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AN ANALYSIS OF THE PRECESSION OF THE ACCRETION DISK OF HERCULES X-1

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ABSTRACT

Hercules X-1 exhibits a 35 day cycle in its X-ray intensity and 1.24 s and 1.7 day periodicities in its pulsar rotation and orbit, respectively. The features of the 35 day clock and the presence of a precessing tilted accretion disk have been discussed by many authors. A ring of matter at a given radius in the accretion disk will precess if it is tilted relative to the orbital plane. Under the assumption of a uniform precession of the disk, the periodic perturbation of the uniform precession has been analyzed by Levine and Jernigan. In this paper we present a complete expression for both uniform and perturbed precession of the disk. The result is compared with observations, and it is in agreement with the analysis of the observations given in the literature.

Subject headings: stars: accretion — stars: individual (Her X-1) — X-rays: binaries

I. INTRODUCTION

The pulsating X-ray binary Her X-1 shows an unusual kind of periodic behavior in the form of an on-off cycle of $\sim$35 days (Tananbaum *et al.* 1972; Giacconi *et al.* 1973). During the 35 day cycle, Her X-1 normally has a high state lasting for $\sim$10 days with a 2–6 keV intensity of $\sim$100 mCb, and a $\sim$5 day secondary high state with peak intensity of $\sim$30 mCb that occurs $\sim$180° out of phase with the primary high state. For the remainder of the 35 day cycle, referred to as the low state, Her X-1 is much fainter at $\sim$5 mCb (Parmar *et al.* 1985).

The 35 day clock mechanism has been discussed by many authors. Kondo, Van Landern, and Wolff (1983) suggested that it is caused by nonlinear oscillations of the normal star, which provide the modulations of the mass flow. Trümper *et al.* (1986) proposed that the clock mechanism resides in the free precession of the neutron star, but the concept of a precessing accretion disk in one form or another is quoted most frequently in discussions of this X-ray binary (Gorecki *et al.* 1982; Levine and Jernigan 1982; Katz and Grandi 1982; McCray *et al.* 1982; Howarth and Wilson 1983). There is evidence that the 35 day cycle is due to a precession of the tilted accretion disk which obscures the X-ray source when the disk is in a certain range of orientation.

Roberts (1974) suggested that an inclined rotational axis for HZ Her could have a dramatic effect on the observed behavior of the binary system. He discussed the possible connections between certain observations and the precession of the axis of HZ Her, calling his model the "slaved disk model." Petterson (1975) suggested that the disk should not just be tilted and precessing, but that it also should be warped. Katz (1973) estimated the precession period of the disk rim due to tidal forces from the companion star. On the assumption of a uniform precession of the disk, Levine and Jernigan (1982) and Katz and Grandi (1982) analyzed certain perturbations.

Based on dynamical analysis, both uniform precession and perturbation of the disk are discussed in this paper. Some comparisons of the theoretical results and the observations are given, and the results are in agreement with the basic observations.

II. DYNAMICAL ANALYSIS

a) Geometry and Parameters

The geometry of the system is given in Figure 1, where M_r, a, and ω are the mass, the radius, and the angular velocity of the ring, respectively, $\omega = \dot{\alpha}$; $M_{\rm opt}$ is the mass of the optical star (HZ Her); R is the distance between HZ Her and Her X-1; M_x is the mass of Her X-1; Ω is the orbital angular velocity ($\Omega = 2\pi/T_{\rm orb}$) of the binary; φ, θ, and α are Euler angles (see Fig. 1); XY represents the orbital plane; and X is the projection of the line of sight into the orbital plane.

The parameters known with a high degree of confidence from X-ray and optical observations are the following: the orbital period $T_{\rm orb} = 1^{\rm d}.70$ (Deeter, Boynton, and Pravdo 1981); the projected maximum radial velocity for Her X-1, $V_x \sin i = 169.05$ km s^{-1} (i is the inclination of the orbital plane to the plane of the sky); the mass function $f_x(M) = M_x^3 \sin^3 i/(M_x + M_{\rm opt})^2 = 0.85\ M_\odot$; the orbital eccentricity $e < 0.0002$; the projected semimajor axis of the orbit, $a_x \sin i = 3.95 \times 10^{11}$ cm; and the inclination $i = 87°$ (Middleditch and Nelson 1976).

The mass ratio, $q = M_x/M_{\rm opt}$, has been determined by various authors, but the derived ratio depends somewhat on the model used: $q = 0.60$ (Middleditch *et al.* 1976); $q = 0.65$ (Koo and Kron 1977); $q = 0.49$ (Hutchings *et al.* 1985). For the approximate calculation below, we shall use a mass ratio of $q = 0.60$. So, $M_{\rm opt} = 2.2\ M_\odot$, $M_x = 1.3\ M_\odot$, $a_{\rm opt} = 2.37 \times 10^{11}$ cm, $R \sin i = 6.32 \times 10^{11}$ cm, and $R = 6.33 \times 10^{11}$cm.

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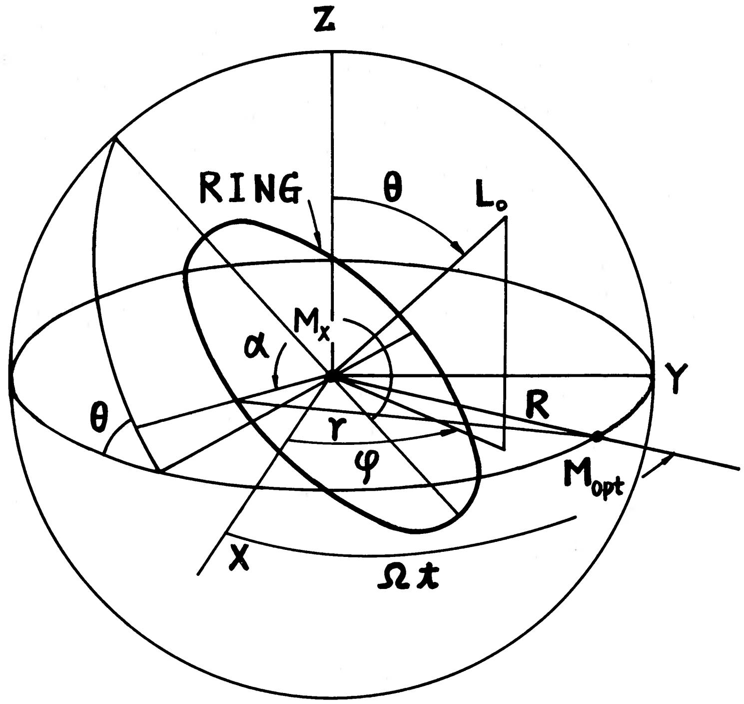

FIG. 1.—The geometry for the calculations

b) Potential Function

Now we analyze the motion of a rigid ring with radius a which is tilted with respect to the binary orbital plane; θ is the angle between the orbital plane and the ring. We take the origin of coordinates as the center of the neutron star, so that HZ Her will move around the origin. The Z-axis is taken as normal to this orbital plane, as shown in Figure 1. The quantity r is the distance between a differential arc $d\alpha$ at the ring and the optical star with the mass M_{opt}:

$$r^2 = R^2 + a^2 + 2Ra[\cos\theta \cos(\Omega t - \varphi)\cos\alpha + \sin(\Omega t - \varphi)\sin\alpha] . \tag{1}$$

The mass of a differential arc is

$$\frac{M_r}{2\pi a}\, a\, d\alpha = \frac{M_r}{2\pi}\, d\alpha ,$$

and the gravitational potential is

$$V = -\int_0^{2\pi} G\,\frac{M_{\text{opt}}(M_r/2\pi)d\alpha}{r} = -\frac{GM_{\text{opt}}M_r}{2\pi}\int_0^{2\pi}\frac{d\alpha}{r} = -\frac{GM_{\text{opt}}M_r}{2\pi}\int_{C/2}^{\pi+C/2}\frac{2\,d\beta}{\sqrt{A - B\sin^2\beta}}$$
$$= -\frac{GM_{\text{opt}}M_r}{2\pi}\frac{4}{\sqrt{A}}\int_0^{\pi/2}\frac{d\beta}{\sqrt{1-(B/A)\sin^2\beta}} = -\frac{GM_{\text{opt}}M_r}{2\pi}\frac{4}{\sqrt{A}}F\left(\sqrt{\frac{B}{A}}\right), \tag{2}$$

where

$$2\beta = \alpha + C , \qquad C = \arctan\frac{\sin(\Omega t - \varphi)}{\cos\theta\cos(\Omega t - \varphi)} ,$$
$$A = R^2 + a^2 + 2Ra[\cos^2\theta\cos^2(\Omega t - \varphi) + \sin^2(\Omega t - \varphi)]^{1/2} , \tag{3}$$
$$B = 4Ra[\cos^2\theta\cos^2(\Omega t - \varphi) + \sin^2(\Omega t - \varphi)]^{1/2} ,$$

and $F(\sqrt{B/A})$ is the complete elliptical integral of the first kind. When $R \gg a$, $B/A \ll 1$, the integration can be written as

$$F\left(\sqrt{\frac{B}{A}}\right) = \frac{\pi}{2}\left[1 + \frac{1}{4}\frac{B}{A} + \frac{9}{64}\left(\frac{B}{A}\right)^2 + \cdots\right]. \tag{4}$$

So

$$V = -\frac{GM_{\text{opt}}M_r}{\sqrt{A}}\left[1 + \frac{1}{4}\frac{B}{A} + \frac{9}{64}\left(\frac{B}{A}\right)^2 + \cdots\right]. \tag{5}$$

Accurate to order $(a/R)^2$, equation (5) becomes

$$V = -\frac{GM_{\rm opt} M_r}{R}\left[\!\!\left[1 + \frac{1}{2}\left(\frac{a}{R}\right)^2 \left\{\frac{3}{2}\left[\cos^2\theta \cos^2(\Omega t - \varphi) + \sin^2(\Omega t - \varphi)\right] - 1\right\}\right]\!\!\right] . \tag{6}$$

c) Euler Equations

When $\dot\alpha = \omega \gg \dot\varphi$, and $\omega \gg \dot\theta$, the kinetic energy of the ring is

$$T = \tfrac{1}{2}M_r a^2(\dot\alpha + \dot\varphi\cos\theta)^2 + \tfrac{1}{4}M_r a^2(\dot\theta^2 + \dot\varphi^2\sin^2\theta) \simeq \tfrac{1}{2}M_r a^2(\dot\alpha + \dot\varphi\cos\theta)^2 . \tag{7}$$

Lagrange's equations are

$$M_r a^2(\dot\alpha + \dot\varphi\cos\theta) \approx M_r a^2\omega = L_0 = \text{const.} , \tag{8a}$$

$$L_0\dot\varphi\sin\theta = -\frac{\partial V}{\partial\theta} , \tag{8b}$$

$$L_0\dot\theta\sin\theta = \frac{\partial V}{\partial\varphi} . \tag{8c}$$

Substituting V and L_0 into equations (8b) and (8c), we have

$$\dot\varphi = -\frac{3}{4}\frac{GM_{\rm opt}}{R^3\omega}\cos\theta[1 + \cos 2(\Omega t - \varphi)] , \qquad \dot\theta = +\frac{3}{4}\frac{GM_{\rm opt}}{R^3\omega}\sin\theta\sin 2(\Omega t - \varphi) . \tag{9}$$

d) The Solutions of Equation (9)

Letting $\tau = \Omega t$. Equation (9) becomes

$$\frac{d\varphi}{d\tau} = -\frac{3}{4}\epsilon\cos\theta[1 + \cos 2(\tau - \varphi)] , \qquad \frac{d\theta}{d\tau} = +\frac{3}{4}\epsilon\sin\theta\sin 2(\tau - \varphi) . \tag{10}$$

Here

$$\epsilon = \frac{GM_{\rm opt}}{R^2\omega\Omega} \tag{11}$$

is a dimensionless quantity.

In our case $\epsilon \ll 1$, we can get the solution by averaging methods (Sanders and Verhulst 1985, p. 33). The first-order approximations are

$$\frac{d\varphi_1}{d\tau} = -\frac{3}{4}\epsilon\frac{1}{2\pi}\int_0^{2\pi}\cos\theta_1[1 + \cos 2(\tau - \varphi)]d\tau = -\frac{3}{4}\epsilon\cos\theta_1 , \qquad \frac{d\theta_1}{d\tau} = 0 , \tag{12}$$

and

$$\varphi_1 = \varphi_0 - (\tfrac{3}{4}\epsilon\cos\theta_0)\tau , \qquad \theta_1 = \theta_0 . \tag{13}$$

The second-order approximations are

$$\begin{aligned} \varphi_2 &= \varphi_0 - [\tfrac{3}{4}\epsilon\cos\theta_0 - \tfrac{9}{64}\epsilon^2(\cos 2\theta_0 + \cos^2\theta_0)]\tau - \tfrac{3}{8}\epsilon\cos\theta_0\sin 2\{[1 + \tfrac{3}{4}\epsilon\cos\theta_0 - \tfrac{9}{64}\epsilon^2(\cos 2\theta_0 + \cos^2\theta_0)]\tau - \varphi_0\} , \\ \theta_2 &= \theta_0 - \tfrac{3}{8}\epsilon\sin\theta_0\cos 2\{[1 + \tfrac{3}{4}\epsilon\cos\theta_0 - \tfrac{9}{64}\epsilon^2(\cos 2\theta_0 + \cos^2\theta_0)]\tau - \varphi_0\} . \end{aligned} \tag{14}$$

Substitute ϵ, τ into equation (14) to obtain

$$\begin{aligned} \varphi_2 = \varphi_0 &- \left[\frac{3}{4}\frac{GM_{\rm opt}}{R^3\omega}\cos\theta_0 - \frac{9}{64}\frac{G^2M_{\rm opt}^2}{R^6\omega^2\Omega}(\cos 2\theta_0 + \cos^2\theta_0)\right]t \\ &- \frac{3}{8}\frac{GM_{\rm opt}}{R^3\omega\Omega}\cos\theta_0\sin 2\left\{\left[\Omega + \frac{3}{4}\frac{GM_{\rm opt}}{R^3\omega}\cos\theta_0 - \frac{9}{64}\frac{G^2M_{\rm opt}^2}{R^6\omega^2\Omega}(\cos 2\theta_0 + \cos^2\theta_0)\right]t - \varphi_0\right\} , \\ \theta_2 = \theta_0 &- \frac{3}{8}\frac{GM_{\rm opt}}{R^3\omega\Omega}\sin\theta_0\cos 2\left\{\left[\Omega + \frac{3}{4}\frac{GM_{\rm opt}}{R^3\omega}\cos\theta_0 - \frac{9}{64}\frac{G^2M_{\rm opt}^2}{R^6\omega^2\Omega}(\cos 2\theta_0 + \cos^2\theta_0)\right]t - \varphi_0\right\} . \end{aligned} \tag{15}$$

Let ω_φ represent the uniform precession angular velocity of the ring:

$$\omega_\varphi \equiv \frac{3}{4}\frac{GM_{\rm opt}}{R^3\omega}\cos\theta_0 - \frac{9}{64}\frac{G^2M_{\rm opt}^2}{R^6\omega^2\Omega}(\cos 2\theta_0 + \cos^2\theta_0) . \tag{16}$$

When the terms which are smaller than ϵ^2 are ignored, the amplitude of the perturbation for θ_2 is

$$\frac{3}{8}\frac{GM_{\rm opt}}{R^3\omega\Omega}\sin\theta_0 = \sin\theta_0\left[\frac{\omega_\varphi}{2\Omega\cos\theta_0}+\frac{9}{128}\epsilon^2\frac{\cos 2\theta_0+\cos^2\theta_0}{\cos\theta_0}\right]\approx\frac{\omega_\varphi}{2\Omega}\tan\theta_0 . \tag{17}$$

The final results are

$$\begin{aligned}\varphi = \varphi_2 &= \varphi_0 - \omega_\varphi t - \frac{1}{2}\frac{\omega_\varphi}{\Omega}\sin 2[(\Omega+\omega_\varphi)t-\varphi_0] ,\\ \theta = \theta_2 &= \theta_0 - \frac{1}{2}\frac{\omega_\varphi}{\Omega}\tan\theta_0\cos 2[(\Omega+\omega_\varphi)t-\varphi_0] .\end{aligned} \tag{18}$$

III. COMPARISON WITH THE OBSERVATIONS

a) The Radius of the Ring

For Her X-1, $\omega_\varphi \approx 2\pi/35^{\rm d}$, so from equation (16) we have

$$\frac{\omega}{\cos\theta_0}\approx\frac{3}{4}\frac{GM_{\rm opt}}{R^3}\frac{35^{\rm d}}{2\pi} . \tag{19}$$

If $\theta_0 \sim 35°$ (see Jones and Forman 1976), using the value given in § II*a*, we have

$$\omega \approx 0.34\times10^{-3} . \tag{20}$$

If the material of the ring moves with Keplerian velocity, then

$$a^3=\frac{G(M_x+M_r)}{\omega^2}\approx\frac{GM_x}{\omega^2} , \tag{21}$$

and

$$a = 1.1\times10^{11}\ {\rm cm} .$$

The radius of the disk was estimated from the optical study by Middleditch and Nelson (1976), $a = 1\times10^{11}$ cm. Our value agrees with it very well.

b) Precession Direction

A detailed numerical comparison of photometric data with the tilted precessing disk model led to the conclusion that the disk precesses with period 35 day in opposite direction to the orbital motion (Crosa and Boynton 1980).

From equation (18),

$$\dot\varphi = -\omega_\varphi - \omega_\varphi\left(1+\frac{\omega_\varphi}{\Omega}\right)\cos 2[(\Omega+\omega_\varphi)t-\varphi_0] . \tag{22}$$

From the second expression given in equation (18), we can see

$$\theta_{\max}=\theta_0+\frac{1}{2}\frac{\omega_\varphi}{\Omega}\tan\theta_0 ,$$

and at this point,

$$\dot\varphi\Big|_{\theta=\theta_{\max}} = +\frac{\omega_\varphi}{\Omega}\omega_\varphi > 0 . \tag{23}$$

But for $\dot\varphi > 0$ it is only limited in the region

$$\cos 2[(\Omega+\omega_\varphi)t-\varphi_0] < -\frac{\Omega}{\omega_\varphi+\Omega}\approx -0.95 . \tag{24}$$

For most situations $\dot\varphi < 0$. This means that the disk precesses in the sense opposite to the orbital motion, and on average it is in agreement with that given by observations.

c) The Change of 35 day Period

No report has been published that the 35 day period has a long time change. This is in agreement with our result.

Boynton, Crosa, and Deeter (1980) reported that over intervals of order 10 cycles the 35 day period changed as: $36^{\rm d}.4 \pm 0^{\rm d}.5$, $35^{\rm d}.2 \pm 0^{\rm d}.2$, $34^{\rm d}.5 \pm 0^{\rm d}.2$, $35^{\rm d}.1 \pm 0^{\rm d}.2$, $36^{\rm d}.2 \pm 0^{\rm d}.2$, and $33^{\rm d}.6 \pm 0^{\rm d}.3$ (1 σ confidence).

From equation (18) we see that for one cycle of precession the perturbation times are

$$\frac{2(\Omega + \omega_\varphi)}{\omega_\varphi} \approx 43.2 ; \tag{25}$$

this means that for about five cycles of the 35 day period, the changes of it make a cycle.

The difference between the long period and the short period comes from the fact that the 0.2 phases [see Eq. (25)] are spent in different ranges. The longest time for the 0.2 phases satisfies

$$\frac{\Delta\varphi}{2} = \omega_\varphi \frac{t_{\max}}{2} - \frac{1}{2}\frac{\omega_\varphi}{\Omega} \sin 2(\omega_\varphi + \Omega) \frac{t_{\max}}{2} ,$$

and the shortest time for the 0.2 phases satisfies

$$\frac{\Delta\varphi}{2} = \omega_\varphi \frac{t_{\min}}{2} + \frac{1}{2}\frac{\omega_\varphi}{\Omega} \sin 2(\omega_\varphi + \Omega) \frac{t_{\min}}{2} .$$

So the maximum difference of the periods is

$$\Delta T = t_{\max} - t_{\min} . \tag{26}$$

Substituting

$$\frac{1}{2}\Delta\varphi = \frac{2\pi}{432} , \qquad \omega_\varphi = \frac{2\pi}{35^{\mathrm{d}}} , \qquad \Omega = \frac{2\pi}{1^{\mathrm{d}}\!.7} ,$$

we get

$$t_{\max} = 0^{\mathrm{d}}\!.431 , \qquad t_{\min} = 0^{\mathrm{d}}\!.080 , \qquad \Delta T = 0^{\mathrm{d}}\!.351 . \tag{27}$$

The changes of the period may come from the disk structure. For example, if the radius of the ring is changed from a to a_1, from equations (21), (16), we get

$$\frac{\omega_{\varphi 1}}{\omega_\varphi} = \left(\frac{a_1}{a}\right)^{3/2} . \tag{28}$$

If the 35 day period changes from 34 to 36 days (about $\pm 2.5\%$), $\omega_{\varphi 1}/\omega_\varphi$ changes from 1.026 to 0.976, respectively, and a_1 changes from 1.12×10^{11} to 1.08×10^{11} cm, respectively. This is just a small change in the structure of the disk.

Indeed, *EXOSAT* observations suggest that a temporary change in the disk structure may have occurred (Parmar *et al.* 1985).

d) The Value and Influence of θ

It was assumed by Levin and Jernigan (1982) that the value of θ is small. This limit is not needed in our calculation.

From the discussion above we can see that the width of the ring responsible for the absorption of the X-rays cannot be large, but the thickness of the ring can be large. For the value of $\theta \sim 30°$, $i \sim 87°$, the shadows of the X-ray illumination are estimated to be $\sim 20°$ (Middleditch, Puetter, and Pennypacker 1985). In fact, most authors use the value of $\theta \sim 30°$ (Jones and Forman 1976).

From equation (18) the maximum change of θ is

$$\Delta\theta = \pm \frac{1}{2}\frac{\omega_\varphi}{\Omega} \tan \theta_0 . \tag{29}$$

For $\theta_0 \sim 35°$, $\Delta\theta \approx \pm 1°$.

The changes of θ can influence the value of the 35 day period, but its important influence is on the turn-on and turn-off scale, the values of $T_{\mathrm{on}}/T_{\mathrm{off}}$ are from 0.33 (Tananbaum *et al.* 1972) to 0.52 (Jones and Forman 1976).

e) Is the Neutron Star in Free Precession?

Jones and Forman (1976) have observed X-ray emission from Her X-1 during the 23 days, the low state, of the 35 day cycle. During this time they observed all the activities normally presented in the high state: regular eclipses, absorption "dips," and 1.24 s pulsations. This fact is contrary to the model of the precession of the neutron star suggested by Trümper *et al.* (1986) and supports the model of the precession of the accretion disk.

IV. CONCLUSION

From our results we can see the following:

1. The change of the precession angular velocity ω_φ and the changes of θ are periodical; there are no secular changes for the period of 35 days.

2. The precession is a common effect for the accretion disk if it is tilted relative to the binary orbital plane as pointed out by other authors (see, e.g. Katz 1973; Levine and Jernigan 1982). The uniform precession has been discussed by Katz (1973). On the assumption that the motion is an approximately uniform precession (the uniform precession is assumed), Levine and Jernigan (1982) explored the dynamics of tilted accretion disks in binary systems and showed that the expected motion of a ring of matter in an

accretion disk is more complicated than simple precession at a uniform rate. Our result for the description of the disk "wobble" is similar to that given by Levine and Jernigan (1982). But there are two important differences:

a) We do not need the assumption of a uniform precession. The uniform precession is obtained as the first-order approximation of our calculation, and is shown in equation (16).

b) In the analysis given by Levine and Jernigan (1982), there was another assumption: θ_0 is small. In our analysis, we do not need this assumption either. Owing to this cause, comparing equation (18) with Levine's equation (8) (Levine and Jernigan 1982), we shall find that there is a difference between the two results, when θ_0 is not small.

3. The motion of the ring discussed either in this paper or in the paper of Levine and Jernigan (1982) is approximated as that of a rigid ring. The actual motion will deviate somewhat from this description since a real accretion disk is a fluid object.

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