THE ASTROPHYSICAL JOURNAL, 337:466–469, 1989 February 1

ROTATING STELLAR COLLAPSE MODEL FOR SUPERNOVA SN 1987A

T. NAKAMURA
Department of Physics, Kyoto University
AND
M. FUKUGITA[1]
Research Institute for Fundamental Physics, Kyoto University
Received 1988 March 1; accepted 1988 July 12

ABSTRACT

We study consequences of a stellar collapse with a moderately rotating core. If one assumes a core rotation corresponding to $q \equiv Jc/GM^2 \approx 3$ with J the total angular momentum and M the gravitational mass of the core, the core collapse is likely to leave protoneutron stars a binary which coalesces in several seconds emitting the gravitational radiation. Characteristically this scenario yields two bursts of the neutrino emission a few seconds apart, one at the time of the collapse and the other at the coalescence, as indicated by the Kamiokande observation for SN 1987A. Some implications for future observations are also mentioned.

Subject headings: neutrinos — stars: collapsed — stars: individual (SN 1987A) — stars: interiors — stars: supernovae

I. INTRODUCTION

The supernova 1987A which emerged in the Large Magellanic Cloud has provided a unique opportunity to explore the physics of a stellar collapse. Particularly unique is the observation of neutrino signals which directly probe the moment of the core collapse (Hirata *et al.* 1987*a*; Bionta *et al.* 1987). The impressive fact with these neutrino events was that the gross features of these events show a good agreement with those expected in the standard view (Arafune and Fukugita 1987; Sato and Suzuki 1987; Bahcall *et al.* 1987; Burrows and Lattimer 1987; Bruenn 1987).

A closer examination of these neutrino events, however, reveals some peculiarities which are not quite expected (Arafune and Fukugita 1987; Sato and Suzuki 1987). The most intriguing among them is that the Kamiokande observation recorded three events 10 s after the first signal with a 7 s intermission between the earlier and the last three events. These last three events correspond to about 25%–30% of the total energy liberation, i.e., $\sim 1 \times 10^{53}$ ergs which is quite a nonnegligible amount. Unfortunately the total number of Kamiokande events is not large enough to extract unambiguously the physics from timing information; especially it is not large enough to assert whether this time gap is really physically meaningful. The IMB observation appears to weaken this peculiarity to some extent. Nevertheless this apparent time gap has worried some people and has tempted them to consider the possibility of unconventional scenarios for SN 1987A.

One of the examples is a scenario with a rotating stellar collapse (Nakamura and Fukugita 1987). This is a possibility conjectured earlier by Ruffini and Wheeler (1971). Nakamura and Fukugita have shown that the observed time gap is naturally explained, if one assumes a moderate rotation of the progenitor core.

In this paper we elaborate the arguments presented in the previous note (Nakamura and Fukugita 1987) and present an analysis on the statistical significance of the presence of the gap. Our statistics argument shows that the probability that the observed neutrino events are explained by the standard picture is at most at the few percent level. This means that the *a priori* probability for observing such an event in the standard model is rather marginal, albeit not excluded.

One of the reasons that makes it worthwhile to pursue this rotating collapse scenario is that it predicts some unconventional characteristics which could be searched for in supernovae in nearby galaxies with experimental techniques which will become available in the near future.

There are also some recent observations suggesting a nonspherical nature of the collapse for SN 1987A. One of them is the observation of linear polarization of the optical light from the supernova, which at maximum amounts to 1%, while decreasing with a time scale of about a month (Schwarz and Mundt 1987; Barrett 1987). This polarization effect can most easily be interpreted by nonspherical shape of the ejecta (Shapiro and Sutherland 1982). The magnitude of the polarization requires an aspect ratio 0.6–0.8 if the shape is oblate (Jeffery 1987), or even less if it is prolate. This nonsphericity suggests that the explosion itself was nonspherical, since we expect that the explosion energy of the core, which is much greater than the thermal and the gravitational energy of the envelope of the progenitor, should dominate the remnant. Such a nonsphericity may well be ascribed to a rotating collapse of the core, models of which have been presented in the calculations by LeBlanc and Wilson (1970) and by Bodenheimer and Woosley (1983).

Another effect suggesting a nonspherical collapse is the mysterious spot three mag dimmer than SN 1987A found by speckle interferometry 30–50 days after the explosion (Nisenson *et al.* 1987; Meikle, Matcher, and Morgan 1987). One can readily see that echo scenarios meet difficulties in explaining the brightness of the spot: The observed brightness requires that the initial UV flux of the supernova was 50 times larger than is generally believed (Felten, Dwek, and Vilgas-Aldrovandi 1987). Thus we are led to suppose that something is ejected from SN 1987A with a velocity $\sim 0.6c$. One of the speculations is a jet formation in a binary model (Rees 1987).

[1] Also at ICEPP, University of Tokyo.

Other authors looked at the possibility of a rotating collapse of the core, which would lead to the formation of a relativistic jet (Piran and Nakamura 1987) or a sling shot (Goldman 1987; Carlson, Glashow, and Sarid 1987). At present it is not clear whether these speculations are really viable. These possibilities, however, at least motivate us to reexamine the rotating collapse model.

In the next section we describe our model for a rotating stellar collapse. In § III we discuss the neutrino observation and its statistical significance. Some implications of the model and remarks pointing toward future observations are given in § IV.

II. THE MODEL

The importance of rotation in a relativistic astrophysical system is conveniently represented by a nondimensional quantity q,

$$q = J/(GM^2/c) , \tag{1}$$

with J the total angular momentum and M the gravitational mass of the core. For example, if $q = 1$, the effect of the angular momentum becomes important only when the size of the core shrinks to 10^6 cm as shown in general relativistic simulations of the collapse of rotating stars (Nakamura 1985; Stark and Piran 1985). The Sun has a surface velocity $v_{\rm rot} \approx 2$ km s^{-1} which corresponds to $q \approx 0.18$. It is known that the giant stars are rotating fast. For instance, the rotating velocity of early B type massive stars are typically 50–200 km s^{-1} (Wolff, Edwards, and Preston 1982), for which q is estimated to be 2–7 for main-sequence stars ($M \approx 20 M_\odot$, $R \approx 8 R_\odot$). The q value for the core is not known. If the rotation is rigid, however, $q_{\rm core} \approx q_{\rm total}$ for the star close to a polytrope with an index 3. In the following we assume the stellar core to be rotating with $q_{\rm core} \approx 3$, which is not an unreasonable value for giant stars responsible for a Type II supernova, and discuss three stages of the stellar collapse in order.

a) Onset of the Collapse

The effect of the angular momentum becomes important only when the size of the core r shrinks to $\lesssim 10^7$ cm, since the size for which the centrifugal force is comparable to the gravitational force is proportional to q^2. In the conventional spherically symmetric collapse model, the size of the unshocked homologous core is of the order of 10^7 cm (e.g., Wilson *et al.* 1975, 1986; Bethe 1984). Hence we expect that the core at bounce is essentially governed by spherically symmetric dynamics. To verify this point we may refer to a simulation by Symbalisty (1984). Our model almost corresponds to model ROT2, in which the rotational energy is initially 4.5% of the total energy. The simulation shows that the collapse does not differ much from a spherical collapse until the radius reaches 50 km and a shock wave forms outside the core. Symbalisty also found that the neutrino sphere was roughly ellipsoidal with a size 60 km along the equator and 30 km along the rotation axis. We then expect that the matter falling in nonhomologously onto the core will liberate a gravitational energy of the order of 10^{52} ergs by neutrino emission (Symbalisty 1984) when the shock reached the neutrino sphere, as in a spherically symmetric collapse.

b) Kelvin Contraction and Disk Fragmentation

In the spherically symmetric model, the core enters the Kelvin-Helmholtz contraction phase, and its radius is expected to be 10^6 cm within 1 s or less after the bounce (e.g., Burrows and Lattimer 1986). The evolution of a rotating core is different. The contraction takes place mainly along the rotation axis due to the centrifugal force. This is clearly seen in the case of model ROT2 by Symbalisty. In this Kelvin phase the released gravitational energy is

$$\frac{(\pi/2 - 1)3GM^2}{5r} \approx 2 \times 10^{52} \text{ ergs} . \tag{2}$$

Using the cooling rate calculated by Burrows and Lattimer, we estimate that the core becomes a disk with a thickness $d \approx 2 \times 10^6$ cm within 0.5 s. We then have a rotating disk–like proto–neutron star with an aspect ratio $r/d \approx 5$.

Such a thin disk is known to be gravitationally unstable irrespective of the equation of state (Goldreich and Lynden-Bell 1965). The most unstable mode of the disk has a wavelength $2\pi d$, and it is likely that it fragments into

$$N = (r/d)^2/\pi^2 \approx 2.5 \tag{3}$$

pieces in a free fall time scale ≈ 0.01 s. Three-dimensional numerical simulations of collapse of rotating isothermal clouds in fact support this result (Boss 1981; Miyama, Hayashi, and Narita 1984). In our case we will have two or three fragments. Since the specific spin angular momentum is reduced by a factor of 3–5 due to this fragmentation, as shown by the simulation of Miyama *et al.*, the effect of angular momentum is not important any more for the Kelvin contraction of each fragment and the two (or three) proto–neutron stars will follow the standard scenario for the spherical case. Then the binding energy of neutron stars $\approx 10^{53}$ ergs will be liberated by neutrino emission after all within the order of 1 s, according to the cooling rate given by Burrows and Lattimer.

At this point we should mention an important difference between the scenario given here and the simulation by Symbalisty. In the latter the author takes an axisymmetric model and account is not taken of the fact that the thin disk fragments. Then the collapse halted at $t = 0.28$ s, where a quasi-rotational equilibrium of the core was established. In our consideration, the disk fragments, which reduces the angular momentum of protoneutron stars that makes further contraction possible.

The main bunch of the neutrino events observed at Kamioka and at IMB should correspond to this phase. We expect that the characteristics of the neutrino emission do not differ much from the conventional case as seen above.

c) Coalescence of Protoneutron Stars

Let us assume for simplicity that a binary system of proto–neutron stars with masses m_1 and m_2 and a separation distance $r \approx 10^7$ cm is formed as a result of the fragmentation. Then the distance r decreases due to the emission of gravitational radiation at the rate of

$$\dot{r} = -7.7 \times 10^5 \text{ cm s}^{-1} \times \frac{\eta(m_1/0.7\ M_\odot)(m_2/0.7\ M_\odot)[(m_1 + m_2)/1.4\ M_\odot]}{(r/10^7 \text{ cm})^3} , \tag{4}$$

with η an efficiency factor. Equation (2) for $\eta = 1$ is derived by assuming that two bodies are point particles. Phase cancellation effect (Nakamura and Sasaki 1981; Nakamura and Oohara 1983) that reduces the efficiency of gravitational radiation should be taken into account for extended objects, and a typical value of η will be 0.1–0.5 for our case. The characteristic time of the decay of the separation length r due to the emission of gravitational radiation is then given by

$$t = 9\ \mathrm{s}\left(\frac{\eta}{0.3}\right)^{-1} \times \frac{(r/10^7\ \mathrm{cm})^4}{(m_1/0.7\ M_\odot)(m_2/0.7\ M_\odot)[(m_1+m_2)/1.4\ M_\odot]} . \quad (5)$$

If the two fragments have almost an equal mass of 0.7 $M_\odot$, two proto–neutron stars will collide after ~ 9 s with a subsonic speed of 2.2×10^8 cm s^{-1}. Eventually a rapidly rotating neutron star of mass 1.4 $M_\odot$ will be formed and the difference of the binding energy up to 10^{53} ergs between proto–neutron stars and a final single neutron star should be liberated in the form of neutrino emission and gravitational radiation. We expect an average neutrino energy of the order of 10 MeV from the radiation law.

If the two fragments have different masses, say, 0.9 $M_\odot$ and 0.5 $M_\odot$, then the binary system enters a stable mass stripping phase under plausible parameters of mass loss and angular momentum loss (Clark and Eardley 1977). In a numerical calculation for the case of two neutron stars of mass 1.3 $M_\odot$ and 0.8 $M_\odot$ the integrated energy of the neutrino emission by mass accretion is estimated to be 2×10^{53} erg, with a time duration of about 2 s (Clark and Eardley 1977). The average energy of neutrinos in the accretion phase is estimated to be ≈ 10 MeV. In any case we expect that a neutrino flux with total energy $\approx 10^{53}$ ergs is liberated almost 10 s after the first burst at the time of coalescence.

We estimate an amplitude of the gravitational radiation during the decay of the orbit of the protoneutron stars to be

$$\langle h \rangle = 6 \times 10^{-21}(55\ \mathrm{kpc}/D)(10^7\ \mathrm{cm}/r)(m_1/0.7\ M_\odot) \times (m_2/0.7\ M_\odot) \quad (6)$$

with a frequency

$$f = 70\ \mathrm{Hz}\ [(m_1+m_2)/1.4\ M_\odot]^{0.5}(r/10^7\ \mathrm{cm})^{-1.5} . \quad (7)$$

The scenario is more complicated if the disk fragments into three bodies, including the possibility that one of them with a small mass escapes from the system with a speed of the order of $\sim (Gm/r)^{1/2}$ which is of the order of a fraction of the speed of light, as speculated by Carlson, Glashow, and Sarid (1987) and Goldman (1987) to explain the mysterious spot. We should comment here that a smaller neutron star has a larger radius, and that tidal disruption will be more important than the ejection effect. A smaller mass neutron star has a larger radius; e.g., a neutron star with $m_3 = 0.1\ M_\odot$ has a radius of the order of $r_3 \simeq 100$ km (see, e.g., Shapiro and Teukolsky 1983). Such a neutron star may be ejected with a velocity of a fraction of the speed of light (~ 0.3–$0.5c$, say) only when it approaches close to the surface of other neutron stars with a mass of the order of $M \approx 1\ M_\odot$. Namely, this happens only when the small mass neutron stars comes within the tidal radius $r(\mathrm{tidal}) = r_3(M/m_3)^{1/3} \simeq 200$ km, within which it receives a strong gravitational tidal force from other neutron stars and cannot be regarded as a self-gravitating system. This would lead to tidal disruption of the small mass neutron star.

III. THE STATISTICAL SIGNIFICANCE OF THE GAP IN THE KAMIOKANDE NEUTRINO EVENTS

The scenario presented in the previous section nicely fits the timing characteristics of the neutrino events observed for SN 1987A in the Kamiokande II detector (Hirata *et al.* 1987*a*). The first eight events were observed in 1.9 s, and the remaining three events came 9.2–10.4 s after the first signal.

Here let us ask how seriously we could take the significance of the 7.3 s time gap. First one might suspect that the last three events of the Kamiokande II could be background events, because the noise of a water Čerenkov detector increases sharply toward the low-energy side. Fortunately the Kamiokande detector is carefully tuned for solar neutrino observation and the property of noise has been well studied. It is shown that the number of background events that mimic an $\gtrsim 8$ MeV neutrino signal in 3 s is 0.1 or less (Hirata *et al.* 1987*b*). Therefore the possibility to ascribe all the last three events within the duration of 3 s to the background is very small.

Next let us ask whether the gap between 1.9 and 9.2 s is significant. An argument sometimes made against the significance of the gap is the Kolmogorov-Smirnov test: Assuming the continuous decay of neutrino luminosity (either exponentially or according to a power law), one obtains a high statistical significance level, typically 0.93 if the Kamiokande data are combined with the IMB data, and one may claim that the gap is not statistically significant. We should note, however, that this significance level hardly changes even if the three events would come later, 1000 s say, after the first signal. This fact demonstrates that the conventional Kolmogorov-Smirnov test is not an adequate one for detecting a subdominant structure of the event.

To estimate the probability of the gap we made Monte Carlo simulation with 10^6 trials. If we assume the constant signal rate for the last 10 s (0.3 events s^{-1}), we find that the probability giving the 7 s gap is 0.027. (The probability giving a 7 s gap anywhere in the 10 s period is 0.054.) This probability is compared with 0.064 and 0.13 for the cases when the gap is 5 s and 6 s, respectively. If the neutrino flux is a decreasing function of time with a time constant of the order of a few seconds (Burrows and Lattimer 1987; Spergel *et al.* 1987), the probability for a 7 s gap is considerably smaller than the above value. If we include the IMB events (adjusting the relative timing by the first signals), the probability giving two events from 5 to 6 s and three events after 9.2 s is found to be 0.0045 in our Monte Carlo simulations assuming the constant neutrino luminosity. (The smallness of this number is ascribed to the last three Kamiokande events.) From a purely statistical viewpoint it is, therefore, more favorable to assume an increase of the neutrino luminosity toward ~ 10 s after the first signal.

We then regard the Kamiokande data to be statistically consistent with the presence of the gap between 1.9 and 9.2 s, which implies that either there is a real time gap in the neutrino flux or that the average energy of neutrinos in this period is too low (< 7 MeV) to allow detection in the Kamiokande detector. We cannot claim that the significance of the gap is more than that shown by this statistical test. Whether the reader considers this gap to be meaningful depends on his judgement of the significance of the statistical analysis. The IMB events do not present much evidence either against or for the existence of the gap, as we demonstrated in the Monte Carlo test for the case of a constant neutrino luminosity.

IV. DISCUSSION AND REMARKS

In this paper we have considered the fate of a stellar collapse with $q = Jc/GM^2 \approx 3$. If this number is smaller than 1, we do not expect anything different from the spherically symmetric collapse. On the other hand, if q is considerably larger than 3, it is likely that the centrifugal force hinders the collapse. Our assumed value, however, seems to be typical of giant stars, and we expect that a supernova according to the present model might occur rather frequently.

Aside from the neutrino emission, the most characteristic prediction of the present scenario is the gravitational radiation (see also Clark and Eardley 1977); following the first collapse signal, regular gravitational waves characteristic of the rotation of binary neutron stars should be observed with the amplitude and frequency (6) and (7), respectively. With the gravitational wave detectors presently proposed, one should be able to detect such an effect for a supernova in a galaxy at least as far as the center of the Virgo cluster. Such a test will be an important challenge for future gravitational wave detectors being planned (e.g., Fairbank 1987). We should also note that this possibility provides a qualified distance indicator to the galaxy (Schutz 1986); the distance can be derived from $\dot{f}/f$ and $\langle h \rangle$. The optical identification of the event is direct in the case of supernova that we have considered here, whereas in the method originally proposed by Schultz, one has to resort to a statistical argument to find the host galaxy.

Finally we should discuss the nature of the newborn neutron star. It is natural to suppose that a rapidly rotating pulsar with a period $P_0 \sim 1$ ms (Bodenheimer and Ostriker 1974) is left behind after the rotating collapse. The total energy liberation rate of the rotational energy due to the magnetic dipole field is given by

$$L_{\text{pulsar}} \sim 4 \times 10^{43} \text{ ergs s}^{-1}\, B_{\perp,12}{}^2 R_6{}^2 \left(\frac{P_0}{1 \text{ ms}}\right)^{-4} \times \left[1 + \frac{t}{16 \text{ yr } (P_0/1 \text{ ms})^2 B_{12}{}^2}\right]^{-2}, \quad (8)$$

where $B_{\perp,12}$ is the strength of the magnetic field perpendicular to the rotation axis in units of 10^{12} G, and R_6 the radius of the neutron star in units of 10^6 km (Pacini 1968). Since the light curve of the SN 1987A 100–200 days after the explosion shows an exponential decrease in accordance with the decay of ^{56}Co, the luminosity which the pulsar injects into the photosphere is restricted to be $\lesssim 10^{39}$ ergs s^{-1} provided that L_{pulsar} is completely thermalized (Bodenheimer and Ostriker 1974). This leads to the limit

$$P_0 > 15 \text{ ms } B_{\perp,12}{}^{-1/2}. \quad (9)$$

We mention the following possibilities which make our expectation consistent with this bound: (1) The pulsar is rapidly rotating with $P_0 \sim 1$ ms, but the magnetic field is either parallel to the rotation axis or weak ($\lesssim 10^{10}$ G). (2) If there is a deviation from axial symmetry of an amount $\epsilon \geq 10^{-3}$ in the moment of inertia of the coalescent neutron star, the spin-down of the pulsar due to gravitational wave emission takes place rapidly. The spin-down time is given by

$$t_{\text{gw}} \sim 9 \times 10^3 (P_0/1 \text{ ms})^4 (\epsilon/10^{-3})^{-2} \text{ s}, \quad (10)$$

i.e., P_0 decreases to ~ 10 ms within a year, if ϵ is of the order of 10^{-3}, due to the gravitational wave emission. This consideration shows that the lack of evidence for a rapidly rotating pulsar does not necessarily provide negative evidence for the rotating collapse scenario.

We would like to thank Piet Hut for suggestions improving the manuscript. This work was supported in part by Grant-in-Aid for Scientific Research (No. 62540188) of the Ministry of Education, Science and Culture.

REFERENCES

Arafune, J., and Fukugita, M. 1987, *Phys. Rev. Letters*, **59**, 367.
Bahcall, J. N., Piran, T., Press, W. H., and Spergel, D. N. 1987, *Nature*, **327**, 682.
Barrett, P. 1987, in *Proc. ESO Workshop on the SN 1987A, Garching*, ed. I. J. Danziger (Garching: ESO), p. 173.
Bethe, H. A. 1984, in *From Nuclei to Stars*, ed. A. Molinari and R. A. Ricci (Amsterdam: North-Holland), p. 147.
Bionta, R. M., *et al.* 1987, *Phys. Rev. Letters*, **58**, 1494.
Bodenheimer, P., and Ostriker, J. P. 1974, *Ap. J.*, **197**, 465.
Bodenheimer, P., and Woosley, S. E. 1983, *Ap. J.*, **269**, 281.
Boss, A. 1981, *Ap. J.*, **250**, 636.
Burrows, A., and Lattimer, J. M. 1986, *Ap. J.*, **307**, 178.
———. 1987, *Ap. J.*, **318**, L63.
Bruenn, S. W. 1987, *Phys. Rev. Letters*, **59**, 938.
Carlson, E. D., Glashow, S. L., and Sarid, U. 1987, preprint.
Clark, J. P. A., and Eardley, D. M. 1977, *Ap. J.*, **215**, 311.
Fairbank, W. M. 1987, in *General Relativity and Gravitation* (GR11), ed. M. A. H. MacCallum (Cambridge: Cambridge University Press), p. 313.
Felten, J. E., Dwek, E., and Viegas-Aldrovandi, M. 1987, preprint.
Goldman, I. 1987, *Astr. Ap.*, **186**, L3.
Goldreich, P., and Lynden-Bell, D. 1965, *M.N.R.A.S.*, **130**, 97.
Hirata, K., *et al.* 1987*a*, *Phys. Rev. Letters*, **58**, 1490.
———. 1987*b*, preprint.
Jeffery, D. J. 1987, *Nature*, **329**, 419.
LeBlanc, J. M., and Wilson, J. R. 1970, *Ap. J.*, **161**, 541.
Meikle, W. P. S., Matcher, S. J., and Morgan, B. L. 1987, *Nature*, **329**, 608.
Miyama, S. M., Hayashi, C., and Narita, S. 1984, *Ap. J.*, **279**, 621.
Nakamura, T. 1985, *Ann. NY Acad. Sci.*, **422**, 56.
Nakamura, T., and Fukugita, M. 1987, in *Proc. ESO Workshop on the SN 1987A, Garching*, ed. I. J. Danziger (Garching: ESO), p. 355.
Nakamura, T., and Oohara, K. 1983, *Phys. Letters*, **98A**, 403.
Nakamura, T., and Sasaki, M. 1981, *Phys. Letters*, **106B**, 69.
Nisenson, P., Papaliolios, C., Karovska, M., and Noyes, R. 1987, *Ap. J. (Letters)*, **320**, L15.
Pacini, F. 1968, *Nature*, **219**, 145.
Piran, T., and Nakamura, T. 1987, *Nature*, **330**, 28.
Rees, M. J. 1987, *Nature*, **328**, 207.
Ruffini, R., and Wheeler, J. A. 1971, in *Proc. of Conference on Space Physics 45-174, Relativistic Cosmology and Space Platforms*, ed. H. V. Hardy (Paris: European Space Research Organisation), p. 52.
Sato, K., and Suzuki, H. 1987, *Phys. Rev. Letters*, **58**, 2722.
Schwarz, H. E., and Mundt, R. 1987, *Astr. Ap.*, **177**, L4.
Schutz, B. F. 1986, *Nature*, **323**, 310.
Shapiro, P. R., and Sutherland, P. G. 1982, *Ap. J.*, **263**, 902.
Shapiro, S. L., and Teukolsky, S. A. 1983, *Black Holes, White Dwarfs, and Neutron Stars* (New York: John Wiley and Sons).
Spergel, D. N., Piran, T., Loeb, A., Goodman, J., and Bahcall, J. N. 1987, *Science*, **237**, 1471.
Stark, R. F., and Piran, T. 1985, *Phys. Rev. Letters*, **55**, 891.
Symbalisty, E. M. D. 1984, *Ap. J.*, **285**, 729.
Wilson, J. R., *et al.* 1975, *Ann. NY Acad. Sci.*, **262**, 54.
———. 1986, *Ann. NY Acad. Sci.*, **470**, 267.
Wolff, S. C., Edwards, S., and Preston, G. W. 1982, *Ap. J.*, **252**, 322.

M. Fukugita: Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan

T. Nakamura: Department of Physics, Kyoto University, Kyoto 606, Japan