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ON THE INTERRELATION BETWEEN THE SURFACE PHOTOMETRIC PARAMETERS AND THE INTERNAL VELOCITIES OF GALAXIES

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ABSTRACT

The interrelations between the surface photometric parameters (luminosity, diameter, surface brightness, concentration index) and the internal velocities (velocity dispersion, rotational velocity) are explored by means of the principal component analysis and the regression line analysis of homogeneous data for 18 elliptical and 28 spiral galaxies of moderate inclinations ($30° \lesssim i \lesssim 65°$) in the Virgo Cluster. We find extremely tight correlations (| correlation coefficients | > 0.98), *both for ellipticals and spirals*, between the visual magnitude V_{26} and a combined parameter $\log w' \equiv \log v D_{26}{}^2$ with v and D_{26} being the internal velocity and the isophotal diameter of galaxies. We infer the physical meaning of $w \equiv (Gw')^{-1}$ to be a sort of phase-space density related to the basic structure of galaxies, and briefly describe its empirical behavior in the diameter versus surface brightness diagram.

Subject headings: galaxies: internal motions — galaxies: photometry

I. INTRODUCTION

The empirical correlations between the internal velocity and the luminosity of galaxies, i.e., the Faber-Jackson relation for ellipticals (Faber and Jackson 1976) and the Tully-Fisher relation for spirals (Tully and Fisher 1977), are widely accepted and applied for the determination of the photometric distances of galaxies (cf. Dressler 1984; Aaronson *et al.* 1986). Recently Dressler *et al.* (1987) found a tight empirical correlation for elliptical galaxies between the central velocity dispersion and the diameter D_n which encloses an integrated surface brightness of a prescribed level ($\mu_B = 20.75$), and demonstrated the applicability of this correlation to the distance estimates, in place of the Faber-Jackson relation. This $D_n - \sigma$ relation was extended to the bulges of disk galaxies by Dressler (1987), to replace the Tully-Fisher relation for galaxies which can be observationally resolved into the bulge and the disk components. As Aaronson, Huchra, and Mould (1979), Dressler (1987), and others suggested, the physical basis of such correlations as above seems to be no more than an expression of the virial theorem in the limit of constant mass-to-light ratio for an isolated self-gravitating stellar system, but the ultimate reasons remain yet unclear.

In the following we are going to reinvestigate the interrelations between the surface photometric parameters and the internal velocity of galaxies, using the homogeneous data set for the Virgo Cluster galaxies, in an effort to elucidate the physical nature of the correlations, rather than to develop an operating method for getting the distances of galaxies.

II. SAMPLE GALAXIES AND DATA SOURCES

The basic sample in the present study are those galaxies in the Virgo Cluster for which homogeneous surface photometric data were published by Watanabe (1983) and by Okamura, Kodaira, and Watanabe (1984). The surface-photometric parameters involved here are angular diameter (D_{26}), mean surface brightness (*SB*), apparent magnitude (V_{26}), and mean concentration index [$X1(P)$] which were defined in the above references. These parameters are derived from the *generalized radial luminosity profiles* which are the face-on luminosity profiles mathematically deduced from the observed surface brightness distributions. The behaviors of the Virgo Cluster galaxies in the multivariable space of these surface photometric parameters alone were already investigated in details (Kodaira, Okamura, and Watanabe 1983; Watanabe, Kodaira, and Okamura 1985).

As for the velocity parameters, the half-width of H I 21 cm line (v_S) is adopted from Richter and Huchtmeier (1984) for spirals, and the central velocity dispersion (v_E) is adopted from Whitmore, McElroy, and Tonry (1985) for ellipticals. Adoption of another data source such as Bottinelli *et al.* (1983) or Davoust, Paturel, and Vauglin (1985) does not affect the essential part of the conclusions of the following study. In order to avoid introducing excessive inclination corrections to the photometric and the velocity parameters of spirals, we restrict the sample to those galaxies having an inclination $30° \lesssim i \lesssim 65°$, with an exception of NGC 4111B ($i = 18°$). The inclination angle (i) is estimated by $\cos i = B/A$, with the apparent axis ratio B/A being adopted from Watanabe (1983). The final sample consists of 28 spirals and 18 ellipticals common to the surface photometric and the velocity data sources. The basic observational data of the sample galaxies are given in Table 1. Although the total number of the sample galaxies is relatively small, the present sample well covers the whole domain of the diameter versus surface brightness diagram in Kodaira, Okamura, and Watanabe (1983). Since the sample galaxies in the Virgo Cluster can be regarded to be at the same distance with an accuracy of about $\pm 10\%$, the apparent magnitude V_{26} can represent the absolute luminosity in the present study, so far as we are concerned with the relative values among the sample galaxies. In spite of this merit, one of demerits of using cluster samples might be possible distortion of H I disks by galaxy interaction, leading to anomalous deviation of a few samples. No inclination corrections due to the internal absorption in spiral galaxies (ΔV_{26}) are applied to V_{26}, because the deviations from the mean correction are small ($|\Delta V_{26} - \overline{\Delta V_{26}}| \leq 0.1$ mag) according to de Vaucouleurs, de Vaucouleurs, and Corwin (1976, hereafter RC2) when the inclination of the sample galaxies is restricted as above. The inclination correction according to Sandage and Tammann (1981,

TABLE 1A

DATA OF ELLIPTICAL GALAXIES

Galaxy	Type	$X1(P)$	log D_{26}	SB	V_{26}	log v_E
N4168..........	E2	0.86	1.67	23.33	11.36	2.26
N4261..........	E2	1.15	1.86	23.35	10.40	2.53
N4339..........	E0	1.09	1.59	23.16	11.58	2.13
N4365..........	E3	1.29	2.10	23.82	9.71	2.42
N4374..........	E1	1.33	2.10	23.40	9.28	2.47
N4387..........	E5	1.33	1.37	22.65	12.17	2.05
N4406..........	E3	0.60	2.29	24.01	8.94	2.41
N4458..........	E0–1	1.39	1.43	22.93	12.17	2.00
N4472..........	E2	1.08	2.26	23.42	8.52	2.50
N4473..........	E5	1.86	1.86	23.21	10.28	2.29
N4489..........	E1	1.07	1.37	22.76	12.27	1.82
N4551..........	E3	0.70	1.37	22.42	11.95	2.08
N4552..........	E0	1.57	1.95	23.23	9.85	2.44
N4564..........	E6	1.94	1.56	22.44	11.00	2.22
N4621..........	E5	1.54	1.92	23.07	9.83	2.35
N4636..........	E0–1	0.62	2.10	23.55	9.42	2.34
N4660..........	E5–6	2.77	1.55	22.33	10.96	2.29
N4486..........	E+0–1p	1.30	2.24	23.75	8.93	2.53

TABLE 1B

DATA OF SPIRAL GALAXIES

Galaxy	T	$X1(P)$	log D_{26}	SB	V_{26}	log v_S
N4351......	2	−0.25	1.53	23.98	12.72	2.34
N4450......	2	0.47	1.85	23.09	10.19	2.29
N4698......	2	1.12	1.71	22.96	10.79	2.38
N4380......	3	−0.63	1.62	23.44	11.71	2.25
N4413......	3	0.12	1.54	23.73	12.40	2.05
N4501......	3	0.09	1.97	23.12	9.64	2.45
N4548......	3	0.10	1.88	23.16	10.16	2.26
N4579......	3	1.00	1.93	22.95	9.66	2.49
N4595......	3	−0.39	1.40	23.07	12.45	2.03
N4232......	4	0.13	1.53	23.10	11.81	2.35
N4321......	4	−0.03	1.99	23.11	9.55	2.39
N4390......	4	−0.80	1.32	23.29	13.07	2.00
N4639......	4	0.38	1.55	22.96	11.57	2.41
N4689......	4	−0.48	1.79	23.51	10.94	2.07
N4254......	5	−0.46	1.79	22.63	10.04	2.28
N4535......	5	−0.78	1.90	23.30	10.16	2.28
N4651......	5	0.50	1.76	23.22	10.78	2.35
N4189......	6	−0.72	1.49	23.06	11.99	2.32
N4498......	6	−0.88	1.53	23.68	12.42	1.98
N4540......	6	−0.34	1.55	23.26	11.90	2.18
N4654......	6	−0.69	1.75	22.96	10.57	2.25
N4519......	7	−0.38	1.50	23.30	12.17	2.20
N4571......	7	−0.77	1.71	23.40	11.24	2.17
N4299......	8	−0.75	1.35	23.08	12.69	2.08
N4411a.....	8	−0.92	1.43	23.99	13.23	1.94
N4411b	8	−0.93	1.50	23.87	12.72	2.10
N4523......	8	−1.24	1.45	24.11	13.25	1.86
I3258.......	9	−0.73	1.39	24.35	13.79	1.87

hereafter RSA) has a stronger dependence on the axial ratio as well as on the morphological type than that in RC2. The results of a recent study about the inclination effects by the present author supports the corrections in RC2 rather than those in RSA (Kodaira and Watanabe 1988). The inclination correction to the velocity v_S is applied in the same way as in Richter and Huchtmeier (1984).

III. PRINCIPAL COMPONENT ANALYSES

The principal component analyses involving the above five parameters are carried out separately for ellipticals and spirals. The resulting eigenvectors and their eigenvalues are given in Table 2. We find one dominant and one marginally significant factors for ellipticals, what indicates the two-parameter nature of ellipticals as found by Dressler *et al.* (1987), Djorgovski and Davis (1987), and others. In the case of spirals, there are two significant factors as was pointed out by Brosche (1973) and others, but the eigenvalues show more continuous distribution indicating more complicated interrelations of parameters than in the case of ellipticals. Table 3 shows the direction cosines of the vectors representing the indivual parameters in the principal component space. The projections of the unit parameter vectors onto the principal planes (Y_{E1}, Y_{E2}) and (Y_{S1}, Y_{S2}) are shown in Figure 1.

In the principal plane for ellipticals, the magnitude vector (V_{26}) is almost antiparallel ($\cos\theta = -0.985$) to the velocity vector (log v_E) and close to the primary axis Y_{E1} ($\cos\theta = 0.979$). It should be noted, however, that the diameter vector (log D_{26}) is located symmetrically to the log v_E vector relative to the V_{26} vector and closer to the principal axis Y_{E1} than the log v_E. This indicates that V_{26} can be better expressed as an empirical function of both log v_E and log D_{26} than as a function of log v_E only. Contrary to this situation, log v_E can be well expressed as an empirical function of V_{26} alone. The

TABLE 2A

EIGENVALUE AND EIGENVECTOR FOR 18 ELLIPTICALS

		PROJECTION				
EIGENVECTOR	EIGENVALUE	log D_{26}	V_{26}	SB	$X1(P)$	log v_E
$Y1$..........	3.556	−0.526	0.507	−0.472	0.151	−0.470
$Y2$..........	1.151	0.050	−0.186	−0.300	0.875	0.326
$Y3$..........	0.175	0.086	0.171	0.718	0.459	−0.486
$Y4$..........	0.118	−0.335	0.578	0.342	0.013	0.661
$Y5$..........	0.000	0.776	0.587	−0.233	0.000	0.000

TABLE 2B

EIGENVALUE AND EIGENVECTOR FOR 28 SPIRALS

		PROJECTION				
EIGENVECTOR	EIGENVALUE	log D_{26}	V_{26}	SB	$X1(P)$	log v_S
$Y1$..........	2.851	0.469	−0.556	−0.361	0.395	0.430
$Y2$..........	1.050	0.574	−0.247	0.684	0.006	−0.376
$Y3$..........	0.679	−0.188	0.163	−0.012	0.880	−0.404
$Y4$..........	0.420	−0.087	0.296	0.570	0.263	0.715
$Y5$..........	0.000	0.639	0.718	−0.276	0.000	0.000

TABLE 3A

UNIT VECTORS IN PRINCIPAL COMPONENT SPACE FOR 18 ELLIPTICALS

Parameter	$Y1$	$Y2$	$Y3$	$Y4$	$Y5$
log D_{26}	−0.991	0.54	0.036	−0.115	0
V_{26}	0.957	−0.199	0.072	0.198	0
SB	−0.890	−0.322	0.301	0.117	0
$X1(P)$	0.284	0.939	0.192	0.004	0
log v_E	−0.886	0.350	−0.204	0.227	0

TABLE 3B

UNIT VECTORS IN PRINCIPAL COMPONENT SPACE FOR 28 SPIRALS

log D_{26}	0.792	−0.588	−0.155	−0.056	0
V_{26}	−0.939	+0.253	0.134	0.192	0
SB	−0.610	−0.701	−0.010	0.369	0
$X1(P)$	0.667	−0.006	0.725	0.170	0
log v_S	0.725	+0.385	−0.333	0.463	0

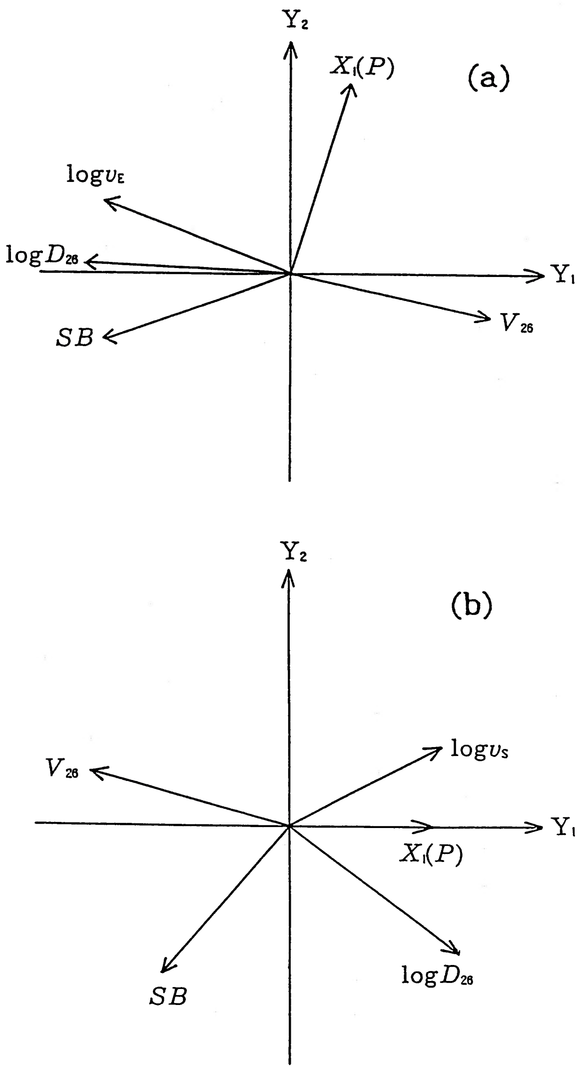

FIG. 1.—Parameter vector projection onto the principal plane: (*a*) ellipticals, and (*b*) spirals.

quantitative evaluation of the effects of combining two parameters, log D_{26} and log v_E, will be made in the next section.

In the case of spirals, the velocity vector (log v_S) is substantially out of (anti-) parallelism ($\cos\theta = -0.731$) to V_{26} vector in the principal plane (Y_{S1}, Y_{S2}), and significantly deviates from the primary axis Y_{S1} ($\cos\theta = 0.725$) in the principal component space. These facts suggest that the velocity-luminosity relation of spirals is much looser than that of ellipticals. The internal velocity log v_S can be expressed as a function mainly of V_{26} whose unit vector is the closest to the primary vector Y_{S1}. An additional argument of *SB* or log D_{26} may slightly improve the empirical function by taking the dispersion along the secondary axis Y_{S2} into account. The magnitude V_{26} of spirals, however, can be poorly expressed by an empirical function of log v_S alone. It seems to be essential for the empirical function of V_{26} of spirals to include the diameter parameter log D_{26} as an argument in addition to log v_S.

IV. REGRESSION LINE ANALYSES

In order to quantitatively evaluate the interrelations among the five parameters indicated in the previous section, we carry out regression line analysis for various linear relations of interest. In the following analyses we exclude two galaxies showing the largest deviation each from the elliptical sample (NGC 4261, NGC 4489) and from the spiral sample (NGC 4351, NGC 4689) to avoid strong influences of possible accidental errors. The results of the simple regression line fitting for velocity-luminosity relations are summarized in Table 4A. The Table shows the resulting coefficients (a, b) of the linear expression,

$$Y = aX + b\,, \tag{1}$$

the dispersion of the fitting (σ_Y), the correlation coefficient (c.c.), and the exponent $\gamma \equiv -a/2.5$ of the conventional velocity-luminosity relation. It is apparent that the simple velocity-luminosity relation is tighter for ellipticals than for spirals.

The results of the regression line fitting for the velocity-diameter and the diameter-luminosity relations in the same fashion as for the velocity-luminosity relation are summarized in Tables 4B and 4C. The dispersion $\sigma = 0.13 \sim 0.15$ of the $D_{26} - v$ relation indicates that this distance estimator, both for ellipticals and spirals, is as good as the $D_n - \sigma$ relation found by Dressler *et al.* (1987) for ellipticals or by Dressler (1987) for the bulges of early-type spirals.

We notice that the log $D_{26} - V_{26}$ relation is tighter than the above two relations, confirming the importance of the diameter in connection with the physical nature of galaxies.

As the next step we try fittings of linear functions of multiple arguments,

$$Y = AX + B(SB) + C \cdot X1(P) + D \tag{2}$$

and

$$Y = AX + B \log D_{26} + C \cdot X1(P) + D \tag{3}$$

TABLE 4A

VELOCITY-LUMINOSITY RELATION $Y = aX + b$

Sample	X	Y	a	b	σ_Y	Correlation Coefficient	γ
16E	log v_E	V_{26}	−7.070	26.625	0.379	−0.948	2.83
	V_{26}	log v_E	−0.1271	3.617	0.051		
26S	log v_S	V_{26}	−5.984	24.704	0.661	−0.847	2.39
	V_{26}	log v_S	−0.1199	3.583	0.094		

TABLE 4B

VELOCITY-DIAMETER RELATION $Y = aX + b$

Sample	X	Y	a	b	σ_Y	Correlation Coefficient
16E	log v_E	log D_{26}	1.827	−2.364	0.126	0.918
	log D_{26}	log v_E	0.461	1.453	0.064	
26S	log v_S	log D_{26}	0.838	−0.214	0.135	0.737
	log D_{26}	log v_S	0.649	1.142	0.119	

TABLE 4C

DIAMETER-LUMINOSITY RELATION $Y = aX + b$

Sample	X	Y	a	b	σ_Y	Correlation Coefficient
16E	log D_{26}	V_{26}	−3.660	17.090	0.255	−0.977
	V_{26}	log D_{26}	−0.261	4.539	0.068	
26S	log D_{26}	V_{26}	−5.954	21.244	0.359	−0.957
	V_{26}	log D_{26}	−0.154	3.407	0.058	

TABLE 5A

REGRESSION LINE FITTING FOR $Y = AX + B \cdot SB + C \cdot X1(P) + D$

Sample	X	Y	A	B	C	D	σ_Y
16E.....	$\log v_E$	V_{26}	−6.614	−0.2198	0.2095	30.389	0.335
	V_{26}	$\log v_E$	−0.1207	+0.0334	0.0538	2.705	0.045
26S.....	$\log v_S$	V_{26}	−5.430	−0.3770	0.0442	14.707	0.654
	V_{26}	$\log v_S$	−0.0674	−0.1284	0.0809	5.996	0.073

TABLE 5B

REGRESSION LINE FITTING FOR $Y = AX + B \log D_{26} + C \cdot X1(P) + D$

Sample	X	Y	A	B	C	D	σ_Y
16E.....	$\log v_E$	V_{26}	−1.555	−3.023	−0.1684	19.718	0.191
	V_{26}	$\log v_E$	−0.0872	+0.1673	+0.0540	2.824	0.045
26S.....	$\log v_S$	V_{26}	−2.290	−4.537	+0.0433	23.984	0.247
	V_{26}	$\log v_S$	−0.1967	−0.6498	+0.0802	5.553	0.072

with X and Y being $\log v$ and V_{26}, or vice versa. Since only two of the three parameters, V_{26}, SB, and $\log D_{26}$ are independent, no fittings are made for functions including all the three parameters. The results are summarized in Tables 5A and 5B. The comparison of Tables 4A, 4C, and 5 reveals that the dispersion $\sigma(V_{26})$ is hardly affected by the inclusion of SB and $X1(P)$ as additional arguments, but significantly decreases by the combination of $\log D_{26}$ and $\log v$ terms, particularly in the case of spirals. This multivariable empirical function itself,

$$V_{26} = A \log v + B \log D_{26} + C \cdot X1(P) + D \ , \quad (4)$$

however, is of little use in determining the distance, because the angular diameter D_{26} is distance-dependent and the value of B is not far from -5 which enters in the term of apparent magnitude V_{26} as distance effect.

V. DISCUSSION

In the preceding sections we have shown that the parameters $\log v$ and $\log D_{26}$ play significant roles in determining the galaxy luminosity both for ellipticals and spirals. A closer inspection of Table 5B further reveals that the coefficients, A and B stand in a special relation $B \risingdotseq 2A$ with an accuracy of better than 3% in the empirical equation (4) *both for ellipticals and spirals*. In order to examine the statistical significance of $B = 2A$, we apply the null-hypothesis test. We find that the hypotheses

$$H_0(1)\colon A = (A + B)/3 \quad \text{and} \quad H_0(2)\colon B = 2(A + B)/3$$

can be accepted at a confidence level higher than 90% in both cases for elliptical and spirals. We further make a regression line analysis for a linear expression

$$V_{26} = p \log (v D_{26}^2) + q \ , \quad (5)$$

to find very tight correlations as are shown in the fourth row of Table 6. The dispersions $\sigma(V_{26})$ of this relation are only marginally larger than those of equation (4).

In order to explore the structure of the multivariable space around the special point of $B = 2A$ in equation (4), we apply the regression line analyses for a linear expression

$$V_{26} = p(A \log v + B \log D_{26}) + q \quad (6)$$

with varying ratio of A to B. In order to avoid redundancy, we restrict the analyses to the case of $B \geq 0$. The results are summarized in Table 6, and the variation of $\sigma(V_{26})$ is visualized in Figure 2 as a function of ratio $A/(|A| + B)$. While the spiral sample clearly shows the best fitting at $B = 2A$, the elliptical sample shows a flat maximum around $B/A = 1 \sim 2$. Although for ellipticals the case of $B = A$ gives even marginally better fit for relation (6) than the case of $B = 2A$, the case of $B = 2A$ becomes the best when the minor effect of $X1(P)$ is taken into account, as found for equation (4) in the previous sections. The various empirical relations among luminosity, diameter, and velocity, as mentioned in the Introduction and shown in Tables 4A, 4B, and 4C, are projections of the above hyperplane onto appropriate two-dimensional planes.

TABLE 6

REGRESSION LINE FITTING FOR $V_{26} = p(A \log v + B \log D_{26}) + q$

		16E				26S			
A	B	p	q	σ	Correlation Coefficient	p	q	σ	Correlation Coefficient
1	0	−7.070	26.625	0.379	−0.948	−5.984	24.704	0.661	−0.847
2	1	−1.876	22.439	0.219	−0.983	−2.274	25.252	0.402	−0.946
1	1	−2.504	20.723	0.202	−0.986	−3.446	24.738	0.295	−0.971
1	2	−1.494	19.287	0.211	−0.984	−2.244	23.794	0.248	−0.980
0	1	−3.661	17.090	0.255	−0.977	−5.954	21.244	0.359	−0.957
−1	2	−2.300	13.525	0.365	−0.952	−3.339	15.077	0.759	−0.793
−1	1	−5.682	7.737	0.586	−0.871	−2.770	9.958	1.184	−0.308
−2	1	−0.549	8.854	1.192	−0.060	+2.207	17.650	1.120	+0.435

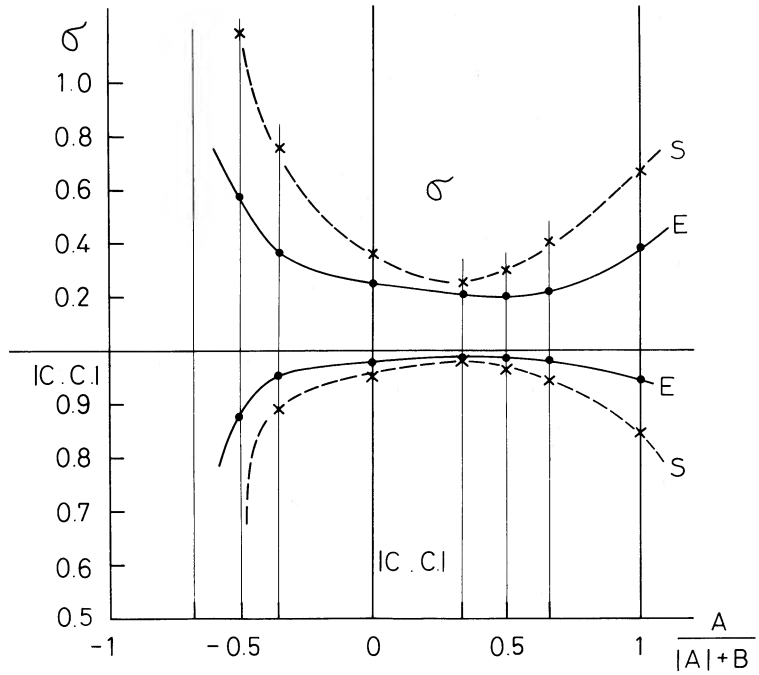

FIG. 2.—Variation in the dispersion (σ) and the correlation coefficient (c.c) as function of the ratio of A to B in the regression line fitting for $V_{26} = p(A \log v + B \log D_{26}) + q$.

We find that the empirical relation between the coefficients in equation (4), $B = 2A$, is highly significant to define the hyperplane thinnest in the V_{26} direction in the multivariable space *both for spirals and ellipticals*, and infer that the combined parameter $w' \equiv v D_{26}^2$ has a physical meaning which may be related to the basic nature of the galaxy structure. This hyperplane for ellipticals might be the same one as was found by Dressler *et al.* (1987) and by Djorgovski and Davis (1987), although it is difficult to justify it due to the difference of used parameters. After some trial we have found that a parameter $w \equiv (Gw')^{-1} = 1/Gv D_{26}^2$, with G being the gravity constant, has the dimension of the phase-space density when we adopt a linear scale for the diameter.

Carlberg (1986) recently discussed the difficulty in the merging process for spirals to become ellipticals using phase-space densities in the core of galaxies. In the course of his discussion, adopting the empirical correlations between the core parameters and the absolute magnitude of ellipticals found by Kormendy (1985) and Lauer (1985), he approximated the maximum phase-space density f_c in the core of elliptical galaxies as a function of M_B; $\log f_c = 0.941 M_B + \text{const}$. The present results suggest that the luminosity is physically related to a parameter of the statistical dynamics, a sort of phase-space density w, *both in ellipticals and spirals*.

Since the dimensional factors in the parameter w (v and D_{26}) represent the global velocities and diameter related to a galaxy, w seems to represent a kind of average density over a large volume of the phase space occupied by a galaxy. When we assume the largest velocity dispersion σ and an isophotal radius R far out enough to contain most of the galaxy luminosity, we can define a kind of average phase-space density

$$\bar{f} \propto \frac{M(R)}{\sigma^3 R^3} \propto \frac{V^2}{G\sigma^3 R^2} = \frac{1}{G\alpha^2 \sigma R^2} = \frac{1}{G\alpha^3 V R^2}, \qquad (7)$$

where $V \equiv \sqrt{GM(R)/R}$ stands for the maximum rotation velocity and $\alpha \equiv \sigma/V$. Then the parameter w is related to $\bar{f}$ by

$$\bar{f} \propto \alpha^3 w_S \simeq \alpha^2 w_E . \qquad (8)$$

In order to evaluate α and to relate $\bar{f}$ to L_{26}, we need theoretical models of galaxies, probably including dark halos. Recent N-body simulations of galaxy formation using sticky particles by Carlberg (1984, 1988) may lead to a possibility of exploring the physical relation between the luminosity and the parameters of statistical dynamics such as phase-space density, suggested by the present study. This task, however, is beyond the scope of the present work, and we restrict the discussion below to the behavior of w in the diameter versus surface brightness diagram (DSBD).

In DSBD of Kodaira, Okamura, and Watanabe (1983), the central lines of the distributions for ellipticals (Y_E) and spirals (Y_S) have slopes $(d \log D_{26}/d \log S_{26})_E \simeq -1.80$ and $(d \log D_{26}/d \log S_{26})_S \simeq +0.575$, respectively, where $-2.5 \log S_{26} \equiv SB$. Using the $V_{26} - \log w$ relation found above and the relation $L_{26} \propto S_{26} D_{26}^2$, we find a remarkable behavior of $w \equiv (Gv D_{26}^2)^{-1}$ that this varies almost log-linearly along both the Y_E and Y_S lines, with a common incremental scale $|\Delta \log w/\Delta \log Y| \simeq 2$, where $(\Delta \log Y)^2 \equiv (\Delta \log S_{26})^2 + (\Delta \log D_{26})^2$. The lines Y_E and Y_S are nearly orthogonal to each other, rotated by about 13° relative to the projection of the principal component axes, Y_1 and Y_2, found by Kodaira, Okamura, and Watanabe (1983), and cross each other at a point, $(\log D_{26})_0 \simeq 1.65$ and $(\log S_{26})_0 \simeq -9.20$, where $(V_{26})_0 \simeq 11$, corresponding to $(L_V)_0 \simeq 10^{10} L_{V,\odot}$ when a distance of 19.5 Mpc is adopted for the Virgo Cluster (cf. Dressler 1987; Prichet and van den Bergh 1987). At this crossing point, the absolute value of the phase space density parameter turns out to be $w_0 \simeq 2.5 \times 10^{-4} m_\odot$ (kpc)$^{-3}$ (km s^{-1})$^{-3}$, and the relative value of the maximum rotation velocity v_S to the central velocity dispersion v_E is found to be $\log \alpha_0 \sim (\log v_S/v_E)_0 \simeq 0.15$. When we adopt this conversion factor α_0, the logarithmic phase space density parameter $\log w$ *linearly* decreases along the Y_S line from the late-type to the early-type spiral galaxies, and continuously decreases along the Y_E line from the normal ellipticals to the cD-type galaxies. The total range of the variation in w amounts up to $\sim 10^2$. The real situation, however, may not be so simple as the above sequential picture, for the actual distribution of galaxies in DSBD shows scatter around the central lines, Y_E and Y_S, in particular in the region around their crossing point.

Further examinations of the tight interrelations discussed here are highly desirable for samples of galaxies other than the Virgo galaxies.

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