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ROTATIONAL DATING OF MIDDLE-AGED STARS

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ABSTRACT

One of the fortunate aspects of angular momentum loss from low-mass (i.e., solar-type) stars is that it is a great equalizer. By the age of about 10^8 yr, the rotational velocity of solar mass stars becomes independent of the initial rotation rate on the main sequence; this is a consequence of the strong dependence of the angular momentum loss rate on the rotation rate. Thus, once this common value is reached, the rotation velocity depends only on time (and the mass of the star). In principle, then, the measured rotation rate of a low-mass star provides an estimate of its age when its mass is known. The observed rotation periods of low-mass stars in the Hyades are used to test this conclusion and to check the validity of the theoretical angular momentum loss rates. The prescription for angular momentum loss by a magnetic stellar wind published by Kawaler in 1988 is consistent with the observed rotation rates for cluster members in the range $[0.60 \leq (B-V) \leq 1.25]$. A mean rotational age for the Hyades of $4.9 \pm 1.1 \times 10^8$ yr follows from this analysis. This rotational age, obtained using a solar-calibrated wind law and the rotation periods of subsolar mass members, agrees closely with the age determined by the more traditional technique of isochrone fitting to the higher mass stars at the turnoff. In general, this technique of rotational dating is *distance-independent* and may also be applied to other clusters of intermediate age (10^8 to several $\times 10^9$ yr) and to individual stars.

Subject heading: clusters: open — stars: evolution — stars: rotation

I. INTRODUCTION

The rotation rates of low-mass stars decrease with time as the result of angular momentum loss. Evidence for spin-down is provided by the observed sharp drop in the mean stellar rotation velocity with increasing spectral type that occurs at about F5 (Kraft 1970). The Kraft (1970) curve reflects the initial distribution of angular momentum with mass for stars hotter than F5, i.e., that mean angular momentum is proportional to M^2 (Kawaler 1987). Low-mass stars fall well below the extrapolated Kraft curve and therefore have undergone angular momentum loss. Indeed, observations of 1 $M_\odot$ stars of known age show that the rotation velocity decreases as $t^{1/2}$ (Skumanich 1972; Soderblom 1983). The initial explanation by Schatzman (1962) was that stellar winds, constrained to corotate with the star by the stellar magnetic field, carry away angular momentum very efficiently. The observed orderliness of this spin-down implies that a fundamental physical process such as Schatzman's magnetic braking is indeed responsible.

Many theoretical studies of the solar wind have addressed the issue of angular momentum loss (i.e., Weber and Davis 1967; Mestel 1968; Belcher and MacGregor 1976; Mestel 1984; Mestel and Spruit 1987). Kawaler (1988, hereafter K88) developed a parameterized angular momentum loss law based on the general form of Mestel (1984) and coupled it to evolving stellar models, and Pinsonneault *et al.* (1989) included the effects of internal angular momentum redistribution on the surface rotation rate. These models successfully reproduced the observed spin-down of the Sun and other low-mass stars, allowing calibration of the parameters of the wind law. The results of K88 and Pinsonneault *et al.* (1989) show two important features. First, by about 10^8 yr, the rotation velocity becomes independent of the initial angular momentum. Second, the time scale for internal angular momentum redistribution is of order 10^8 yr, so that low-mass stars rotate (and spin down) roughly as solid bodies by an age of a few $\times 10^8$ yr.

These two points suggest an exciting application of magnetic braking theory. The rotation periods of low-mass stars depends only on time and knowable stellar parameters such as mass and radius. Thus, a calibrated angular momentum loss law provides a tool for determining stellar ages without dependence on stellar distance.

This *Letter* adapts the magnetic braking model of K88 to the task of determining stellar ages. Section II briefly reviews this law and the assumptions and calibrations used in its formulation. The angular momentum loss rate is then integrated to derive a relation between rotation period and age (and stellar parameters). In § III, this $P_{\rm rot}(t, M, R, I)$ relation is converted to a period-age-color relation using simple stellar models. Section IV tests the technique of rotational dating by application to the low-mass stars in the Hyades; this *Letter* concludes with a discussion of the application of this method to individual stars.

II. THE MAGNETIC BRAKING LAW

In the angular momentum loss model of K88, the stellar wind (mass-loss rate $\dot{M}_{14}$ in units of 10^{-14} $M_\odot$ yr^{-1}) is assumed to corotate with the star out of the Alfvén radius r_A, at which point it detaches from the magnetic field and its angular momentum is lost to the star. The magnetic field strength is assumed proportional to the rotation rate, and the wind velocity is set to the escape velocity at the Alfvén radius. With these assumptions, the rate of angular momentum loss per unit time is

$$\frac{dJ}{dt} = -K_w(n)\Omega^{1+(4n/3)}\left(\frac{R}{R_\odot}\right)^{2-n}(\dot{M}_{14})^{1-(2n/3)}\left(\frac{M}{M_\odot}\right)^{-n/3}, \quad (1)$$

where the constant term

$$K_w(n) \approx 2.04 \times 10^{33} f_\odot(n)(1.43 \times 10^9)^n \quad (2)$$

is determined by the basic physical assumptions in the model and calibration of various quantities to their solar values. The value of n is related to the magnetic field configuration; $n = 2$ corresponds to a radial field, while $n = 3/7$ corresponds to a dipole field. The factor $f_{\odot}(n)$ is the solar calibration factor, which adjusts equation (1) so that solar models spin down to the solar rotation rate at the solar age for a given value of n. A value for $f_{\odot}$ is given by

$$f_{\odot} = 2.95 \times 10^{(3-2n)} \tag{3}$$

which is an approximation to Table 1 of K88 for solid body rotation. For details of the derivation of equations (1) and (2), see K88.

Evolutionary models of main-sequence stars, such as in K88, show that radius and moment of inertia change very slowly with time once a low-mass star arrives on the main sequence. Also, studies of rotation velocities in young clusters demonstrate that after about 10^8 yr, low-mass stars spin down roughly as solid bodies (Pinsonneault *et al.* 1989; K88). Hence, with the assumption that stars rotate as solid bodies, then $J = I\Omega$, and equation (1) can be integrated directly to obtain the rotation period as a function of time, valid for $t > 10^8$ yr:

$$P_{\rm rot}^{4n/3} = \left[\frac{4n}{3}(2\pi)^{4n/3}\frac{K_w(n)}{I}\left(\frac{R}{R_{\odot}}\right)^{2-n} \times (\dot{M}_{14})^{1-2n/3}\left(\frac{M}{M_{\odot}}\right)^{-n/3}\right]t + P_0^{4n/3}, \tag{4}$$

where P_0 is the initial rotation period. A value of $n = 1.5$ in equation (4) reproduces the $t^{-1/2}$ spin-down law for 1 $M_{\odot}$ *and* conveniently eliminates the dependence on the mass-loss rate. Hence a value of $n = 1.5$ is adopted, noting that for $n = 1$ or 2, the dependence on the mass-loss rate remains weak. With these simplifications, equations (2) and (4) become a relationship between rotation period, age, and stellar mass, radius, and moment of inertia:

$$P_{\rm rot}^2 = 3.724 \times 10^{-5}\left[\left(\frac{I}{I_{\odot}}\right)^{-1}\left(\frac{R}{R_{\odot}}\right)^{1/2}\left(\frac{M}{M_{\odot}}\right)^{-1/2}\right]t + P_0^2 . \tag{5}$$

The Sun provides a good example of the small effect of the initial rotation period P_0 on the rotation period at large times. In the T Tauri phase, observed rotation velocities are often in the 10–20 km s^{-1} range (e.g., Hartmann *et al.* 1986); when projected to the ZAMS, these stars would rotate with velocities about 100 km s^{-1} (Stauffer and Hartmann 1986). In 1 $M_{\odot}$ models of K88, models that rotate at 20 km s^{-1} in the T Tauri phase have begun magnetic braking by the time they reach the ZAMS their peak rotation velocity is 65 km s^{-1} ($P_{\rm rot} = 0\overset{d}{.}70$) at an age of 2.2×10^7 yr, but is reduced to 25 km s^{-1} ($P_{\rm rot} = 1\overset{d}{.}8$) by an age of 5.0×10^7 yr; these numbers agree with observed velocities of young stars. Therefore a representative value for the initial rotation period of 1 day is adopted. With this value, the time dependent part of equation (5) is equal to P_0^2 after only 6.4×10^6 yr of angular momentum loss. The initial rotation term decreases the rotation rate by only 3% after 10^8 yr of angular momentum loss; by 3×10^8 yr, the contribution is less than 1%. Therefore, after about 10^8 yr, low-mass stars essentially forget their initial rotation rates, and the subsequent history of the surface rotation is determined solely by the magnetic braking rate. This integration explains why K88 and Pinsonneault *et al.* (1989) find the rotation velocity of evolving solar models to be independent of the initial angular momentum. The initial rotation period can therefore be safely ignored in this analysis.

III. THE PERIOD-AGE-COLOR RELATION

Rarely are stellar masses or radii (or moments of inertia, for that manner) determined directly. Therefore, a more useful form of equation (5) would express these quantities in terms of observable parameters such as color. This can be achieved by expressing the radius and moment of inertia in terms of the stellar mass using stellar models. As an illustrative example for this *Letter*, consider the approximate main-sequence homology relationships near 1 $M_{\odot}$,

$$R = R_{\odot}\left(\frac{M}{M_{\odot}}\right)^r \tag{6a}$$

and

$$I = I_{\odot}\left(\frac{M}{M_{\odot}}\right)^i . \tag{6b}$$

Expressing the rotation period in days (P_d), and the age in Gyr (t_9), equation (5) becomes

$$P_d{}^2 = (12.55)^2 t_9\left(\frac{M}{M_{\odot}}\right)^{-i+[(r-1)/2]} \tag{7}$$

which relates the rotation period to age and stellar mass (where we have dropped the P_0 term). In logarithmic form,

$$\log(P_d) = 0.5\log(t_9) + 0.5\left(-i + \frac{r-1}{2}\right)\log\left(\frac{M}{M_{\odot}}\right) + 1.099 . \tag{8}$$

Since mass is rarely the observed quantity, the $(B-V)$ color can be used as a mass surrogate; a mass-color relationship of the form

$$\log\left(\frac{M}{M_{\odot}}\right) = K_{BV} + S_{BV}(B-V) \tag{9}$$

has been adopted. Using the stellar models from K88 ($Z = 0.02$) with the color calibrations described by Green (1988) gives $K_{BV} = 0.275(\pm 0.005)$ and $S_{BV} = -0.410(\pm 0.010)$. These values show little dependence on times for ages between 3×10^7 and 5×10^9 yr in the mass range of interest. Therefore, using $(B-V)$ as a mass surrogate in equation (8) produces the following period-age-color relation:

$$\log(P_d) = 0.5\log(t_9) - 0.205\left(-i + \frac{r-1}{2}\right)(B-V) + 0.824 . \tag{10}$$

Thus the observed rotation period and color of a low-mass star yield its age. Equation (10), derived assuming $n = 1.5$ to reproduce the empirically determined $t^{-1/2}$ spin-down rate for 1 $M_{\odot}$ stars, effectively extends this law to include stars over a large range of colors (masses). This value of $n = 1.5$ corresponds to a magnetic field geometry that is intermediate between radial and dipole (Kawaler 1988). Other choices of n are possible, allowing derivation of analogs to equation (10) for various field configurations.

Fundamental stellar data derived for main-sequence stars in binary systems (Popper 1980) give a value for r that is very close to 1 for stars at and below 1 $M_{\odot}$. These radii agree well with the models of K88; thus, we use stellar models that match the observed radius for a given mass to obtain a value of about

1.9 for the exponent i in equation (6b). With these representative values, equation (10) becomes

$$\log (P_d) = 0.5 \log (t_9) + 0.390(B-V) + 0.824 , \quad (11)$$

which is the adopted period-age-color relation for low-mass stars. The internal error in $\log (t_9)$ from equation (11) is a about 0.08. In order of importance, this error comes from the following factors: (1) the term $f_{\odot}$, reflecting our lack of knowledge of the angular momentum redistribution history of low-mass stars; (2) the uncertainties in the homology exponents i and r; and (3) the color-mass calibration.

Since the rotation velocity v_{rot} is equal to the radius multiplied by the angular rotation velocity, equation (11) can be converted to a rotation velocity-age-color relation by application of equation (6a):

$$\log (v_{rot}) = 0.5 \log (t_9) - 0.390(B-V) + 0.880 , \quad (12)$$

where v_{rot} is measured in km s^{-1}. Note however that equation (12) is implicitly more dependent on the stellar radius. As such, it is more sensitive to uncertainties in the adopted mass-radius relation.

IV. APPLICATION: THE AGE OF THE HYADES

The rotation periods, determined photometrically by Radick *et al.* (1987), and colors for a sample of low-mass stars in the Hyades provide a stringent test of the rotational dating technique. Rotation periods are preferable because, while there are many measurements of rotation velocities of Hyades stars, the velocity-age-color relation has larger uncertainties. In addition, the measured velocities are reduced by the sine of the inclination of the rotation axis, leading to additional uncertainties.

Lines of constant age (rotation isochrones), computed using equation (11), are superposed over the data from Radick *et al.* (1987) in Figure 1. The data parallel the isochrones very closely for $(B-V) > 0.59$; the star vB 190, which shows anomalously fast rotation (see the comments in Radick *et al.* 1987) has not

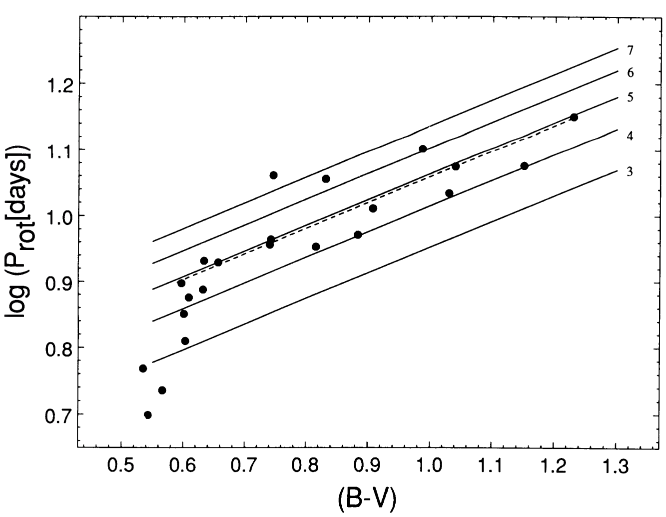

FIG. 1.—Rotation periods (in days) for low-mass stars in the Hyades, measured by Radick *et al.* (1987), as a function of $(B-V)$. Rotational isochrones are shown as solid lines and are labeled with the age in units of 10^8 yr. The dashed line shows the best linear fit with slope 0.39 to stars with $(B-V) > 0.59$; this fit corresponds to an age of 4.9×10^8 yr, with a statistical uncertainty of 1.1×10^8 yr.

been included in this analysis. The rotational age of the Hyades can be read off of Figure 1 as approximately 5×10^8 yr. A more precise estimate of the rotational age is obtained by making a linear fit to the observed rotation periods. Varying both the slope and the intercept of the fit, the best fit for $(B-V) \geq 0.60$ is

$$\log (P_{rot}) = 0.402(B-V) + 0.652 .$$

The intercept of this relation is a combination of the constant in equation (11) and the logarithmic age of the stars in the Hyades; thus the above fit yields a mean age for these stars of 4.5×10^8 yr.

An important point to note is that the slope of the $P[(B-V)]$ relation is very close to that expected from the theory of angular momentum loss (0.39). This indicates that the value of the intercept obtained while imposing a slope of 0.39 on the fit to the data yields a mean age of $(4.9 \pm 1.1) \times 10^8$ yr for the low-mass stars in the Hyades for which rotation periods have been measured; age determinations for individual stars with $(B-V) \geq 0.60$ range from 4×10^8 to 7×10^8 yr. These ages are actually skewed slightly toward larger values by any (few) initially very slow rotators that are still spinning down, but at a slower rate than the average. Nevertheless, the rotational ages over a wide range of colors agree quite well with more traditional determinations of the age of the Hyades.

In his study of the spin-down of solar mass stars, Skumanich (1972) included solar mass stars in the Hyades, with an assumed age of 4×10^8 yr. With Skumanich's data alone, the rotational age for the Hyades would not be precisely independent of other dating techniques because of the use of the $t^{-1/2}$ law in deriving equation (10). However, Soderblom (1983) used data from cluster and field stars in confirming the $t^{-1/2}$ law for solar mass stars. In fact, removing the Hyades data from Soderblom's sample would not significantly alter the spin-down law he derives. These empirical determinations of the $t^{-1/2}$ law only consider stars in a narrow range of $(B-V)$ centered on the solar value. Here we show that the magnetic braking law correctly predicts the relationship between rotation period and color at a given time over a wide range of color; thus, we may date stars over a larger range of color than considered by Skumanich (1972) or Soderblom (1983). The derived rotational age of the Hyades is therefore a valid test of the theory of magnetic braking for low-mass stars.

V. CONCLUSIONS

The integrated angular momentum loss law gives a consistent description of the rotation of low-mass stars in the Hyades with $(B-V) \geq 0.60$. The derived rotational age of low-mass stars in the Hyades is very close to the turnoff age of between 3 and 5×10^8 yr determined by G. DaCosta and P. Demarque (private communication) using the Revised Yale Isochrones (Green, Demarque, and King 1987). This agreement indicates that the magnetic braking law is valid for stars redder than about $(B-V) = 0.6$, i.e., less massive than about 1.1 $M_{\odot}$. Above this mass, where the observations shown in Figure 1 depart from a single isochrone, some of the assumptions of the braking model begin to break down. Compared to lower mass stars, the character of magnetic field generation and modulation, and the structure of the stellar wind begin to change for stars with $(B-V) > 0.60$. These higher mass stars have begun to climb back up to the Kraft curve.

In general, the rotational age is a completely independent age estimate, as it employs low-mass stars, while isochrone fitting relies on the more massive stars near the turnoff. Furthermore, *the rotational age is independent of distance.* Rotational dating should work for Population I clusters with ages between 10^8 and several $\times$ 10^9 yr, without recalibration. This procedure can be applied to individual field stars, with an internal uncertainty of 20% or so. More sophisticated application of the wind law to models of evolving rotating stars will enable us to extend the isochrones to higher mass (bluer) stars and therefore decrease further the uncertainty in the derived rotational age.

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