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ACTIVE GALACTIC NUCLEI. III. ACCRETION FLOW IN AN EXTERNALLY SUPPLIED CLUSTER OF BLACK HOLES

A. G. PACHOLCZYK
Steward Observatory, University of Arizona, Tucson, and Specola Vaticana, Castel Gandolfo

T. F. STEPINSKI
Lunar and Planetary Laboratory, University of Arizona, Tucson

AND

W. R. STOEGER
Vatican Observatory Research Group, Steward Observatory, University of Arizona, Tucson
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ABSTRACT

This third paper in the series modeling QSOs and AGNs as clusters of accreting black holes studies the accretion flow within an externally supplied cluster. Significant radiation will be emitted by the cluster core, but the black holes in the outer halo, where the flow is considered spherically symmetric, will not contribute much to the overall luminosity of the source because of their large velocities relative to the infalling gas and therefore their small accretion radii. As a result, the scenario discussed in Paper I will refer to the cluster cores, rather than to entire clusters. This will steepen the high-frequency region of the spectrum unless inverse Compton scattering is effective. In many cases accretion flow in the central part of the cluster will be optically thick to electron scattering, resulting in a spectrum featuring optically thick radiative component in addition to power-law regimes. The fitting of these spectra to QSO and AGN observations is discussed, and application to 3C 273 is worked out as an example.

Subject headings: black holes — galaxies: nuclei — quasars

I. INTRODUCTION

In the first paper of this series, Pacholczyk and Stoeger (1986, hereafter Paper I), we introduced the black hole cluster model of quasars (QSOs) and active galactic nuclei (AGNs) in a highly idealized form. We elaborated in some detail the strong motivations from developing cluster models of this type, stressing their intimate connection with the single black hole model, set down some preliminary assumptions, and then proceeded to fit the model to individual QSOs and AGNs on the basis of those assumptions. As we emphasized in Paper I, this simplified picture undoubtedly does not adequately describe the dominant underlying physical processes in these classes of objects. However, the results presented there indicate that models of this type are very promising, from several different points of view. In particular, the general properties of such clusters of compact objects have no difficulty in fitting the general features of quasar and AGN spectra, energy output and variability, and seem to be able to explain very naturally—through the greater variety and richness of the underlying gravitational environment itself—the distinguishing characteristics of the different classes of such sources. Moreover, from a dynamical and evolutionary perspective, the lifetime of such clusters—before they end in a single supermassive black hole—is long enough to make them realistic alternatives to that model and to assure us that such objects can exist in sufficient numbers and therefore should be observed.

Finally, it has been pointed out in (Pacholczyk and Stepinski 1988, Paper II) that such an accreting cluster of black holes is a natural site for a system of shock structures capable of accelerating particles to energies adequate to explain the observed properties of synchrotron and inverse-Compton radiation from QSOs and AGNs.

The principal assumptions of our idealized black hole cluster model as given in Paper I are (1) each AGN or QSO is powered by a relaxed cluster of accreting black holes with a mass spectrum $f(m) = Bm^{-\beta} dm$; (2) each black hole has a "standard" geometrically thin, optically thick accretion disk surrounding it, i.e., an accretion disk as described by Shakura and Sunyaev (1973) and Novikov and Thorne (1973); (3) the accretion disks are randomly oriented with respect to the line of sight; (4) the accretion $\dot{M}$ for each black hole is assumed to be $\dot{M} = 0.3\dot{M}_{\rm crit}$, where $\dot{M}_{\rm crit} = 3 \times 10^{-8}$ m (in solar masses per year); (5) accreting material is supplied by the galaxy in which the cluster is embedded or by stars near or within the cluster itself or both.

By assuming that the specific accretion rate onto each black hole is the same, and that the same thin disk model applies to each, we were able to fit our cluster model to the spectrum and spectral luminosity of any QSO or AGN in a very simple way—so that its mass spectrum reflects the radiative power spectrum, and the number of black holes contained in it reflects the luminosity. The radius of the cluster was determined by the variability of the source.

In this paper we move forward to modify and strengthen this idealized cluster model in a number of ways, focusing primarily on a proper and consistent treatment of the accretion flow onto the entire cluster, and within it—as it filters through it to feed all the component black holes—and on the changes which must be introduced in the cluster model as a consequence of that. We shall include a number of important considerations, each in a relatively simple way, in order to give ourselves some idea what suitable zeroth-order cluster models of QSOs and AGNs are really like. We shall find that they will be very similar to those of Paper I, with all the efficiently radiating holes confined to the cluster core.

We shall here concentrate on investigating externally supplied clusters of black holes. Internally supplied clusters will be dealt with in a subsequent paper. When most of the accreting material comes from outside the cluster, the accretion rates of the individual holes must be described consistently with the characteristics of the overall flow filtering through the cluster. We cannot just assume that $\dot{M}_h = 0.3\dot{M}_{crit}$, as we did in Paper I. It turns out that these considerations alter our description of externally supplied clusters as QSO and AGN sources. In particular, we find that the regions of each cluster away from its core will not contribute significantly to the radiation from it. This is due to "the velocity problem" and, in some cases, to "the temperature problem." The relative velocity of almost all the holes outside the core with respect to the accretion flow will be too high for them to feed efficiently from it, hence the velocity problem. At the same time, if there is sufficient X-ray flux originating anywhere in the cluster, as there often will be, the infalling gas will often be preheated to those X-ray temperatures, further inhibiting its accretion onto the component holes, and we have the temperature problem. Our treatment will be limited to clusters for which the flow onto the core is subcritical. We shall discuss the almost critical cases, however, for which there will be an optically thick region within the cluster.

We first describe, in § II, the two main flow regimes within clusters—the "cluster core," from which most of the radiation will originate, and the much more extensive region, or halo, surrounding the core, and we examine this halo and show that the holes there will always be underfed by the accretion flow. The structure of the black hole cluster is described in § III, and the core is dealt with in § IV, in a very similar way to our preliminary treatment of the entire cluster in Paper I. In § V we discuss the optically thick regions of the cluster and construct a model for those. We discuss our improved externally supplied cluster models in § VI and motivate our future work on internally supplied and supercritical core clusters.

II. ACCRETION IN THE HALO: THE VELOCITY PROBLEM

Throughout regions of the cluster away from the center—particularly those near the edge—the flow will be spherically symmetric, as we are assuming the external flow is. It will also be subcritical—and, as long as it is not approaching the critical accretion rate $\dot{M}_{0_{crit}}$, its velocity will be of the order of the free-fall velocity $v_{ff} = (2GM_0/r)^{1/2}$, where M_0 is the mass of the cluster and r is the radius from its center.

The accretion rate $\dot{M}_h$ onto individual black holes is given by

$$\dot{M}_h = \pi R_A^2 \rho |\boldsymbol{v}_h - \boldsymbol{v}| , \tag{1}$$

where ρ is a density of flow far from an individual hole, $\boldsymbol{v}$ is the velocity of the accretion flow with respect to the cluster, $\boldsymbol{v}_h$ is the velocity of the hole with respect to the cluster (notice that $\boldsymbol{v}_h - \boldsymbol{v}$ is a vector difference), and

$$R_A = \frac{2GM_h}{a^2 + (\boldsymbol{v}_h - \boldsymbol{v})^2} \tag{2}$$

is the accretion radius for the hole and a is the sound speed of the gas. Combining equations (1) and (2), we have simply

$$\dot{M}_h = \frac{4\pi G^2 M_h^2}{[a^2 + (\boldsymbol{v}_h - \boldsymbol{v})^2]^2} \rho |\boldsymbol{v}_h - \boldsymbol{v}| . \tag{3}$$

Of course, the accretion rate onto the entire cluster is

$$\dot{M}_c = 4\pi r^2 \rho v , \tag{4}$$

and so in equation (3) ρ can be expressed in terms of $\dot{M}_c$, r, and v. It can be shown that, despite some of the accreting material falling into black hole sinks in the outer region of the cluster, $\dot{M}_c$ will remain approximately constant until deep inside (see below).

We shall define the "cluster halo" to be that part of the cluster away from the center where the flow remains essentially spherically symmetric on average and subcritical (with respect to the total mass contained inside a given radius) and where the accretion rate onto individual holes can be expressed by equation (3), with the density given by equation (4).

The "cluster core" will be that region inside the halo where spherically symmetric flow breaks down and where the accretion rate onto individual holes is not dictated by equations (3) and (4). We shall describe in greater detail later the structure of our cluster. Assuming equipartition and virialization—or at least an advanced stage of evolution toward these two conditions—holes within the cluster will be mass segregated, with larger mass holes concentrated toward the center. The central region of a cluster, then, will be dominated by the large black holes (there will still be a lot of smaller ones, too, of course—these will be smoothly distributed throughout the entire cluster), and all the material not accreted by holes in the halo will end up in the core. It is obvious that the central region will be chaotic—the accretion flow there will be far from spherically symmetric, even though it started out that way, due to the churning movement of all the larger black holes. The unaccreted material will be "sloshing around" in this deep potential well.

It is impossible to give a detailed prescription of where the "cluster core" begins and an adequate treatment of the processes which determine the radiation emanating from it. What is clear is that, as long as the flow is still subcritical with respect to the total mass inside a given radius, all of the inflowing material not absorbed by holes in the cluster halo must be accreted by the holes in the cluster core, with the corresponding emission of radiation. For our models, we shall define the "cluster core" observationally, by the radius of variability R_{var}. Outside this core defined by R_{var}, as we shall see, very little radiation is produced by the component black holes. Within this core, we shall not use equation (3) for $\dot{M}_h$, as we do in the halo. Instead, we shall assume that all the holes—big and small alike—accrete the available material in proportion to their masses, i.e., that their specific accretion rates are the same and equal to $\dot{m}_{core}$, the "specific accretion rate" onto the core, as long as $\dot{m}_{core}$ is subcritical.

This then is the way we define and treat the "cluster halo" and the "cluster core." Notice that these definitions are not related to other definitions of "core" and "halo" in the literature on clusters, and as such have nothing to do with gravitational core collapse and other dynamical issues. In studying the halo, we shall see that it will accrete very little material in the very subcritical case. Let us examine the reason for this more carefully and substantiate the same conclusion also for the nearly critical cases, in which there is significant radiation pressure to slow the flow from its free-fall velocity v_{ff}. Before focusing directly on this issue, it is important first to understand the preheating and cooling of the accreting gas within the cluster.

The problem of accretion with preheating and cooling of the infalling gas by X-rays produced in this process is, in general, a very complex problem. It has been treated among others by Ostriker *et al.* (1976), Cowie, Ostriker, and Stark (1978), Bisnovatyi-Kogan and Blinnikov (1980), and Krolik and London (1983). They considered the spherically symmetric case with Compton heating and bremsstrahlung and Compton cooling. In addition, Krolik and London (1983) added a supplemental term to heating to represent the complicated heating and cooling at low temperatures and to allow the description of thermal equilibrium at 10^4 K without affecting processes at temperatures larger than 10^6 K. Ostriker *et al.* (1976) argued that for high values of the ratio of the cluster luminosity L to the critical Eddington luminosity $L_E = 1.3 \times 10^{38} M_0/M_\odot$ ergs s^{-1}, and high efficiencies the accretion flow cannot become transonic steadily and smoothly when the sonic point is assumed to be at the temperatures close to the outside temperature (temperature at infinity) of $\sim 10^4$ K. Even for low efficiency, but high luminosites L/L_E, heating inside the sonic radius results in recurrent flares, as was pointed out by Cowie, Ostriker, and Stark (1978), although the average luminosity is equal to that expected from a stationary flow. Bisnovatyi-Kogan and Blinnikov (1980) were able to construct numerically stationary solutions for high values of L/L_E and of the efficiency; these solutions smoothly match the asymptotics at infinity for any luminosity lower then L_E and for an arbitrary value of the efficiency. In their solutions the sonic temperature T_s, that is, the temperature $T_s = T(R_s)$ at the sonic radius R_s, is close to T_x, the temperature equivalent to the maximum energy E_m possessed by the X-ray flux ($\cong E_m/4k$), and not to $T_r = 10^4$ K, the temperature at the recombination radius R_r, for high efficiencies and luminosities. The sonic radius is always positive for any value of efficiency, and it goes to zero in the Eddington limit $L \to L_E$. The results of Bisnovatyi-Kogan and Blinnikov (1980) are in good agreement with the time-averaged luminosities found by Cowie, Ostriker, and Stark (1978). No detailed investigation of the stability of Bisnovatyi-Kogan and Blinnikov's solutions were carried out by these authors. They mention only that crude estimates indicate a possible thermal instability against isobaric perturbations of short wavelength resulting in the disruption of the accretion flow into small blobs of cold matter embedded in a hot medium. It remains unknown what effects on the development of the instability factors such as thermal conduction, magnetic fields, or radiative friction will have and how they might inhibit the instability in real situations. Krolik and London (1983) corroborated the point that the accreting gas may find a sonic radius temperature T_s very different from the temperature at infinity assumed to be of the order of $T_r \cong 10^4$ K, for high values of luminosity and efficiency, if there exists a bound on the highest radiative equilibrium temperature, which should in general be determined by Comptonization. Krolik and London also extended the work into the range of high masses of the central black hole modeling a QSO or an AGN. Their outside temperatures T_{out} (at $R = 1R_{out}$ and also $10R_{out}$, where $R_{out} = 3.8(M_0/10^8\ M_\odot)T_{C8}^{-1}$ pc, with T_{C8}, the Comptonization temperature, in units of 10^8 K) were high (from 10^6 to 10^8 K), with $T_x = 10^8$ K. They found that solutions with high luminosities and efficiencies may be thermally unstable and that the instability may disrupt single-phase steady accretion flow. They also estimated the instability conditions for flows with initial temperatures of 10^4 K. They confirm in principle the determination of regions of steady flow in the luminosity-efficiency plane done by Cowie, Ostriker, and Stark (1978).

We will adopt here the picture characterized by isothermal flow into the accretion radius R_{AC} of the cluster, a flow with increasing temperature between R_{AC} and R_x, the radius at which the temperature of the flow is T_x, the temperature equivalent to the maximum energy E_m possessed by the X-ray flux, and a second region of isothermal flow for $r < R_x$. Bisnovaty-Kogan and Blinnikov (1980) chose $E_m = 10$ keV for their case; our E_m will have to be chosen differently, based on the spectrum being emitted by our cluster and its X-ray cutoff. At any rate, like the authors cited above, we shall have the radius R_x, generally speaking, located outside our cluster. The accretion flow for $r \leq R_x$ will be isothermal, with a velocity close to free fall for small accretion rates:

$$v \cong \delta v_{ff} = \delta \sqrt{\frac{2GM_0}{r}}\,, \tag{5}$$

where $\delta = (1 - L/L_E)^{1/2}$, L is the luminosity from inside r, and L_E is the critical luminosity for the mass inside r. Because of our conclusions above, both L and L_E can be taken with reference to the cluster core. Now we can return to the principal issue and show why holes in the cluster halo will accrete very little of the inflowing material.

The velocity v_h of a given hole will also be of the same order as v_{ff}, except for holes in highly elliptical or radial orbits. For such situations, there will be segments of the orbit during which v_h will be very small, and other segments during which the relative velocity of the hole with respect to the fluid $v_{rel} = |\boldsymbol{v}_h - \boldsymbol{v}|$ is small. In all other cases the relative velocity between a given hole and the flow will be of the order of v_{ff} and will be supersonic. One can show that the sonic point of the accretion flow will be outside the core radius R_{var} and usually outside the radius of the entire cluster. We write the "specific accretion rate" $\dot{m} = \dot{M}_h/\dot{M}_{h_{crit}}$ for a given component hole, from equation (3) with equations (4) and (5) as

$$\dot{m} = \frac{1}{4}\left(\frac{R_0}{r}\right)^2 \frac{m}{m_0}\frac{v_{rel}}{v}\left[\frac{v_0^2}{a^2 + (\boldsymbol{v}_h - \boldsymbol{v})^2}\right]^2 \dot{m}_c\,, \tag{6}$$

where m and m_0 are the masses of an individual hole and of the cluster, respectively, in solar masses, $\dot{m}_c$ is the specific accretion rate of the cluster, and v_0 is the free-fall velocity at R_0. We see clearly from this that, as long as $a^2 + (\boldsymbol{v}_h - \boldsymbol{v})^2$ is larger than, or of the same order as, v_0^2, $\dot{m}$ is smaller than $\dot{m}_c$; often $\dot{m} \ll \dot{m}_c$. The maximum possible specific accretion rate onto a hole takes place for radial or elliptical orbits when $v_{rel} \cong a$; then we have from equation (6)

$$\dot{m}_{max} = 2.46 \times 10^{-24}\,\frac{1-\delta^2}{\delta}\left(\frac{M_0}{R_0 T}\right)^{3/2}\left(\frac{R_0}{r}\right)^{3/2}\frac{m}{m_0}\,. \tag{7}$$

For example, for a hole with mass $m = 1$ in a cluster with $M_0 = 10^8$ solar masses, $R_0 = 10^{15}$ cm, $T = 10^6$ K, and $L/L_{crit} = 0.8$, this maximum specific accretion rate is close to 0.5 at $r = R_0$ and larger at smaller radii. Only for certain portions of radial or highly elliptical orbits will "the velocity bracket" be much larger than 1, as needed for $\dot{m}$ to be of the same order as $\dot{m}_c$. The time interval in which a hole accretes nearly critically taken as a percentage of hole's period is a good indicator of inefficiency of the accretion process in a halo. The value of this percentage in the first approximation depends only on mass of the hole and degree of criticality L/L_{crit}, and can be calculated to be of the order of 10^{-3}. This is precisely why component black holes in the halo will not accrete efficiently from the overall flow. We call this conclusion the "velocity problem." In "the velocity bracket" we also see the origin of the "temperature problem." If the temperature, and thus the sound speed a, of the flow is too high, then—even if $v_{\rm rel}$ is small—accretion will be small.

We can see from this analysis that, even when a is less than $v_{\rm rel}$, the percentage of time during which holes in radial orbits will be accreting efficiently is very small. If we were to put many more holes into the halo to obtain enough radiation by this process, we should have to increase their number in the core proportionately. The radiation from these holes in the core would still overwhelm that from the cluster halo. Finally, from equation (7), it is clear that max ($\dot{m}$) will approach the critical value of 1 for small holes and for reasonable values of r, only if $T < 10^6$ K. But if X-rays are being produced in the cluster, $T > 10^7$ K, assuming a balance between bremsstrahlung cooling and bremsstrahlung and Compton heating, which render the flow within the cluster isothermal at T, the temperature equivalent to the maximum X-ray energy (see Bisnovatyi-Kogan and Blinnikov 1980). Thus, in general we do not expect significant radiation from accretion onto the black holes in the halo of the clusters, for subcritical accretion regimes. Even when $\dot{M}_c$ nears criticality at a given radius, with respect to the mass contained therein, slowing v significantly, the velocity of individual holes v_h and the sound speed a will still be very high.

The above conclusions concerning the relatively small accretion rate onto black holes in the halo of our cluster and the correspondingly insignificant or small contribution of these objects to the overall radiation from the cluster are the results of the velocity problem and of the temperature problem discussed above in a fairly general manner. We would expect these conclusions to hold almost independently of the details of the internal structure or mass distribution of our cluster, and therefore the bulk of the cluster's radiation to come from its core. We shall examine this radiation from the core in § IV. We shall also examine the case of a cluster of many small holes acting as an outer region (halo) around one large massive hole acting as a core. We will see that such a limiting case can plausibly model some of the X-ray emitting active galactic nuclei.

III. STRUCTURE OF THE BLACK HOLE CLUSTER

As we have already mentioned, each AGN or QSO is considered to be powered by a relaxed cluster of accreting black holes with a mass spectrum

$$f(m)dm = \mathscr{B}m^{-\beta}\,dm\,, \qquad m \in (m_1, m_2)\,, \tag{8}$$

where m is the mass of an individual hole in solar units, $\mathscr{B}$ and β are constants, and m_1 and m_2 are the lower and upper limits of the mass distribution. The lower limit is firm as it is related to the smallest black hole which can form, of the order of one solar mass. The upper limit is related to the largest mass within the cluster, and it can be expressed in terms of $\mathscr{B}$ and β as follows:

$$m_2 = \left(\frac{\mathscr{B}}{\beta - 1}\right)^{1/(\beta-1)}, \qquad \beta \neq 1\,. \tag{9}$$

This expression is derived from the assumption that there is only one largest mass in the distribution. The total mass of the cluster, the total number of holes in the cluster, and other quantities related to the number of holes in the cluster and their masses can be calculated from equation (8). There are basically two parameters: $\mathscr{B}$ and β which will be fixed from the spectra. Figure 1 shows some properties of the distribution given by equation (8) as functions of parameters $\mathscr{B}$ and β. For the purpose of this presentation we divide all holes in the cluster into two classes: "small," such that $f(m) > 1$ and "large," such that $f(m) < 1$. Figure 1 shows how much of the cluster mass is concentrated in "small" holes, in "large" holes, in the largest hole, and also the ratio of the number of "small" holes to "large" ones. As we can see from Figure 1, in the case

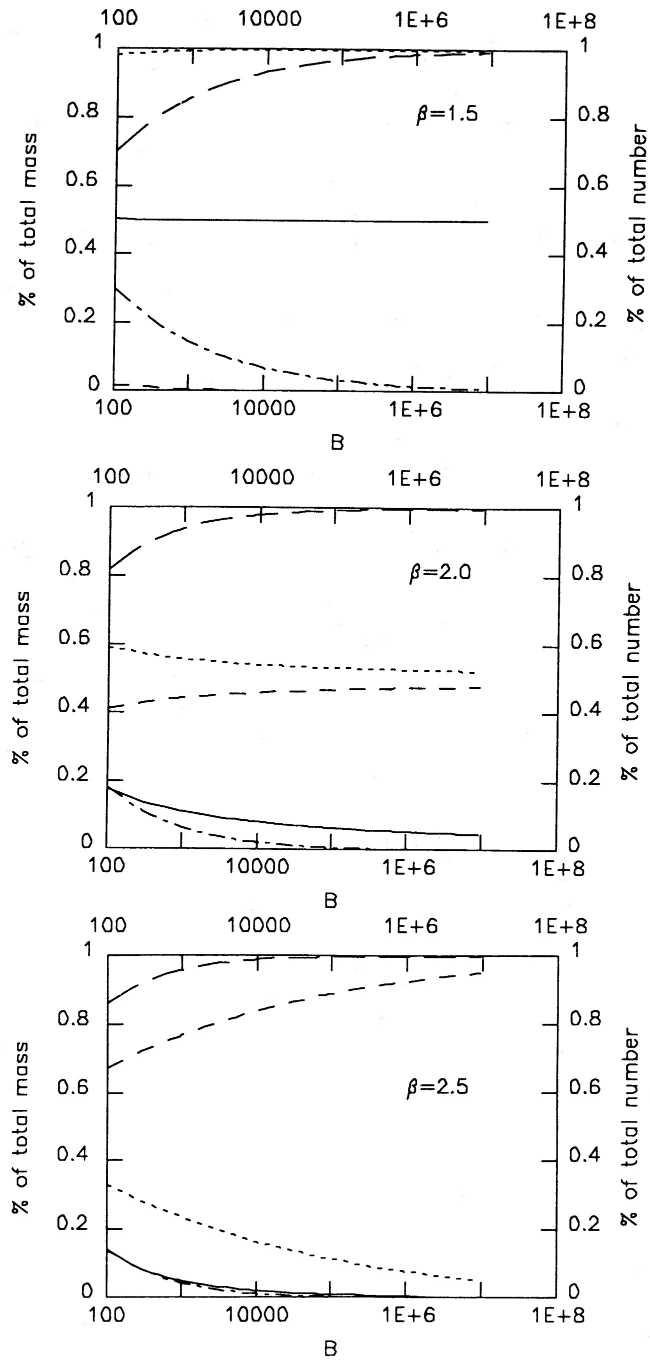

FIG. 1.—Properties of the distribution of mass inside the cluster. The value of the parameter $\mathscr{B}$ is on the x-axis, and the percentage of total cluster mass or percentage of total number of holes in the cluster is on the y-axis. The solid line represents the largest mass in the cluster, the dotted line represents mass contained in "large" masses; the dashed line represents mass contained in "small" masses. Number of "large" holes is represented by dotted-dashed line, and number of "small" holes is represented by long-dashed line. Three cases for different values of β are shown.

of small β ($\beta = 1.5$) half of the cluster mass is concentrated in the largest hole with the rest of the cluster mass distributed among a few "large" holes. The vast majority of holes are "small," but they do not contribute significantly to the cluster mass. For large β ($\beta = 2.5$) the picture is different; the largest hole contributes only marginally to the total cluster mass, and most of the cluster mass comes from "small" holes.

We will assume that the holes are segregated by mass within the cluster, that is, larger objects occupying more central locations and smaller ones extending to larger radii. For the purpose of this paper, we will take a spatially homogeneous distribution of holes of a given mass throughout the portion of the cluster up to a certain "confinement radius" $R_m(m)$. There will be no object of that mass m outside the confinement radius. The dependence of the confinement radius R_m on mass will be assumed to be of a power-law type

$$R_m(m) = R_0\left(\frac{m}{m_1}\right)^{-\kappa} , \tag{10}$$

where κ is a constant and R_0 is the radius of the cluster. If the cluster is virialized and equipartition has been achieved, it can be shown that $\kappa = \frac{1}{2}$. If this is not the case, $\kappa < \frac{1}{2}$. With no mass segregation, of course, $\kappa = 0$.

Now we are in a position to calculate the spatial mass distribution within the cluster, which we shall need later in the paper. First, we have to calculate how many holes with a given mass (between m and $m + dm$) are in the spherical shell at radius $r, r + dr$. This number is given by

$$N(m, r)dm\,dr = \frac{f(m)dm}{4\pi/3R_m^3}\, 4\pi r^2\, H(R_m - r)dr , \tag{11}$$

where H is the Heaviside function accounting for the fact that different masses extend to different radii in the cluster. Having $N(m, r)$, we can calculate the total mass of the shell $r, r + dr$ by integrating over all masses present in this shell

$$m_s(r)dr = \int_{m_1}^{m_2} mN(m, r)dr\,dm . \tag{12}$$

The next quantity we have to calculate is the mass interior to a given radius r. This mass (in solar units) is given by integrating masses of spherical shells (given by eq. [12]) from the origin up to the given radius and is used in the equation of motion to calculate the free-fall velocity of the accretion flow.

Constructing a sink function from equations (3) and (11), we can easily show that the accretion rate $\dot{M}_c(r)$ in the cluster halo is approximately constant with r. Figure 2 shows this result very clearly. The cluster radius $R_0 = 1$. As the flows of a given $\mathscr{B}$ move toward smaller radii, we see what percentage of $\dot{M}_c$ still remains. $\dot{M}_c$ is essentially constant, and the cluster acts as a sieve or filter, most of the accreting material passing through the outer layers to be accreted by the black holes in the core.

IV. RADIATION FROM THE CORE

From our analysis in § II, we conclude that in our black hole clusters, the bulk of the radiation must come from the cluster core as we have defined it in § II, where the accretion can be much more efficient than in the halo. Since we know so little about what happens in the core, we have defined it observationally, letting the variability of the given source set the limit on the core radius, instead of using it to limit the size of the entire cluster. This is obviously the step to take, given that the cluster halo contributes so little radiation. In fact, the primary result of what we have done so far in this paper is the modification of the results of Paper I so that they apply to cluster cores, instead of to the entire clusters themselves.

With one important modification the results of Paper I can be taken over and applied to cluster cores—the total mass, M_0, the number of holes N, and the radius $R_{\rm var}$ are now taken to apply to the core and not to the cluster, onto which the accretion is spherically symmetric. By the time the flow reaches the core radius $R_c = R_{\rm var}$, it is already chaotic and far from spherically symmetric, so that the inefficient accretion regimes in the halo no longer apply. Exactly what the details of accretion in the core are is unknown and constitutes a very complex problem. We assume that they are roughly equivalent to all the holes in the core radiating in the disk mode with $\dot{m} = \dot{m}_{\rm core}$ for each, as we explained above.

The important modification referred to above is that the distribution of masses in the core is not the same as that of the

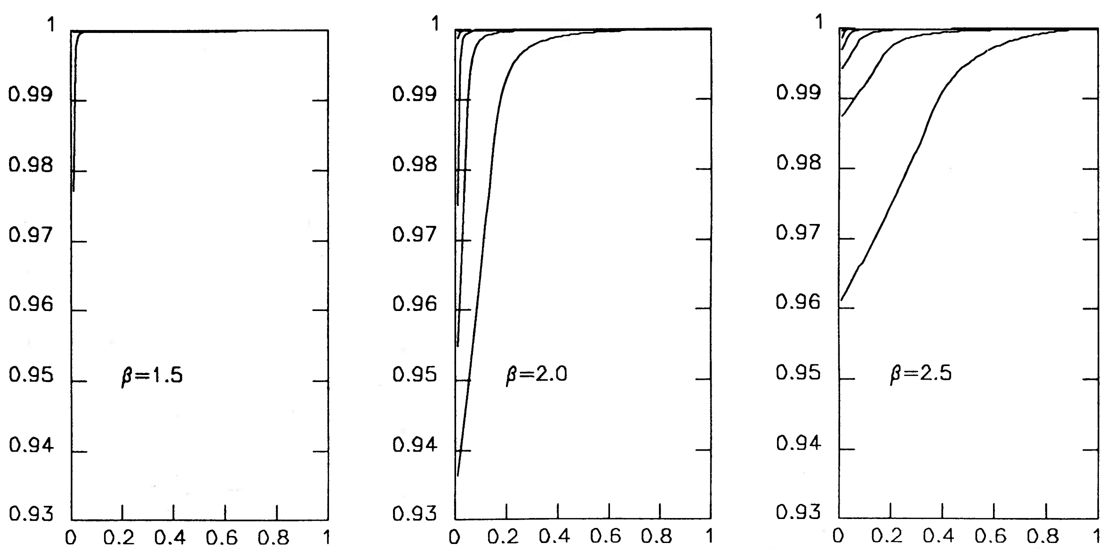

FIG. 2.—The value of accretion flow $\dot{M}_c$ in units of the flow at the radius of the cluster $\dot{M}_{c0}$ plotted against the radius in units of cluster radius. The lowest line on every figure corresponds to $\mathscr{B} = 100$, the highest corresponds to $\mathscr{B} = 10^7$. For $\beta = 1.5$ the flow is practically constant for every $\mathscr{B}$, for larger values of β the flow decreases in central parts of the cluster.

entire cluster of which it is a part. In fact, it is not a simple power law, if that for the cluster is. Obviously, too, even though the cluster halo contributes little to the overall emission from the source, it will be very important in discussing its dynamics and evolution (its lifetime). The extent of the cluster beyond the core, for example, must be taken into account. The confinement radii for black holes of different masses will also be determined by the cluster radius R_0, according to equation (10). The fact that R_0 will be larger than R_c will ensure that the cluster will have a longer lifetime than that of a cluster the size of the core. The related issue of gravitational core collapse, however, must still be addressed (Stoeger 1985).

In Paper I, with the assumptions that all the accreting disks are of the "outer regime" type (Shakura and Sunyaev 1973; Novikov and Thorne 1973), and thus the bulk of the radiation from any disk is at a frequency near that corresponding to the inner-edge temperature, we found that for a variety of Seyferts, QSOs, and BL Lacs their power-law spectra between the optical and the X-ray are fitted by black hole clusters with β between 1.8 and 2.05, and $M_{\rm tot}$ between 6×10^3 and 5×10^8 solar masses. We assumed an individual black hole specific accretion rate of $\dot{m} = 0.3$. We also found that a cluster would have a reasonable long lifetime for variability radii not too small ($R_{\rm var} > 10^{14}$ cm or 55 light-minutes). With the same assumptions and with modifications explained above, these values of β and of $M_{\rm tot}$ will now apply to "cluster cores" as we have defined them in this paper. The "velocity problem" prevents us from self-consistent description of an entire cluster with radius $R_0 = R_{\rm var}$ in this way. It is rather "the core" embedded within a larger, more extensive cluster which now models the AGNs and QSOs.

The distribution of mass in the core is obtained by integrating the number of holes with a given mass m in a spherical shell at r, given by equation (11) over r up to the core radius R_c. The result is

$$f_c(m)dm = \mathscr{B}\zeta^{-3\kappa}m_1^{-3\kappa}m^{-\beta+3\kappa}\,dm\ , \tag{13}$$

for masses smaller than $m_1\zeta$, and $\mathscr{B}m^{-\beta}$ thereafter; $\zeta = (R_0/R_c)^{1/\kappa}$.

To obtain the emission spectrum of the cluster core we must integrate over the core distribution (13) the emission from a single accretion disk associated with mass m:

$$I_{\nu d} = I_{0d}\,m^{4/3}\dot{m}^{2/3}\nu^{1/3}H(\nu_m - \nu)\ , \tag{14}$$

where

$$\nu_m = \nu_{m0}\,m^{-1/4}\dot{m}^{1/4} \tag{15}$$

(see eqs. [7] and [6a] in Paper I).

The situation is now more complex than considered in Paper I since the integral must be weighted by the core mass distribution (13) rather than the cluster mass distribution (8). We have

$$\begin{aligned} I_{\nu c} &= \int_{m_1}^{m_2} I_\nu f_c(m)dm \\ &= \int_{m_1}^{\zeta m_1} I_{\nu d}\mathscr{B}\zeta^{-3\kappa}m_1^{-3\kappa}m^{-\beta+3\kappa}\,dm + \int_{\zeta m_1}^{m_2} I_{\nu d}\mathscr{B}m^{-\beta}\,dm\ . \end{aligned} \tag{16}$$

The result of this integration is a function of frequency ν; we will represent it as function of $x = \nu/\nu_{m_1}$, where $\nu_{m_1} = \nu_m(m_1)$ (eq. [15]).

At frequencies higher than ν_{m_1} (that is, for $x > 1$) the intensity $I_{\nu c}$ vanishes. The frequency ν_{m_1} (or $x = 1$) is the upper limit to radiation from the cluster core; the smallest holes of mass m_1 contribute to the radiation at ν_{m_1}. At frequencies lower than $\nu_{m_2} = \nu_m(m_2)$, where m_2 is the upper end of the mass distribution, the spectrum is proportional to $\nu^{1/3}$; the main contribution to radiation comes from the outer portions of accretion disks surrounding the largest masses in the cluster core. In the region of frequencies between $\nu_{m_2} = \nu_m(m_2)$ and $\nu_{m_1}\zeta = \nu_m(m_1\zeta)$, that is for x between ν_{m_2}/ν_{m_1} and $\zeta^{-1/4}$, the solution $I_{\nu c}$ corresponds to the case considered in Paper I. The emitted spectrum has the form

$$\frac{I_{\nu c}}{I_{0c}} = x^{-9+4\beta} - \left(\frac{R_c}{R_0}\right)^3\left[k - (k-1)\left(\frac{R_c}{R_0}\right)^{(\beta-7/3-3\kappa)/\kappa}\right]x^{1/3}\ , \tag{17}$$

where

$$I_{0c} = I_{0d}\,\frac{\mathscr{B}}{7/3-\beta}\,\dot{m}^{3/4}m_1^{9/4-\beta}\,\nu_{m0}^{1/3} \tag{18}$$

is independent of κ, and

$$k(\beta, \kappa) = \frac{7/3-\beta}{7/3-\beta+3\kappa}\ . \tag{19}$$

When $R_c = R_0$, $\zeta = 1$, $\nu_{m_1}\zeta = \nu_{m_1}$, the core is identical with the entire cluster considered in Paper I; the frequency region in which the solution (16) is valid extends from ν_{m_2} to υ_{m_1} and is independent of mass segregation given by κ.

When $R_c < R_0$ the upper end of this frequency range, $\nu_{m_1}\zeta$, depends on mass segregation and decreases with κ. If the core is small enough for the confinement radius R_{m_2} of the largest masses to be equal to the core radius R_c, $\nu_{m_2} = \nu_{m_1}\zeta$, and the range of applicability of the solution (17) vanishes. Note that in every case the spectral index of the solution (17) does not depend on mass segregation.

At higher frequencies, in the range from $\nu_{m_1}\zeta$ to ν_{m_1} (x within $\zeta^{-1/4}$ and 1), the cluster spectrum is steeper:

$$\frac{I_{\nu c}}{I_{0c}} = k\left(\frac{R_c}{R_0}\right)^3(x^{-9+4\beta-12\kappa} - x^{1/3}) \tag{20}$$

(except when no mass segregation takes place, $\kappa = 0$). For a typical value of $\kappa = 0.5$ (for a virialized cluster with equipartition) the spectral index is $-15 + 4\beta$. This steepening of the core emission at higher frequencies (with respect to the emission from the cluster discussed in Paper I) is due to a deficiency in the number of small holes in the core. The large number of them are now located outside the core. This deficiency becomes significant for holes with masses smaller than $m_1\zeta = m_1(R_0/R_c)^{1/\kappa}$, for which the confinement radius equals the core radius and all larger masses are confined to the core.

The above described frequency spectra of cluster core emission are illustrated in Figure 3 for several values of β and R_c/R_0, and for $\kappa = 0.5$. Note the increase of the steeper region of the spectrum with decreasing core size. These features of the radiation spectra emitted by our cluster cores complicate their fitting to actual QSO and AGN source spectra. We shall postpone a detailed discussion of fitting to a subsequent paper until we have studied several other relevant issues in our models, including internal mass supply and spectrum flattening by absorption and inverse Compton scattering. Here we simply point out that, if we fit the spectra of the model, as given in

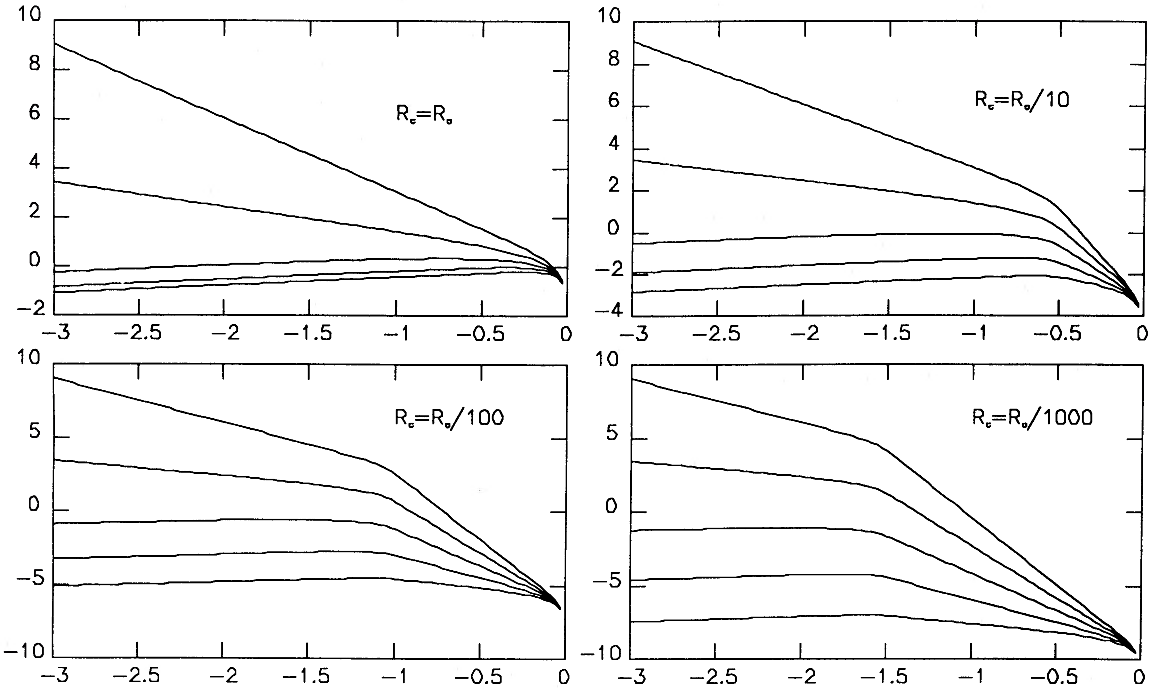

FIG. 3.—The frequency spectra of cluster core emission for several values of R_c/R_0 and β. The x-axis represents $\log(\nu/\nu_{m_1})$, the y-axis represents $\log(I_{\nu c}/I_{0c})$, and the index κ is set to be 0.5. The highest curve on every picture represents the frequency spectrum for the case $\beta = 1.5$; lower curves represents cases $\beta = 2.0$, $\beta = 2.5$, $\beta = 3.0$, and $\beta = 3.5$, respectively.

equations (17) and (20), to a single power law derived from the data, as in Paper I, we obtain a pair of values for β and $\mathscr{B}(R_c/R_0)^3$, or for β and $\mathscr{B}[(R_c^3 - R_\tau^3)/R_0^3]$ if a central optically thick component of radius R_τ is present in the core (see § V below). In the totally optically thin case, we cannot separately relate $\mathscr{B}$ and R_0, the cluster radius. This, however, is possible when a central optically thick component is present in the cluster spectrum.

V. THE OPTICALLY THICK REGIME

The discussion of the emission spectrum from the core of a cluster of accreting black holes was presented in the preceding section under the assumption that the cluster is optically thin throughout. We must relax this assumption now and consider at what radius R_τ within the cluster the accretion flow becomes optically thick and what contribution such an optically thick central component will make to the spectrum. In such situations the QSO or AGN is modeled by a single spherically symmetric component of radius R_τ plus the distribution of component black holes for $R_\tau < R < R_{\rm core} = R_{\rm var}$. It turns out that these considerations severely alter our cluster model.

There are two radii important for this optically thick region. The first, obviously, is R_τ. The second is $R_{\rm trap}$, the radius inside which radiation is swept irretrievably into the central holes by the optically thick flow. The radius R_τ is determined by the requirement that the optical depth from infinity to this radius be equal to unity:

$$\tau_s = \int_\infty^{R_\tau} \kappa_\tau \rho\, dr = \int_\infty^{R_\tau} \frac{\kappa_\tau \dot{M}_c}{4\pi r^2 v}\, dr = 1 , \tag{21}$$

where v is given by solution to the equation of motion. The integration in equation (21) will be some very complicated function of R. Instead of doing this, we shall assume for simplicity that R_τ is not much different from what it would be if all the mass of the core were placed in a single black hole. Then

$$R_\tau = \left(\frac{\dot{m}_c}{\eta}\right)^2 \left(1 - \frac{L}{L_{\rm E}}\right)^{-1} R_{\rm S} , \tag{22}$$

where η is the efficiency of the accretion process (0.06 for Schwarzschild holes in the disk mode), and $R_{\rm S}$ is the Schwarzschild radius of the core. If we take $\dot{m}_c = 0.3$, for instance, and use the relationship $L/L_{\rm E} = \dot{m}_c$ as well, then

$$R_\tau = 1.05 \times 10^7 \mathscr{B} \int_{m_1}^{m_2} m^{1-\beta}\, dm . \tag{23}$$

The trapping radius below which the mean diffusion velocity of radiation, $c/\tau_{\rm T}$, is less than free-fall velocity, due to large Thomson scattering optical depth $\tau_{\rm T}$ (see, e.g., Begelman 1978) can be shown to be approximately given by

$$R_{\rm trap} = \frac{\dot{m}_c R_{\rm S}}{2\eta} \cong 7.38 \times 10^5 \int_{m_1}^{m_2} m^{1-\beta}\, dm , \tag{24}$$

where $\dot{m}_c = 0.3$ has been assumed.

Now what we need to do is to calculate the total luminosity produced by the black holes between $R_{\rm trap}$ and R_τ, assume a spectrum on the basis of the opacity mechanism dominant there (depending on the temperature and the density), and calculate the photospheric temperature of this central optically thick component. First of all, using equations (10) and (11), L between $R_{\rm trap}$ and R_τ will be given by

$$L_{R_\tau} = \int_{R_{\rm trap}}^{R_\tau} \int_0^\infty \int_{m_1}^{m_2} \frac{\mathscr{B} m^{-\beta} I_{\nu d}\, dm\, d\nu}{(4\pi/3)(m_1/m)^{3/2} R_0^3}\, 4\pi r^2 H\!\left(\frac{m_1}{r^2} R_0^2 - m\right) dr , \tag{25}$$

where $I_{\nu d}$ is given by equation (14). Working out the integrations in equation (25) and neglecting all but the dominant contributions, we obtain for $2 < \beta < 3.5$

$$L_{R_\tau} \cong f(\beta)\mathscr{B}\dot{m}m_1^{2-\beta}\frac{R_\tau^{2\beta-4} - R_{\rm trap}^{2\beta-4}}{R_0^{2\beta-4}}, \quad (26)$$

where

$$f(\beta) = \frac{9.88 \times 10^{39}}{(23/6 - \beta)(14 - 4\beta)(2\beta - 4)(1.44)^{1.74\beta}}. \quad (27)$$

Now we assume that, in this optically thick region, $\kappa_{\rm ff} < \kappa_{\rm es}$ and $\rho = Cr^{-3/2}$, as given by $\dot{M}_c = 4\pi\rho r^2 v$, with $\dot{M}_c \simeq$ constant and velocity v approximately equal to free-fall velocity; and $C = 1.85 \times 10^2 m_r^{1/2}\dot{m}_c$. Using $\kappa_{\rm es} = 0.4$ cm^2 g^{-1} and $\kappa_{\rm ff}$ given by the formula (Zel'dovich and Shakura 1969)

$$\kappa_{\rm ff} = \frac{1.14 \times 10^{56}}{(k/h)^3 T^{7/2}\chi^3}(1 - e^{-\chi}) \text{ cm}^2 \text{ g}^{-1}, \quad \chi = \frac{h\nu}{kT}, \quad (28)$$

we can calculate the spectrum from a optically thick isothermal atmosphere

$$F_\nu = 4.59 \times 10^{-11} C^{-2/5} T^{23/10} \frac{\chi^{12/5} e^{-\chi}}{(1 - e^{-\chi})^{4/5}}, \quad (29)$$

and integrating over frequencies

$$F = \int_0^\infty F_\nu d\nu \cong 3C^{-2/5} T^{33/10}. \quad (30)$$

Using equation (30) and the relationship between L and F ($L = 4\pi R^2 F$), we have the expression for the temperature

$$T = \tfrac{1}{3} C^{4/33} R_\tau^{-20/33} L_{R_\tau}^{10/33}. \quad (31)$$

Now if we go back and look at equation (26), which depends on $\mathscr{B}_1 = \mathscr{B}[(R_\tau^{2\beta-4} - R_{\rm trap}^{2\beta-4})/R_0^{2\beta-4}]$, we can see that the data will constrain this parameter. In particular, if the spectrum (29) peaks near the optical or X-ray regions, $\mathscr{B}$ and R_0 must be such that

$$L_{R_\tau} < \int_{\nu_0}^{\nu_x} I_0 \nu^{-\alpha_{ox}} d\nu, \qquad F_{\nu_\tau} < I_0 \nu^{-\alpha_{ox}}, \quad (32)$$

and the flux from the holes outside R_τ must be fitted to the difference between $I_0 \nu^{-\alpha_{ox}}$ and F_{ν_τ}. If, on the other hand, the spectrum (29) peaks at frequencies lower than those of the optical, data from those frequencies will constrain L_{R_τ} and therefore $\mathscr{B}_1$. This constraint, together with that described in § IV, will separately determine $\mathscr{B}$ and R_0.

Taking 3C 273 as an example with the values determined in Paper I: $I_0(2500 \text{ Å}) = 2.6 \times 10^{31}$, $\alpha_{ox} = 1.22$, $\beta = 1.95$, $M_{\rm tot} = 9.1 \times 10^7$, and $R_{\rm var} = 1.1 \times 10^{15}$, let us see what sort of an optically thick component we might have. In the absence of a detailed fitting to our cluster core model, we assume $R_c/R_0 = 0.1$—that is the cluster radius $R_0 = 1.1 \times 10^{16}$ cm. We also assume that because of this, $\beta = 2.2$ and $\mathscr{B} = 1.0 \times 10^8$ (for 3C 273, $\mathscr{B}$ in Paper I was 1.26×10^7). If β goes up, $\mathscr{B}$ must also go up. Then $m_{\rm core} = 2.12 \times 10^8$ and $R_{S_{\rm core}} = 6.25 \times 10^{13}$ cm. For $\dot{m} = 0.3$ as assumed in Paper I, R_τ equals 2.23×10^{15}, which is greater than $R_{\rm var}$, and there would be no optically thin core. If we choose $\dot{m} = 0.2$, then $R_\tau = 8.68 \times 10^{14}$ and $R_{\rm trap} = 1.05 \times 10^{14}$ cm. Then, from equation (41) $L_{R_\tau} = 1.36 \times 10^{46}$ ergs s^{-1}, which is less than $L_{\rm bol}$ of 3C 273, which is greater than 1.0×10^{47} (Bassani, Dean, and Sembay 1983). Also, equation (45) gives $T = 1.38 \times 10^5$ K, corresponding to a maximum radiation frequency of $\sim 2.87 \times 10^{15}$ Hz, which is in the ultraviolet. This gives a rough idea of what we might expect, on the basis of our model, for the optically thick central component of a source like 3C 273. It also illustrates how parameters like $\dot{m}$ may be constrained by the data.

VI. SUMMARY AND CONCLUSIONS

Our discussion of the radiation from a cluster of black holes with accreting disks indicated that the outer portion of such a cluster is not an efficient emitter if the source of the accreted material is external to the cluster. The reason for this is the rather small value for the accretion radius of a hole located in the outer region ("halo"), resulting from a large relative velocities between the holes and the spherically symmetric flow of the infalling gas (the "velocity problem"). This is a fairly general conclusion which does not depend on the mass spectrum or the spatial distribution of masses or on the other details of the cluster model.

In an externally supplied cluster the bulk of the radiation, however, originates in its core, where the flow is chaotic and far from spherically symmetric, and where the infalling matter is apportioned among all the objects in the core. As shown in § IV, the resulting spectrum of the core radiation will be the same as that of the corresponding full cluster model (Paper I) for lower frequencies, but steeper for higher frequencies (X-rays), depending on the model (see Fig. 3). The smaller R_c is relative to R_0, the steeper the high-frequency spectrum will be and the more it will extend into lower frequencies. This results from the diminished contributions of the smaller mass holes, the majority of which are located in the halo and therefore accrete at a lower rate due to the velocity problem discussed in § II.

Relying on the results of § II, we can see that a limiting case (with respect to the index β of the power-law distribution and corresponding to a small β) of a cluster of N small (say, $m = 1$) holes accreting in a "halo" mode, and surrounding one large, massive central hole acting as a core can also plausibly model some of the AGNs. The small holes accrete at a critical rate only during a time interval Δt on their orbits (§ II). Because $\Delta t/T$ does not depend on parameters of the orbit we can say that, on average $(\Delta t/T)f(m)$ holes of mass m radiate critically at any given time. The spectrum from the entire cluster is

$$I_{\nu\,\rm cluster} = 10^{14} m_{\rm core}^{4/3} \nu^{1/3} H(\nu_{\rm core} - \nu) + N\frac{\Delta t}{T} 10^{14} m_1^{4/3} \nu^{1/3} H(\nu_1 - \nu), \quad (33)$$

where $m_{\rm core}$ is the mass of core, $\cong 10^8$, $\nu_{\rm core}$ is the maximum frequency emitted from the core, $\cong 3 \times 10^{15}$ Hz; N is the number of small holes; m_1 is the mass of a small hole $\cong 1$; and ν_1 is the maximum frequency emitted from small hole $\cong 3 \times 10^{17}$ Hz. In this calculation $\Delta t/T$ is a constant $\cong 0.004$. Substituting all these values into equation (33) yields a spectrum which varies as $\nu^{1/3}$ with a discontinuity at $\nu = \nu_{\rm core} \cong 3 \times 10^{15}$ Hz. The value of the jump at the discontinuity depends on the number of small holes N. Observations indicate a typical value of the jump of the order of 10^2–10^3 yielding $N \cong 10^{11}$–10^{10} and the total cluster mass of the order of 10^{11}–10^{10} solar masses.

The accretion radius of a small hole can be expressed in the form:

$$R_A = \frac{1}{\epsilon^2\delta^2}\frac{m}{m_0} r, \quad (34)$$

where $\epsilon\delta$ is the ratio of the relative velocity of the hole with respect to the fluid to the free-fall velocity; R_A depends on the position r where critical accretion takes place, which in turn depends on L/L_{crit}. For $L/L_{crit} \cong 0.1$ critical accretion takes place at $r \cong 0.1R_0$, and for $L/L_{crit} \cong 0.9$ at $r \cong R_0$. For a hole with $m = 1$ in either case R_A is of the order of $10^{-3}R_0$. At any given moment there are $\sim 0.004N$ holes accreting critically, from a volume of accretion $\cong 0.004NR_A^3 \simeq 4 \times 10^{-12}NR_0^3$. For $N \cong 10^8$ the accretion volume is smaller than the cluster volume by a factor of 4×10^{-4}, but for $N \cong 10^{11}$ the accretion volume is $\sim 40\%$ of the cluster volume. We see that such a cluster of small holes accreting in the "halo" regime can effectively consume much of the infalling gas, the velocity problem notwithstanding.

Another result of critical importance to these cluster core models is that in many cases there will be a radius R_τ at which the accretion flow becomes optically thick to the outside observer. The model must then be modified to include a central spherically symmetric source of radius R_τ, emitting a total luminosity L_{R_τ} with a spectrum F_{ν_τ} at a photospheric temperature T_{s_τ}. This makes data fitting more complicated, but it also ensures that the model is well constrained by the data, as pointed out in § V. There we also showed how L_{R_τ} is determined and derived an expression for F_{ν_τ}, for a $\rho = Cr^{-3/2}$ isothermal atmosphere with $\kappa_{ff} < \kappa_{es}$. This will usually be the opacity in these situations and, as we pointed out in § II, the flow will be isothermal, or nearly so, throughout the cluster. With L_{R_τ} and F_{ν_τ}, we then derived T_{s_τ}, and worked out what we would expect for 3C 273. We found that for $\dot{m}_c = 0.2$ we would have an optically thick central component radiating greater than 1.0×10^{46} ergs s^{-1}, peaking in the ultraviolet and having $R_\tau = 9 \times 10^{14}$ cm. In fact, it seems likely that these models will often yield a optically thick, UV-radiating central component. This is interesting, since many QSOs and AGNs are strong UV emitters and our models situate the site for some of this emission (more UV will come from the accretion disk at $R > R_\tau$) in a central component, inside the dominant source of the X-rays.

Recently Guilbert and Rees (1988) pointed out that in QSOs and AGNs there should, generally speaking, be a significant amount of cold, optically thick material in or near the core. The presence of this material may help to explain, through the reprocessing of hard radiation, some of the perturbations on the pure power-law spectrum which is characteristic of many of these sources. They are concerned almost exclusively with models in which the bulk of the X-rays and γ-rays are produced by nonthermal mechanisms very close to the core. This is different from our cluster models, principally because the bulk of harder radiation in the cluster will be produced either by black hole accretion disks or by shocks (Paper II), away from the central part of the core. This is especially true if there is a central optically thick component as described in § V. However, Guilbert and Rees also clearly remark, in agreement with our analysis, that for objects having $L > 0.1L_{crit}$ an optically thick central region is inevitable, irrespective of the nonthermal mechanisms, except where AGNs are powered by electromagnetic extraction of the central black hole's spin energy.

Finally, it is important to point out that the principal features of the radiation from the externally supplied cluster of black holes are not significantly altered by the details of the model of the accretion disks surrounding the individual holes (Shakura and Sunyaev 1973; Novikov and Thorne 1973). More detailed discussion of this point will be included in a future paper.

Our primary concern in future papers of this series will be to go beyond the paradigm of an externally supplied cluster and examine the two most important scenarios for internal supply. The first of these is tidal disruption of stars within the cluster by individual black holes, and the second is the transfer of gas from stars to the black holes to which they are bound (in binary or multiple systems), leading to a multiplicity of X-ray sources similar to those observed in our own Galaxy. Thus, we shall describe our cluster as containing both stars and black holes, which obey the same power-law mass scales, from 0.1 to 2 solar masses dominated by the stars, and from 2 solar mass and above dominated by the black holes. For the scenario relying on tidal disruption, we shall estimate the amount of material liberated by this mechanism and then determine the efficiency of the accretion process at various radii within the cluster. Such an estimation has been recently attempted by Hagio and Yokoyama (1988) using a cluster of stars and 10^4 solar mass black holes in which tidal disruption is the major source of material supply and where the radiation spectrum reflects the spectrum of accretion rates onto the individual holes, rather than their mass spectrum. Again, as in the case of an externally supplied cluster, the "velocity problem" will dominate the "halo," and its radiative contribution will be far smaller than that of the core. For the scenario dominated by "binary systems" we shall estimate the number of black holes of various sizes which are bound to at least one star which supplies substantial material and then determine the characteristics of a cluster of such systems which would model a QSO or an AGN. In this case the "velocity problem" will not dominate our discussion of the "halo," as a large percentage of the mass lost by a star bound to a black hole will be accreted by the black hole.

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A. G. Pacholczyk: Steward Observatory, University of Arizona, Tucson, AZ 85721

T. F. Stepinski: University of Arizona, Space Sciences Building, Tucson, AZ 85721

W. R. Stoeger, S. J.: Vatican Observatory Research Group, Steward Observatory, University of Arizona, Tucson, AZ 85721