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A ROTATING, MAGNETIC, RADIATION-DRIVEN WIND MODEL FOR WOLF-RAYET STARS

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ABSTRACT

We apply a radiation-driven wind model that incorporates the effects of rotation and an open magnetic field to Wolf-Rayet stars in order to address the wind momentum problem. For the flow in the equatorial region we study the dependence of the mass-loss rate and terminal velocity on the rotation rate and surface magnetic field. The transition from a purely radiatively driven wind to a rotationally and magnetically driven wind is investigated for the case in which the stellar luminosity is consistent with Maeder's mass-luminosity relation. If the equatorial wind is to explain both the observed high mass-loss rate and fast terminal speed of Wolf-Rayet stars, the star must have a field strength of 10^4 G and be rotating at a rate more than 85% maximum velocity. Such a model gives rise to a "spin-down" problem because the magnetic braking will decrease the rotation rate on a time scale of only 10^4 yr. We therefore develop an alternative picture for Wolf-Rayet winds in which there is a slower but denser equatorial flow and a fast radiation-driven wind at higher latitudes. This axisymmetric model requires reinterpretation of the observed radio flux and the P Cygni profiles. We present wind models for V444 Cyg, CV Ser, and γ^2 Vel using luminosities that are consistent with interior theory. If the stars have field of ~1500 G and rotate at rates greater than ~85% maximum, they can satisfy the radio and UV observations, hence explain the momentum problem, and also overcome the spin-down problem.

Subject headings: magnetic fields — stars: rotation — stars: winds — stars: Wolf-Rayet

I. INTRODUCTION

The winds from Wolf-Rayet stars are characterized by large mass-loss rates ($\dot{M} = 0.1$–$1 \times 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$) and high terminal velocities ($v_\infty = 1000$–$3000\ \mathrm{km\ s}^{-1}$; Abbott *et al.* 1986). This combination makes the wind momentum flux $\dot{M}v_\infty$ extremely large and leads to now well-recognized "wind momentum problem" (Barlow, Smith, and Willis 1981; Cassinelli 1982). The problem can be demonstrated by setting the total photon momentum flux (L/c) to the final observed momentum flux ($\dot{M}v_\infty$); we find the "single scattering" maximum mass-loss rate $\dot{M}_{\max} = (L/c)/v_\infty$ (Cassinelli and Castor 1973). Typically, $\dot{M}v_\infty$ is about 5–50L/c for the Wolf-Rayet stars (Abbott *et al.* 1986; Barlow *et al.* 1981), yet the best that radiation-driven wind theory for Wolf-Rayet stars can do, even with multiple scattering of the photons, is about $\dot{M}v_\infty = 5L/c$ (Friend and Castor 1983; Abott 1987, private communication).

How well determined are the quantities $\dot{M}$, v_∞, and L, which define the momentum fluxes? The terminal velocities and mass-loss rates are two quantities about which we have good information. The terminal velocities are obtained from the sharp blueward edge of the ultraviolet P Cygni resonance lines that sample the wind along the line of sight toward the star. The mass-loss rates are found from the radio free-free emission of the wind (Abbott *et al.* 1986). The radius where the optical depth equals unity is typically 1000 stellar radii from the star, and the wind may be considered to be at its terminal velocity with the density falling off as r^{-2}. It is possible that there is a contribution to the radio flux from nonthermal emission. However, in this paper, we choose to make the usual assumption that the radio flux is from free-free emission.

The total luminosity of Wolf-Rayet stars is the most problematical quantity. To determine the luminosity observationally, we need a measurement of the observable flux, the stellar distance, and an estimate of the fraction of the stellar luminosity that occurs in the unobservable extreme ultraviolet. The most uncertain of these quantities is the latter which can be estimated for ordinary hot stars if we have some knowledge of the star's effective temperature. However, for Wolf-Rayet stars, it has been known for a long time that the concept of effective temperature is an ambiguous one, because the winds are so optically thick that the atmosphere is extended and the "photosphere" is not clearly defined (Chandrasekhar 1934; Cassinelli 1971; Castor 1974). The energy distribution of Wolf-Rayet stars tends to be flat, as is expected for a star with an extended atmosphere (van Blerkom 1971; van der Hucht *et al.* 1979). The "characteristic temperature" for these stars has been estimated to be in the range from 20,000 (Willis 1980) to greater than 80,000 K (Cherepashchuk, Eaton, and Khaliullin 1984; hereafter CEK). Evidence for the very high temperatures has been derived from the observational study of the eclipsing binary V444 Cyg.

Schmultz, Hamann, and Wessolowski (1988*b*) have recently used expanding model atmosphere methods to derive the luminosities, core temperatures, and mass-loss rates for 30 Wolf-Rayet stars. Assuming a standard velocity law for the winds, they derived equivalent widths for He I and He II lines. From a comparison with observed line strengths, and by fitting the continuum to the observational absolute visual magnitude, they have determined that Wolf-Rayet luminosities are in the range 1×10^5–$7 \times 10^5\ L_\odot$. For V444 Cygni, in particular, they derive a luminosity significantly lower than the one implied by the high-temperature estimate of CEK.

The luminosity of Wolf-Rayet stars can also be estimated from stellar interior considerations, using the evolutionary models calculated by Maeder (1980, 1983) and Prantzos *et al.* (1986). These models account for the effects of mass loss on the evolution of very massive stars. The models explain reasonably well the ratio of red supergiants to Wolf-Rayet stars, and also explain the peculiar abundances associated with the WC, WN and WO classes of Wolf-Rayet stars. The underlying cores of the Wolf-Rayet stars are thought to be near the helium-burning main-sequence, and the overall lifetime of a Wolf-Rayet star is thought to be on the order of 10^5 yr. For our consideration of the momentum problem, the crucial result is the mass-luminosity relation. From Maeder (1983), we have

$$\log (L/L_{\odot}) = 1.5 \log (M/M_{\odot}) + 3.8 . \quad (1)$$

The luminosities predicted by equation (1) generally are higher than the luminosities obtained by Schmutz, Hamann, and Wessolowski (1988*b*).

Even within the large uncertainty in the stellar parameters, it is very difficult to understand the large momentum fluxes in the winds of Wolf-Rayet stars. The line-driven wind theory of Castor, Abbott, and Klein (1975; hereafter CAK) as modified by Friend and Abbott (1986) has been successful in describing the wind from an O star where the stellar parameters are fairly well known. Application of this model to standard Wolf-Rayet star parameters produces a mass-loss rate (and momentum flux) that is too small by about an order of magnitude. Nonetheless, given the uncertainty in the luminosity of the Wolf-Rayet stars and the adjustable parameters in line-driven wind theory, the radiation-driven wind models cannot be ruled out. For example, it is commonly assumed that radiation-driven models can never yield momentum fluxes in excess of the single scattering limit. However, CAK–type models, which ignore the effects of overlapping lines, can yield arbitrarily high momentum fluxes if the luminosity is large enough or if the radiation force parameters are modified (Friend, Poe, and Cassinelli 1988). Recently, Pauldrach *et al.* (1985) have found that a sufficiently high luminosity would allow radiation to drive the wind of V444 Cyg. However, the luminosity that they required is a factor of $\sim$2.5 times higher than the luminosity given by Maeder's mass-luminosity relation for Wolf-Rayet stars and is a factor of 4 times higher than observed by Schmutz *et al.* (1988*b*). It is therefore reasonable to question whether radiation alone can drive the winds. The momentum problem is a serious one, and additional wind-driving mechanisms should be considered.

Maeder (1985) has suggested that the high mass-loss rates from Wolf-Rayet stars could be produced by pulsational instabilities of the helium core due to the ϵ-mechanism. In this model, Wolf-Rayet stars are marginally unstable, losing just enough mass to remain quasi-homogeneous and stable, while the change in the chemical composition of the core tends to push the stars toward instability. Maeder showed that the mass-loss rate required to maintain the balance is similar to that which is observed. However, his suggestion addresses only the mass-loss part of the momentum problem. It is also necessary to drive this large mass flux to high terminal velocity.

An alternative explanation of Wolf-Rayet star winds is the magnetic loop model of Underhill and Fahey (1987, and references therein) which does not presume high mass-loss rates; therefore, in their view, a momentum problem does not exist. In their picture, the radio emission is nonthermal and is produced by gyroresonance of electrons in a strong closed magnetic field. The outflowing wind is presumed to be much less dense than the confined material. It is not clear whether their scenario, which involves extensions of closed loops out to several stellar radii, is dynamically possible.

There are two other stellar properties that are known to be possible causes of fast massive winds: stellar rotation and corotating magnetic fields. Together these properties can increase the mass-loss rates by transferring the star's rotational angular momentum to the wind. Wolf-Rayet stars could be rapid rotators. At the onset of the Wolf-Rayet phase, the helium core of a formerly massive star contracts to initiate the triple-α process (Maeder 1983). If the core conserves angular momentum, it must spin-up, possibly to a rotation rate near the "critical" value, where the centrifugal force equals gravity at the surface. Wolf-Rayet stars could also have large magnetic fields on their surfaces, since these stars are the exposed convective cores of former O stars, which may have contained strong magnetic fields. Both a high rotation rate and a large magnetic field could be hidden from direct observation because the winds are optically thick and the deeper, nearly hydrostatic part of the atmosphere of Wolf-Rayet stars apparently cannot be seen.

A rotating magnetic model to explain the momentum problem of Wolf-Rayet winds was first presented by Hartmann and Cassinelli (1981), as described by Cassinelli (1982). They applied the fast magnetic rotator (FMR) model of Hartmann and MacGregor (1982; see also Belcher and MacGregor 1976) to Wolf-Rayet stars. Hartmann and Cassinelli found that rotation rates near the critical value and surface magnetic field strengths of $\sim$10 kG are required to fit the observed $\dot{M}$ and v_{∞} in the equatorial plane. This FMR model ignored the radiation force in spectral lines, however, and even if radiation alone cannot drive the wind, a complete wind model for Wolf-Rayet stars should include the line radiation force.

A more complete rotating magnetic wind model was constructed by Friend and MacGregor (1984), who combined the Weber and Davis (1967) description of a rotating, magnetic solar wind with the CAK theory for a line-driven wind. They showed that for O star parameters, rotation increases the mass-loss rate for all field strengths and decreases the terminal velocity for small field strengths. For a given rotation rate, increasing the magnetic field strength (B_0) increases v_{∞}. Both the FMR model and the Friend and MacGregor model indicate that the addition of rotation and a magnetic field can produce a wind that is denser and faster than the CAK theory would predict.

Very recently, Nerney and Suess (1987) have considered a modification to the FMR model, which accounts for the line radiation force in the limit that all lines are optically thick. In this limit, the line force is proportional to the velocity gradient, so this model is essentially the same as the FMR model of Hartmann and Cassinelli (1981), except for an extra factor which reduces the inertial term in the equation of motion. This factor is a measure of the strength of the line radiation force, which, for reasonable values, slightly changes the wind model predictions. Nerney and Suess reach basically the same conclusions as Hartmann and Cassinelli, namely that a very large magnetic field and rotation rate are needed to explain the winds from Wolf-Rayet stars. Their treatment of the radiation force is unnecessarily restrictive, however, since all of the lines cannot be optically thick, and a general distribution of line strengths can be treated by using the CAK approach. Since their line force is proportional to the velocity gradient, the topology of their solutions is completely different from the general case that was treated by Friend and MacGregor (1984). Nerney and Suess claim that it is impossible to find consistent radiation-driven wind models, for a distribution of optical depths in the lines, for large magnetic fields and rotation rates. This claim is incorrect, since the Friend and MacGregor model produces complete and unique solutions for a given set of stellar parameters, even those with very large magnetic fields and rotation rates.

None of these rotating wind models for Wolf-Rayet stars have considered the effects of the large asymmetry of the wind that should be produced by rapid rotation. Asymmetries can have a major effect on predicted radio fluxes and P Cygni profiles. As a result, basic properties such as mass-loss rates that have been derived using spherically symmetric models must be reconsidered.

In this paper, we will improve the Friend and MacGregor model by adding the correction for treating the star as a uniform disk instead of a point source of radiation (Friend and Abbott 1986). The model is essentially the same one that was recently applied to Be stars (Poe and Friend 1986). The basic stellar parameters we have to specify are the mass (M), the radius of the star (R), the luminosity (L), the equatorial rotational velocity (v_{rot}), and the radial surface magnetic field strength (B_0). For a given set of these parameters, a unique wind solution must be found that passes through two CAK–type critical points. Once this solution is found (by the shooting method; see Poe and Friend), we then have the mass-loss rate, terminal velocity, and radial and azimuthal velocity profiles. The approach taken in this paper is to study the dependence on magnetic forces. We then ask what combination of parameters can give a plausible fit to general Wolf-Rayet wind properties. We shall use three specific stars as a guide in this study, but our goal here is *not* to develop the best possible fit to those stars; that will be done in subsequent papers. Here we shall find that our asymmetric model provides a possible explanation for several perplexing problems, including the wind momentum problem, associated with Wolf Rayet stars in general.

A survey of magnetic and rotational effects is carried out using our "Wolf-Rayet test model" in § II. In § III, we discuss the various possibilities for driving Wolf-Rayet winds in the light of our new axisymmetric model. We present results for two models, the combined force model and a high-luminosity purely radiative model, that fit major observational constraints on Wolf-Rayet winds in § IV. We summarize our conclusions in the final section.

II. THE TRANSITION FROM A PURELY RADIATIVELY DRIVEN WIND TO A MAGNETICALLY DRIVEN WIND

a) *The Test Model*

In this section we consider the effects on winds of adding centrifugal and magnetic forces. In the weak field slow rotation case, a star will have a mass-loss rate and wind velocity that is well described by the recent Friend and Abbott (1986) update of the CAK theory. Using the stellar luminosity for a star that satisfies the interior theory mass-luminosity relation (eq. [1]), we find the classic problem that the predicted wind momentum, $\dot{M}v_\infty$, is far below the observational estimates. In the limit of very fast rotation and very large surface fields, we will recover the Hartmann and Cassinelli (1981) FMR results. In this section we explore the transition between these two limiting cases.

As a test model let us consider rather typical parameters for Wolf-Rayet stars (that are taken to correspond roughly to estimated values for the WC 8 component of CV Ser): $M = 13$ $M_\odot$ (Massey 1981); $\dot{M} = 3 \times 10^{-5}$ $M_\odot$ yr^{-1} (Abbott *et al.* 1986); $v_\infty = 2900$ km s^{-1} (Howarth, Willis, and Stickland 1982). From Maeder's (1983) mass-luminosity relation we get $\Gamma = 0.36$ or $L = 3 \times 10^5$ $L_\odot$. As a first estimate of the stellar radius we choose, $R = 8$ $R_\odot$.

In this section we treat these values as plausible and typical for Wolf-Rayet stars. Observations can show rather wide variations for the Wolf-Rayet properties. For example, for the Wolf-Rayet star CV Ser, the mass-loss rate is estimated from *IUE* observations to be 7.2×10^{-5} $M_\odot$ yr^{-1} by Howarth Willis, and Stickland (1982), while different interpretations of radio observation upper limits yield less than 3×10^{-5} $M_\odot$ yr^{-1} (Abbott *et al.* 1986) or less than 7×10^{-5} $M_\odot$ yr^{-1} (van der Hucht Cassinelli, and Williams 1986). The terminal velocity estimates for the star range from 2300 km s^{-1} (Torres Conti, and Massey 1986) to 2900 ± 300 km s^{-1} (Abbott *et al.* 1986). We shall see that any of these values can be explained with appropriate changes of the theoretical model parameters.

Let us first consider the radiation-driven wind limit, as described in Friend and Abbott (1986) and CAK. The density at $8R_\odot$ is adusted until the electron scattering optical depth, τ_{es}, is equal to $\frac{2}{3}$, resulting in $\rho_0 = 1.0 \times 10^{-9}$ g cm^{-3}. The two radiation force constants k and α of CAK theory are taken from the tables of Abbott (1982), using the highest temperature and middle density values (see Pauldrach *et al.* 1985), yielding $k = 0.18$ and $\alpha = 0.61$. The quantity k is related to the number of optically thick lines and α is a measure of the relative number of optically thin and thick lines. For this test model, the density near the critical point was found to be between the middle and high density values in Abbott's table. If the upper density values in Abbott's table were used, k will increase by a factor ~ 2. Since in the CAK theory, the mass-loss rate is proportional to $k^{1/\alpha}$, the mass-loss rate would increase by a factor of 3.

Using the Friend and Abbott (FA) theory for our test star, we get $\dot{M} = 1.8 \times 10^{-6}$ $M_\odot$ yr^{-1} and $v_\infty = 1900$ km s^{-1}. These values are to be compared with typical $\dot{M}$ and v_∞ estimates for Wolf-Rayet stars of 3×10^{-5} $M_\odot$ yr^{-1} and 2000–3000 km s^{-1}. It would be possible to increase the predicted terminal velocity by simply reducing our estimate for R, because the terminal velocity is proportional to the surface escape speed. However, we choose not to do that here and will consider the effects of such adjustments in our discussion of three specific stars in § IV.

Now let us consider the effect of adding rotational and magnetic forces to our test model. Figure 1 shows the effect of v_{rot} on the mass-loss rates for various surface magnetic field strengths. Note the sharp rise in $\dot{M}$ around $v_{rot} = 350$ km s^{-1}. Even a weak (500 G) field can make a big difference near this rise by increasing the azimuthal velocity of the wind near the star, which increases the centrifugal acceleration there, thus mimicking a higher rotation rate (Poe 1989). Also shown in Figure 1 is the "observed $\dot{M}$." A rotational velocity near 85% of the critical velocity is required to match the observed $\dot{M}$ with this set of stellar parameters.

The mass-loss rate shown in Figure 1 is an estimate of the total mass-loss rate from the star. The quantity that is calculated is the mass-loss rate per solid angle at the equator $(d\dot{M}/d\Omega)_{eq}$ $(= \rho_{eq} v_{eq} r_{eq}^2)$. We find that $4\pi\,(d\dot{M}/d\Omega)_{eq} \equiv \dot{M}_{eq}$ is a good estimate of the total mass-loss rate for our models. To show this, let us assume that the zero rotation rate model represents the flow from the polar region and the rotational model gives the rate of flow from the equator. If we assume that $d\dot{M}/d\Omega$ varies with polar angle as $d\dot{M}/d\Omega = (d\dot{M}/d\Omega)_{pole}$ $+ [(d\dot{M}/d\Omega)_{eq} - (d\dot{M}/d\Omega)_{pole}] \times \sin^n \theta$, we find that the total mass-loss rate

$$\dot{M} \equiv \int_0^{4\pi} \frac{d\dot{M}}{d\Omega}\, d\Omega \qquad (2)$$

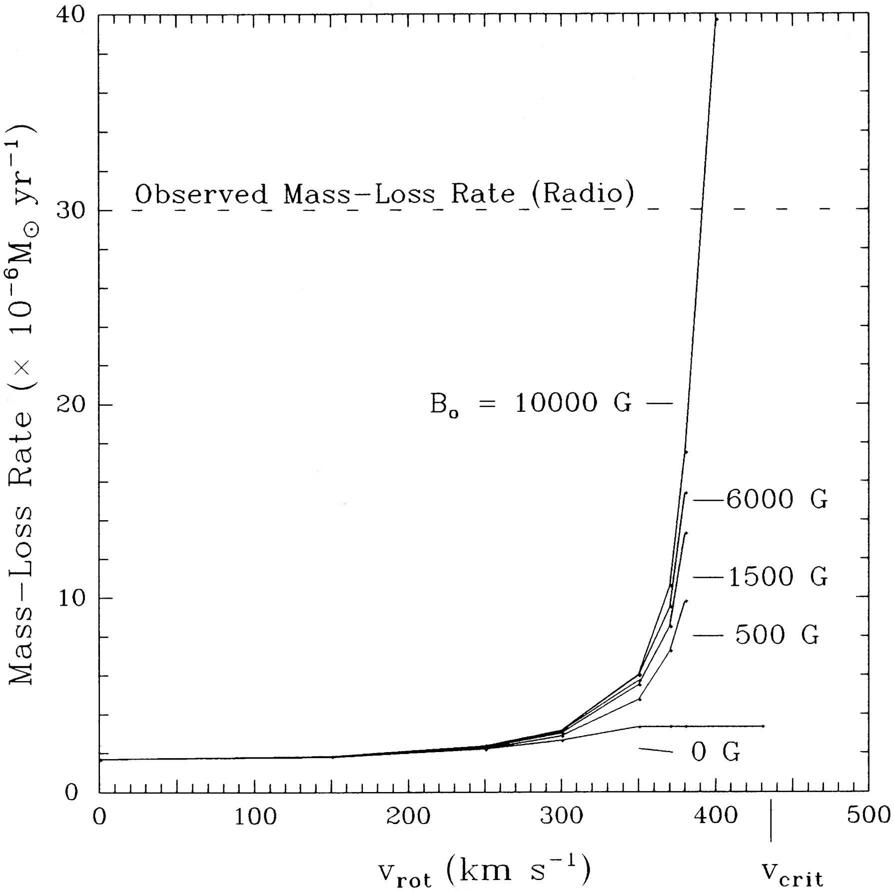


FIG. 1.—Dependence of the equatorial mass-loss rates ($4\pi\, d\dot{M}/d\Omega$) on the rotation rate of the star. Each curve is labeled with the value of B_0, the radial magnetic field at the base of the wind. The mass-loss rate of $3.0 \times 10^{-5}\, M_\odot\, yr^{-1}$, typical of the values inferred from radio observations, is shown by the horizontal dashed line. Such high mass-loss rates are obtainable only for rotational velocities larger than 80% of the the critical velocity. The value of this critical velocity is indicated on the horizontal axis.

is easily integrated. For example, for $n = 1$ we get

$$\dot{M} = \dot{M}_{eq}\left[\frac{\pi}{4} + \left(1 - \frac{\pi}{4}\right)\frac{\dot{M}_{pole}}{\dot{M}_{eq}}\right], \quad (3)$$

where $\dot{M}_{pole} = 4\pi\rho_{pole} v_{pole} r^2_{pole}$. In the limit $\dot{M}_{eq} \gg \dot{M}_{pole}$ the equatorial mass-loss rates given in this paper exceed the total derived from equation (3) by $\sim$20%. If the density in the wind is strongly concentrated toward the equator, then n will be much larger than 1. For a case in which $n = 9$ and $\dot{M}_{eq} = 5 \times \dot{M}_{pole}$, the total mass-loss rate would be about half the equatorial value. Our conclusion that $\dot{M}_{eq}$ is a reasonably good estimate of the total mass-loss rate differs from the conclusion of Hartmann and MacGregor (1982). They assumed that the mass flow was confined to an opening angle of 10° from the equator, while in our Wolf-Rayet models there is substantial mass flow even from the polar zones.

The effect of v_{rot} on the terminal velocity for various values of B_0 is shown in Figure 2. For small field strengths, increasing the rotation rate will decrease v_∞. This occurs because the effective escape speed on the equator is reduced, and v_∞ in radiation-driven wind theory is proportional to the escape speed. However, for large B_0, the terminal velocity can increase with rotation rate because of the increased magnetic force at large radii where the field lines become curved. When the rotation rate becomes quite large ($\geq$300 km s^{-1}), the mass flux increases significantly and v_∞ decreases as a function of v_{rot} for all field strengths. Figures 1 and 2 also show the typical observational estimates of the mass-loss rates and terminal velocities for Wolf-Rayet stars. Note that to fit these values with the equatorial flow from our test model would require a field strength of 10^4 G and a rotation rate of $\sim$90% the critical value.

b) *Comparison to Fast Magnetic Rotator Theory*

If the luminosity of Wolf-Rayet stars is given by Maeder's mass-luminosity relation (eq. [1]), then a very rapid rotation rate and large surface magnetic field are required to fit the observed $\dot{M}$ and v_∞ along the equator. A similar result was described by Cassinelli (1982) concerning the fast magnetic rotator (FMR) model as applied to Wolf-Rayet stars. To study the transition from the line-driven model to the FMR model, we solved the equations for a rotationally and magnetically driven wind similar to the Weber and Davis (1967) solar model as given by Belcher and MacGregor (1976) and approximated the FMR solution as in Hartmann and MacGregor (1982). For the FMR model, the two critical points in the wind solution are particularly simple to find. The slow (inner) critical point becomes the sonic point, and the fast (outer) critical point becomes the point where the flow velocity is equal to the

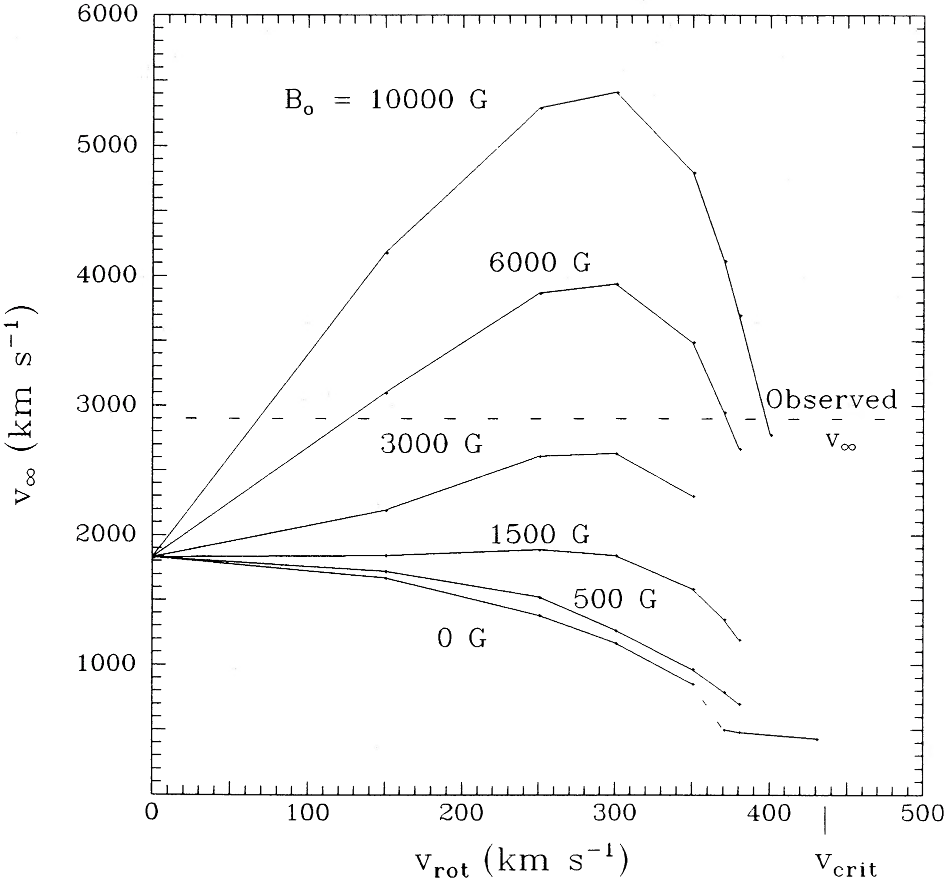


FIG. 2.—Dependence of the equatorial wind terminal velocity, v_∞, on the rotational speed of the star. Each curve is labeled with the value of B_0 in gauss. Horizontal dashed line is a typical observed terminal velocity of a Wolf-Rayet star wind that is derived from ultraviolet P Cygni profiles. The break in the zero magnetic field curve at $v_{rot} = 350$ km s^{-1} is due to the sudden drop in the terminal velocity at the maximum mass-loss limit (see text).

Michel velocity, v_M, given by

$$v_M^3 = \beta^2 \frac{GM(1-\Gamma)}{R^3} \frac{(R^2 B_0)^2}{\dot{M}_{eq}}, \quad (4)$$

where β is the ratio of rotational velocity to the critical velocity (approximately unity), B_0 is the surface magnetic field strength, and Γ is the ratio of the continuum radiation pressure to gravity. This fast point occurs far out in the wind so that the Michel velocity is nearly equal to v_∞. For a given set of stellar parameters, equation (4) then gives a relation between v_∞ and $\dot{M}$. To find $\dot{M}$ for the FMR model, we first have to locate the inner critical point. Assuming that the magnetic field enforces solid-body rotation, the inner critical point, r_{c1} (or sonic point), is given by (see eq. [10] of Hartmann and MacGregor)

$$\frac{r_{c1}}{R} = \beta^{-2/3}. \quad (5)$$

Conservation of energy in the subsonic regions can be used to calculate the radial velocity at R (see eq. [13] of Hartmann and MacGregor). The density at R has to be assumed. The mass-loss rate in the FMR model depends only on the rotation rate and is independent of the assumed magnetic field strength for the fields that are strong enough to enforce solid-body rotation inside the sonic point. The upper limit to $\dot{M}$ in the FMR model, for a given set of stellar parameters, occurs at $\beta = 1$ when, according to equation (5), the sonic point moves into the star. For our test model, in which $\rho_0 = 1 \times 10^{-9}$ g cm^{-3}, this maximum mass-loss rate is $1.2 \times 10^{-4}\ M_\odot$ yr^{-1}.

Figure 3 shows how $\dot{M}$ various with v_∞ for our solutions with the test model parameters. The solid curves show the behavior of our solutions as the rotation rate increases, while holding B_0 constant (as in Figs. 1 and 2). The dotted curves are the lines of constant rotation rate. The straight dashed lines show the behavior of the Michel velocity as determined from equation (4) using $\beta = 1$. Note that for low rotation rates, the $\dot{M}$ for our models increases slowly with the rotation rate, while at high rotation rates our results for $\dot{M}$ versus v_∞ eventually merges with the Michel velocity lines. We should point out, however, that even in the high rotational velocity regime, the mass-loss rates that our combined force model produces can be significantly larger than predicted by pure FMR theory. For example, a rotation rate of $\beta = 0.8$ in the FMR model would give a mass-loss rate of only $5 \times 10^{-9}\ M_\odot$ yr^{-1}, while, as is seen in Figure 3, it would lead to a mass-loss rate of $6 \times 10^{-6}\ M_\odot$ yr^{-1} in our model. This is because the radiation forces in our model provide a minimal mass-loss rate ($\sim 2 \times 10^{-6}\ M_\odot$ yr^{-1} for the test model); rotation and magnetic fields only increase it from there, as is shown in Figure 3. Only when β is greater than ~ 0.96 does the FMR mass-loss rate become nearly equal to that of our model. Clearly the line radiation force should not be ignored in considering the winds of rapidly rotating stars with strong magnetic fields. Perhaps the most important conclusion from Figure 3 is that the combined force

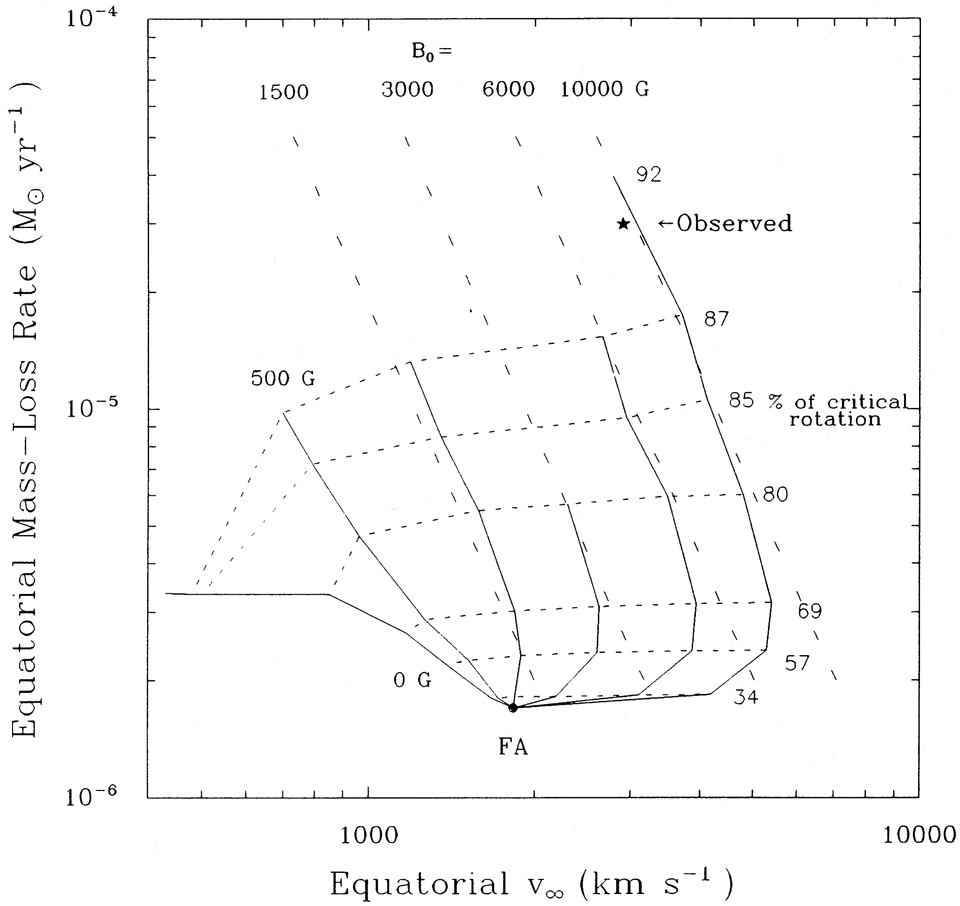


FIG. 3.—Dependence of the equatorial mass-loss rate ($4\pi\, d\dot{M}/d\Omega$) and terminal velocity on the rotation rate and magnetic field. Solid lines are the model results. Star symbol gives the mass-loss rate and terminal velocity adopted for CV Ser. The point marked "FA" gives the values of $\dot{M}$ and v_∞ expected from the Friend and Abbott (1986) line-driven wind model. Long dashed lines running diagonally across the diagram give the results expected from the fast magnetic rotator model (FMR) of Hartmann and Cassinelli (1981); each line is labeled with the assumed surface magnetic field. Short dashed lines running roughly horizontally across the figure connect points with equal rotation rates; the percentage of maximal rotation speed ($\beta \times 100$) is indicated on each of these lines.

model can give rise to mass-loss rates on the order of $10^{-5}\, M_\odot$ yr^{-1} even for fields significantly less than 10^4 G, but the flow would have a greatly reduced equatorial terminal velocity.

Figures 1, 2 and 3 also show the zero magnetic field strength model. Increasing the rotation rate leads to a larger mass-loss rate and lower terminal velocity of the wind. The direction of the zero field line in Figure 3 corresponds to $\dot{M} v_\infty =$ constant, where the constant is set by the Friend and Abbott (FA) model. So the initial increase in $\dot{M}$ can be explained as follows. The rotation reduces the effective surface gravity and escape speed. This reduces the wind speed. The radiation field can transfer the momentum flux $(\dot{M} v_\infty)_{\rm FA}$ to the wind. Therefore the mass flux from the equator can increase inversely proportional to the escape speed. There is a limit, however, which is explained in detail by Poe (1987, 1989). As the rotation rate increases, the mass-loss rate increases only as high as the value derived by Castor (1979) for rotating radiation-driven winds. Castor used the CAK radiation force which assumes a central point source of continuum radiation. He found that increased rotation does not increase the mass-loss rate, but merely decreases the flow speed. For the point source case the critical point moves to larger radii as the rotation rate is increased. In our models, the solution curve that leads to continuous flow to infinity shifts from one that has a critical point near the star to a solution like that of Castor (1979) at very rapid rotation. The net effect is that for the case with no magnetic field we cannot drive a mass-loss rate larger than a value near Castor's result. The flat portion shown in Figure 3 for the zero B field curve corresponds to this limiting $\dot{M}$, and the last point shown is for a rotation rate of $\sim 99\%$ maximum. It is the largest value of the mass-loss rate that can be driven by rotation and radiation alone and is about a factor of 2 larger than the mass-loss rate from the zero rotation rate model.

c) *Spin-down Times*

We have confirmed that the fast, massive winds of the Wolf-Rayet stars could, in principle, be explained by fast rotator models. However, can such models be ruled out by other considerations? We have argued before that large rotation rates and magnetic fields might be hidden from direct observation by the dense wind. However, another important consideration is the time scale for the star to lose its angular momentum through magnetic braking. We can estimate a time scale for angular momentum loss using the expression from Friend and MacGregor (1984);

$$\tau_{\rm spin} = \frac{3}{5}\left(\frac{M}{\dot{M}}\right)\left(\frac{R}{r_{\rm A}}\right)^2 , \qquad (6)$$

where r_A is the Alfvén radius. This equation assumes the star is rotating as a solid body, and that might be a reasonable assumption for Wolf-Rayet stars since they are thought to be the remnant convective cores of O stars.

For the best fit models assuming Maeder's luminosity, we find that $\tau_{spin} = 3000$ yr for our test model. This is much less than the nominal lifetime of a Wolf-Rayet star of 10^5 yr or more. A Wolf-Rayet star could not maintain the high rotation rate needed to produce its large mass-loss rate if at the same time it must have a very large magnetic field to achieve the observed terminal velocities (see Fig. 2).

III. A TWO-COMPONENT ASYMMETRIC WIND MODEL

a) *Introduction*

The results of the previous section have shown that is possible to use rotation and magnetic fields to overcome the Wolf-Rayet momentum problem. However, this has the effect of introducing an equally perplexing "spin-down" problem. Here we suggest that the new problem can be overcome by considering a plausible alternative to the FMR picture. The primary difficulty can be traced to the fact that we have been requiring a large magnetic field to drive a *high-speed* flow in the equatorial plane. The flow from the polar regions, on the other hand, should be largely unaffected by rotation and a large field is not required to drive a fast wind. Radiation pressure alone is sufficient to produce a high-speed flow from the polar regions. Let us consider, then, the consequences of a model in which the observer's line-of-sight intersects the high-speed wind from the polar regions, while the slower equatorial zone provides a region of higher density that can affect the radio continuum and optical line emission.

b) *The Radio Flux*

Consider the observational determination of the mass-loss rates from the radio measurements of the free-free emission in the wind (Abbott *et al.* 1986). For rapidly rotating stars, the emission will be predominantly from the high-density regions in the equatorial plane. To estimate the flux at 6 cm from our axisymmetric model, we assume that the flow is purely radial in the region where the radio emission is formed and that the radial dependence of the density is proportional to $1/r^2$. The density distribution versus polar angle, θ, is assumed to be of the form

$$\rho(r, \theta) = \{\rho_{\rm pole}(r_0) + [\rho_{\rm eq}(r_0) - \rho_{\rm pole}(r_0)] \sin^n \theta\}(r_0/r)^2 , \quad (7)$$

where $\rho_{\rm pole}$ and $\rho_{\rm eq}$ are the radial distribution of density along the pole and equator, respectively, evaluated at a large radius r_0.

Wright and Barlow (1975) showed that the radio flux caused by free-free emission in a spherically symmetric wind is proportional to an integral of $1 - e^{-\tau(p)}$ through the wind, where p is the impact parameter and τ is the optical depth along p. The integral for our axisymmetric model is very similar to the Wright and Barlow integral except that the optical depth is modified by the geometry assumed in equation (7). For our model, seen pole-on, the radio flux (in janskys) is given by

$$S_\nu({\rm Jy}) = 23.2\left(\frac{1}{D}\right)^2\left[\left(\frac{Z}{\mu}\right)\left(\frac{\dot{M}}{v_\infty}\right)_{\rm pole}\right]^{4/3} G^{2/3} , \quad (8)$$

where D is the distance to the star in kiloparsecs, Z is the ionic charge, μ is the mean atomic weight per nucleon, and the ratio $(\dot{M}/v_\infty)$ has replaced the density. The geometrical correction factor, G, is the ratio of the optical depth along p for our model to the optical depth along p for a spherically symmetric model. It is given by

$$G = 1 + \begin{pmatrix} 4 \\ \text{or} \\ 8/\pi \end{pmatrix}\left[\frac{(n+1)!!}{(n+2)!!}\right]\left(\frac{\Delta\rho}{\rho}\right) + 2\left[\frac{(2n+1)!!}{(2n+2)!!}\right]\left(\frac{\Delta\rho}{\rho}\right)^2 , \quad (9)$$

where

$$\frac{\Delta\rho}{\rho} = \frac{\rho_{\rm eq} - \rho_{\rm pole}}{\rho_{\rm pole}} = \frac{(\dot{M}/v_\infty)_{\rm eq} - (\dot{M}/v_\infty)_{\rm pole}}{(\dot{M}/v_\infty)_{\rm pole}} , \quad (10)$$

and $n!! = n(n-2)(n-4)\cdots$. If n is even, the first coefficient in equation (9) is 4, and, if n is odd, the coefficient is $8/\pi$. For the case where $n = 1$, G reduces to

$$G = 1 + \frac{16}{3\pi}\left(\frac{\Delta\rho}{\rho}\right) + \frac{3}{4}\left(\frac{\Delta\rho}{\rho}\right)^2 . \quad (11)$$

Note that for $\Delta\rho = 0$, the model is spherically symmetric and we recover the Wright and Barlow formula.

For our test model let us take from Abbott *et al.* (1986) the values for Z, μ, D, and the value for S_ν that they tabulate for CV Ser. Let us assume that v_∞ along the polar regions is the observed v_∞ (2900 km s^{-1}) and that $\dot{M}_{\rm pole}$ is the value from our zero rotation and zero magnetic field model ($1.8 \times 10^{-6}\ M_\odot$ yr^{-1}). To match the radio observations requires that

$$\frac{S_\nu}{S_\nu({\rm obs})} = \left[\left(\frac{\dot{M}}{v_\infty}\right)_{\rm pole}\left(\frac{\dot{M}}{v_\infty}\right)_{\rm obs}^{-1}\right]^{4/3} G^{2/3} \sim 1 , \quad (12)$$

where $(\dot{M}/v_\infty)_{\rm obs}$ is the quoted value in Abbott *et al.* assuming a spherically symmetric model. In our model, $(\dot{M}/v_\infty)_{\rm pole}$ is much smaller than $(\dot{M}/v_\infty)_{\rm obs}$, therefore, the value of G must be large to obtain the observed radio flux. For our test model, $G = 278$.

For a given $v_{\rm rot}$ and B_0, our model produces an $\dot{M}$ and v_∞ from which $\Delta\rho/\rho$ and G can be calculated. Assuming $n = 1$, Figure 4 shows the normalized radio flux from equation (12) as the rotation rate of the star is varied, for field strengths of 500, 1500, and 10,000 G. At low rotation rates, the density in the wind is too small to produce the observed radio flux. Only at high rotation rates will the density in the equator be large enough. For the $B_0 = 500$ G model, $\beta \approx 0.87$ is required to obtain the observed radio flux. It is interesting to note in Figure 4 that a large radio flux can be obtained even with a relatively small magnetic field. This fact will be used to help explain the spin-down problem.

Van der Hucht, Cassinelli, and Williams (1986) have argued that the mass-loss rates of W-R stars are even larger than have been derived by Abbott *et al.* (1986) because of ionization effects and the larger mean mass per electron in WC stellar winds. Similarly, Schmutz and Hamann (1986) found that the dominate stage of ionization at radio optical depth unity for most Wolf-Rayet stars is He II, not He III as assumed by Abbott *et al.* Although both of these studies suggest that the value of Z/μ in equation (8) is lower than in Abbott *et al.*, in this paper we have chosen to use the value of Z/μ from Abbott *et al.* If the correct value of Z/μ is actually smaller, then we have to increase the value of G in equation (8) to obtain the observed radio flux. As shown in Figure 4, the radio flux (and hence G) increases very rapidly with increasing rotation rate.

If the density of the wind is concentrated toward the equator, n will be larger than 1. The coefficents in equation (9) become smaller with increasing n. This result is due to the decreasing number of electrons in the mid-latitudes of the wind

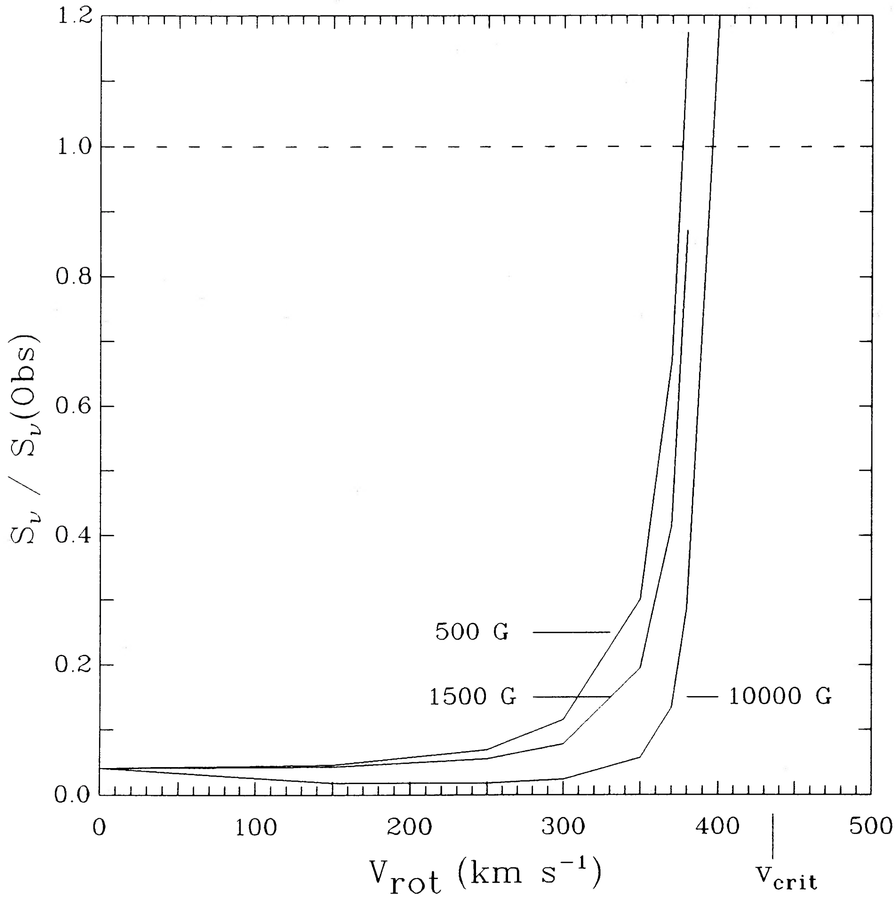


FIG. 4.—The free-free radio flux at 6 cm, expressed as a ratio to the observed value, vs. the rotation rate for the two-component Wolf-Rayet model seen pole-on assuming $n = 1$ in eq. (9). Each curve is labeled with the assumed magnetic field at the base of the wind, B_0. In the low rotational velocity limit, corresponding to FA in Fig. 3, the predicted radio flux is only 3% of the observed value. For large rotation rates, it is seen that there is a wide range of magnetic fields such that large radio fluxes can be produced.

as the concentration toward the equator is increased. So for larger n, the values of $\Delta\rho/\rho$ have to increase. For example, in order to obtain $G = 278$ of our test model with $n = 9$, $\Delta\rho/\rho$ has to increase by a factor of 1.5 compared with the $n = 1$ case. The $B_0 = 500$ G model in Figure 4 will have to be rotating at $\beta \approx 0.90$ to satisfy the radio flux.

c) *Resolution of the Spin-down and Momentum Problems*

Figure 5 illustrates a possible resolution of the spin-down problem in stars that can also produce the large radio fluxes of Wolf-Rayet stars. Figure 5 contains the model data as Figure 3, but it also contains a straight line, on which the model stars could produce the observed radio flux if seen pole-on. On that isoradio flux line are indicated the spin-down times of the models. For Figure 5, we have calculated the radio flux as in equation (12) and as shown in Figure 4. To estimate the spin-down times we need an estimate of the Alfvén radius. For this we used equation (20) of Hartmann and MacGregor (1982), which assumes that the winds are in the FMR regime, therefore

$$\left(\frac{r_A}{R}\right)^2 = \frac{3}{4\beta^2 Z_P}\left(\frac{v_\infty}{a}\right)^2_{Eq} + \frac{1}{\beta^2} + \frac{1}{2}, \qquad (13)$$

where a is the isothermal sound speed, and Z_P is the Parker radius,

$$Z_P = \frac{GM(1-\Gamma)}{2Ra^2}, \qquad (14)$$

which is ~ 100 for our model. The magnetic field enters equation (13) through the terminal velocity, from equation (4). For small fields, the Alfvén radius is close to the star and the spin-down time increases inversely with $\dot{M}$. For large fields the spin-down time is a much stronger function of $\dot{M}$. This is because r_A in equation (13) is proportional to v_A in the strong field case, so the spin-down time is inversely proportional to $\dot{M}v_\infty^2$. Now if we use the fact that the observed radio flux is proportional to $\dot{M}/v_\infty$ in the large field limit, the spin-down time is inversely proportional to $\dot{M}^3$. Figure 5 shows that for stars that satisfy the radio flux and that have surface fields less than $\sim 10^3$ G, the spin-down times can be longer than $\sim 10^5$ yr.

Another interesting result of our moderate field two-component model is that the mass-loss rate is much smaller than the rate inferred from the spherically symmetric model. Consider the test model in Figure 5 that has $\log(\tau_{spin}) = 5.1$. The equatorial mass-loss rate is $\dot{M} = 7 \times 10^{-6}\ M_\odot$ yr^{-1} (which, as argued before, is close to the "true" $\dot{M}$; see § IIa). This value is well below the quoted rate of $3 \times 10^{-5}\ M_\odot$ yr^{-1} that was derived assuming a spherically symmetric model. Nevertheless, the smaller mass-loss rate is sufficient because the ratio flux depends on the density of the wind, which is proportional to $\dot{M}/v_\infty$. Thus, since the terminal velocity on the equator is low, owing to the rapid rotation, the mass-loss rate can be relatively low to produce a given radio flux. The mass-loss rate that is inferred from the radio flux must be very large

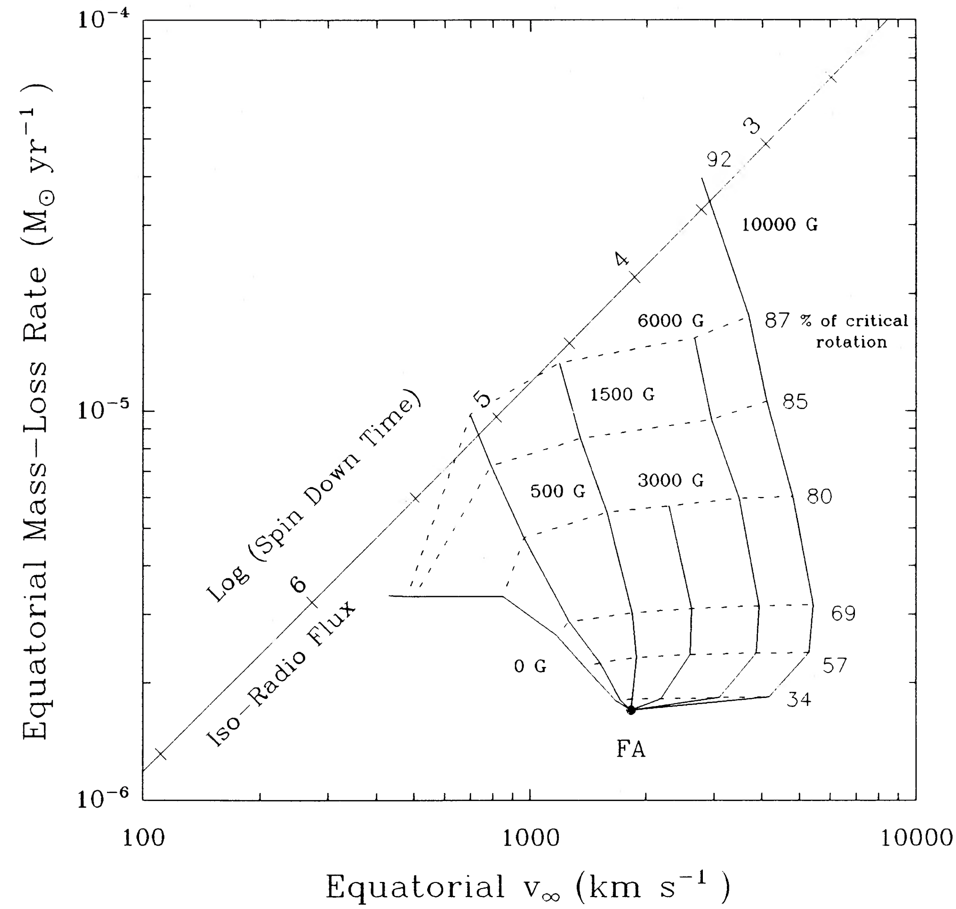


FIG. 5.—Illustrates the spin-down problem and its possible resolution. The diagonal solid line gives values of equatorial mass-loss rate $\dot{M}$ and v_∞ that satisfy the radio flux observations. As in Fig. 4 the (500 G, 87%) and (10,000 G, 92%) pair of B_0 and β both fit the radio flux, but with different spin-down times. Along the isoradio flux line are given several values of the logarithm of the spin-down time in years. The models assume that the wind density varies with polar angle in proportion to sin $\theta(n = 1)$ in eq. (7).

only if the outflow is assumed to be occurring at a large velocity.

Note also what our two-component model implies about the momentum problem. The product of $\dot{M}$ and v_∞ determines the wind momentum flux, and in our model these are never both large in the same latitude in the wind. Therefore, the momentum flux in the wind is comparable to the momentum flux in the radiation field (although it is actually the centrifugal force which produces most of the equatorial momentum flux). The momentum problem only exists when the radio fluxes are interpreted in terms of spherically symmetric models, while in fact the radio observations demand a large density, but not necessarily a large momentum flux.

We have shown that a two-component picture for Wolf-Rayet winds can explain both the "momentum problem" and the spin-down problem. The model requires that the equatorial wind be dense, but flowing at a velocity significantly slower than that from the polar zone.

d) *Inclination Effects*

The radio flux, S_ν, was derived for our model under the assumption that the star is viewed pole on. This radio flux should be reasonably accurate even for inclinations approaching 90° as can be inferred from the study of Schmid-Burgk (1981). He assumes that the meridional density distribution is a rotated ellipse. This distribution is simiar to ours for the case $n \geq 9$. He shows that the flux is insensitive to the aspect angle. For example, applying our test model polar and equatorial values to Schmid-Burgk's density distribution, the radio flux from this model seen edge-on ($i = 90°$) is ~ 0.6 of the radio flux seen from the pole-on orientation. Schmid-Burgk further concludes that the mass-loss rates that are derived from the spherical approximation are correct in spite of the wind asymmetry. However, this latter conclusion requires the additional assumption that v_∞ is the same at all polar angles. In our model, v_∞ is a strong function of polar angle, being fast at the pole and slow at the equator. So we get the larger density in the equatorial regions but at a specific mass-loss rate that is small at the equator. The mass-loss rate of our model is smaller than would be inferred from Abbott *et al.* (1986) using a spherically symmetric assumption.

From our model one should expect that observations would show a wide range of terminal velocity from one star to another because of the different inclination angles of their polar axes relative to the line of sight. Torres, Conti, and Massey (1986) show a plot of terminal velocities versus WC spectral class. There does in fact appear to be a rather wide range for v_∞ even within one spectral class. Barlow, Roche, and Aitken (1988) have noted that for Wolf-Rayet stars in binary systems, the shortward-shifted absorption edge of a P Cygni profile can be masked by the saturated profile from the higher terminal velocity flow from the comparison O star. For γ^2 Vel, Barlow, Smith, and Willis (1981) derived a terminal velocity of 2000 ± 200 km s^{-1} from *IUE* observations, which could be contaminated by the O star. Using infrared lines that should form in the outer wind of the Wolf-Rayet star in γ^2 Vel, Barlow *et al.* (1988) derive $v_\infty = 1500 \pm 200$ km s^{-1}. Similar large differences in terminal velocity estimates have been derived for V444 Cyg by Barlow *et al.* (1981) and Underhill and Fahey (1987). This terminal velocity problem for V444 Cyg is under investigation by van der Hucht, Cassinelli, and Meade (1988). Another property of the resonance P Cygni lines in W-R stars that may also be relevant to our model is the shape of shortward edge of the absorption trough. Theoretically, the shortward edges of saturated P Cygni lines are expected to be vertical if the lines are formed in a spherical monotonic outflow, but observationally the edges extend over a finite interval. The violet edge of C IV line in the spectrum of HD 93131, for example, has a width of 540 ± 150 km s^{-1} (Willis 1982; Lucy 1983). V444 Cygni shows a range of 560 ± 200 km s^{-1} (van der Hucht, Cassinelli, and Meade 1988). Lucy (1983) has explained the width of the violet edges in the context of a chaotic wind model, one having a nonmonotonic velocity distribution. In the context of that model the width indicates that wind velocities for Wolf-Rayet stars have been overestimated by ~ 600 km s^{-1}. However, in the context of our asymmetric model, the width could in part be due to the fact that different lines of sight to the photosphere intercept different maximal flow speeds because of the latitude dependence of the wind. We intend to investigate such a possibility in the future; however, we feel at the present time we can argue, at the very least, that observations do not rule out the possibility of a latitude-dependent terminal velocity.

While focusing just on the effects of a density enhancement in the equatorial plane, Rumpl (1980) showed that a variety of P Cygni profiles could be formed in an asymmetric wind. He found that an enhanced mass-loss rate in the equatorial regions produces P Cygni profiles for HD 50896 which are in better agreement with the observed profiles than those calculated assuming spherical symmetry.

It might appear that the low polar mass-loss rates of our model would enable us to see the hydrostatic surfaces of Wolf-Rayet stars if they are observed near the pole-on orientation, since the mass-loss rates we find are more characteristic of O stars. However, we may not be able to see the surface, since the smaller radii of Wolf-Rayet stars means that the *density* is much higher at their surfaces. The optical depth of a Wolf-Rayet wind is then much higher than in an O star with the same, $\dot{M}$, and the surface may still be unobservable. Our models should probably assume a higher optical depth on the pole, but we found that changing the optical depth had very little effect on the mass-loss rate and the density at the sonic point. Fitzpatrick (1982) and Massey (1980) have discussed the narrow absorption lines that are seen in the spectra of some Wolf-Rayet stars, which they interpret as being "photospheric" absorption. While this interpretation is considered doubtful because many of these stars are known to be binaries with an O star companion, some evidence may exist indicating that some absorption lines may be intrinsic to the Wolf-Rayet star (Schmutz, Hamann, and Wessolowski 1988*a*). Also, even for the case of a single star, we should expect equatorial regions to produce broad, strong emission lines, which should make it difficult to see any narrow absorption features superposed against the emission. In addition, the rapid rotation required by our model would broaden and weaken any intrinsic photospheric absorption lines.

e) Inclination Effects in Binary Systems

Our two-component explanation of the momentum problem requires that the line of sight intercept some of the high-speed wind material emerging from the polar regions. There are estimates of the inclination angle to the three Wolf-Rayet stars V444 Cyg, CV Ser, and γ^2 Vel that are discussed in detail in the next section. Given the inclination angle and observed line-of-sight terminal velocity gives us some information on the dependence of the wind polar angle. For V444 Cyg, the inclination angle is 72° (Massey 1981) and the "observed" terminal velocity is 2500 km s^{-1}. Although there is some possibility that the velocity is an overestimate because the O star wind could be responsible for part of the P Cygni line absorption. In any case, we might require that the low-velocity equatorial flow be confined roughly to the zone $i = 75°$–$105°$. Alternatively, the rotational axis of the W-R star and the orbital axis might not coalign because there has been insufficient time for tidal forces to enforce alignment of the axes during the rather short Wolf-Rayet phase of the star. For the other two stars, the inclination constraints pose even less of a problem for our model. For CV Ser, there are only wind eclipses (i.e., the photosphere is not eclipsed), as has been shown by Massey and Niemela (1981) and Eaton, Cherepashchuk, and Khaliullin (1985). The binary system γ^2 Vel shows only wind eclipses at UV wavelengths (Howarth, Willis, and Stickland 1982)

f) The Base Magnetic Field

We have seen that the two-component model allows for an explanation of the Wolf-Rayet winds using a magnetic field that is significantly smaller than in the FMR model. A smaller field can work because we now do not require the equatorial flow to reach speeds as high as those occurring in the polar zones. The lower velocity produces a larger density in the equatorial zone of the wind and thereby provides an explanation of the very large radio flux from Wolf-Rayet stars. The smaller field resolves the short spin-down problem of the FMR model.

Given that a smaller field is better than a larger one for explaining the Wolf-Rayet stars, why should we have *any* magnetic field in the model? Figure 5 shows the line for the zero magnetic field case. Unless the inferred density in the wind is significantly less than the Abbott *et al.* (1986) model predicts, the zero magnetic field model is not quite sufficient to explain the large radio fluxes associated with the Wolf-Rayet stars. All improvements to the Abbott *et al.* model tends to push the isoradio flux line in Figure 5 away from the zero magnetic line. Our numerical studies of the Wolf-Rayet stars suggest that they have fields ranging from several hundred to several thousand gauss if the observations are to be explained with our two-component picture.

A recent theoretical study by Maheswaran and Cassinelli (1988) has led to additional support for magnetic fields of ~ 1000 G in rapidly rotating Wolf-Rayet stars. They have found that Eddington-Vogt circulation currents in the stellar envelope lead to a minimal magnetic field that can penetrate into the outer atmosphere. Weak fields are dominated and suppressed by the circulation. The minimal field that they have derived for CV Ser is ~ 1500 G, quite close to the value we find is needed to overcome the Wolf-Rayet wind and spin-down problems.

g) *Summary of the Two-Component Model*

Models 1, 2, and 3 in Table 1 show the results for our test star. The first two columns in Table 1 are the model number and the star name. The next five columns are the assumed basic stellar parameters $R/R_\odot$, $M/M_\odot$, Γ, B(G), and $\beta = v_{rot}/v_{crit}$. Column eight is the spin-down time scale in years (eq. [6]) for the rotating models. The last five columns give the terminal velocity (in km s^{-1}) along the polar and equatorial regions, the mass-loss rate ($\times 10^{-6}$ $M_\odot$ yr^{-1}) along the polar and equatorial regions, and calculated radio flux relative to the observed radio flux.

Model 1 is a purely radiation-driven wind model assuming the luminosity from Maeder's mass-luminosity relation (eq. [1]). The radio flux is much smaller than the observed flux because the low luminosity cannot drive a dense enough wind. Model 2 in Table 1 shows our two-component model again assuming the luminosity from Meader's relation. This model has a fast polar wind and a dense equatorial wind. The spin-down time of 3×10^5 yr is consistent with estimated Wolf-Rayet star lifetimes.

Model 3 increases the luminosity until the "observed" mass-loss rate is matched. The radio flux is too large in this model because the terminal velocity is too small and the density is too high. The terminal velocity can be increased by reducing the stellar radius, which we will do in the next section.

IV. APPLICATION OF THE TWO-COMPONENT WIND-MODEL TO SPECIFIC WOLF-RAYET STARS

a) *V444 Cyg*

This totally eclipsing binary system (WN 7 +O7) has been studied in great detail to derive information concerning the wind density structure (CEK). We can therefore compare the density distributions from the models with these observations. The basic parameters that we have adopted for the W-R star in V444 Cyg are $M_{WN} = 10$ $M_\odot$ (Massey 1981), $\dot{M} = 1.2 \times 10^{-5}$ $M_\odot$ yr^{-1} (Abbott *et al.* 1986), $v_\infty = 2500$ km s^{-1} (CEK) and $i = 72°$ (Massey 1981). From the variation in opacity with wavelength, CEK have estimated the temperature in the wind. At the "photosphere" where $\tau_{es} = \frac{2}{3}$ and $R = 2.9$ $R_\odot$, the temperature is between 80,000 and 100,000 K, resulting in a luminosity of 2×10^5 to 5×10^5 $L_\odot$. The lower luminosity is in agreement with Maeder's mass-luminosity relation (eq. [1]). The higher luminosity was used by Pauldrach *et al.* (1985) to fit the observed velocity law successfully. We shall consider both models, but with special emphasis placed on fitting the observations using a low-luminosity model.

Since the velocity at $\tau_{es} = \frac{2}{3}$ is supersonic, we must start the integration inside R. We varied both the starting radius and density until we found a good fit to the density structure for the given rotation rate and field strength. Small changes in the starting radius and density did not affect the results. All of these solutions start at $R = 2.3$ $R_\odot$ with $\rho_0 = 1 \times 10^{-9}$ g cm^{-3} for the high-luminosity model and $\rho_0 = 5 \times 10^{-7}$ g cm^{-3} for the low-luminosity model. The CAK constants k and α are 0.25 and 0.65, respectively, from an extrapolation of the high-density values in Abbott's (1982) table to higher effective temperatures.

For the low-luminosity case, Figure 6 shows the velocity and density structure from a high magnetic field model and compares them with the results of CEK. For a rotation rate of 84% and a surface magnetic field strength of 10 kG, a good fit to data is obtained. This result is similar to that discussed for the test model, in that when the luminosity is obtained from Maeder's mass-luminosity relation, large rotation rates and magnetic field strengths are required to fit the observations. However, as explained before, this type of model has too short a spin-down time ($\sim$4000 yr).

We have also computed a two-component model for V444 Cyg, as shown in Figure 7. For the low-luminosity parameters, the polar wind reaches a terminal velocity of $\sim$4000 km s^{-1} with a mass-loss rate of 1.2×10^{-6} $M_\odot$ yr^{-1}. Figure 7 also shows results for the equatorial wind model which has a magnetic field of 1500 G and a rotational velocity of 85% of critical speed. This produces an equatorial wind with $\dot{M} = 7 \times 10^{-6}$ $M_\odot$ yr^{-1} and $v_\infty = 1070$ km s^{-1}. This combination of polar and equatorial wind models can satisfy the radio flux condition

TABLE 1

MODEL RESULTS

Number	Star	R ($R_\odot$)	M ($M_\odot$)	Γ	B (G)	β	log τ (yr)	$v_{\infty, pole}$ (km s^{-1})	$v_{\infty, eq}$ (km s^{-1})	$\dot{M}_{pole}$[a]	$\dot{M}_{eq}$[a]	S_ν[b]
1.........	WR (test)	8.0	13	0.36	0	0	...	1900	1900	1.8	1.8	0.041
2.........	WR (test)	8.0	13	0.36	500	0.87	5.3	1900	700	1.8	10.	1.1
3.........	WR (test)	8.0	13	0.90	0	0	...	540	540	35.	35.	12.
4.........	V444 Cyg	2.3	10	0.32	0	0	...	4000	4000	1.2	1.2	0.025
5.........	V444 Cyg	2.3	10	0.32	1500	0.85	5.5	4000	1100	1.2	7.0	1.2
6.........	V444 Cyg	2.3	10	0.80	0	0	...	2500	2500	15	15	1.3
7.........	CV Ser	3.9	13	0.36	0	0	...	2900	2900	1.6	1.6	0.020
8.........	CV Ser	3.0	13	0.26	1500	0.92	5.3	3400	990	1.5	11.	0.92
9.........	CV Ser	0.5	13	0.92	0	0	...	2900	2900	27.	27.	0.87
10.........	γ^2 Vel	13.0	20	0.45	0	0	...	1550	1550	4.4	4.4	0.018
11.........	γ^2 Vel	5.0	20	0.45	3000	0.92	4.7	2900	950	3.8	61	0.88
12.........	γ^2 Vel	1.0	20	0.963	0	0	...	1500	1500	89	89	1.0

NOTE.—Observational estimates for stellar parameters:
V444 Cyg: $v_\infty = 2500$, $S_\nu = 0.30$ mJy, $\dot{M} = 1.2 \times 10^{-5}$, $\Gamma = 0.32$.
CV Ser: $v_\infty = 2900$, $S_\nu < 0.40$ mJy, $\dot{M} < 3.0 \times 10^{-5}$, $\Gamma = 0.36$.
γ^2 Vel: $v_\infty = 1500$, $S_\nu = 29.0$ mJy, $\dot{M} = 8.8 \times 10^{-5}$, $\Gamma = 0.45$.
The Alfvén radius, r_A/R, for No. 2 is 2.00; No. 5 is 1.65; No. 8 is 1.85; No. 11 is 1.98.
[a] $\dot{M} = 4\pi\, d\dot{M}/d\Omega$ in 10^{-6} $M_\odot$ yr^{-1}.
[b] Calculated radio flux in units of the observed flux computed from eq. (9) assuming $n = 1$.

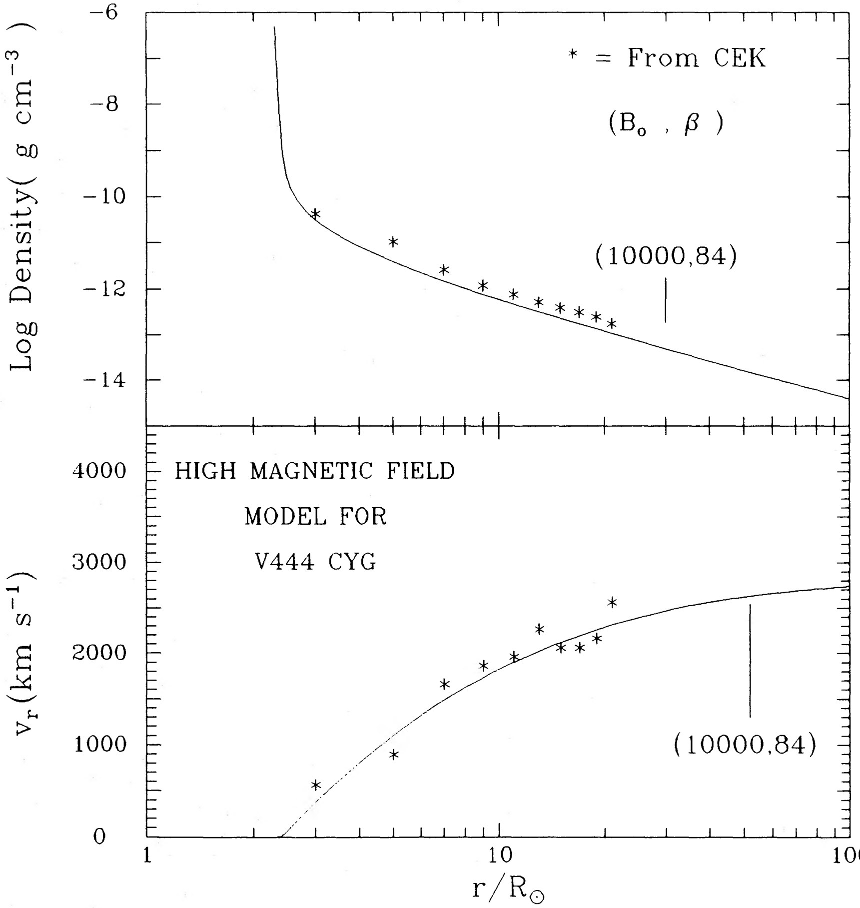


FIG. 6

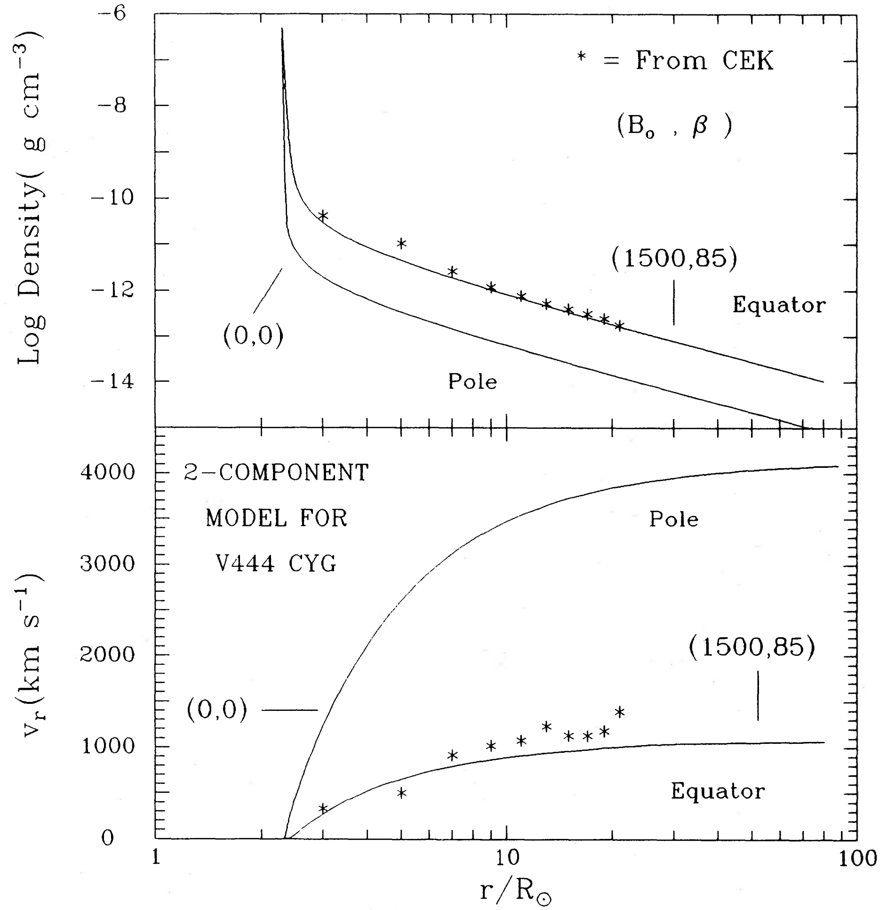


FIG. 7

FIG. 6.—Density and velocity distribution vs. radial distance for V444 Cyg. Asterisks show the distributions derived from observations of CEK of this eclipsing star, assuming $\dot{M} = 4\pi\rho v r^2$. These are compared with the theoretical distributions in the equatorial plane of a rapidly rotating, high magnetic field model, for which $B = 10^4$ G and $\beta = 84\%$ maximum. For this model a luminosity that satisfies Maeder's mass-luminosity relation is assumed.

FIG. 7.—Density and velocity distributions along the polar axis and along the equatorial axis for our two-component model of V444 Cyg. The magnetic field and rotation rate for the equatorial model is 1500 G and 84% maximum. Asterisks are the values inferred from the observations of CEK. The terminal velocity along the polar axis is larger than the observed value (3000 km s^{-1}), while that along the equator is lower than the observed. The combination of the two zones is able to explain the observed radio flux, and the model has a sufficiently long spin-down time of 3×10^5 yr.

of equation (9), with $G = 358$, and a spin-down time of $\sim 300,000$ yr. The comparison with the CEK observational data is also shown in Figure 7. The interpretation is not so straightforward because CEK assumed that the wind is spherically symmetric, and they derived a velocity distribution by combining the mass-loss rate with the inferred density distribution. We have assumed in Figure 7 that the observationally inferred $\rho(r)$ is basically that of the equatorial zone. The "observed" terminal velocity of 2500 km s^{-1} indicates that the inclination of the Wolf-Rayet must be less than 90°, but we have not yet carried out a detailed comparison with CEK using the observed value for the inclination angle.

Table 1 shows results for three models for V444 Cyg: model 4 is the low-luminosity zero rotation rate model shown as the polar model in Figure 7; model 5 is our two-component model; and model 6 is a high-luminosity model like that of Pauldrach *et al.* (1985).

As we noted earlier, there are observational uncertainties in the estimates of $\dot{M}$, L and v_∞ for V444 Cyg. We have here illustrated a model that has a wide range in velocity as a function of inclination angle. If the terminal velocity is in the range of 1500–1800 km s^{-1} instead of our adopted 2500 km s^{-1}, then it will be possible to fit the star with a smaller rotation rate and with a less extreme pole to equator contrast.

b) CV Ser

CV Ser is a partially eclipsing binary system of a WC 8 and a late O star. Using the mass of 13 $M_\odot$ and Maeder's mass luminosity relation we get $L = 3 \times 10^5\ L_\odot$ or $\Gamma = 0.36$; many of the other stellar parameters are the same as for the test model. The test model of § II presented results for CV Ser, assuming it to have a radius of 8 $R_\odot$. However, that choice led to a polar wind velocity of 1900 km s^{-1}. We have computed other models for CV Ser, adjusting the radius of the star so that the terminal velocity along the pole is larger than 2900 km s^{-1}. As seen from the observer's aspect angle, the observed terminal velocity of 2900 km s^{-1} should be intermediate between the polar and equatorial value. We reduce the radius to 3 $R_\odot$ which gives a polar wind (i.e., a zero rotation rate and zero magnetic field strength model) that has $\dot{M} = 1.5 \times 10^{-6}\ M_\odot$ yr^{-1} and $v_\infty = 3400$ km s^{-1}. For the equatorial region we assume, for simplicity, the same radius as the polar model and a magnetic field strength of 1500 G. A rotation rate of 92% of critical velocity will produce a flow with $\dot{M} = 1.1 \times 10^{-5}\ M_\odot$ yr^{-1} and $v_\infty = 990$ km s^{-1}. The geometric correction factor (G) in equation (8) is 480 for this model, which will satisfy to within 10% the radio flux condition in equation (9). The spin-down time scale for this model is $\sim 300,000$ yr, sufficiently long for a Wolf-Rayet star. The velocity and density distributions of the polar and equatorial zones are shown in Figure 8. This low-luminosity model, when viewed at some inclination less than 90°, satisfies the major observational constraints on Wolf-Rayet winds.

Table 1 summarizes the models for CV Ser. In model 7, the observed terminal velocity requires a radius of 3.9 $R_\odot$, but using Maeder's luminosity we get a radio flux that is deficient by a factor of 50. Model 8 is the two-component model, discussed above. Model 9 is a high-luminosity Friend and Abbott model that can also explain the radio flux, but it requires $\Gamma = 0.90$ and a photospheric radius of only 0.5 $R_\odot$. It has been pointed out to us by M. Barlow (private communication) that the actual terminal velocity of the line-of-sight flow from CV Ser could be significantly less than the 2900 km s^{-1} adopted here. Torres, Conti, and Massey (1986) estimate a terminal velocity of 2300 km s^{-1} from a correlation between the optical line width and excitation potential. Even 2300 km s^{-1} might be too high since the extrapolation has no solid theoretical underpinning. The velocity could be as low as 2000 km s^{-1}. If such is the case the requirements on both the two-component model and the high-luminosity line-driven wind model can be reduced. The velocity is proportional to the escape speed, so, for example, a larger radius could be used. It is useful to note that our model could explain even the more difficult case of a large line-of-sight terminal velocity. The smaller the observed line-of-sight velocity, the less extreme must be our pole equator contrast.

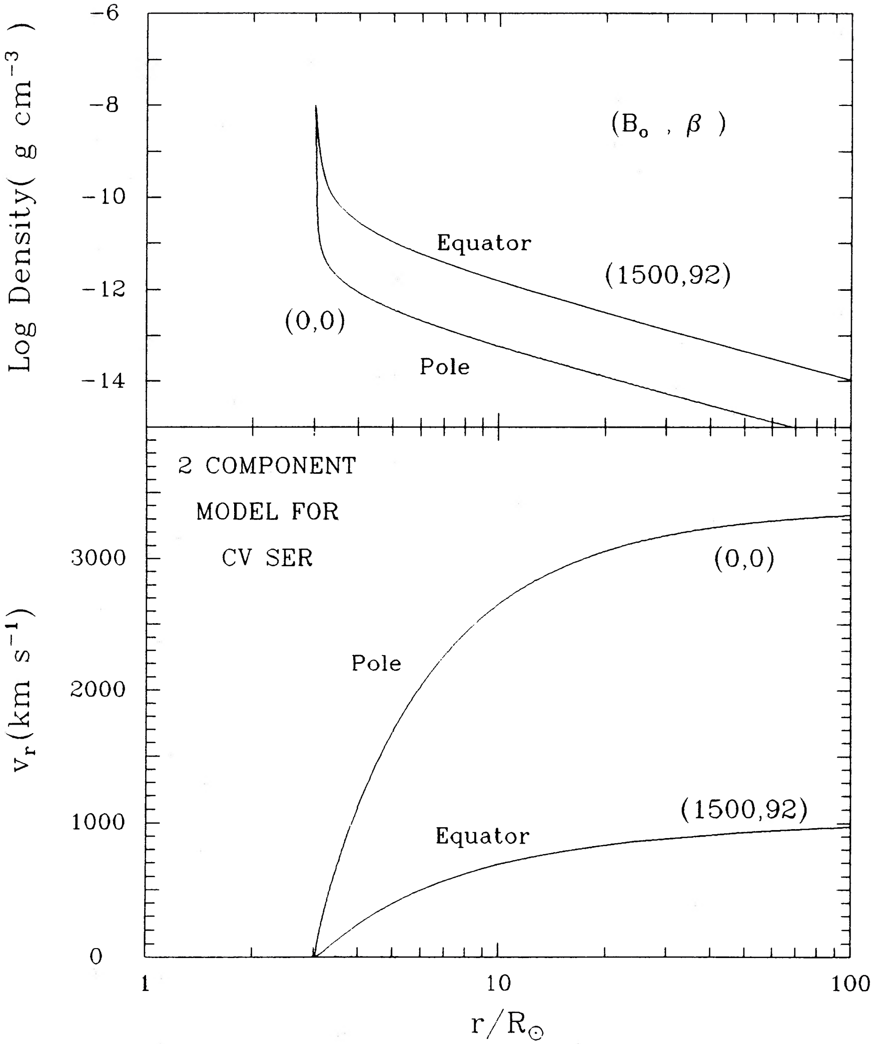
c) γ^2 Vel

The WC 8 star γ^2 Vel has an extremely large inferred mass-loss rate of $\sim 8.8 \times 10^{-5}\ M_\odot\ yr^{-1}$ derived from its large radio flux, and its wind speed inferred from infrared lines (Barlow, Roche, and Aitken 1988). The mass of γ^2 Vel is known from the orbital solution ($M = 20\ M_\odot$) and Maeder's relation (eq. [1]) gives $\Gamma = 0.45$. The values of k and α are the same as for the test model, and the density at the surface is 1×10^{-8} g cm^{-3}. As with CV Ser, the radius, R, is not known. We adjusted the radius so as to match the terminal velocity of 1500 km s^{-1}. For the zero rotation rate model in Table 1 (model 10), this leads to $R = 13\ R_\odot$. The radio flux for this model is too low by about a factor of 50. For our two-component model (model 11), we decreased the stellar radius to 5 $R_\odot$ so that the polar terminal velocity is significantly larger than the observed terminal velocity as is appropriate for this case in which we are seeing the star at a large inclination angle. This two-component model is shown in Figure 9. Using a rotation rate of 92% of the critical rate, we obtain a radio flux of 88% of the observed flux, presumably well within observational uncertainty. The geometrical correction factor for this model is $G = 1560$. The high-luminosity Friend and Abbott model (model 12) that matches both the radio flux and the terminal velocity has an extremely high value for Γ of 0.963!

V. CONCLUSIONS

We have shown that a rotating, magnetic, radiation-driven wind model can reproduce the observed radio fluxes of Wolf-Rayet stars and can explain the observed high terminal speeds provided that we are not observing the equatorial region. Rotation, with a weak magnetic field, can increase the density along the equator enough to produce the observed radio flux. Such a two-component model does not encounter the problems with spin-down times due to magnetic braking. This model solves the "momentum problem," since mass flux and velocity are never large in the same part of the wind. The model also uses a luminosity that is consistent with what is expected from interior theory (with $\Gamma \approx 0.32$ to 0.45). Our rotating magnetic model requires that the surface of Wolf-Rayet stars rotate at rates near the critical rate. Although these rotation speeds are large, they cannot be ruled out observationally because the broad line profiles and apparent absence of photospheric lines prevent us from deriving rotational velocities from observations.

We have heard arguments that Wolf-Rayet stars in binary systems, such as the three we investigated here, cannot be rapid rotators because their rotation should be tidally locked into the much slower revolution rate. There are several good reasons to doubt that this is the case. First the Wolf-Rayet



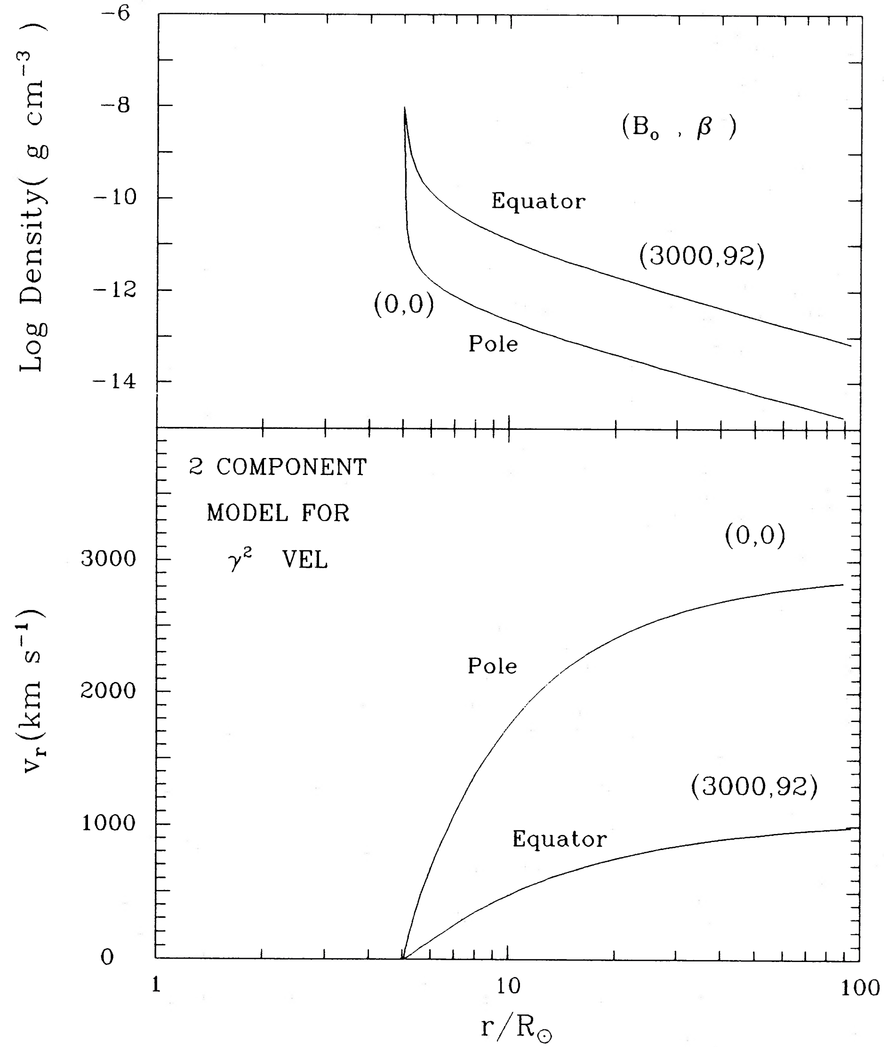


FIG. 8.—Results for the two component model for CV Ser. The polar wind reaches a terminal velocity of 3400 km s^{-1}, while the equatorial terminal speed is 900 km s^{-1}. The model uses the luminosity consistent with interior theory, a magnetic field of 1500 G and a rotation rate of 90% maximum. The model satisfies the radio observations and has a spin-down time of 3×10^5 yr.

FIG. 9.—The two-component model for γ^2 Vel. The polar and equatorial wind terminal speeds are 2900 and 950 km s^{-1} respectively. The model assumes a luminosity consistent with stellar interior theory, a magnetic field of 3000 G, and a rotation rate of 92% maximum. The predicted radio flux is nearly as large as the observed flux. The star has a spin-down time time of 50,000 yr.

stars are in their final stages of evolution. The overall collapse that would be associated with the initiation of helium burning would, because of conservation of angular momentum, have unlocked any preexisting synchronous rotation. Second, the W-R stars were the cores of massive stars, and it is not clear that this core would have decelerated synchronously with the envelope because of tidal effects. Finally, the lifetimes of the W-R phase is very short, on the order of 10^5 yr, and tidal locking at these final stages would probably not occur.

The alternative to having large rotation rates is to have very high luminosities implying $\Gamma > 0.9$, as has been proposed by Pauldrach *et al.* (1985). However, there is very little evidence, both observationally or theoretically, to support this high luminosity. For the case of V444 Cyg, CEK derived an approximate color temperature from their analysis of the eclipses. Interpretation of this temperature in terms of the luminosity is questionable. In addition, the new results by Schmutz *et al.* (1988*b*) indicate that the luminosities of Wolf-Rayet stars are much lower than required by the radiation-driven wind models. Without $\Gamma > 0.9$ or major modifications to the theory, radiation-driven wind models alone cannot drive the Wolf-Rayet star winds. Friend, Poe, and Cassinelli (1988) present alternative ways to increase the radiation force within the framework of the Friend and Abbott model.

All of these approaches can solve the "wind momentum problem," but at a price. They involve assuming a value for a stellar parameter that appears to be extreme and is unsubstantiated by observational evidence. The lack of firmer observational constraints for the Wolf-Rayet winds means that all of these models must be seriously considered until further evidence is found that confirms or refutes them.

It is also necessary, from the theoretical point of view, to see if the parameters used in these models can be justified in terms of stellar interior theory. Constraints from interior theory concerning magnetic fields in Wolf-Rayet winds have been discussed by Maheswaran and Cassinelli (1988). They find that the circulation currents generated by rapid stellar rotation will tend to submerge the field in a Wolf-Rayet star if it is less than $\sim 10^3$ G. Thus the minimal field that can withstand the effects of circulation are consistent with those needed in our models. Other important interior considerations concern the maximal allowed values for Γ for Wolf-Rayet stars and the maximal rotation speeds of exposed stellar cores.

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