	0.07	0.92	0.67	0.57	1.68	0.25	0.28	0.58	0.36	0.24	0.84	0.68	0.32	0.70	0.59	0.73	0.81	0.68	0.90	0.69	0.58	1.44	1.21	1.18	0.80	0.68	0.40
<i>m</i> tot	12.90	13.20	12.40	13.40	13.40	12.50	12.40	12.90	14.00	12.90	13.70	14 20	13.60	14.00	11.90	13.30	11.70	13.70	12.40	12.60	12.90	14.20	14.10	13.80	13.80	12.80	14.20	13.50	14.20	13.30	14.10	13.70	13.80	13.20	13.90	12.80	13.60	14.00
$\sigma_v \ (km \ s^{-1})$	681. 700	/ oo. 141.	170.	478.	76.	225.	344. 249.	294.	151.	706.	248.	238. 564	499.	125.	172.	172.	157. 950	200. 14.	281.	213.	84.	711. 939	298.	161.	47.	228.	264. 206	388.	198.	323.	245.	212.	134.	136.	299.	344.	57. 740	73.
$\frac{cz}{cm \ s^{-1}}$	6550. 6240	0049. 6278.	5724. 9608	11163.	7224.	5179.	5441. 7213.	Ţ 582.	4755.	11266.	4675.	4442. 11960	11209.	8158.	4002.	8328.	3151. 2267	4446.	3927.	4222.	4276.	10439. QA53	9382.	9503.	9553.	10346.	10052. 0801	9615.	9458.	10093.	7056.	9159.	10107.	9933.	11006.	9807.	9518.	9940. 11123.
61950 (k	29 42 21 46	21 40 27 53	31 13 30 56	33 27	31 28	31 51	30 32 31 26	35 2	31 34	30 25	34 12	53 54 20 49	28 57	29 57	35 21	35 59	36 41 25 97	30 14	31 7	29 13	36 50	30 57 33 36	27 56	30 46	31 27	28 53 25 53	27 29 28 26	28 48 28 48	36 2	29 47	31 20	35 8	34 46	36 9	3250	36 15	33 41 36 4	30 43
Å 1950	3 8.0	3 10.6	314.3 316.5	3 18.1	3 18.6	3 19.7	3 26.9	3 33.0	3 36.7	3 40.5	344.9	348.1	3 55.2	3 58.7	4 0.7	4 6.5	4 15.0	4 26.4	4 26.7	4 27.4	4 34.1	436.4	4 47.7	5 18.9	519.2	5 25.2	540.0	5 47.5	6 8.6	6 11.2	6 14.1	6 16.1	6 19.1	620.1	625.2	635.3	6 46 4	6 50.7
mem	5	3 4 3 1	4 1	5 1	5	6 v	م م	5 1	3 1	80	 	 	 -	3 1	7 1	5 1	4 •	* ~~	7 1	3 1	ر م		, m	3 1	3	9	т т п	4 1 1	3	5 1	3]	3	4 1	6 1	4 1	7	יי די יי מי	າ — າ ຕາ
D N	74	61 77	79 70	80	81	82	84 84	85	86	87	80	68 00	92 92	94	95	96	97 00	100	101	102	104	105	107	108	109	110	111	114	115	116	117	118	119	121	122	123	124	126
() ()																			· · ·																		_	
$4/L_{B(M_{\odot}/L)}$	23.	٥ [.] .	69. 573	63.	89.	57.	140. 49.	280.	Ŀ.	66.	20.2	192 193	21.0	21.	36.	36.		21.	136.	567.	142.	32.	00	84.	99.	84.	30	63.	95	5.	ы.	9.	ч.	81	31.	533.	13. 13.	571
A C				. –	Ű			Ξ		61	CN 2	Ň T	4 V.		П		Ψ,	-	4.	2	4				-		Цч	5 00	10	2]	ñ	39	H	1			-	- ~
log Å M/M _© (I	12.53	9.68	11.86 13.53	13.33	14.00 6	12.93	13.26	13.60 13	10.67	13.57 2	12.95 2	13.91 24	14.67 5	13.36 2	13.07 1	13.28	13.57 (12.37	12.21	14.73 2	15.26 4	13.78	12.62	12.40	12.74 7	12.79	13.08 IJ	14.09 8	14.26 19	12.65 21	12.36 3	13.77 39	11.94 1	.14.19 12	12.30	13.63	11.96	14.89 3
$rac{t_c/t_0}{M/M_\odot} rac{log}{M}$	0.98 12.53	0.00 9.68	0.64 11.86	0.07 13.33 1	0.20 14.00 6	0.27 12.93	0.19 13.26	0.26 13.60 1	0.00 10.67	0.19 13.57 2	0.36 12.95 2		0.04 14.67 5	0.07 13.36 2	0.39 13.07 1	0.07 13.28	0.19 13.57 (0.47 12.37	0.37 12.21	0.06 14.73 2	0.07 15.26 4	0.09 13.78 1	0.07 12.62 1	0.06 12.40	0.12 12.74 7	0.10 12.79	0.05 13.08 11	0.04 14.09 8	0.08 14.26 19	0.03 12.65 21	0.13 12.36 3	0.18 13.77 39	0.08 11.94 1'	0.17 .14.19 12	0.32 12.30	0.15 13.63	0.98 11.96	0.07 14.89 3
$rac{R_h}{Mpc} rac{t_c/t_0}{M/M_\odot} rac{log}{M}$	0.93 0.98 12.53	0.00 0.00 9.68	0.42 0.64 11.86 0.30 0.06 13.53	0.29 0.07 13.33 1	0.99 0.20 14.00 6	0.53 0.27 12.93	0.55 0.19 13.26 0.54 0.40 12.60	0.88 0.26 13.60 1	0.00 0.00 10.67	0.70 0.19 13.57 2	0.66 0.36 12.95 2		0.58 0.04 -14.67 5	0.32 0.07 13.36 2	0.77 0.39 13.07 1	0.30 0.07 13.28	0.69 0.19 13.57 (0.51 0.47 12.37	0.38 0.37 12.21	0.81 0.06 14.73 2	2.08 0.07 15.26 4	0.51 0.09 13.78 1	0.17 0.07 12.62 1	0.13 0.06 12.40	0.27 0.12 12.74 7	0.24 0.10 12.79	0.18 0.05 13.08 1	0.38 0.04 14.09 8	0.68 0.08 14.26 19	0.10 0.03 12.65 21	0.21 0.13 12.36 3	0.78 0.18 13.77 39	0.11 0.08 11.94 1'	1.06 0.17 .14.19 12	0.37 0.32 12.30	0.62 0.15 13.63	0.60 0.98 11.96 0 ff 0 21 12 86 1	1.00 0.07 14.89 3
$egin{array}{c c c c c c c c c } R_p & R_h & t_{ m c}/t_0 & log & M \ (Mpc) & (Mpc) & M/M_{\odot} & (l) \end{array}$	0.96 0.93 0.98 12.53	0.08 0.00 0.00 9.68	0.45 0.42 0.64 11.86 0.51 0.30 0.06 13.53	0.50 0.29 0.07 13.33 1	0.89 0.99 0.20 14.00 6	0.69 0.53 0.27 12.93	0.54 0.55 0.19 13.26 0.52 0.54 0.40 12.60	0.76 0.88 0.26 13.60 1	0.36 0.00 0.00 10.67	0.74 0.70 0.19 13.57 2	0.59 0.66 0.36 12.95 2		1.00 0.58 0.04 14.67 5	0.61 0.32 0.07 13.36 2	0.91 0.77 0.39 13.07 1	0.45 0.30 0.07 13.28	0.96 0.69 0.19 13.57 6	1.12 0.90 0.18 13.96 1 0.60 0.51 0.47 12.37	0.34 0.38 0.37 12.21	0.83 0.81 0.06 14.73 2	1.83 2.08 0.07 15.26 4	0.84 0.51 0.09 13.78 1 0.81 0.74 0.38 13.06 1	0.22 0.17 0.07 12.62	0.26 0.13 0.06 12.40	0.35 0.27 0.12 12.74 7	0.20 0.24 0.10 12.79	0.25 0.18 0.05 13.08 11	1.14 0.38 0.04 14.09 8	0.65 0.68 0.08 14.26 19	0.09 0.10 0.03 12.65 21	0.38 0.21 0.13 12.36 3	0.75 0.78 0.18 13.77 39	0.16 0.11 0.08 11.94 1'	0.97 1.06 0.17 .14.19 12	0.46 0.37 0.32 12.30	0.73 0.62 0.15 13.63	0.67 0.60 0.98 11.96 0.40 0.55 0.31 12.86 1	1.09 1.00 0.07 14.89 3
$rac{m_{tot}}{(Mpc)} rac{R_{p}}{(Mpc)} rac{t_{c}/t_{0}}{M/M_{\odot}} rac{10}{(l}$	13.00 0.96 0.93 0.98 12.53	13.90 0.77 0.37 0.17 12.83 13.40 0.08 0.00 0.00 9.68	13.90 0.45 0.42 0.64 11.86 13.70 0.51 0.30 0.06 13.53	12.60 0.50 0.29 0.07 13.33 1	12.60 0.89 0.99 0.20 14.00 6	13.30 0.69 0.53 0.27 12.93	13.50 0.54 0.55 0.19 13.26 1 13.20 0.52 0.54 0.40 12.60	14.10 0.76 0.88 0.26 13.60 1	13.40 0.36 0.00 0.00 10.67	12.60 0.74 0.70 0.19 13.57 2		13.90 0.78 0.94 0.20 13.91 26	12.30 1.00 0.58 0.04 14.67 5	14.10 0.61 0.32 0.07 13.36 2	13.60 0.91 0.77 0.39 13.07 1	13.80 0.45 0.30 0.07 13.28	13.10 0.96 0.69 0.19 13.57 (13.20 1.12 0.90 0.10 19.30 1 12.90 0.60 0.51 0.47 12.37	13.70 0.34 0.38 0.37 12.21	14.00 0.83 0.81 0.06 14.73 2	14.20 1.83 2.08 0.07 15.26 4	12.90 0.84 0.51 0.09 13.78 1 13.30 0.81 0.74 0.38 13.06 1	11.80 0.22 0.17 0.07 12.62 1	12.20 0.26 0.13 0.06 12.40	13.70 0.35 0.27 0.12 12.74 7	13.90 0.20 0.24 0.10 12.79	13.20 0.25 0.18 0.05 13.08 11 13.10 0.70 0.36 0.04 14.07 5	12.80 1.14 0.38 0.04 14.09 8	13.70 0.65 0.68 0.08 14.26 19	12.90 0.09 0.10 0.03 12.65 21	11.70 0.38 0.21 0.13 12.36 3	14.20 0.75 0.78 0.18 13.77 39	14.20 0.16 0.11 0.08 11.94 1'	13.60 0.97 1.06 0.17 14.19 12	13.60 0.46 0.37 0.32 12.30	13.80 0.73 0.62 0.15 13.63	14.20 0.67 0.60 0.98 11.96 12.80 0.40 0.55 0.21 12.86 1	12.40 1.09 1.00 0.07 14.89 3
$\sigma_{v} \qquad m_{tot} \qquad R_{p} \qquad R_{h} \qquad t_{c}/t_{0} log \qquad M/M_{\odot} $	51. 13.00 0.96 0.93 0.98 12.53	115. 13.90 0.77 0.57 0.17 12.85 69. 13.40 0.08 0.00 0.00 9.68	35. 13.90 0.45 0.42 0.64 11.86 287 13.70 0.51 0.30 0.06 13.53	229. 12.60 0.50 0.29 0.07 13.33 1	270. 12.60 0.89 0.99 0.20 14.00 €	107. 13.30 0.69 0.53 0.27 12.93	154. 13.50 0.54 0.55 0.19 13.26 73. 13.20 0.52 0.54 0.40 12.60	181. 14.10 0.76 0.88 0.26 13.60 1	106. 13.40 0.36 0.00 0.00 10.67	196. 12.60 0.74 0.70 0.19 13.57 2	98. 13.80 0.59 0.66 0.36 12.95 2	248. 13.90 0.78 0.94 0.20 13.91 24	ZIZ. 13.30 0.00 0.39 0.20 13.19 4 765 12.30 1.00 0.58 0.04 14.67 5	228. 14.10 0.61 0.32 0.07 13.36 2	105. 13.60 0.91 0.77 0.39 13.07 1	215. 13.80 0.45 0.30 0.07 13.28	196. 13.10 0.96 0.69 0.19 13.57 (2/0. 13.20 1.12 0.90 0.16 13.30 1 58. 12.90 0.60 0.51 0.47 12.37	56. 13.70 0.34 0.38 0.37 12.21 ¢	691. 14.00 0.83 0.81 0.06 14.73 2	789. 14.20 1.83 2.08 0.07 15.26 4	293. 12.90 0.84 0.51 0.09 13.78 1 105 13.30 0.81 0.74 0.38 13.06 1	133. 11.80 0.22 0.17 0.07 12.62 1	117. 12.20 0.26 0.13 0.06 12.40	121. 13.70 0.35 0.27 0.12 12.74 7	135. 13.90 0.20 0.24 0.10 12.79	217. 13.20 0.25 0.18 0.05 13.08 11	479. 12.80 1.14 0.38 0.04 14.09 8	436. 13.70 0.65 0.68 0.08 14.26 19	175. 12.90 0.09 0.10 0.03 12.65 21	87. 11.70 0.38 0.21 0.13 12.36 3	233. 14.20 0.75 0.78 0.18 13.77 39	76. 14.20 0.16 0.11 0.08 11.94 1 [']	325. 13.60 0.97 1.06 0.17 ,14.19 12	62. 13.60 0.46 0.37 0.32 12.30	222. 13.80 0.73 0.62 0.15 13.63	15. 14.20 0.67 0.60 0.98 11.96 07 13.80 0.40 0.55 0.31 13.86 1	746. 12.40 0.49 0.33 0.31 12.60 3
cz σ_v m_{tot} R_p R_h t_c/t_0 log A $km \ s^{-1}$) $(km \ s^{-1})$ (Mpc) (Mpc) M/M_{\odot} (l)	7515. 51. 13.00 0.96 0.93 0.98 12.53	7958. 115. 13.90 0.77 0.57 0.17 12.85 3130. 69. 13.40 0.08 0.00 0.00 9.68	4235. 35. 13.90 0.45 0.42 0.64 11.86 6001 987 13.70 0.51 0.30 0.06 13.53	6586. 229. 12.60 0.50 0.29 0.07 13.33 1	6786. 270. 12.60 0.89 0.99 0.20 14.00 6	8057. 107. 13.30 0.69 0.53 0.27 12.93	8129. 154. 13.50 0.54 0.55 0.19 13.26 5703 73. 13.20 0.52 0.54 0.40 12.60	6405. 181. 14.10 0.76 0.88 0.26 13.60 1	6535. 106. 13.40 0.36 0.00 0.00 10.67	6745. 196. 12.60 0.74 0.70 0.19 13.57 2	6035. 98. 13.80 0.59 0.66 0.36 12.95 2	6084. 248. 13.90 0.78 0.94 0.20 13.91 26 2604 210 12.00 0.68 0.00 0.5 12.70 4	0094. ZIZ. 10.90 U.00 U.99 U.20 L0.19 4 0576 765 12.30 1.00 0.58 0.04 14.67 5	8838. 228. 14.10 0.61 0.32 0.07 13.36 2	7579. 105. 13.60 0.91 0.77 0.39 13.07 1	9745. 215. 13.80 0.45 0.30 0.07 13.28 1	10433. 196. 13.10 0.96 0.69 0.19 13.57 (10209. 270. 13.20 1.12 0.90 0.10 13.37 1 6002 58 12.90 0.60 0.51 0.47 12.37	2553. 56. 13.70 0.34 0.38 0.37 12.21 4	10156. 691. 14.00 0.83 0.81 0.06 14.73 2	11901. 789. 14.20 1.83 2.08 0.07 15.26 4	9558. 293. 12.90 0.84 0.51 0.09 13.78 1 8488 105 13.30 0.81 0.74 0.38 13.06 1	3236. 133. 11.80 0.22 0.17 0.07 12.62 1	3433. 117. 12.20 0.26 0.13 0.06 12.40	3262. 121. 13.70 0.35 0.27 0.12 12.74 7	7835. 135. 13.90 0.20 0.24 0.10 12.79	3284. 217. 13.20 0.25 0.18 0.05 13.08 11 8508 482 13.10 0.70 0.26 0.01 14.07 5	7124. 479. 12.80 1.14 0.38 0.04 14.09 8	7775. 436. 13.70 0.65 0.68 0.08 14.26 19	3861. 175. 12.90 0.09 0.10 0.03 12.65 21	3915. 87. 11.70 0.38 0.21 0.13 12.36 3	9727. 233. 14.20 0.75 0.78 0.18 13.77 39	7617. 76. 14.20 0.16 0.11 0.08 11.94 1'	8201. 325. 13.60 0.97 1.06 0.17 14.19 12	7038. 62. 13.60 0.46 0.37 0.32 12.30	7730. 222. 13.80 0.73 0.62 0.15 13.63	8360. 15. 14.20 0.67 0.60 0.98 11.96 4871 07 13.80 0.40 0.55 0.31 13.86 1	4011. 91. 12.00 0.49 0.00 0.01 12.00 1 7174. 746. 12.40 1.09 1.00 0.07 14.89 3
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	36 46 7515. 51. 13.00 0.96 0.93 0.98 12.53	29 32 7958. 115. 15.90 0.77 0.57 0.11 12.55 35 55 3130. 69. 13.40 0.08 0.00 0.00 9.68	32 50 4235. 35. 13.90 0.45 0.42 0.64 11.86 36 10 6001 - 287 13.70 0.51 0.30 0.06 13.53	30 12 6586. 229. 12.60 0.50 0.29 0.07 13.33 1	33 46 6786. 270. 12.60 0.89 0.99 0.20 14.00 €	30 16 8057. 107. 13.30 0.69 0.53 0.27 12.93	34 8 8129. 154. 13.50 0.54 0.55 0.19 13.26 1 32 5 6703 73 13.20 0.52 0.54 0.40 12.60	35 29 6405. 181. 14.10 0.76 0.88 0.26 13.60 1	32 31 6535. 106. 13.40 0.36 0.00 0.00 10.67	36 27 6745. 196. 12.60 0.74 0.70 0.19 13.57 2	34 56 6035. 98. 13.80 0.59 0.66 0.36 12.95 2	32 30 6084. 248. 13.90 0.78 0.94 0.20 13.91 24 20 F 2604 243 13.00 0.78 0.94 0.20 13.91 24	20 2 2024. 212. 12.90 0.00 0.99 0.29 12.19 4 28 53 9576 765 12.30 1.00 0.58 0.04 14.67 5	37 9 8838. 228. 14.10 0.61 0.32 0.07 13.36 2	36 28 7579. 105. 13.60 0.91 0.77 0.39 13.07 1	28 37 9745. 215. 13.80 0.45 0.30 0.07 13.28	34 29 10433. 196. 13.10 0.96 0.69 0.19 13.57 (25 41 10209. 213. 13.20 1.12 0.90 0.16 13.30 1 28 31 6902 58 12.90 0.60 0.51 0.47 12.37	33 22 2553. 56. 13.70 0.34 0.38 0.37 12.21	34 6 10156. 691. 14.00 0.83 0.81 0.06 14.73 2	36 27 11901. 789. 14.20 1.83 2.08 0.07 15.26 4	33 27 9558. 293. 12.90 0.84 0.51 0.09 13.78 1 35 32 6486 105 13 30 0.81 0.74 0.38 13.06 1	32 30 3236. 133. 11.80 0.22 0.17 0.07 12.62 1	28 4 3433. 117. 12.20 0.26 0.13 0.06 12.40	30 55 3262. 121. 13.70 0.35 0.27 0.12 12.74 7	32 9 7835. 135. 13.90 0.20 0.24 0.10 12.79	29 55 3284. 217. 13.20 0.25 0.18 0.05 13.08 11	20 22 0000. TOU. 19.10 0.10 0.00 0.01 17.01 0 31 36 7124. 479. 12.80 1.14 0.38 0.04 14.09 8	33 6 7775. 436. 13.70 0.65 0.68 0.08 14.26 19	29 31 3861. 175. 12.90 0.09 0.10 0.03 12.65 21	29 8 3915. 87. 11.70 0.38 0.21 0.13 12.36 3	34 55 9727. 233. 14.20 0.75 0.78 0.18 13.77 39	29 5 7617. 76. 14.20 0.16 0.11 0.08 11.94 1'	28 28 8201. 325. 13.60 0.97 1.06 0.17 .14.19 12	32 20 7038. 62. 13.60 0.46 0.37 0.32 12.30	28 10 7730. 222. 13.80 0.73 0.62 0.15 13.63	36 26 8360. 15. 14.20 0.67 0.60 0.98 11.96 25 25 4871 07 12 80 0.40 0.55 0.21 12 86 1	20 20 20 4011. 91. 12.00 0.49 0.00 0.01 12.00 1 20 20 20 20 20 20 20 20 20 20 20 20 20
$lpha_{ m 1950}$ $\delta_{ m 1950}$ cz $\sigma_{ m v}$ m_{tot} R_{p} R_{h} t_{c}/t_{0} log \overline{M} (Mpc) (Mpc) M/M_{\odot} (l)	8 46.4 36 46 7515. 51. 13.00 0.96 0.93 0.98 12.53	846.9 29 32 7958. 115. 13.90 0.17 0.51 0.11 12.05 857.4 35 55 3130. 69. 13.40 0.08 0.00 0.00 9.68	9 6.8 32 50 4235. 35. 13.90 0.45 0.42 0.64 11.86 0.10 5 36 10 6001 987 13.70 0.51 0.30 0.06 13.53	910.9 30 12 6586. 229. 12.60 0.50 0.29 0.07 13.33 1	918.6 33 46 6786. 270. 12.60 0.89 0.99 0.20 14.00 6	9.25.2 30 16 8057. 107. 13.30 0.69 0.53 0.27 12.93	930.9 34 8 8129. 154. 13.50 0.54 0.55 0.19 13.26 J 0327 32 5 6703 73. 13.20 0.52 0.54 0.40 12.60	933.8 35 29 6405. 181. 14.10 0.76 0.88 0.26 13.60 1	9 36.9 32 31 6535. 106. 13.40 0.36 0.00 0.00 10.67	940.5 36 27 6745. 196. 12.60 0.74 0.70 0.19 13.57 2	9 42.7 34 56 6035. 98. 13.80 0.59 0.66 0.36 12.95 2	105.732306084, 248 , $13.900.78094$, $0.2013.9126$	11 Z.O 20 2 2034. ZIZ. 13.30 0.00 0.39 0.20 12.19 4 11 7.0 28 53 0576 765 12.30 1.00 0.58 0.04 14.67 5	11 7.8 37 9 8838. 228. 14.10 0.61 0.32 0.07 13.36 2	1114.8 36 28 7579. 105. 13.60 0.91 0.77 0.39 13.07 1	11 15.7 28 37 9745. 215. 13.80 0.45 0.30 0.07 13.28	1118.8 34 29 10433. 196. 13.10 0.96 0.69 0.19 13.57 (11 24.3 33 41 10209. 213. 13.20 1.12 0.90 0.10 13.30 1 11 28 6 28 31 6002 58 12.90 0.60 0.51 0.47 12.37	1131.2 33 22 2553. 56. 13.70 0.34 0.38 0.37 12.21 ϵ	11 36.9 34 6 10156. 691. 14.00 0.83 0.81 0.06 14.73 2	11 37.3 36 27 11901. 789. 14.20 1.83 2.08 0.07 15.26 4	1142.8 33 27 9558. 293. 12.90 0.84 0.51 0.09 13.78 1 1140.9 35 99 6486 105 13.30 0.81 0.74 0.38 13.06 1	11 54.3 32 30 3236. 133. 11.80 0.22 0.17 0.07 12.62 1	11 55.6 28 4 3433. 117. 12.20 0.26 0.13 0.06 12.40	1156.1 30 55 3262. 121. 13.70 0.35 0.27 0.12 12.74 7	1157.9 32 9 7835. 135. 13.90 0.20 0.24 0.10 12.79	12 0.0 29 55 3284. 217. 13.20 0.25 0.18 0.05 13.08 11	12 4.6 31 36 7124. 479. 12.80 1.14 0.38 0.04 14.09 8	12 4.9 33 6 7775. 436. 13.70 0.65 0.68 0.08 14.26 19	12 6.4 29 31 3861. 175. 12.90 0.09 0.10 0.03 12.65 21	1210.4 29 8 3915. 87. 11.70 0.38 0.21 0.13 12.36 3	1211.5 34 55 9727. 233. 14.20 0.75 0.78 0.18 13.77 39	1217.2 29 5 7617. 76. 14.20 0.16 0.11 0.08 11.94 1 [']	12 19.3 28 28 8201. 325. 13.60 0.97 1.06 0.17 ,14.19 12	12 35.0 32 20 7038. 62. 13.60 0.46 0.37 0.32 12.30	12 35.1 28 10 7730. 222. 13.80 0.73 0.62 0.15 13.63	12 57.7 36 26 8360. 15. 14.20 0.67 0.60 0.98 11.96 13 4 4 25 25 4671 07 12 80 0.40 0.55 0.21 12 86 1	13 4.4 28 56 7174 746 12.00 0.49 0.00 0.01 12.00 1 13 4.4 28 56 7174 746 12.40 1.09 1.00 0.07 14.89 3
$rac{V_{mem}}{V^{mem}} = rac{\sigma_{1950}}{(km \ s^{-1})} \ rac{\sigma_v}{(km \ s^{-1})} \ rac{m_{tot}}{(km \ s^{-1})} \ rac{R_p}{(Mpc)} \ rac{R_h}{Mpc} \ rac{t_c/t_0}{M/M_\odot} \ rac{1}{N_O} \ rac{1}{N_$	6 8 46.4 36 46 7515. 51. 13.00 0.96 0.93 0.98 12.53	4 846.9 29 32 7958. 115. 13.90 0.77 0.37 0.17 12.55 3 857.4 35 55 3130. 69. 13.40 0.08 0.00 0.00 9.68	3 9 6.8 32 50 4235. 35. 13.90 0.45 0.42 0.64 11.86 4 0.105 36 10 6001 987 13.70 0.51 0.30 0.06 13.53	5 910.9 30 12 6586. 229. 12.60 0.50 0.29 0.07 13.33 1	5 918.6 33 46 6786. 270. 12.60 0.89 0.99 0.20 14.00 6	5 9.25.2 30 16 8057. 107. 13.30 0.69 0.53 0.27 12.93	5 930.9 34 8 8129. 154. 13.50 0.54 0.55 0.19 13.26 1 3 0327 32 5 6703 73 13.20 0.52 0.54 0.40 12.60	3 933.8 35 29 6405. 181. 14.10 0.76 0.88 0.26 13.60 1	4 936.9 32 31 6535. 106. 13.40 0.36 0.00 0.00 10.67	6 940.5 36 27 6745. 196. 12.60 0.74 0.70 0.19 13.57 2	3 942.7 34 56 6035. 98. 13.80 0.59 0.66 0.36 12.95 2	3 10 5.7 32 30 6084. 248. 13.90 0.78 0.94 0.20 13.91 24 5 11 5 7 25 7 250 1 21	3 11 2.0 30 3 6394. 212. 13.30 0.06 0.39 0.29 1.17 3 11 7.0 28 53 9576 765 12.30 1.00 0.58 0.04 14.67 5	3 11 7.8 37 9 8838, 228, 14.10 0.61 0.32 0.07 13.36 2	4 1114.8 36 28 7579. 105. 13.60 0.91 0.77 0.39 13.07 1	3 1115.7 28 37 9745. 215. 13.80 0.45 0.30 0.07 13.28	8 1118.8 34 29 10433. 196. 13.10 0.96 0.69 0.19 13.57 (2 13.00 0.7 27 10000 077 10000 110 000 110 000 110 000 1	5 11 24.5 53 41 10209. 273. 15.20 1.12 0.90 0.13 13.96 1 5 11 28 6 28 31 6902 58. 12.90 0.60 0.51 0.47 12.37	3 1131.2 33 22 2553. 56. 13.70 0.34 0.38 0.37 12.21	3 1136.9 34 6 10156. 691. 14.00 0.83 0.81 0.06 14.73 2	3 1137.3 36 27 11901. 789. 14.20 1.83 2.08 0.07 15.26 4	8 1142.8 33 27 9558. 293. 12.90 0.84 0.51 0.09 13.78 1 4 1140.9 25 29 6486 105 13.20 0.81 0.74 0.38 13.06 1	8 1154.3 32 30 3236. 133. 11.80 0.22 0.17 0.07 12.62 1	6 1155.6 28 4 3433. 117. 12.20 0.26 0.13 0.06 12.40	3 1156.1 30 55 3262. 121. 13.70 0.35 0.27 0.12 12.74 7	3 1157.9 32 9 7835. 135. 13.90 0.20 0.24 0.10 12.79	4 12 0.0 29 55 3284. 217. 13.20 0.25 0.18 0.05 13.08 11 4 13 3 4 99 9508 483 1310 0.70 0.36 0.04 14.07 5	6 12 4.6 31 36 7124. 479. 12.80 1.14 0.38 0.04 14.09 8	4 12 4.9 33 6 7775. 436. 13.70 0.65 0.68 0.08 14.26 19	3 12 6.4 29 31 3861. 175. 12.90 0.09 0.10 0.03 12.65 21	7 1210.4 29 8 3915. 87. 11.70 0.38 0.21 0.13 12.36 3	3 1211.5 34 55 9727. 233. 14.20 0.75 0.78 0.18 13.77 39	3 1217.2 29 5 7617. 76. 14.20 0.16 0.11 0.08 11.94 1 [']	4 1219.3 28 28 8201. 325. 13.60 0.97 1.06 0.17 ,14.19 12	4 12 35.0 32 20 7038. 62. 13.60 0.46 0.37 0.32 12.30	4 1235.1 28 10 7730. 222. 13.80 0.73 0.62 0.15 13.63	3 1257.7 36 26 8360. 15. 14.20 0.67 0.60 0.98 11.96 2 13 44 25 25 427 07 12 50 040 055 021 12 56 1	0 10 13 4.4 28 56 7174. 746. 12.40 1.09 1.00 0.07 14.89 3

 $\ensuremath{\textcircled{O}}$ American Astronomical Society $\ \bullet$ Provided by the NASA Astrophysics Data System

TABLE 6	

IMEDIAN VALUE	S OF PHYSIC	AL FARAMETERS	
Parameter	Median	1st Quartile	3d Quartile
All Gr	oups, N _{group}	_{os} = 92	
$\sigma_{\rm n} ({\rm km s^{-1}}) \dots$	209	111	290
$\log (M/M_{\odot})$	13.30	12.72	13.84
$M/L_{B(0)}(M_{\odot}/L_{\odot})$	186	70	583
t_{o}/t_{0}	0.06	0.03	0.10
\tilde{R}_{L} (Mpc)	0.51	0.23	0.78
$R_p''(Mpc)$	0.67	0.45	0.84
Rich G	roups, N _{grou}	_{1ps} = 36	
$\sigma_{\rm u} ({\rm km \ s^{-1}})$	228	145	344
$\log (M/M_{\odot})$	13.59	12.96	13.96
$\widetilde{M/L}_{B(0)}(\widetilde{M}_{\odot}/L_{\odot})$	175	76	583
t_c/t_0	0.06	0.03	0.10
\tilde{R}_{μ} (Mpc)	0.52	0.34	0.80
$R_p''(Mpc)$	0.69	0.56	0.96

between the spread in our catalog and the spread obtained for simulations (Heisler, Tremaine, and Bahcall 1985) is probably not significant. It is thus reasonable to conclude that the intrinsic scatter in mass-to-light ratio is small.

b) Distance Dependence of Group Properties

Distance dependence of group properties is one indication of selection bias in a group finding algorithm. Here we examine the 92 group sample for distance dependence.

Figure 7 shows V_L (dashed curve) along with the median σ_v (filled circles) as a function of redshift. For each redshift bin, the vertical bars span the range from the first to the third quartile of the σ_v distribution. The median velocity dispersion of groups is far from any limit imposed by the selection parameter and is nearly independent of redshift.

Figures 8a-8d shows σ_v , log $(M/L_{B(0)})$, log t_c , and R_h as a function of the mean redshift of the group. Figure 8d shows a clear increase in the envelope of R_h with redshift; we identify the largest groups only at the largest distances. However, there are also many small groups at these distances. In contrast, the simulated catalog contains only large groups at large distances. No strong distance-dependent effects are present for σ_v (Fig. 8a) or for log $(M/L_{B(0)})$ (Fig. 8b). The crossing time (Fig. 8c) is also distance-independent; a bias here would be surprising because the algorithm is designed to keep time scales distance-independent.

c) Groups and Abell Clusters

There are seven Abell clusters in the region of redshift space covered by the survey: the algorithm identifies all seven. Table 7 includes the group number, the richness class, the number of members (two values) σ_v (two values), and $M/L_{B(0)}$ for each of the clusters. The first values of $N_{\rm mem}$ and σ_v come from this analysis; the second values come from Zabludoff, Huchra, and Geller (1989) and typically include velocities for galaxies fainter than those in the redshift survey sample. Other properties for these clusters (except Coma) are in Table 5. Considering the differences in sampling, the velocity dispersions we obtain for A1185, A1228, A1267, and Coma are in reasonable agreement with those from Zabludoff *et al.*

For A1257 and A779, the values of σ_v obtained here are substantially smaller than those obtained by Zabludoff, Huchra, and Geller (1989). In both cases the exclusion of a small number of members (one of the two additional members in A1257 and three of the 15 additional members in A779) in the larger Zabludoff *et al.* samples brings the dispersions into agreement. The sensitivity of the dispersion to the details of the membership assignment underscores the need to obtain complete, magnitude-limited samples for reliable determination of cluster velocity dispersions. Only with such controlled sampling can objective methods for membership selection be applied. In the analysis below, we take the values from this analysis; we thus preserve the uniformity of the selection criteria over the entire catalog.

Arrows above the histograms in Figures 6a-6f mark the values we obtain for σ_v , $M/L_{B(0)}$, M, T_c , R_h , and R_p for the seven Abell clusters in the sample. In the $M/L_{B(0)}$ histogram the values for the Abell clusters straddle the peak of the distribution; the median $M/L_{B(0)}$ for these clusters is similar to the one for all of the groups in the catalog. For the other quantities (except t_c) the medians for the clusters are larger than those for the catalog as a whole.

The substantial overlap in the properties of groups and Abell clusters suggests that the distinction between groups and clusters is artificial. The survey includes a continuum of systems. In fact there are many groups which share the characteristics of the Abell clusters and some which appear to be even more "cluster-like" than systems included by Abell. Abell does claim that his selection is biased for velocities $\leq 6000 \text{ km s}^{-1}$; however, most of the systems we identify are at velocities $\geq 6000 \text{ km s}^{-1}$ (see § VIe). Possible explanations for the apparent discrepancies in the identification of rich systems include (1) incompleteness in the Abell catalog, (2) biases in the selec-

TABLE 7 Abell Clusters

Abell Cluster	Group Number	Abell Richness Class	N _{mem} ^a	σ_v^a (km s ⁻¹)	${M/L_{B(0)}}^{ m a}$ (M_{\odot}/L_{\odot})	N _{mem} ^b	σ_v^{b} (km s ⁻¹)
Coma	70	2	139	868	357	234	1016
A1185	29	1	13	765	581	16	873
A1228	34	1	8	196	61	8	188
A779	10	0	5	270	689	20	627
A1257	35	0	6	275	190	8	470
A1267	36	0	6	223	169	6	217
A2162	116	0	5	323	283		•••

^a This paper.

^b Zabludoff, Huchra, and Geller 1989.







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FIG. 7.—Selection parameter V_L (dashed line) and the median σ_v (filled circles) as a function of redshift. The bars span the range from the first to the third quartile.



FIG. 8.—Redshift dependence of group properties: (a) σ_v , (b) log $(M/L_{B(0)})$, (c) log (t_c/t_0) , (d) log R_h

tion of clusters from the distribution of galaxies on the sky rather than from a redshift survey, (3) differences in the morphological composition of Abell clusters and "groups" (in the sense that the Abell clusters are richer in early-type galaxies), and (4) differences in the luminosity function for galaxies in groups and clusters. Some of these issues may be important for attempts to understand the relationship between the correlation function for rich clusters and the correlation function for the galaxy distribution.

d) Groups and Large-Scale Structure

The cone diagram in Figure 4b shows the position of all of the groups in the survey, including those which lie near the survey limits. The crosses mark the positions of Abell clusters. The groups trace out the large-scale structure evident in Figure 1. At first glance, the groups appear to be nearly uniformly distributed within the sheets (we will soon report a more detailed analysis of the distribution of groups; Ramella, Geller, and Huchra 1989).

e) Comparison with Other Group Catalogs

Previous complete group catalogs (Tully 1987; HG82; GH83) are generally shallower than the catalog derived from this survey. The region of overlap between this and other catalogs is, thus, small. In order to make a comparison with groups identified in other complete redshift surveys, we applied the search algorithm once again to the original CfA survey with $m_{B(0)} \leq 14.5$ (Huchra *et al.* 1989). The group selection parameters and the luminosity function parameters are the ones used to produce the catalog discussed in § V (Table 4) and differ from those in GH83. Here we take a 300 km s⁻¹ Virgo infall into account.

For the 124 groups in the $m_{B(0)} \le 14.5$ catalog, the median velocity dispersion is $\sigma_v = 131$ km s⁻¹ and the median $M/L_{B(0)}$ is 250. These results agree very well with those obtained by NW. NW argue correctly that the V_0 used by GH83 is too large; it is so large that groups include members on both the near and far edges of a void. On the other hand, the difference

between the scaling in equation (4) and the scaling used by NW does not affect the results. In both cases the steps in velocity are small enough to avoid the problem in GH83.

The median velocity dispersions differ by about the quartile range for the 14.5 and 15.5 (Table 6) group catalogs. However, the median mass-to-light ratios are well within the quartile ranges. A natural first reaction is that the difference in velocity dispersion is an artifact introduced by the group identification algorithm. We now demonstrate that the difference in the physical parameters of the groups in the two samples is a physical effect related to the properties of large-scale structure. Tully (1987) has previously suggested such a link.

For the 14.5 survey, the effective depth (the maximum redshift at which an L^* galaxy is included in the survey) is ~5400 km s⁻¹; for the 15.5 survey, the effective depth is ~8500 km s⁻¹. Figure 9 shows the distribution of groups in the two surveys as a function of the mean velocity of the system. In the 14.5 survey, the median redshift of groups is 40% of the effective depth; in the 15.5 survey, the median redshift is 85% of the effective depth. As a consequence, most (58%) of the groups in the 15.5 survey contain three galaxies brighter than L^* . However, only 23% of the groups in the 14.5 survey have three members brighter than L^* . The systems in the two catalogs have significantly different intrinsic luminosities; for the luminosity function parameters of equation (12), 25% of the luminosity of a group is in galaxies brighter than L^* .

The difference in the median velocity dispersions for the two catalogs reflects the lack of a fair sample. The observed bubbleor spongelike structures in the 14.5 survey tend to be thinner (compare the nearby structures in Fig. 1 with the more distant ones) than the structures in the 15.5 survey. The thicker structures are undersampled or absent from the 14.5 survey. The 14.5 survey, although it covers about the same volume in redshift space as the 15.5 survey, is not sensitive to the largest structures present in the 15.5 survey. The effective depth of the 14.5 sample is just about equal to the diameter of the largest void in the 15.5 sample. The large angular scale coverage of the 14.5 survey does not compensate for the greater depth of the



FIG. 9.—Number of groups as a function of redshift for the 14.5 sample (*light line*) and for the 15.5 sample (*heavy line*). The vertical lines mark the depths at which the apparent magnitude limits of the two surveys correspond to the absolute magnitudes $M_{B(0)}^* - 1$, $M_{B(0)}^*$ and $M_{B(0)}^* + 1$, respectively.

15.5 survey. Much of the information about large-scale structure in the 14.5 survey is redundant (see de Lapparent, Geller, and Huchra 1988).

VII. CONCLUSIONS

We apply an objective group identification algorithm (Huchra and Geller 1982) to the Center for Astrophysics redshift survey complete to $m_{B(0)} = 15.5$ over the right ascension range $8^{h} \le \alpha \le 17^{h}$ and declination range $26^{\circ}.5 \le \delta < 38^{\circ}.5$. We extract a catalog of 128 groups with three or more members; 92 of these groups constitute our "statistical sample."

We use simulations of the geometry of large-scale structure to choose the selection for the identification algorithm. We identify a range where the survey contains more rich ($N_{\rm mem} \ge$ 5) groups than the simulation and where the groups in the survey are stable to variation in the selection parameters. The groups have $\delta \rho / \rho = 80$ and $V_0 = 350$ km s⁻¹. For this group catalog, comparison with the simulations indicates that $\geq 30\%$ of the groups with three or four members are probably accidental superpositions. These groups are a consequence of the geometry of large-scale structure; they are not necessarily the expected "fingers" in redshift space associated with physical systems of galaxies. Of course, it is not possible to distinguish the real groups from the accidental superpositions on the basis of these data.

One interesting issue beyond the scope of this paper is the behavior of the correlation function for individual galaxies which are not identified as group members. In other words, does the removal of "fingers" in velocity space remove the distortion observed in the correlation function, $\xi(R_p, \pi)$, of projected separation r_p and relative pairwise line-of-sight velocity π ? For both the 14.5 and 15.5 samples the median velocity dispersion of groups is smaller than the expected \sim 70% of the rms pairwise peculiar velocity (de Lapparent, Geller, and Huchra 1988; Davis and Peebles 1983). The discrepancy may be merely a result of the different weighting of large systems with large velocity dispersions in the calculations of rms pairwise peculiar velocity relative to median group velocity dispersion.

The median velocity dispersion for the 36 groups in the "statistical sample" with five more members is $\sigma_v = 228$ km s⁻¹, and the median $M/L_{B(0)} = 178h M_{\odot}/L_{\odot}$. The median

parameters for the 92 group sample are similar for the sample with $N_{\rm mem} \ge 5$. For this sample the critical mass-to-light ratio is $M/L_{\rm crit} = 1360h \ M_{\odot}/L_{\odot}$. If the mass-to-light ratios of groups are characteristic of the universe as a whole, the cosmological mean mass density Ω is equal to $0.13e^{\pm 0.9}$. The "error" in Ω comes from the quartile ranges in Table 6 and reflects the large spread in group mass-to-light ratios.

The 128 group sample contains seven Abell clusters; the physical properties of these clusters overlap substantially with those of groups. In fact, the distinction between groups and clusters is not generally apparent on the basis of selection of systems in redshift space. The similarity of group and cluster properties raises fundamental questions about the reliability of cluster catalogs as guides to systems with similar physical properties.

Comparison of the distribution of group centers with the distribution of all of the galaxies in the survey shows qualitatively that groups trace the large-scale structure in the region. We plan to examine this issue quantitatively by calculating the correlation function for the group centers. Perhaps comparison of the correlation functions for individual galaxies, groups, and clusters will lead to a resolution of the problems posed by the high amplitude of the cluster correlation function (see Bahcall and Soneira 1983; Geller 1987).

The physical properties of groups may be related to the details of the large-scale structure in the region. Groups extracted from the earlier CfA survey complete to $m_{B(0)} = 14.5$ have a significantly lower median velocity dispersion $\sigma_v = 131$ km s^{-1} in agreement with the previous results of NW. About 58% of the groups in the 15.5 survey contain three or more galaxies brighter than L*; only 23% of the groups in the 14.5 survey have three or more bright members. The difference in the group catalogs is probably largely a result of the location of large-scale structures relative to the survey limits. The inhomogeneity of group properties probably reflects the large-scale inhomogeneity of the galaxy distribution.

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