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THE PROBABILITY OF ARC LENSING BY CLUSTERS OF GALAXIES

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ABSTRACT

We examine the probability that clusters of galaxies act as gravitational lenses and create extended detectable arcs, or unusually elongated images, from distant sources. It is assumed that the clusters are isothermal spheres, and the sources are galaxies modeled as exponential disks. We consider detection thresholds in image shape (axial ratio, A, and angular extent θ), and in brightness (apparent surface brightness, s, or apparent magnitude, m). For galaxies of a given size, these thresholds define an axisymmetric source volume behind the cluster extending to a maximum redshift z_m, within which a galaxy must fall to be lensed by the cluster into an arc. The number of detectable sources is then determined by integration over the number density of visible galaxies in this volume. We find that for a cluster like Abell 370 ($\sigma \approx 1500$ km s^{-1} and $z_l \approx 0.37$), the probability of observing giant arcs like those already found ($A \geq 15$, $\theta \geq 90°$) is about 1 in 200 for $z_m = 0.72$, and approaches 1 in 10 if the cluster is inspected down to a B_j surface brightness of 26.5 mag arcsec^{-2}. Many more clusters should show distorted galaxy images which are unusually elongated perpendicular to the radius vector to the cluster center. For example, if nearby clusters of $\sigma \geq 1000$ are inspected to a limiting B_j magnitude of 26.5, roughly 1 in 3 (1 in 16) should show a distorted galaxy with axial ratio ≥ 5 (≥ 10).

Subject headings: galaxies: clustering — gravitational lenses

I. INTRODUCTION

Large arcs near the centers of two high-redshift clusters were first reported by Lynds and Petrosian (1987). Since then several other arcs have been discovered (Notalle 1988, Koo 1988). The large arc in Abell 370 has been studied observationally in some detail (Lynds and Petrosian 1989; Miller 1988; Soucail *et al.* 1988; Fort *et al.* 1988). A plethora of theoretical speculation has also surrounded the arcs (see, for example, Blandford and Kovner 1988; Katz 1988; Begelman and Blandford 1988; Braun and Dekel 1988). The foremost hypothesis being considered explaining the arcs, motivated by their positions near the centers of large clusters of galaxies and their extreme circularity, is that they are formed by the gravitational lens affect of a cluster on a background object (e.g., Paczyński 1987). Several works have since appeared analyzing the possible ways the lens could generate an optical image like those seen in the observed arcs (Blandford and Kovner 1987; Grossman and Narayan 1988). Support for the lens model comes from an indication, based on the emission feature identified as an O II line, that the arc in Abell 370 has a redshift $z \approx 0.72$, greater than the redshift of the cluster, $z = 0.37$ (Soucail *et al.* 1988; Lynds and Petrosian 1989).

The first attempt to estimate the probability of these arcs being caused by gravitational lensing was made by Kovner (1987), who made a brief argument based on the angular scale of the asymmetric lens needed to suppress opposing images. Grossman and Narayan (1988) estimated probabilities from the results of their attempt to model the observed image character. They gave the probability of clusters showing arcs as 15%. In the present work, we give a more complete treatment which incorporates a surface brightness limit of an arc search survey, and the relative probabilities of arcs of different sizes and shapes.

The probability treatment we give here is an application of the detection volume formalism described in Nemiroff (1989). We define the volume behind the lens within which a source must fall to be distorted into an arc as seen by the observer. This is the source volume. The boundaries of this volume are defined by the given detection criteria, such as how bright and how distorted the image is demanded to be. The distortion criteria define a radius from the observer-lens axis, closer than which stronger lensing effects would be seen. Revolving this radius about the observer-lens axis defines a disk, and summing the disks at each distance behind the lens defines a detection volume. The volume extends from the lens out to where the sources are too dim to be detected. Integration over the number density of observable sources within this volume then gives the number of sources falling in its interior. Since this number is usually found to be less than one, the result of this integral is taken to be the probability of an appropriate source falling into this volume and hence generating the image that obeys the detection criteria.

In § II we first consider the radius of the source volume, as a function of redshift, implied by the arc shape detection thresholds (axial ratio and angular extent), for galaxies of a given size. We then consider the surface brightness threshold that defines the far boundary of the source volume, and set up the integral that will yield the probability of lensing. In § III we apply our analysis to the known astrophysical parameters of arc systems and derive the relevant probability estimates. Our results are summarized and discussed in § IV.

II. DETECTION CRITERIA AND PROBABILITY ANALYSIS

a) Notations and Basic Relations

We assume that a cluster of galaxies is a smooth, spherical, isothermal lens acting alone. Such a lens has simple qualities (see Turner, Ostriker, and Gott 1984) which allow such a detailed probability analysis. We speculate that the probability analysis for lenses with a slight deviation from spherical symmetry would yield similar results, although it would be significantly more complex. The isothermal lens has been considered

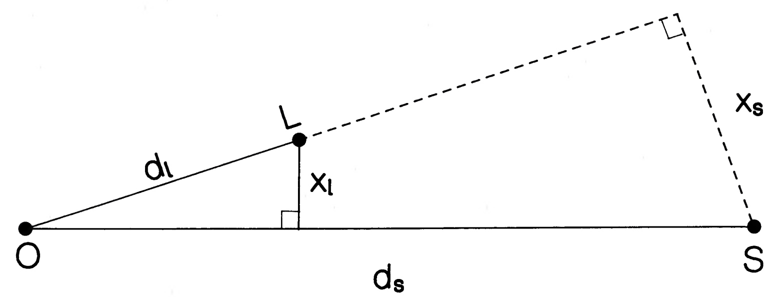


FIG. 1.—Side view of the lensing geometry of the observer, lens, and source. The angle of deflection is greatly distorted. Here O represents the observer, L represents the lens, and S represents the source. The distances are angular diameter distances.

to be a reasonable approximation for clusters of galaxies, whose central velocity dispersions are known to be fairly constant (West, Dekel, and Oemler 1987, Fig. 15, and references therein).

We assume hereafter a flat Einstein–de Sitter universe where the comoving metric distance to an object at redshift z is

$$r = 2cH_0^{-1}[1 - (1 + z)^{-1/2}] . \tag{1}$$

The observer-lens-source geometry is shown schematically in Figure 1. In this figure d_l and d_s represent the *angular-diameter distances* to the lens and to the source, respectively, and x_l and x_s are the proper distances between the lens and the observer-source line and between the source and the observer-lens line, respectively. In the "filled beam" approximation, these angular-diameter distances are related to the comoving distances by $d(z) = r(1 + z)^{-1}$. With angular-diameter distances, angular measures involving perpendicular proper distances obey the principles of Euclidean geometry, so that

$$x_s/x_l = d_s/d_l . \tag{2}$$

The comoving length along the transverse directions is given by $x(1 + z)$.

b) *Axial Ratio*

We assume the source is intrinsically homogeneous and circular with a proper radius R. The appearance of the source on the image plane, in the absence of gravitational lensing, is an undistorted circle of angular radius R/d_s at an angular distance x_l/d_l from the cluster center, as shown in Figure 2. It is evident that

$$\sin(\theta/2) = (R/d_s)(d_l/x_l) . \tag{3}$$

After lensing, the largest image is shifted away from the center of the cluster by an increment of angular distance (see Turner, Ostriker, and Gott 1984)

$$\phi = 4\pi\sigma^2 c^{-2}(1 - r_l/r_s) . \tag{4}$$

Projecting the edges of the unlensed disk in the image plane out to its new distance from the cluster center, and recognizing that isothermal lenses do not distort the angular width of images, it is clear that the minor axis of the image equals the unlensed radius of the galaxy,

$$b = 2R/d_s . \tag{5}$$

The major axis is elongated, however, to an angular length of approximately

$$a \approx \theta(x_l/d_l + \phi) . \tag{6}$$

For a given lens (σ and z_l), and a given arc axial ratio $A \equiv a/b$, one can solve equations (1)–(6) numerically for x_s as a function of the source parameters R and z_s. The quantity R may itself be a function of the absolute luminosity L and z (see § III). The source volume (e.g., Nemiroff 1989) for galaxies of radius R that produce arcs of axial ratios larger than A is an axisymmetric volume about the line of sight behind the cluster, with a proper radius which is given by the above solution for x_s. We name this radius $x_A(R, z)$. The comoving radius $(1 + z)x_A(R, z)$ is shown in Figure 3 for typical spiral galaxies with a radius $R(L_*, z)$ (the radius which we assume for an L_* galaxy, see below) and for the arc parameters $A \geq 15$ and $\sigma = 1500$ km s^{-1}. Rotation of the x_A curve in Figure 3 about the observer-lens axis creates the boundary of the detection volume within which a source must fall to create an image showing an $A \geq 15$.

Note that x_A drops suddenly to zero for z less than a certain redshift z_0 because for $z_l < z < z_0$ it is impossible for the lens to

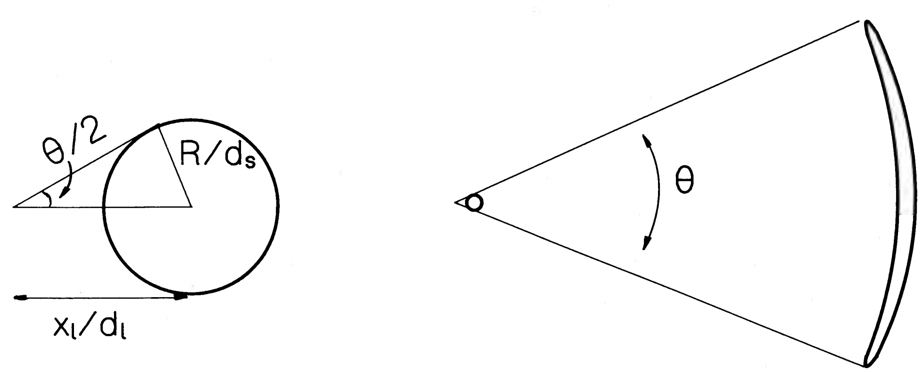


FIG. 2.—The observer's sky. A circular source is distorted by the lens into an arc. (a) What the observer would see in the absence of any lens effect; (b) the largest, most elongated image of the original circular source as created by the isothermal lens.

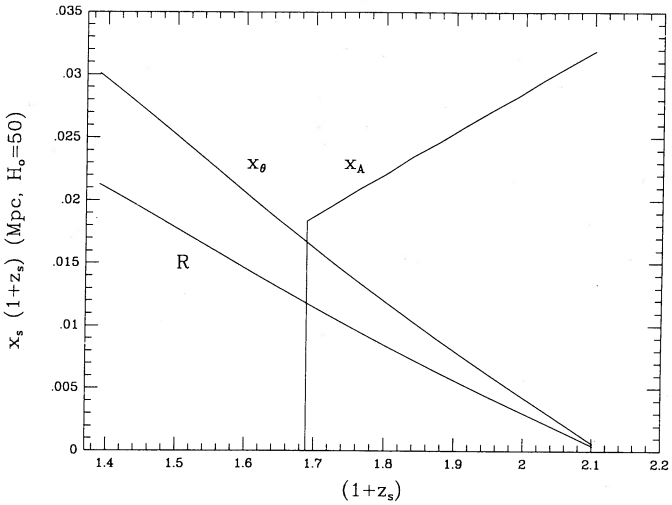


FIG. 3.—Plot of $(1 + z)x_s$, the comoving radius of the axisymmetric source volume, and $(1 + z)R$, the comoving radius of an L_* galaxy, vs. $(1 + z)$, the redshift of the source, for a galaxy of luminosity L_*. Both detection radii x_θ, based on the angular distortion of the arcs, and x_A, from the axial ratio of the arcs, are shown. The radius x_A may be zero when the lens is incapable of distorting a galaxy of luminosity L_* to an axial ratio greater than A. Any source of luminosity $L > L_*$ placed inside this boundary would be distorted by the lens to create an image visible to the observer as having an axial ratio greater than or equal to A. This plot was generated for $z_l = 0.37$, $\sigma = 1500$ km s^{-1}, $\theta = \pi/2$, and $A = 15$.

distort the finite-sized source into the required A or more. This can be seen in Figure 3.

The case of small angular extent, where $\sin(\theta/2) \simeq \theta/2$, can be solved explicitly. In this case θ can be eliminated so that, from equations (5) and (6), $A \simeq 1 + \phi d_l/x_l$, from which x_l can be isolated and R cancels out. Using equations (4) and (2), one then obtains

$$x_A(z) \simeq 4\pi\sigma^2 c^{-2} d_s(1 - r_l/r_s)(A - 1)^{-1} . \tag{7}$$

For distant sources, $z \gg z_l$, the (proper, noncomoving) source volume approaches a cone independent of R, of solid angle

$$\pi(x_s/d_s)^2 \simeq 2 \times 10^{-4} \deg^2 (\sigma/1000 \text{ km s}^{-1})^4 (A - 1)^{-2} . \tag{8}$$

This simple relation might give incite into the more sophisticated and complete computations described below.

For small galaxies the apparent axial ratio might be affected by the finite resolution, p, which is imposed by atmospheric seeing limitations or by pixel size. If $2R/d_s \leq p$, then the actual measured minor axis would be $b = p$ instead of the above. The above result for $x_A(R, z)$ would change such that A would be replaced by $Ap/(2R/d_s)$. The arc-discovery survey of Lynds and Petrosian (1989) used pixel sizes of less than 1". Our procedure has been tested for a pixel size of $p = 1''$, with the results changing by less than 20%. Larger pixel sizes will have a large effect on the results.

c) Angular Extent

Some of the observed arcs span more than one-fourth of a circle ($\theta \geq 90°$). We take this angular extent to be a separate detection criterion. From Figure 1, a threshold in θ corresponds to a source-volume radius of

$$x_\theta(R) = R/\sin(\theta/2) . \tag{9}$$

This radius is also shown in Figure 3 for a galaxy of $L = L_*$, and $\theta = \pi/2$.

In general, when both axial ratio and angular extent are applied as shape detection criteria, with any additional similar thresholds, the radius of the source volume behind a given cluster is given by

$$x(R, z; A, \theta, \ldots) = \min [x_A(R, z), x_\theta(R, z), \ldots] . \tag{10}$$

d) Brightness

Another important limiting detection criterion is surface brightness. The apparent surface brightness, s, of a galaxy of intrinsic surface brightness S, at a redshift z, can be written independently of the specific cosmological model by

$$s(z) = 10^{-K(z)/2.5} S/(1 + z)^4 , \tag{11}$$

where $K(z)$ is the K-correction. This relation defines a limiting redshift $z_m(S; s)$ for a source S that would be detectable above a threshold s.

Alternatively one can consider a magnitude limit. The associated apparent luminosity, l, is related to the intrinsic luminosity L, via

$$l(z) = 10^{-K(z)/2.5} L/[4\pi(1 + z)^2 r^2(z)] , \tag{12}$$

which defines an analogous limiting redshift for the source volume, $z_m(L; l)$.

e) Integration over Sources

Assume that the distribution of sources is given by a luminosity function $\Phi(L, z)$ (which can be translated to an analogous diameter function, or surface brightness function). The total number of detectable sources per lens (of velocity dispersion $\geq \sigma$ and redshift $\leq z_l$) is then given by the integral over comoving volume and galaxy properties

$$N(A, \theta, s, \ldots) = \int_0^\infty dL \int_{z_l}^{z_m} dr(z)\Phi(L, z) \times \pi(1 + z)^2 x^2[R(L, z), z; A, \theta, \ldots] , \tag{13}$$

where $x(1 + z)$ is the comoving radius of the detection volume.

This number N is directly related to the probability of arc lensing. When N is equal to unity for a given cluster, the volume enclosed by the detection boundaries is equal to the average volume per object. By Poissonian statistics, this cluster has a

$$P = (1 - e^{-N}) \tag{14}$$

chance of showing an arc, which is 63% when N is unity. When P is small the quantity $1/P$ is then the number of clusters one must inspect to have a 63% chance of finding at least one arc that fulfills the detection criteria.

III. APPLICATION TO OBSERVATIONS

a) Assumed Relations and $z_m(s)$

Certain input astrophysical relations concerning galaxies are needed in order to estimate $z_m(S, s)$ and the resultant probabilities. These are the luminosity function, $\Phi(L, z)$, the absolute surface brightness function of the source galaxies $S(L, z)$, the intrinsic radius-luminosity relation $R(L, z)$, and the K-correction $K(z)$. The following estimates will be based on given assumed approximate relations, but the same analysis can be

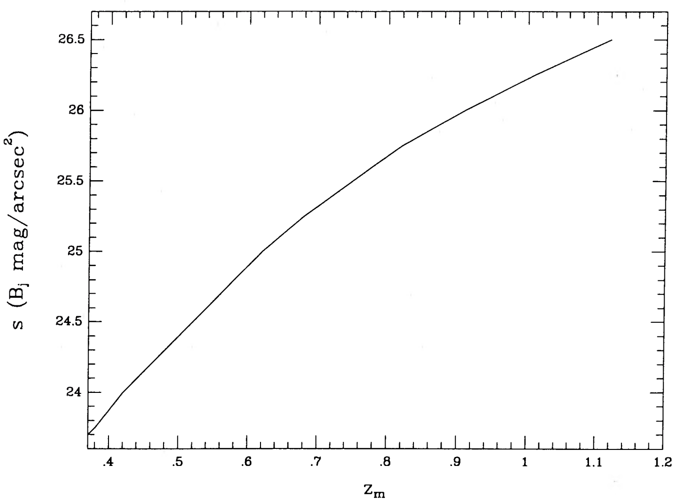


FIG. 4.—Plot of the limiting surface brightness of a survey and the maximum limiting source redshift this implies, for the cosmological scenario described in the text. The relation at $z > 1$ is extrapolated from observational data at smaller redshifts.

carried out using any other, more detailed, relations.

In the following analysis, all magnitudes relate to the B_j magnitude system (frequently used in dim galaxy surveys, e.g., Tyson 1988). Translations to other magnitude systems are listed in Schweizer (1976). The Hubble parameter is taken to be $H_0 = 50\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

The sources are assumed to be normal early-type spiral field galaxies. We take the present-day luminosity function to be the Schechter function

$$\Phi(L, z = 0) = \Phi_0 L^{-\alpha} e^{-L} , \quad (15)$$

where L is in units of L_*, M_* (absolute magnitude) is taken to be -21.15, $\Phi_0 = 0.009\ \mathrm{Mpc^{-3}}$, and $\alpha = 1.25$ (Schechter 1976; Broadhurst, Ellis, and Shanks 1989). We adopt the evolutionary scenario of Broadhurst, Ellis, and Shanks (1989) such that the luminosity-dependent evolution of the number density of galaxies is given by

$$\log \Phi(L, z) = \log \Phi\,(L, 0) + (0.1z + 0.2z^2) \log [\Phi(L, 0)/\Phi(L_{\max}, 0)] , \quad (16)$$

where $L_{\max}$ is the bright end of the luminosity function and corresponds to $M_{\max} = -23.5$.

The K-correction we take from King and Ellis (1985) to be

$$K(z) = -0.05 + 2.35z + 2.55z^2 - 4.89z^3 + 1.85z^4 , \quad (17)$$

in the range $0 < z < 1.25$.

We model our source field galaxies as disks following the relations of Freeman (1970). The surface brightness profiles of these disks decay exponentially, such that

$$S = S_0 e^{-R/R_g} . \quad (18)$$

We assume that all galaxies brighter than some $L_{\min}$ have the same central surface brightness S_0, corresponding to 21.42 mag arcsec^{-2} (Schweizer 1976). The characteristic radius depends on luminosity via

$$R_g = 10^{(-17.16 - M)/5}\ \mathrm{kpc} , \quad (19)$$

(Freeman 1970). The surface brightness is assumed to be negligible for dwarf galaxies of $L \leq L_{\min}$, where $L_{\min}$ corresponds to $M = -18$ (e.g., the Magellenic Clouds; see Kormandy 1985; Binggeli 1985). Intrinsic surface brightness for galaxies may itself be a function of redshift, so that the simple empirical picture we assume may be incomplete.

The quantity $R(L, z)$ is determined in the following way: for each galaxy of a given magnitude L, (or magnitude M), R_g is defined by equation (19); S is defined from z and the surface brightness limit of the arc survey in equation (11). Then, by equation (18), a unique R is defined. Because all galaxies brighter than $L_{\min}$ are assumed to have the same S_0, equation (11) defines a common redshift, z_m, where the surface brightnesses of the centers of all galaxies falls below the minimum detectable surface brightness. The precise relationship between this s and z_m is shown in Figure 4.

b) *Probability of Giant Arcs*

We now estimate the probability of observational surveys measuring giant arcs like those which have already been discovered. This involves lensing clusters at redshifts of $z_l = 0.37$, and with velocity dispersions of about $\sigma = 1500\ \mathrm{km\ s^{-1}}$. This velocity dispersion for A370 is calculated from equations (7) and (4) assuming $z = 0.72$ and $\phi = 25''$. We assume the radius of the arc is all due to ϕ, and not x_l/d_l because the large axial ratio of the arcs imply strong amplification.

The shape detection thresholds for the observed arcs are taken to be $\theta = 90°$ and $A = 15$. If we take the tentative redshift of $z = 0.72$ for the arc of A370 to be a lower limit of z_m, we then obtain, from equations (13) and (14), that the probability of this cluster showing such an arc is roughly 1 in 200. A similar lower limit is obtained if we use the average apparent surface brightness of the arc ($B_j \approx 25.2$) to define the back end of the detection volume. Lynds and Petrosian (1989) have estimated that 8% of the clusters they surveyed showed arcs. This is consistent with our estimates if we assume that they were sensitive to galaxies with lower surface brightness than A370, down to ~ 26.5 mag arcsec^{-2}.

Other observers should have different surface brightness detection thresholds. We therefore plot the number of clusters one must inspect as a function of the detection threshold in surface brightness in Figure 5. Figure 5*a* not only shows $1/P$ for the detection parameters we deemed relevant for the arc in A370 ($A > 15$, $\theta > \pi/2$), but for other less stringent detection parameters as well. Figures 5*b* and 5*c* show the relation between $1/P$ and A and θ. One can see that if an observer can detect galaxies out to a redshift of 1 ($s \approx 26.5$ mag arcsec^{-2}), then one has an excellent chance of discovering arcs in most clusters. One cannot take our results for $z > 1$ too seriously because the K-correction and $\Phi(L, z)$ are not necessarily applicable to these distances.

c) *Probability of Small, Elongated Images*

In the special case where we apply an axial-ratio threshold as the only shape detection criterion, and assume a magnitude (instead of surface brightness) limit, we can make use of number counts like those of Tyson and Jarvis (1979). We assume for a rough estimate that all the galaxies counted have redshifts greater than the cluster lens and that $r_l/r_s = \frac{1}{2}$. At 24th magnitude, $\sim 10^4$ galaxies were recorded per square degree. If we now multiply this number by the angular area of equation (8), we see that roughly 1 in 30 (160) clusters of velocity dispersion above 1000 km s^{-1} are expected to show an unusually distorted galaxy with axial ratio greater than 5 (10). Incorporating an assumed constant logarithmic slope 0.4 of the number counts versus magnitude relation found by Tyson and Jarvis (1979), we find that if all 26.5 magnitude galaxies could be detected, 1 in 3 (16) clusters would be expected to show an unusually elongated galaxy of axial ratio 5 (10).

Clearly, arcs of smaller angular extent are more common, with the probability increasing in proportion to $\sin^{-2}(\theta/2)$. Galaxies with smaller axial ratio are also more common, with the probability of detection increasing approximately with A^{-2}. The exact probability of finding an arc above a given θ and A can be obtained from numerical integration of equation (13), and some results are summarized in Figure 5.

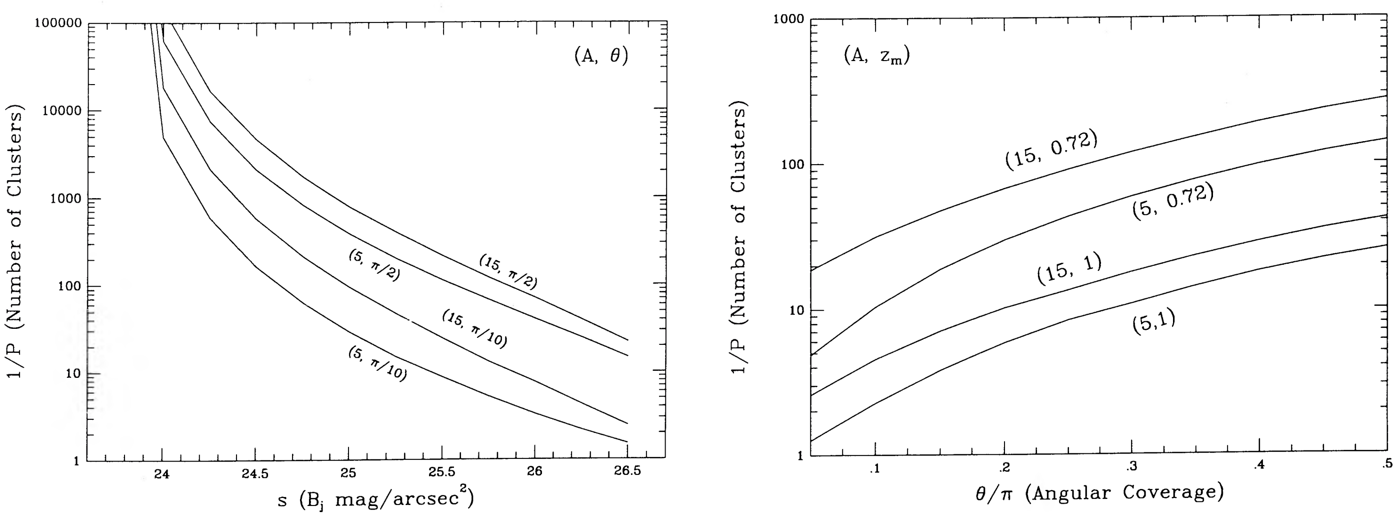
IV. DISCUSSION

From the above analysis we see that the creation of detectable arcs by clusters of galaxies is not improbable. Therefore a survey of the order of tens of nearby rich clusters ($\sigma \geq 1000$ km s^{-1}) to deep surface brightness (26.5 mag arcsec^{-2}) should yield detection of arc structures similar to those observed, caused by gravitational lensing.

We also predict that much more common than arcs must be unusually elongated galaxy images. Many nearby rich clusters inspected to deep surface brightness would yield detection of an elongated galaxy. These elongated galaxies should look somewhat different from normal edge-on spiral galaxies, since they should not have dominant central bulges and should be somewhat curved along a circle. These elongated galaxies are also expected to have their major axes perpendicular to the line connecting them to the cluster center.

Our analysis also predicts that the most probable clusters to act as gravitational lenses, per cluster, are the nearest clusters, since they have the greatest source volumes behind them. However, considering all clusters, it is more probable to find arcs around distant clusters than around close ones. To first order, the probability increases with the square of the distance to the clusters because, while the source volume behind clusters decreases with distance in proportion to the metric distance to the cluster, the number of clusters increases with the proper distance cubed. Another selection effect which works in favor of finding arcs in high-redshift clusters is that these clusters are usually studied very carefully to greater depths.

The sources are most likely to be placed at large redshifts, near $z_m(s)$. Even if the angular-extent threshold is dominant at



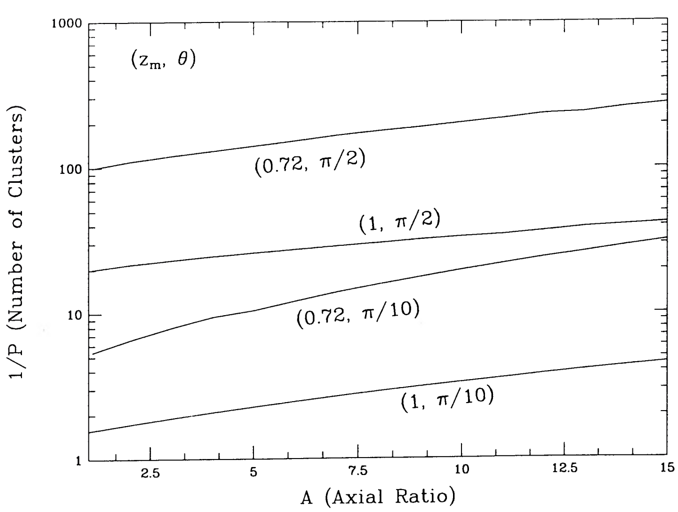


FIG. 5.—Plots of $1/P$, the number of candidate clusters lenses of velocity dispersion $\sigma \geq 1500$ and redshift $z_l \leq 0.37$, one must inspect for the detection of arclike images to become likely, where the detection thresholds listed are enforced. Here θ refers to the angular coverage of the arc over a circle, A is the axial ratio of the resultant image, and z_m is the maximum redshift where candidate source galaxies are visible.

determining the radius of the source volume at large distances, which means that the (proper) source volume is cylindrical, the increase of source density with $(1 + z)^3$ would make sources with high z somewhat more probable than those with small z. Selection effects would work in the opposite direction, however: the closer sources would be brighter and have higher surface brightnesses, thus making them more probable to observe than those of high z.

The detection of elongated galaxies would also provide a measure for the velocity dispersions of clusters of galaxies independent of the internal galaxy motions. The observer need only measure the redshifts of the cluster and the elongated galaxy, as well as noting the angular distance between them, to be able to use equation (4) and solve for the cluster velocity dispersion.

If only a few clusters have been found with greatly extended arcs, then one would expect tens of clusters to show elongated galaxies. These elongated galaxies should be able to be detected with present technology, and hence the cluster lenses should not only become the most numerous form of gravitational lensing, but a leading, independent method of mass determination for clusters of galaxies.

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