

## A MAP-BASED DETERMINATION OF THE NATURE OF BETA DELPHINI

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### ABSTRACT

The Beta Delphini binary system presents a stringent test of the theory of stellar evolution. Improved parallax and component masses are found for its giant (F5 III and F5 IV) stars. A study of the evolutionary status of the system indicates it to be 1.9 Gyr (1.9 billion years) old and to have a metallicity of approximately 1.5 times that of the Sun. The perturbation due to the 26.6 yr orbital motion is clearly shown in this 2.2 yr study and allows the most precise determination of the relative masses of the component stars to date. The next few months present an unusual opportunity for orbital study as the system passes through periastron. Two of the reference stars are found to have distances of less than 100 parsecs.

*Subject headings:* stars: binaries — stars: individual ( $\beta$  Del) — stars visual multiples

### I. BACKGROUND

Consisting of an F5 III and an F5 IV stars (Edwards 1976), with a well-determined orbit (Finsen and Worley 1970),  $\beta$  Del affords an unusual opportunity to obtain data on the post-main-sequence evolution of stars. In his discussion of the mass-luminosity relationship, Heintz (1978) lists several binary stars that contain luminosity class III giant stars, but only one for which the listed values do not depend upon a dynamical parallax,  $\beta$  Del. However, the accepted masses for this system, which is beyond the effective range of previous parallax techniques, have been called into question by van de Kamp (1954) (who performed the original study in 1938) and Underhill (1963).

Discovered by Burnham in 1873, the components of  $\beta$  Del have been followed through more than four full orbits and are still observed regularly by both visual (Worley 1988) and interferometric techniques (McAlister 1988). High-quality visual orbits have been published by Aitken (1932) (computed in 1912), Finsen (1938), and Couteau (1962).

The single-line system also reveals a radial velocity semi-amplitude of  $3.6 \text{ km s}^{-1}$  (Underhill 1963), and both Underhill and Abt and Levy (1976) have estimated the mass function. Underhill points out that the accepted orbit and parallax yield an overmassive system and calls for continued observation. Her 12 yr of observations show a clear rate of change in the radial velocity from which she estimates a ratio (mass 1/mass 2) of 3.63. Abt and Levy began their observations 11 yr later covering the next 4 yr. Combining their observations with those of Underhill they find a mass function, which when combined with the Couteau orbit, yields a ratio of 2.13 to 1. However, Abt and Levy point out that additional spectroscopic observations are needed for a definitive spectroscopic orbit.

Studies of the composite spectrum of the system, by Christy and Walker (1969) and by Edwards (1976), yield very similar MK classifications for the components. Both find the primary to be of luminosity class III and the secondary to be of class IV. However, Christy and Walker assign a spectral class of F6 to

the primary and secondary stars, while the later Edwards study assigns an F5 spectral type to both stars.

Hauck (1986) points out that  $\beta$  Del exhibits an enhanced value of  $\Delta m_2$ , and that this may indicate an enhanced [Fe/H] value. We will return to this below. Olsen (1980) classifies the spectra of the brightest reference star in this region, AO 723 = BD 14°4367, as “Am/p, g/cF5.”

Because of its obvious importance, several observatories have attempted to determine the parallax of the system with somewhat discordant results, Table 1. Van de Kamp (1938) computed the only previous astrometric mass ratio known to us. However, the result is not in accord with that found here, or by Underhill, or by Abt and Levy. In a later recalculation, using a revised parallax, van de Kamp (1954) obtained a total mass of  $4.77 M_{\odot}$  (with a standard error we calculate to be approximately  $\pm 70\%$ ) and a mass function (mass B/total mass) of 0.538 with a formal error of approximately 0.024. Thus van de Kamp finds B to be more massive than A. He rates his results for this binary as among those of lowest astrometric quality and calls for better determinations of all the observed quantities and “above all to work for substantially more accurate parallax determinations.”

### II. METHOD AND DATA

While the general characteristics of main-sequence stars are fairly well understood, there are little data for giant stars (Heintz 1978). However, there are a number of binary star systems, with at least one giant member and with well-determined orbits (Finsen and Worley 1970) that lie beyond the distance to which previous astrometric studies were effective. The recently developed multichannel astrometric detector (MAP) (Gatewood 1987) and new optical system of the Thaw refractor (Gatewood *et al.* 1986) of the Allegheny Observatory combine to give that instrument a precision of several thousandths of an arcsec per observation (Gatewood *et al.* 1988; Gatewood 1989). Thus the new instrumentation may be used to measure the trigonometric parallax of such binaries and

TABLE 1  
PARALLAX DETERMINATIONS OF  $\beta$  DEL

Observatory	Parallax	Standard Error
Allegheny .....	0 <sup>o</sup> 020	0 <sup>o</sup> 010
Allegheny/MAP .....	0.0345	0.0017
McCormick .....	0.008 <sup>a</sup>	0.012
Mount Wilson .....	0.037	0.012
Sproul .....	0.026	0.016
Yerkes .....	0.043	0.009

<sup>a</sup> Value excluded from calculations.

Adopted absolute parallax =  $0.0344 \pm 0.0016$ .

Table 1 includes all of the parallax determinations of  $\beta$  Del. The first parallax determination (Beardsley *et al.* 1971) has been corrected to absolute, in accordance with the magnitude of the reference stars and the galactic latitude of the region (van Altena 1974), by the addition of 0<sup>o</sup>003. Of the listed parallaxes, only the Allegheny/MAP (from the present study) and Sproul values were calculated with the effects of the orbital motion taken into account. The McCormick value is clearly discordant. Excluding the McCormick value, the weighted mean of the photographically determined parallaxes is 0<sup>o</sup>0328, one standard error below the value obtained with the MAP. The adopted value is derived using the indicated weights.

It should be noted that modern photographic parallaxes generally have standard errors of 0<sup>o</sup>004 to 0<sup>o</sup>006 and that those quoted above were measured and computed by older techniques than those generally in use today.

therefore to determine the masses of a class of stars for which knowledge is limited.

Observation of this region with the MAP was started in the summer of 1986, shortly after the installation of the new objective lens and continued through 1988. Altogether 36 MAP observations were obtained in just over 2 yr.

The reduction of the photometric-phase measurements of the MAP to astrometric positions has been described by Gatewood (1987), and the transformation of these to the star con-

stants listed below is similar to that described by Eichhorn and Jefferys (1971) and by Gatewood (1972), except that equations (2) of the latter document are used instead of equations (7). This latter phase of the reductions is known as the central overlap technique (Gatewood and Russell 1974; Murray 1983; Eichhorn 1988).

Photographic observations of the region include a 1934 plate taken with the Thaw refractor's 76 cm photographic objective and one acquired with the new red-light objective lens in late 1987. All of the stars listed in Table 2 were measured on these plates using the Observatory's new PDS (Theiss) measuring machine (glass scales presently define the machine metric, the laser interferometer not yet being available), and the data from the early plate were used to improve the proper motion determination for  $\beta$  Del.

### III. ASTROMETRIC RESULTS

Table 2 is a continuation of a series of parallaxes now under observation with the Thaw refractor/MAP system. In that series each reference star is subject to the same analysis as the target object. This approach provides the information necessary for the reduction (and future improvement in that reduction) of the relative parallaxes to absolute distances. The positions and motions resulting from the current study are listed in the last four columns of Table 2, over their standard errors. The system of the positions and motions is that of the AGK3 but are for the epoch and equinox J2000. The errors are given in the units of the parameter to which they pertain and are strictly internal at the epoch 1987.0. They do not include allowance for the zero-point or orientational uncertainty of the reference system's positions or motions.

Photometric data listed in Table 2, for all but the three brightest reference stars, are preliminary and are from a paper by Castelaz and Persinger (1989). Castelaz and Persinger give the *UBVIR* and DDO magnitudes for the reference stars in 10

TABLE 2  
ASTROMETRIC PARAMETERS FOR STARS IN THE REGION OF  $\beta$  DELPHINI

AO No.	d	V (mag)	B - V	abs $\pi$ (mas)	R.A. (2000)	PM (r.a.)	Decl. (2000)	PM (Decl.)
719.....	2	8.55	0.38	7.8	20 <sup>h</sup> 36 <sup>m</sup> 36 <sup>s</sup> 38463	-0 <sup>o</sup> 000982	14 <sup>o</sup> 37' 5 <sup>o</sup> 8854	-0 <sup>o</sup> 00774
				1.1	0.00009	0.000081	0.0012	0.00111
720.....	2	10.30	1.05	1.8	20 36 43.75056	-0.001590	14 29 21.9525	-0.02655
				1.3	0.00013	0.000119	0.0020	0.00170
721.....	2	8.68	0.61	10.5	20 36 45.08329	-0.001640	14 42 58.0160	-0.04266
				1.0	0.00008	0.000072	0.0011	0.00103
722.....	2	10.08	0.96	3.1	20 36 57.31096	-0.000518	14 31 35.2324	-0.00661
				1.6	0.00012	0.000114	0.0017	0.00158
723.....	2	8.28	0.46	3.6	20 37 3.73420	0.001659	14 51 12.8047	0.02475
				0.9	0.00006	0.000060	0.0009	0.00084
724.....	2	9.43	0.77	2.3	20 37 29.04020	0.000673	14 46 17.9387	0.00724
				0.9	0.00007	0.000064	0.0010	0.00089
725.....	2	3.58	0.49	34.5	20 37 32.86735	0.007214	14 35 42.6440	-0.03349
				1.7	0.00011	0.000095	0.0015	0.00132
726.....	2	10.20	0.04	0.3	20 38 2.95909	0.000095	14 22 50.5580	-0.00501
				1.7	0.00014	0.000134	0.0019	0.00176
727.....	2	10.59	0.86	1.7	20 38 33.06867	0.000685	14 50 47.0813	0.01967
				1.5	0.00011	0.000104	0.0016	0.00146
728.....	2	9.97	0.46	11.5	20 39 2.54717	0.000102	14 34 28.4333	0.00815
				1.1	0.00009	0.000081	0.0012	0.00111

Column "d" denotes the device used to gather the astrometric data. Several different devices are used in the Allegheny Observatory program. A "2" indicates that the data was obtained with the MAP.

The absolute parallaxes listed above were obtained by adding an adjustment to absolute of 5.2 milliarcseconds. All standard errors, for example those of the positions, are strictly internal and do not allow for the zero point errors of the reference system. The precession for +50 yr, at the target object, is 2.338 minutes of time in R.A. and 10<sup>o</sup>53 in Decl.

of the regions now under study with the Thaw/MAP and derived photometric spectral classes for the observed stars. Values listed to one decimal point were estimated from blue and red bandpass plates and have an estimated error of  $\pm 0.1$  magnitudes.

While parallax was included in the last two iterations, no constraints were placed on their weighted mean. Thus the system of equations converged on the relative, not the absolute, parallaxes. For the mean magnitude and galactic latitude of the region, van Altena (1974) predicts an adjustment of  $+0''.0056$  for the reduction of the relative parallaxes to absolute.

As pointed out by van Altena, the most reliable way to determine the difference between the relative and the absolute parallaxes is to determine the mean spectroscopic parallax of each reference star and average the differences. The spectra of four of the reference stars were sufficiently exposed (and not overlapped) on a plate acquired with the Case 24 inch (61 cm) Schmidt telescope, using the  $10^\circ$  objective prism, to allow Stephenson to determine their spectral classification. The spectral dispersion provided by the  $10^\circ$  prism is  $108 \text{ \AA mm}^{-1}$  at H, high enough to allow reasonably accurate spectral classification on the MK system, as discussed by Bidelman (1966). Spectral plates for approximately 100 MK spectral-type standard stars were available.

The spectra of AO 721 was overlapped with that of two other stars enough to make the luminosity class, either a G5 IV or V, rather uncertain. Castelaz and Persinger agree on the temperature of the star and express the same uncertainty about the luminosity class. Since the value adopted here, G5 IV, leads to a spectroscopic parallax substantially smaller than that measured, it is possible that the object is in fact on the evolutionary track from a V to a IV.

The image of AO 724 was underexposed and yielded an uncertain G8 IV. Castelaz and Persinger assign a value of G8 III which is adopted here. AO 719 and 723 were not measured by Castelaz and Persinger, but they were by Stephenson. He lists them as F5 V and F5 II respectively. As noted above, Olsen lists the latter as an Am/p, g/cF5. The parallax derived here suggests that the F5 II calibration may in fact be too bright.

That there seems to be little or no color excess in this region is in general agreement with the results of Burstein and Heiles (1982). We have adopted the absolute magnitudes derived by

interpolating from the tabular values listed in *Astrophysical Quantities* (Allen 1973), and we have not included absorption in the derivation of any of the spectroscopic parallaxes.

Column (3) of Table 3 lists the adopted spectral types for the nine reference stars in this region. Unless discussed above, these are from Castelaz and Persinger. Column (4) lists the absolute parallaxes predicted for these stars. Column (5) gives the observed relative parallaxes, and the individual estimates of the value necessary to adjust the relative parallaxes to absolute are listed in column (6). The average of column (6) is 5.20 mas (in excellent agreement with that estimated by van Altena) with a standard error of one measure of 2.02 mas. The residuals about this value are listed in column (7).

There are of course two sources of the residuals listed in the last column, that of the relative parallaxes and that of the spectroscopic parallaxes. The average estimated error given in Table 2 for the nine reference stars listed in Table 3 is 1.23 mas. This implies that the spectroscopic parallaxes have an average error of approximately 1.6 mas or approximately 31% of their average value. This is very close to the value predicted by Castelaz (1989) and indicates that the parallax errors given in Table 2 are in fact good approximations of their external errors (Gatewood 1989).

Without the discordant McCormick measurement, the mean of the previous parallax determinations (Table 1) for  $\beta$  Del ( $0.0328 \pm 0''.0055$ ) is well within its error of the value determined here. Combining this with the present result, the weighted mean of all studies to date is  $0.0344 \pm 0''.0016$ . This parallax and the apparent magnitudes of the components yield absolute visual magnitudes of 1.64 and 2.60 with standard errors of 0.1 mag, respectively. These values are in good agreement with the accepted luminosity characteristics of the two components.

To illustrate the strength of the photocentric perturbation caused by the orbital motion of this system, we have plotted the residuals to a least-squares adjustment in Figure 1. The combined two axis adjustment included (1) position; (2) proper motion; and (3) parallax. It did not include a term for the photocentric semimajor axis. Because of the current direction of the orbital motion, almost all of the effect is in declination. Figure 2 is a plot of the residuals of a dual axis adjustment allowing for the above motions and that suggested by the Cousteau orbit. Apparently the astrometric perturbation is satisfied by this orbit.

TABLE 3  
SPECTROSCOPIC AND RELATIVE PARALLAXES

AO No. (1)	BD No. (2)	Adopted Spectral Type (3)	Spectroscopic Parallax (mas) (4)	Relative Parallax (mas) (5)	Adjustment to Absolute (mas) (6)	Residual (mas) (7)
719.....	14°4359	F5 V	10.5	2.6	7.9	2.7
720.....		K3 III	0.9	-3.4	4.3	-0.9
721.....	14°4362	G5 IV	8.7	5.3	3.5	-1.7
722.....	14°4366	K1-2 III	1.0	-2.1	3.1	-2.1
723.....	14°4367	F5 II	0.9	-1.6	2.5	-2.7
724.....	14°4368	G8 III	1.8	-2.9	4.7	-0.5
726.....	13°4472	A8 V	2.7	-4.9	7.6	2.4
727.....		K3 IV	3.3	-3.5	6.8	1.6
728.....	14°4375	G7 V	12.7	6.3	6.4	1.2

Average adjustment to absolute = 5.20 mas.  
Standard deviation one comparison = 2.02 mas.  
Standard error of the mean = 0.67 mas.

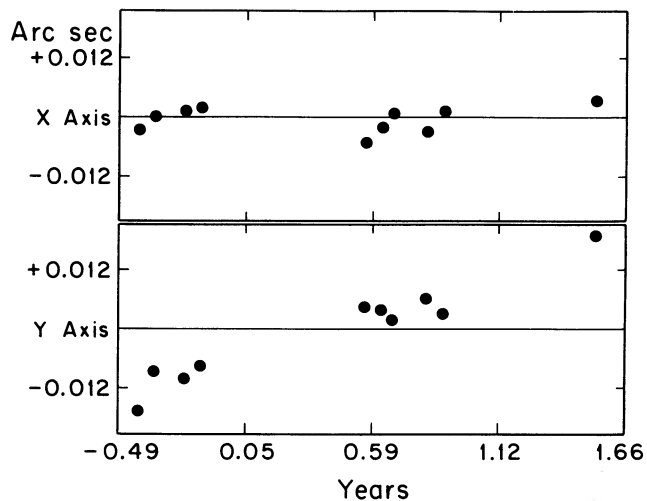


FIG. 1

FIG. 1.—Above are shown the MAP residuals for  $\beta$  Del (AO 725). We have plotted normal points of four observations each to illustrate more clearly the orbital motion of the photocenter.

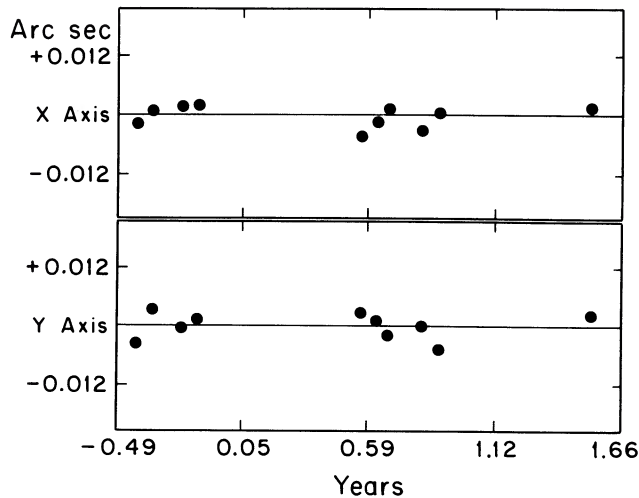


FIG. 2

FIG. 2.—The normal points illustrate the residual motion of the photo-center, after correction for the motion implied by the Cousteau orbit.

Using the weighted mean parallax from Table 1 and adopting the Cousteau orbit, we find a total mass of  $3.71 \pm 0.57$  solar masses for the  $\beta$  Del system, Table 4. The individual masses are  $2.11 \pm 0.33$  and  $1.60 \pm 0.25 M_{\odot}$  for the A and B components, respectively.

VI. AGE AND EVOLUTIONARY STATUS OF  $\beta$  DEL

The  $\beta$  Del binary system presents a stringent test of the theory of stellar evolution. Since the two stars in the system must have a common origin, it is reasonable to assume that they both have the same age and chemical composition. If theory is correct, the two stars must then satisfy two different sets of constraints simultaneously. One set of constraints relates to their luminosities and surface temperatures, the other relates to their masses. In other words, the two components A and B of  $\beta$  Del must (1) when plotted in the HR-Diagram, both lie on the same theoretical isochrone, compatible with their chemical composition; (2) have orbital masses which both agree, within the error of observations, with the masses predicted for their position on the theoretical isochrone.

TABLE 4  
THE BETA DELPHINI SYSTEM

Parameter	Value	Standard Error
$M_v$ A .....	1.64	$\pm 0.10$
$M_v$ B .....	2.60	0.10
Total mass .....	3.71	0.57
$\beta$ .....	0.292	0.108
Mass function .....	0.431	0.012
Mass A .....	2.11	0.33
Mass B .....	1.60	0.25

All masses are in solar units.

In the discussion below, we compare the observations to the revised Yale isochrones (RYI; Green, Demarque, and King 1987), and we show that the age and evolutionary status of the  $\beta$  Del system can be derived from the newly observed parameters in a manner that is consistent with both tests (1) and (2). Our analysis also shows that a fully consistent interpretation of the  $\beta$  Del system would not have been possible on the basis of the previous data and emphasizes the need for high-quality parallaxes and orbital data for binary stars if a sufficiently rigorous test of the theory of stellar evolution is to be made.

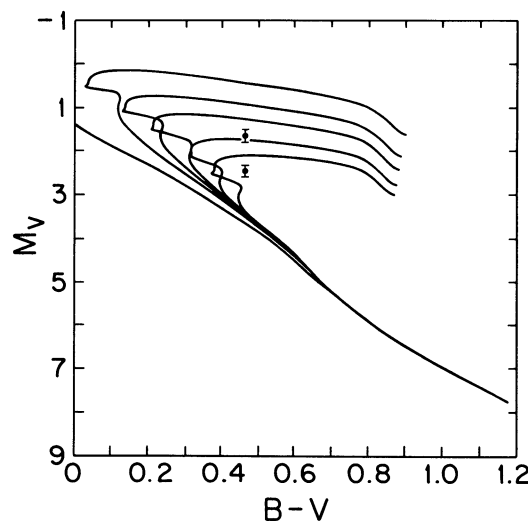


FIG. 3.—Theoretical isochrones are shown for solar metallicity. From the top of the figure the isochrones are for 0.75, 1.0, 1.5, 2.0, 2.5, and 0.0 (ZAMS) Gyr. Positioning along the isochrones is based upon luminosity and temperature. The masses derived from this fit are then compared with those derived from the orbital motion of the system.

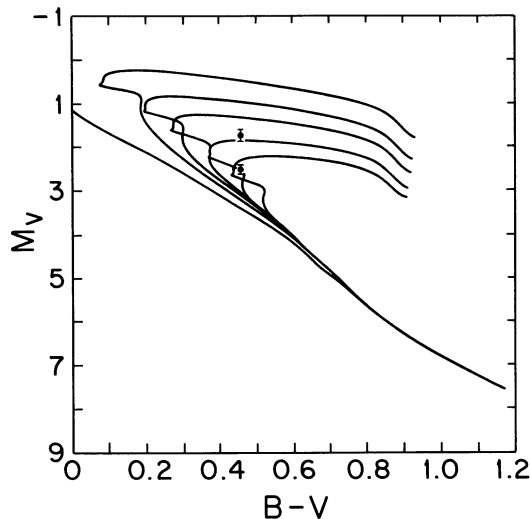


FIG. 4

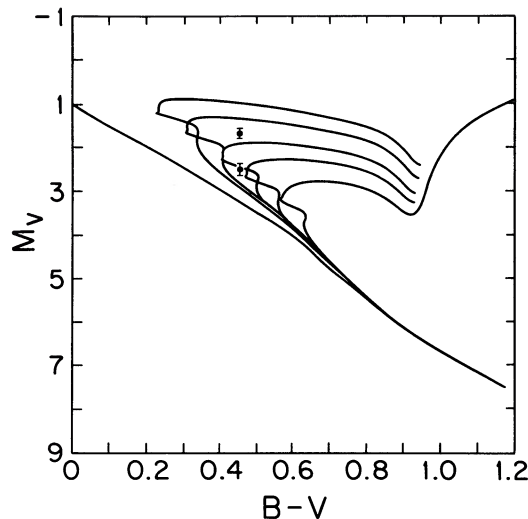


FIG. 5

FIG. 4.—Theoretical isochrones are shown for 1.5 times solar metallicity. The derived parallax and photometry yield points for each component that is within its error of the 2.0 Gyr isochrone. The masses predicted by the model for these values are within approximately  $1\sigma$  of the values found from the orbit and perturbation.

FIG. 5.—Theoretical isochrones are shown for 2.0 times solar metallicity

In Table 5, we list the spectral classes of  $\beta$  Del A and B, the masses and absolute magnitudes based on the old parallax listed by Heintz (1978), and the new masses and absolute magnitudes from this study. The previous values (van de Kamp 1954) of the masses and luminosities derived from astrometric techniques are impossible to fit with one isochrone, regardless of the range of the parameters. The most severe problem is provided by the value of the masses, in particular by the fact that the secondary is found to be more massive than the primary.

Figures 3, 4, and 5 illustrate the fit of the data from the MAP to theoretical isochrones for  $Z = 1.0 Z_{\odot}$ ,  $Z = 1.5 Z_{\odot}$ , and  $Z = 2.0 Z_{\odot}$ , where  $Z_{\odot} = 0.02$ . We find the agreement in the H-R diagram, although not unreasonable in any of the three cases, to be much better for the model with  $Z = 1.5 Z_{\odot}$ . For these parameters, the age of  $\beta$  Del is approximately 1.9 Gyr, and  $M_A = 1.7$  and  $M_B = 1.5 M_{\odot}$ . The latter are somewhat less than the values found here, but the total mass and that of each component is within approximately  $1\sigma$  of the astrometrically determined values.

Uncertainties in the extent of overmixing beyond the edge of the convective core are well known to affect sensitively the shape of the isochrone and the luminosity function at this phase of evolution (Prather and Demarque 1974). Work now

in progress (Sofia *et al.* 1987), which incorporates the effects of internal rotation in stellar evolution following the approach of Endal and Sofia (1981), will provide the means of improving theoretical models to the point of making possible much more detailed comparisons with observations in all phases of stellar evolution near the main sequence. Other improvements in the physics of stellar interiors, such as in the equation of state and opacities, are also in progress.

The  $\beta$  Del system will pass through periastron some time in 1989, an event that means most of those who track its orbit. But because those efforts will undoubtedly lead to further analysis, the authors encourage the observation or reobservation of all of the system's characteristics.

The development and implementation of the MAP and the refurbishment of the scientific facilities and plant of the Allegheny Observatory, which made this paper possible, have extended over a period of more than a decade. Both the National Science Foundation, most recently through grant AST-8617642, and the National Aeronautics and Space Administration, through grant NAG 253, have supported the effort continuously throughout this period. Some of the references used in this study were derived from SIMBAD, database of the Strasbourg, France, Astronomical Data Center.

Lee A. Breakiron, now with the US Naval Observatory, was instrumental in the choice of, and preparation of, this region for observation. The senior author's efforts benefited from discussions with his teacher, Joost Kiewiet de Jonge, and from the assistance of his student, Inwoo Han. Timothy Persinger, James Prosser, and Thomas Reiland were instrumental in the development of the equipment and the acquisition and reduction of the data. Some of the observers who helped to bring success amidst the annoyances which emanate from telescopic equipment under development were J. Bradburn and S. Goughnour and D. Burkhard, T. Finnegan, R. Poole, A. Keeley, J. Kirby, L. Large, T. McLaughlin, D. Peart, P. Sasson, and T. Worek.

TABLE 5  
BETA DELPHINI HR 7882

STUDY	ABSOLUTE MAGNITUDE		ORBITAL MASS		ISOCHRONE MASS (1.5)	
	$M_V$ A	$M_V$ B	A	B	A	B
Old .....	1.47	2.37	2.2	2.6	1.93	1.75
New .....	1.64	2.60	2.11	1.60	1.70	1.51
	$\pm 0.1$	$\pm 0.1$	$\pm 0.33$	$\pm 0.25$		

All masses and metallicities are in solar units.

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