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LIMITS ON DUST IN DAMPED LYMAN-ALPHA SYSTEMS AND THE OBSCURATION OF QUASARS

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ABSTRACT

The damped Lyα systems discovered in the spectra of quasars at high redshifts are natural places to search for dust. They have column densities of neutral hydrogen greater than 10^{20} cm^{-2} and may be protogalaxies or galactic disks in an early, gas-rich phase. We compare the spectra of quasars in the Wolfe *et al.* survey that have damped Lyα with those that do not have damped Lyα to obtain statistical information about the reddening by dust. Our results are given in terms of the dimensionless dust-to-gas ratio $k \equiv 10^{21}(\tau_B/N_H)$ cm^{-2}, where τ_B is the optical depth in the B band in the rest frame of an absorber and N_H is the column density of neutral hydrogen. Using nonparametric tests, we find, at the 95% confidence level, $k \leq 0.41$ (GAL), $k \leq 0.29$ (LMC), and $k \leq 0.19$ (SMC), depending on whether the extinction curve is assumed to have the same shape as that in the Milky Way or the Large or Small Magellanic Clouds. Our upper limits on the dust-to-gas ratio in the damped Lyα systems are half the observed value in the Milky Way but are several times larger than the observed values in the Magellanic Clouds. We also develop a new method to set limits on the mean and the variance of the optical depth along random lines of sight. This includes a correction for the effect, emphasized by Ostriker and Heisler, that highly obscured quasars are less likely to be included in optically selected samples than quasars with little dust in the foreground. Our results for the mean optical depth in the B band of the observer can be approximated by the formula $\bar{\tau}_B(z) = 0.4\tau_*[(1+z)^{5/2} - 1]$; using the upper limits on the dust-to-gas ratio in the damped Lyα systems, we obtain $\tau_* \leq 0.07$ (GAL), $\tau_* \leq 0.06$ (LMC), and $\tau_* \leq 0.05$ (SMC). For comparison, the first models by Ostriker and Heisler predict $\tau_* = 0.4$ or 0.8, while their more recent models predict $\tau_* = 0.16$. We conclude that neither set of models is consistent with our limits and that the apparent cutoff in the counts of quasars at $z \simeq 3$ is probably not caused by dust in the damped Lyα systems. All the limits derived in this paper could be reduced significantly or a positive detection could be made by determining more accurately the spectral indices of the quasars in the Wolfe *et al.* survey.

Subject headings: cosmology — galaxies: intergalactic medium — quasars

I. INTRODUCTION

Obscuration by dust could seriously affect our view of objects at cosmological distances. The dust might be smoothly distributed in an intergalactic medium or it might be located in galaxies and protogalaxies. In either case, the observed spectra of quasars at high redshifts should appear steeper than those at low redshifts unless the reddening is offset by evolution in the emitted spectra. There is a very weak correlation between the spectral indices and the redshifts of quasars in the Richstone-Schmidt (1980) survey, from which Wright (1981) derived a mean visual extinction of 0.85 ± 0.5 mag at $z = 3$. This can also be interpreted as an upper limit of about $A_V \leq 2$ at the 95% confidence level. A stronger limit, from a different sample of quasars, has recently been reported by Wright and Malkan (1987). If the dust were located in galaxies or protogalaxies, it might also produce broad absorption features, corresponding to a bump in the extinction curve at 2200 Å, or other deviations from power laws in the observed spectra of background quasars. Searches for dust by this method have so far not led to any convincing detections (McKee and Petrosian 1974; Jura 1977; Boisse and Bergeron 1988, and references therein). The upper limits that can be set on the obscuration do, however, require strong assumptions about the extinction curve of the dust and/or the emitted spectra of the quasars. For example, the extinction curve of the Small Magellanic Cloud (SMC), which may be typical of galaxies with low metallicities, would not produce absorption features in the spectra of background quasars. In this paper, we introduce a new method to set limits on the abundance of dust that is free of many of the assumptions required in previous searches.

An important analysis of the obscuration and reddening of quasars by dust in foreground galaxies has been presented by Ostriker and Heisler (1984). In particular, they suggested that the apparent cutoff in the counts of quasars at redshifts near 3 might be the result of obscuration rather than evolution. Another key idea in their paper is that, even though the obscuration may be large, the mean reddening in an optically selected sample of quasars may be small. This occurs because the lines of sight that do not encounter galaxies will be unreddened, while those that do encounter galaxies will usually have enough obscuration to prevent quasars from appearing in the sample. To illustrate these effects, Ostriker and Heisler presented a simple model in which all galaxies had the same optical depth, independent of orientation and impact parameter ("hard-edged" galaxies). The extinction they adopted was several times larger than that of galaxies at the present epoch. Wright (1986) refined the model to allow for random orientations of the disks and nonuniform distributions of dust within them ("soft-edged" galaxies). He concluded that dust in foreground galaxies cannot significantly affect the counts of quasars at high redshifts without also producing observable amounts of reddening. Heisler and Ostriker (1988) have recently modified their original models, taking into account the refinements by Wright and reducing the adopted values of the parameters that control the extinction within each galaxy. The new models consequently predict weaker effects than the old models. We learned of the second paper by Heisler and Ostriker after most of our work on this problem had been completed.

To address some of the issues raised by Ostriker and Heisler (1984), it would be advantageous to know which quasars have galaxies along the lines of sight and which do not. Wolfe *et al.* (1986, hereafter WTSC) have recently completed a survey of 68 quasars with high redshifts to search for Lyα absorption lines with large equivalent widths. They find that, in about 20% of the spectra, one or more lines are broadened by radiation damping. The column densities of neutral hydrogen in the damped Lyα systems, 10^{20}–10^{22} cm^{-2}, are comparable to or slightly larger than those in nearby galactic disks, and the total mass of neutral hydrogen is similar to that in all forms of luminous matter at the present epoch. On the basis of these and other observations, WTSC suggest that the damped Lyα systems are in fact the disks of galaxies in an early, gas-rich phase. If so, they are ideal for testing the suggestions made by Ostriker and Heisler. But whatever the real nature of the damped Lyα systems, some measure of the abundance of dust within them would be valuable. In § II, we compare the spectra of quasars in the WTSC survey that have damped Lyα with those that do not to set limits on the dust-to-gas ratio in the damped Lyα systems. We then use these results in § III to set limits on the mean and variance of the optical depth along random lines of sight. Our conclusions are summarized in § IV.

II. LIMITS ON THE DUST-TO-GAS RATIO

We are interested here in the changes in the continuum spectra of quasars caused by any dust in the damped Lyα systems. To establish some notation, we consider a quasar at a redshift z_e with one or more absorbers at redshifts z_a along the same line of sight. The wavelengths of photons at emission, absorption, and observation are related by

$$\lambda_o = (1 + z_a)\lambda_a = (1 + z_e)\lambda_e , \tag{1}$$

and the emitted and observed spectra are related by

$$f_o(\lambda_o) = \frac{f_e(\lambda_e) \exp [-\Sigma_a \tau_a(\lambda_a)]}{4\pi(1 + z_e)d_L{}^2(z_e)} , \tag{2}$$

where $d_L(z_e)$ is the luminosity distance of the quasar and $\tau_a(\lambda_a)$ is the optical depth of the ath absorber. By definition, $f_e(\lambda_e)d\lambda_e$ is the emitted power in the range of wavelengths $(\lambda_e, \lambda_e + d\lambda_e)$, and $f_o(\lambda_o)d\lambda_o$ is the observed flux in the range of wavelengths $(\lambda_o, \lambda_o + d\lambda_o)$. We now define the indices of the two spectra by $\alpha_o \equiv d \ln f_o/d \ln \lambda_o$ and $\alpha_e \equiv d \ln f_e/d \ln \lambda_e$. Differentiating equation (2) and using equation (1) gives

$$\Delta\alpha \equiv \alpha_o - \alpha_e = -\sum_a d\tau_a/d \ln \lambda_a . \tag{3}$$

In addition to any reddening caused by the damped Lyα systems, this expression might include contributions from the more numerous Lyα forest systems and/or smoothly distributed dust. However, even if the extra terms were present, they would not affect the statistical results derived below. We therefore display only the contributions to $\Delta\alpha$ from the damped Lyα systems.

The column density of dust is proportional to the optical depth at a fixed wavelength in the rest frame of an absorber. We choose the B band and write

$$\tau_a(\lambda) = \tau_{Ba}\,\xi(\lambda) , \qquad \xi(\lambda) \equiv A(\lambda)/A(4400 \text{ Å}) , \tag{4}$$

where $A(\lambda)$ is the extinction in magnitudes. We now define a dimensionless dust-to-gas ratio k_a through the expression

$$\tau_{Ba} \equiv k_a N_{Ha}/10^{21} \text{ cm}^{-2} , \tag{5}$$

where N_{Ha} is the column density of neutral hydrogen in the ath absorber. When radiation damping is important in the Lyα absorption line, its equivalent width in the rest frame W_a is uniquely related to N_{Ha} by

$$N_{Ha} = 1.88 \times 10^{18}(W_a/\text{Å})^2 \text{ cm}^{-2} , \tag{6}$$

(Spitzer 1978). Combining equations (3) through (6), we obtain

$$\Delta\alpha = -0.19 \sum_a k_a(W_a/10 \text{ Å})^2\, d\xi(\lambda_a)/d \ln \lambda_a . \tag{7}$$

Since the properties of any dust grains in the damped Lyα systems are not known, we use the mean extinction curves for the Milky Way (Savage and Mathis 1979), the LMC (Koornneef and Code 1981; Nandy *et al.* 1981), and the SMC (Prevot *et al.* 1984). These are shown in Figure 1. The three curves are similar at optical wavelengths, but their slopes differ in the range of interest here: $1000 \text{ Å} \le \lambda_a \le 1600$ Å. (The extinction curves for the LMC and SMC have been extrapolated over part of this range.) At the relevant wavelengths, $d\xi(\lambda_a)/d \ln \lambda_a$ is negative, and $\Delta\alpha$ is therefore positive.

The use of equation (7) to determine the dust-to-gas ratio in a particular damped Lyα system would require knowledge of both the observed and emitted spectra of the background quasar. We avoid this difficulty by comparing the observed spectra that have damped Lyα with the observed spectra that do not have damped Lyα in a large sample of quasars. The corresponding spectral indices are denoted by $\alpha_o{}^d$ and $\alpha_o{}^u$. Our basic assumption is that the emitted spectra of the quasars do not depend on whether damped Lyα systems happen to lie along the lines of sight to Earth. In other words, the spectral indices $\alpha_e{}^d$ and $\alpha_e{}^u$ are assumed to have been drawn at random from the same parent population. We also assume, for the moment, that all damped Lyα systems have the same dust-to-gas ratio, denoted by k. For each spectrum with damped Lyα and a given value of k, we compute the increment $\Delta\alpha$ from equation (7). The distribution of $\alpha_o{}^d - \Delta\alpha$ is then compared with the distribution of $\alpha_o{}^u$ using a Mann-Whitney U-test. This

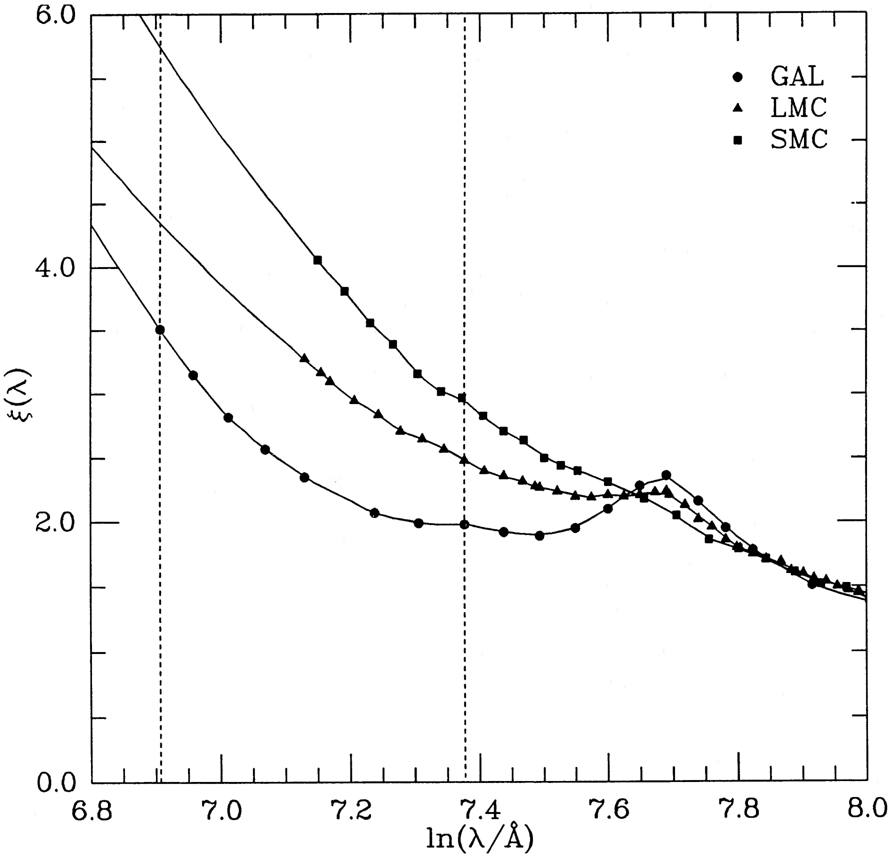

FIG. 1.—Nondimensional extinction curves used in the calculation of the dust-to-gas ratio. The data points were taken from Savage and Mathis (1979) for the Milky Way, Koornneef and Code (1981) and Nandy *et al.* (1981) for the LMC, and Prevot *et al.* (1984) for the SMC. The extrapolations at short wavelengths are all of the form $\xi(\lambda) \propto \lambda^{-1}$. The vertical dashed lines, at $\lambda = 1000$ Å and $\lambda = 1600$ Å, indicate the range of wavelengths relevant to our study.

gives the probability, with the assumed value of k, that $\alpha_o^d - \Delta\alpha$ and α_o^u, and therefore α_e^d and α_e^u, were drawn at random from the same parent population. A nonparametric test is appropriate because the distribution of spectral indices is not known in advance and may be affected by observational errors. The U-test is the strongest nonparametric test for differences in "central tendency," but a Kolmogorov-Smirnov test could also be used (see, for example, Sachs 1984).

We now give a brief description of the U-test in a notation suitable for our problem. For a given value of k, we compute the statistic

$$U(k) = n_d n_u + \tfrac{1}{2} n_d(n_d + 1) - R_d(k) , \tag{8}$$

where n_d and n_u are, respectively, the numbers of spectra in which damped Lyα is present and absent, and $R_d(k)$ is the sum of the ranks of the indices $\alpha_o^d - \Delta\alpha$ in the combined sample $\{\alpha_o^u\} \cup \{\alpha_o^d - \Delta\alpha\}$. We repeat the calculations for many different dust-to-gas ratios to determine the dependence of U on k. The sampling distribution of U, and therefore k, is known under the null hypothesis that $\alpha_o^d - \Delta\alpha$ and α_o^u were drawn from the same population; Mann and Whitney (1947) give a recursion relation for the exact probabilities for samples of arbitrary sizes. When n_d and n_u are both larger than 10, the distribution of U is approximately normal with a mean $n_d n_u/2$ and a variance $n_d n_u(n_d + n_u + 1)/12$. In this case, the probability that the dust-to-gas ratio is less than k takes the form

$$P(<k) = \int_{x(0)}^{x(k)} dx \exp\left(-\tfrac{1}{2}x^2\right) \bigg/ \int_{x(0)}^{x(\infty)} dx \exp\left(-\tfrac{1}{2}x^2\right) , \tag{9}$$

where

$$x(k) = \frac{\sqrt{12}U(k) - \sqrt{3}n_d n_u}{(n_d n_u)^{1/2}(n_d + n_u + 1)^{1/2}} . \tag{10}$$

The limits of integration and denominator of equation (9) ensure that $P(<k)$ vanishes for $k = 0$, as required on physical grounds, and that $P(<k)$ approaches unity for $k \to \infty$, as required by the definition of a probability.[1] We use $P(<k)$ to set confidence limits on the dust-to-gas ratio in the damped Lyα systems.

[1] If one were to contemplate negative dust-to-gas ratios, the lower limits of integration in eq. (9) would need to be replaced by $x(-\infty)$. This would lead to more restrictive confidence limits on k; for example, all the 95% limits derived below would become 97.5% limits.

At this point, some comments about our method may be worthwhile. First, the quasars without damped Lyα in their spectra (the "control sample") must be selected and observed in the same way as the quasars with damped Lyα in their spectra. Second, dust in an intergalactic medium or in the Lyα forest systems would redden the spectra of all quasars by an amount that depends only on redshift. Unless the presence or absence of damped Lyα is correlated with the redshifts of the quasars, the extra reddening will affect the distribution of $\alpha_o^d - \Delta\alpha$ in the same way as it affects the distribution of α_o^u. Thus our results only pertain to any dust in the damped Lyα systems. Finally, we consider the bias discussed by Ostriker and Heisler (1984); absorbers with large quantities of dust may be underrepresented in a sample derived from optically selected quasars. We avoid this problem altogether by assuming that the dust-to-gas ratio is the same in each of the damped Lyα systems. To the extent that our assumption is true, the "missing" quasars will not affect our estimate of the dust-to-gas ratio. There may in fact be variations in the dust-to-gas ratios in the damped Lyα systems, but these are likely to be smaller than the variations in the column densities of neutral hydrogen, which span nearly two orders of magnitude. In the next section, where we calculate the mean and variance of the optical depth along random lines of sight, some corrections must be made for the Ostriker-Heisler effect. We argue in the last section that our results are not sensitive to modest variations in the dust-to-gas ratio.

We now apply the method outlined above to the survey by WTSC. They obtained spectra in the region 3200 Å $< \lambda_o <$ 5200 Å at a resolution of 10 Å for 68 quasars with apparent magnitudes brighter than $V_l \simeq 18.5$, and redshifts in the range $2.27 \leq z_e \leq 3.40$. The presence or absence of damped Lyα was not considered in the selection of the quasars. The spectra were corrected for instrumental sensitivity and atmospheric refraction, but because some of the quasars were observed through small apertures and large air masses, there are random errors in the spectrophotometry; we return to this point later. WTSC used an iterative technique to fit a continuum to each spectrum and to locate absorption features shortward of Lyα emission with signal-to-noise ratios greater than 4. Features with rest frame equivalent widths greater than 5 Å were considered to be candidates for damped Lyα, and about half of these were later observed with spectral resolutions of 1–3 Å (Turnshek *et al.* 1989, and references therein). A single damped Lyα line was confirmed by Voigt-profile fitting in each of 11 of the intermediate-resolution spectra, and two damped Lyα lines were confirmed in each of two of the intermediate-resolution spectra; most of the other candidates turned out to be blends of Lyα forest lines. The redshifts of the damped Lyα systems lie in the range $1.78 \leq z_a \leq 2.80$, and their equivalent widths lie in the range 7.3 Å $\leq W_a \leq$ 52 Å. (We use the classification of the features in Table 3 of WTSC and the column densities communicated to us by Wolfe 1987. These are the same as the more recent results given by Turnshek *et al.* 1989, except for revisions in the three weakest lines, which have a negligible effect on our results.)

To estimate the dust-to-gas ratio, we compare the 13 spectra from the WTSC survey in which the presence of damped Lyα has been confirmed with the 38 spectra in which all absorption features have rest frame equivalent widths less than 5 Å. Damped Lyα is, with a reasonable degree of certainty, absent in the second set of spectra. We have checked that the distributions of the redshifts and magnitudes of the quasars in these subsamples are statistically indistinguishable. The presence or absence of damped Lyα has not yet been confirmed in the remaining 17 spectra from the WTSC survey, which we omit from the analysis in this section. Any damped Lyα systems at redshifts below 1.63, corresponding to $\lambda_o <$ 3200 Å, will occur with equal probability along the lines of sight to all quasars and therefore will not bias our estimates of the dust-to-gas ratio. Ideally, we would determine the spectral indices redward of Lyα emission, but this is not possible for the quasars with the highest redshifts. Figure 2 shows the blue parts of the continuum spectra derived by WTSC with $\log f_o$ plotted against $\log \lambda_e$ on a relative scale. We have determined the spectral indices by least-squares fits of the form $\log f_o = \alpha_o \log \lambda_o + \text{const}$ over a common range of wavelengths in the rest frames of the quasars: 1000 Å $\leq \lambda_e \leq$ 1150 Å. The values of $\log f_o$ were weighted in proportion to the squares of the signal-to-noise ratios given by WTSC. Three of the spectra (Q0458−020, Q1312+043, and Q1318−113), indicated by the circles in Figure 2, were omitted because they did not extend to the lower wavelength limit. The upper wavelength limit was chosen so as to avoid the blue wings of strong Lyα emission. Figure 3 shows the distributions of the spectral indices in the two subsamples. They are both reasonably symmetrical and there is no obvious tendency for the spectra with damped Lyα to be steeper than the spectra without damped Lyα.[2]

The results of the U-tests are shown in Figure 4. Here, the probability that the dust-to-gas ratio in the damped Lyα systems is less than k is plotted against k for the Galactic, LMC, and SMC extinction curves.[3] In the computation of the increments $\Delta\alpha$, the derivatives $d\xi(\lambda_a)/d\ln\lambda_a$ were evaluated at wavelengths corresponding to the range over which the spectral indices were fitted, 1000 Å $\leq (1 + z_a)\lambda_a/(1 + z_e) \leq$ 1150 Å, and the column densities $N_{\mathrm{H}a}$ were taken from Wolfe (1987). The most probable value of k, where $dP(<k)/dk$ has a maximum, is always near zero. Thus no reddening is observed in the damped Lyα systems. We find that the 95% confidence limits on the dust-to-gas ratio, using the three different extinction curves, are

$$k \leq 0.41\ (\mathrm{GAL})\,, \qquad k \leq 0.29\ (\mathrm{LMC})\,, \qquad \text{and} \qquad k \leq 0.19\ (\mathrm{SMC})\,. \tag{11}$$

These would be reduced by 20%–30% if the outlying point (Q2206−199N) in the lower panel of Figure 3 were omitted. For comparison, the observed reddenings and column densities in the three galaxies imply $k = 0.79$ (GAL), $k = 0.19$ (LMC), and $k = 0.05$ (SMC). [We have used the relation $k = 9.2 \times 10^{20}(1 + R_V)(E_{B-V}/N_{\mathrm{H}})\ \mathrm{cm}^{-2}$, with $R_V = 3.1$ and the values of E_{B-V}/N_{H} given by Bohlin, Savage, and Drake 1978, Koornneef 1982, and Lequeux *et al.* 1984.] Thus our upper limits on the dust-to-gas ratio in the damped Lyα systems are half the observed value in the Milky Way but are several times larger than the observed values in the Magellanic Clouds. We find very similar results when a Kolmogorov-Smirnov test is used instead of the U-test.

Our limits on the dust-to-gas ratio in the damped Lyα systems would be stronger if the distributions of the spectral indices were narrower. In other studies, where the spectra of quasars are fitted over much larger ranges of wavelength, the dispersion is

[2] The distributions could be compared using the mean spectral indices but these are extremely sensitive to outlying points. We find $\langle\alpha_o^d\rangle - \langle\alpha_o^u\rangle = +0.33 \pm 0.86$ when all the quasars are included but $\langle\alpha_o^d\rangle - \langle\alpha_o^u\rangle = -0.34 \pm 0.50$ when Q2206−199N is omitted. This is one of the reasons we have adopted a nonparametric test.

[3] For the subsamples in our study, with $n_d = 12$ and $n_u = 36$, eq. (9) differs from the exact probabilities by less than 0.3%.

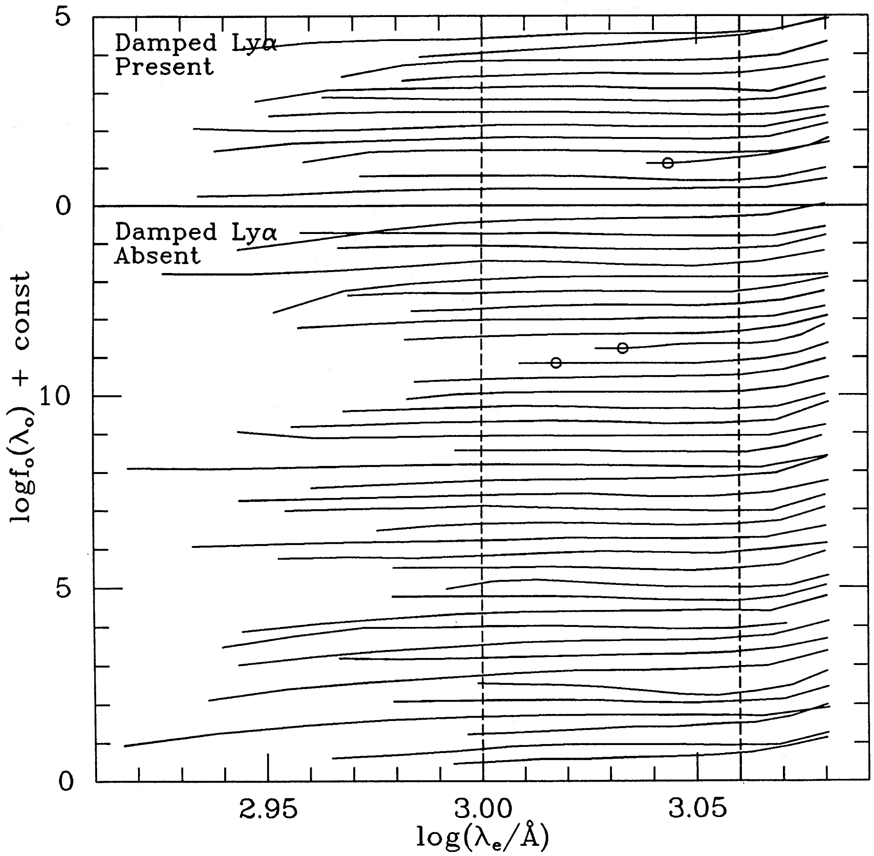

FIG. 2.—Continuum spectra of quasars in the WTSC sample with observed flux plotted against emitted wavelength. The upper panel includes the spectra in which damped Lyα has been confirmed, and the lower panel includes the spectra in which all absorption features have rest frame equivalent widths less than 5 Å. The vertical dashed lines indicate the range of wavelengths over which the spectral indices have been fitted. The spectra marked with circles are not used in our estimates of the dust-to-gas ratio.

$\langle(\alpha_o - \bar{\alpha}_o)^2\rangle^{1/2} \simeq 0.5$ (Richstone and Schmidt 1980). This is probably close to the intrinsic dispersion or "cosmic scatter" $\langle(\alpha_e - \bar{\alpha}_e)^2\rangle^{1/2}$. To understand the large dispersion found in our study, $\langle(\alpha_o - \bar{\alpha}_o)^2\rangle^{1/2} \simeq 2$, we consider three effects: (1) variations in galactic reddening, (2) errors in the spectrophotometry, and (3) blanketing by Lyα forest lines. The procedures adopted by WTSC for the selection and observation of the quasars ensure that each effect is random with regard to the presence or absence of damped Lyα. Variations in galactic reddening are small because most of the quasars are at high galactic latitudes; we find that the contribution to $\langle(\alpha_o - \bar{\alpha}_o)^2\rangle^{1/2}$ is less than 0.1. Errors in the spectrophotometry, although not important for the main objectives of the WTSC survey, could affect our determinations of the spectral indices. We can make a direct estimate of the errors because two of the quasars in the WTSC survey (Q0805+046 and Q0941+261) have also been observed by Steidel and Sargent (1987). In one case the spectral indices are nearly identical, and in the other case they differ by 1.2. Blanketing by Lyα forest lines causes wiggles and a depression blueward of Lyα emission in the apparent continua of low-resolution spectra. The wiggles, which are essentially stochastic, can make a significant contribution to $\langle(\alpha_o - \bar{\alpha}_o)^2\rangle^{1/2}$ when the spectral indices are determined over a small range of wavelengths.

We now consider in some detail the effects of blanketing by Lyα forest lines. From the basic definitions, one can show that the observed and emitted spectra of a quasar are related by

$$f_o(\lambda_o) \propto f_e(\lambda_e)\left[1 - \frac{1}{\lambda_a}(1 + z)\int_0^{W_r} dW\, W n(W, z)\right]. \tag{12}$$

In this expression, $\lambda_a = \lambda_o/(1 + z) = 1216$ Å is the wavelength of Lyα absorption, $n(W, z)$ is the number of lines per unit equivalent width and per unit redshift, and W_r is the equivalent width below which the lines are not resolved. Since the second term in the brackets is much smaller than unity, we have, to a good approximation,

$$\ln f_o(\lambda_o) = \ln f_e(\lambda_e) - \frac{1}{\lambda_a}(1 + z)\int_0^{W_r} dW\, W n(W, z) + \text{const}. \tag{13}$$

The difference between the indices of the observed and emitted spectra is given by a regression of $\ln[f_o(\lambda_o)/f_e(\lambda_e)]$ against $\ln \lambda_o$ over a finite range of wavelengths $\lambda_1 \leq \lambda_o \leq \lambda_2$. The resulting expression for $\Delta\alpha$, which involves sums over discrete values of $\ln \lambda_o$, is then

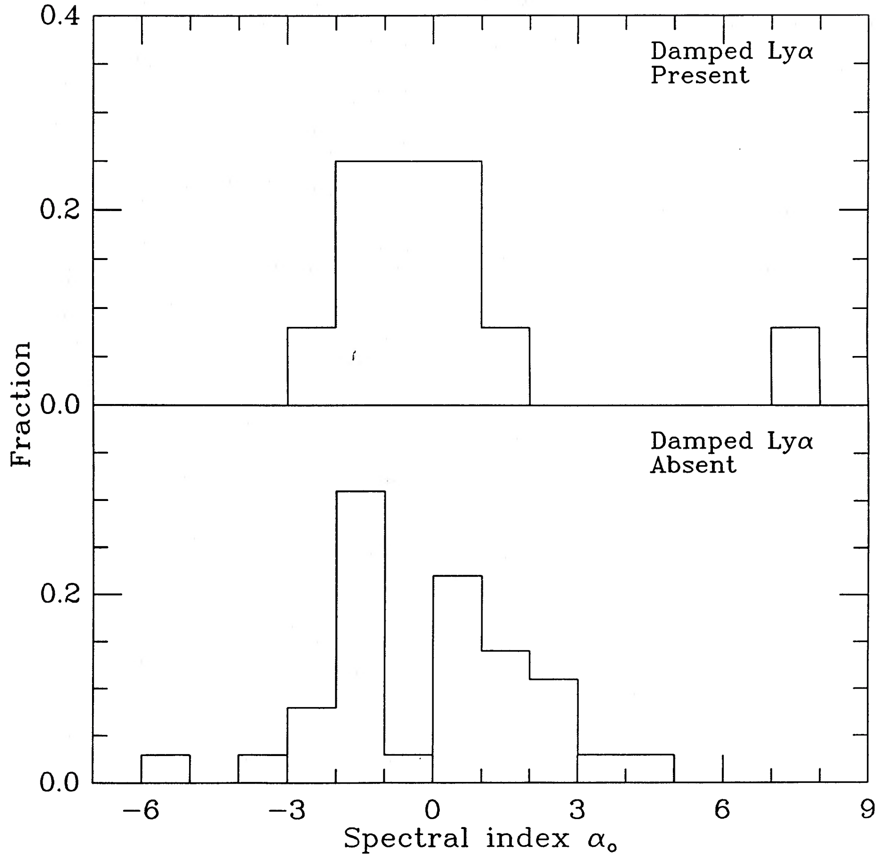

FIG. 3.—Histograms of the observed spectral indices of quasars in the WTSC sample. The upper panel includes the spectra in which damped Lyα has been confirmed, and the lower panel includes the spectra in which all absorption features have rest frame equivalent widths less than 5 Å (after omitting those marked with circles in Fig. 2). The large dispersion in the spectral indices is discussed in § II.

simplified by taking the continuum limit. After some lengthy but straightforward calculations, we obtain

$$\Delta\alpha = \alpha_o - \alpha_e = \frac{-(12/\lambda_a)}{[\ln(\lambda_2/\lambda_1)]^3} \int_{(\lambda_1/\lambda_a)-1}^{(\lambda_2/\lambda_a)-1} dz \ln[\lambda_a(1+z)/(\lambda_1\lambda_2)^{1/2}] \int_0^{W_r} dW W n(W, z) . \tag{14}$$

The mean increment over many lines of sight $\overline{\Delta\alpha}$ is given by the same expression with $\bar{n}(W, z)$ substituted for $n(W, z)$. Since any correlations between the equivalent widths and redshifts of Lyα lines along different lines of sight are completely negligible, we can write

$$\langle [n(W, z) - \bar{n}(W, z)][n(W', z') - \bar{n}(W', z')] \rangle = \bar{n}(W, z)\delta(W - W')\delta(z - z') , \tag{15}$$

where δ denotes the usual delta function. The variance of $\Delta\alpha$, computed from equation (14), is therefore

$$\langle (\Delta\alpha - \overline{\Delta\alpha})^2 \rangle = \frac{(12/\lambda_a)^2}{[\ln(\lambda_2/\lambda_1)]^6} \int_{(\lambda_1/\lambda_a)-1}^{(\lambda_2/\lambda_a)-1} dz \{\ln[\lambda_a(1+z)/(\lambda_1\lambda_2)^{1/2}]\}^2 \int_0^{W_r} dW W^2 \bar{n}(W, z) . \tag{16}$$

This is the contribution to $\langle (\alpha_o - \bar{\alpha}_o)^2 \rangle$ from blanketing by Lyα forest lines at a fixed value of α_e.

To make further progress, we require the mean density of Lyα forest lines. The standard expression is

$$\bar{n}(W, z) = (n_o/W_*)(1+z)^\gamma \exp(-W/W_*) , \tag{17}$$

and the parameters appropriate for a mixture of Lyα-only and Lyα-metal systems are $n_o = 16$, $\gamma = 1.8$, and $W_* = 0.36$ Å (Sargent *et al.* 1980; Murdoch *et al.* 1986). The list of absorption features in the WTSC survey is virtually complete for $W_a \geq 5$ Å and has very few entries for $W_a \leq 1$ Å (Wolfe, Turnshek, and Smith 1989); thus, we adopt $W_r = 3$ Å as the effective limit of resolution. Our determinations of the spectral indices were made over the range of observed wavelengths from $\lambda_1 = 1000(1 + z_e)$ Å to $\lambda_2 = 1150$ $(1 + z_e)$ Å. Inserting these quantities in equations (14) and (16), we obtain $\overline{\Delta\alpha} = -0.6$ and $\langle (\Delta\alpha - \overline{\Delta\alpha})^2 \rangle^{1/2} = 0.7$, with only a weak dependence on the emission redshift z_e. The sum, in quadrature, of the contributions to $\langle (\alpha_0 - \bar{\alpha}_0)^2 \rangle^{1/2}$ from cosmic scatter, variations in galactic absorption, and blanketing by Lyα forest lines, is 0.9. We are fairly certain that the remainder of the dispersion in the spectral indices is caused by errors in the spectrophotometry. The dispersion could be reduced in future studies by obtaining

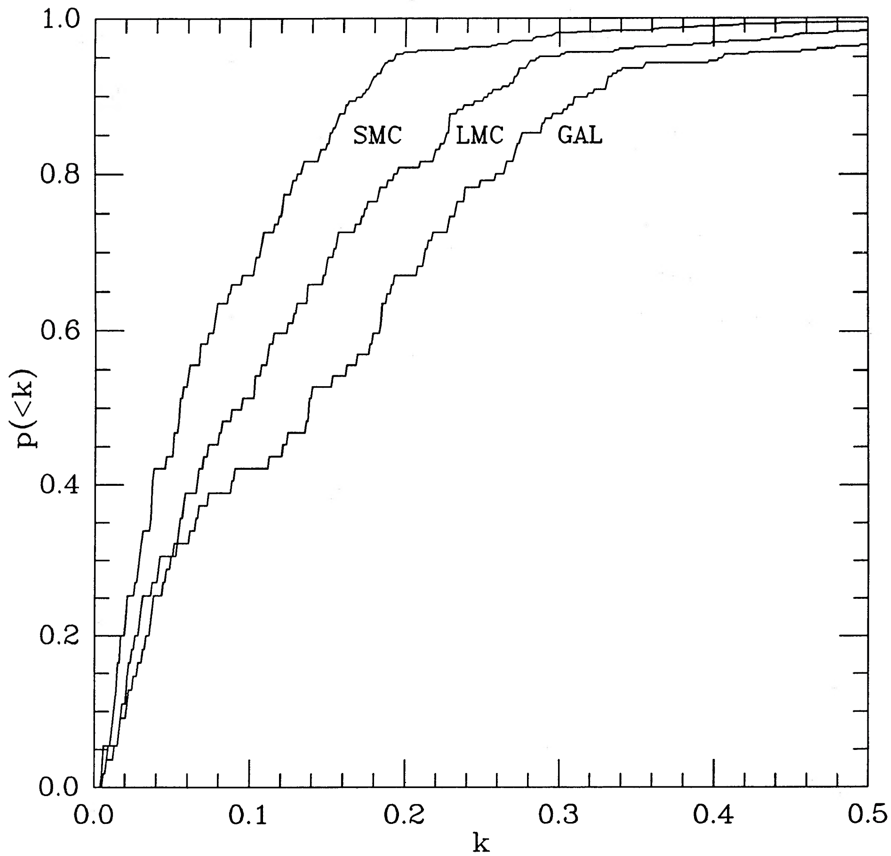

FIG. 4.—Probability derived from the U-test that the dust-to-gas ratio in the damped Lyα systems is less than k. Results are shown for the Galactic, LMC, and SMC extinction curves. The most probable values of k are always near zero and the 95% confidence limits are given in eq. (11).

spectra with accurate flux calibration over a large range of wavelengths to the red of Lyα emission. In this way, our limits on the dust-to-gas ratio in the damped Lyα systems could be improved or a positive detection could be made.

III. STATISTICAL PROPERTIES OF THE ABSORBERS

We compute in this section some statistical properties of any obscuration caused by the damped Lyα systems. In particular, we are interested in the mean optical depth as a function of redshift along random lines of sight and the variance about the mean optical depth. Since no reddening has been detected in the damped Lyα systems, our results must be interpreted as upper limits on $\bar{\tau}(z)$ and $\langle \Delta\tau^2(z) \rangle$. The method we use to compute these quantities is similar to the method of characteristic functions introduced by Wright (1986). We define $\rho(\tau, z)d\tau\, dz$ as the mean number of absorbers along random lines of sight with optical depths in the interval $(\tau, \tau + d\tau)$ and redshifts in the interval $(z, z + dz)$. Furthermore, we define $p(\tau \mid z)d\tau$ as the probability that the total optical depth of all absorbers with redshifts less than z along random lines of sight lies in the interval $(\tau, \tau + d\tau)$. Throughout this section, the optical depths, with or without subscripts, are assumed to be evaluated at a fixed wavelength in the rest frame of the observer (usually the B band). The function $\rho(\tau, z)$ is directly related to the observed or assumed properties of the absorbers, while the function $p(\tau \mid z)$ is needed to compute $\bar{\tau}(z)$ and $\langle \Delta\tau^2(z) \rangle$.

The relation between $p(\tau \mid z)$ and $\rho(\tau, z)$ can be derived as follows. We first introduce an auxiliary function $q(\tau \mid z, z + \delta z)$, defined as the probability that the optical depth is τ for all absorbers with redshifts in the small interval $(z, z + \delta z)$. On the scales of interest here, any correlations between the redshifts of the absorbers should be negligible. We can therefore write the probability for the interval $(0, z + \delta z)$ as a convolution of the probabilities for the intervals $(0, z)$ and $(z, z + \delta z)$, i.e.,

$$p(\tau \mid z + \delta z) = \int_0^\tau d\tau' p(\tau - \tau' \mid z) q(\tau' \mid z, z + \delta z) . \tag{18}$$

To satisfy this equation, $q(\tau \mid z, z + \delta z)$ must approach a delta function $\delta(\tau)$ in the limit $\delta z \to 0$. We now assume that the probability of finding one absorber in the interval $(z, z + \delta z)$ is an infinitesimal of order δz and that the probability of finding more than one absorber is an infinitesimal of order $(\delta z)^2$; thus

$$q(\tau \mid z, z + \delta z) \propto \delta(\tau) + \rho(\tau, z)\delta z + O(\delta z)^2 . \tag{19}$$

The constant of proportionality is fixed by the normalization of $q(\tau \mid z, z + \delta z)$ with respect to τ, which implies

$$q(\tau \mid z, z + \delta z) = \delta(\tau)\left[1 - \delta z \int_0^\infty d\tau' \rho(\tau', z)\right] + \rho(\tau, z)\delta z + O(\delta z)^2 . \tag{20}$$

Substituting this result into equation (18) and taking the limit $\delta z \to 0$, we obtain the differential equation

$$\frac{\partial p(\tau \mid z)}{\partial z} = \int_0^\tau d\tau' p(\tau - \tau' \mid z)\rho(\tau', z) - p(\tau \mid z) \int_0^\infty d\tau' \rho(\tau', z) . \tag{21}$$

The required boundary condition is $p(\tau \mid 0) = \delta(\tau)$.

Equation (21) can be solved by taking Fourier transforms on the variable τ. We adopt the convention

$$\tilde{f}(s) = \int_{-\infty}^{+\infty} d\tau f(\tau) \exp(2\pi i s \tau) . \tag{22}$$

Since $p(\tau \mid z)$ and $\rho(\tau, z)$ must vanish for negative values of τ, we obtain

$$\frac{\partial \tilde{p}(s \mid z)}{\partial z} = \tilde{p}(s \mid z)[\tilde{\rho}(s, z) - \tilde{\rho}(0, z)] . \tag{23}$$

The integration over z, with $\tilde{p}(s \mid 0) = 1$, gives

$$\tilde{p}(s \mid z) = \exp\left\{\int_0^z dz'[\tilde{\rho}(s, z') - \tilde{\rho}(0, z')]\right\} , \tag{24}$$

and the inverse transform gives

$$p(\tau \mid z) = \int_{-\infty}^{+\infty} ds \exp\left\{-2\pi i s \tau + \int_0^z dz'[\tilde{\rho}(s, z') - \tilde{\rho}(0, z')]\right\} . \tag{25}$$

It is easy to verify that $p(\tau \mid z)$ is always normalized to unity. One can also show that the nth moment of the optical depth as a function of redshift is given by

$$\langle \tau^n(z) \rangle \equiv \int_0^\infty d\tau \tau^n p(\tau \mid z) = \frac{1}{(2\pi i)^n}\left[\frac{\partial^n}{\partial s^n} \tilde{p}(s \mid z)\right]_{s=0} . \tag{26}$$

The formulae derived above are quite general and can be used in a variety of ways. Wright (1986) and Heisler and Ostriker (1988) calculate $p(\tau \mid z)$ using a specific model for the extinction by galactic disks (with identical, nonevolving, exponential profiles). Since the damped Lyα systems may not conform to this model, we use the observed column densities and redshifts to calculate $\rho(\tau, z)$ and hence $p(\tau \mid z)$. For the moment, we ignore the fact that the absorbers are only detected at redshifts greater than 1.63. Thus we write

$$\rho(\tau, z) = \frac{1}{Q(z)} \sum_a \delta(\tau - \tau_a)\delta(z - z_a) , \tag{27}$$

where $Q(z)$ is the number of lines of sight that extend to a redshift z and the sum is over all absorbers in the sample. In this case, equation (24) gives

$$\tilde{p}(s \mid z) = \exp\left\{\sum_a \frac{\theta(z - z_a)}{Q(z_a)} [\exp(2\pi i s \tau_a) - 1]\right\} , \tag{28}$$

where theta is the unit step function, defined as $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$. From equations (26) and (28), the mean optical depth is

$$\bar{\tau}(z) = \sum_a \frac{\tau_a \theta(z - z_a)}{Q(z_a)} , \tag{29}$$

and the variance about the mean is

$$\langle \Delta\tau^2(z) \rangle = \sum_a \frac{\tau_a{}^2 \theta(z - z_a)}{Q(z_a)} . \tag{30}$$

We now consider the normalization factors $Q(z_a)$ that appear in the denominators of equations (27) through (30). Were it not for the Ostriker-Heisler effect, we could simply write $Q(z_a) = \sum_e \theta(z_e - z_a)$, the number of quasars in the sample with redshifts z_e greater than z_a. However $Q(z_a)$ will be smaller and $\bar{\tau}(z)$ and $\langle \Delta\tau^2(z) \rangle$ will be correspondingly larger when obscuration is taken into account. In the models proposed by Heisler and Ostriker (1988), the mean optical depth along random lines of sight is twice the mean optical depth along lines of sight to optically selected quasars. We correct for this bias in a self-consistent way by introducing a weighting factor $w(z_e)$, defined as the probability that a quasar with a redshift z_e, even though obscured, remains brighter than the limiting magnitude of the sample. In other words, $1 - w(z_e)$ is the fraction of quasars at a redshift z_e that are "missing" from the sample. The corrected normalization factor, appropriate for random lines of sight, is then

$$Q(z_a) = \sum_e w(z_e)\theta(z_e - z_a) , \tag{31}$$

where the sum is over all quasars in the sample.

The weighting factors $w(z_e)$ are closely related to the true luminosity function of quasars. We define $\phi_t(L, z)dL$ as the number of quasars per unit comoving volume at a redshift z with absolute luminosities in the range $(L, L + dL)$. For each quasar in the sample, we compute the total optical depth of all absorbers along the line of sight, $\tau_e = \sum_a^e \tau_a$. The corresponding weighting factor is then proportional to the number of quasars with the same redshift and extinction that are bright enough to have been included in the sample; thus

$$w(z_e) = \int_{L_l(\tau_e, z_e)}^{\infty} dL\phi_t(L, z_e) \Big/ \int_{L_l(0, z_e)}^{\infty} dL\phi_t(L, z_e) . \tag{32}$$

The denominator ensures that $w(z_e) = 1$ for $\tau_e = 0$, and the limit of integration $L_l(\tau, z)$ is the absolute luminosity of a quasar with an optical depth τ and a redshift z that appears at the limiting magnitude of the sample. For $q_0 = 0.5$ and $H_0 = 50$ km s^{-1} Mpc^{-1} and luminosities in the B band, the relation is

$$L_l(\tau, z) = e^{\tau}[(1 + z) - (1 + z)^{1/2}]^2 \text{ dex } \{20.32 + 0.4[K(z) - B_l]\} L_\odot . \tag{33}$$

We use $B_l = 18.0$, the approximate limiting magnitude of the WTSC survey, and $K(z) = -0.1$, a good approximation to the K-correction for quasars with $z < 3$ (see Fig. 6 of Koo and Kron 1988). One can show that, although both $\phi_t(L, z)$ and $L_l(\tau, z)$ depend on q_0 and H_0, the $w(z_e)$ do not.

Most determinations of the quasar luminosity function are based on the assumption that space is transparent. However, if obscuration is important, the true luminosity function $\phi_t(L, z)$ required in the calculation of $w(z_e)$ will differ from the observed luminosity function $\phi_o(L, z)$. For a fixed optical depth τ, the relation is $\phi_o(L, z) = e^{\tau}\phi_t(e^{\tau}L, z)$, where the first factor of e^{τ} is the Jacobian of the transformation between the luminosities determined with and without extinction. In the general case, we know only the probability that the optical depth is τ; thus

$$\phi_o(L, z) = \int_0^{\infty} d\tau e^{\tau}\phi_t(e^{\tau}L, z)p(\tau \mid z) . \tag{34}$$

Taking the Fourier transform on the variable ln L, we obtain the simple result

$$\tilde{\phi}_o(s, z) = \tilde{\phi}_t(s, z)\tilde{p}\left(\frac{1}{2\pi i} - s \mid z\right) . \tag{35}$$

The true luminosity function is therefore

$$\phi_t(L, z) = \int_{-\infty}^{+\infty} dsL^{-2\pi i s}\tilde{\phi}_o(s, z) \Big/ \tilde{p}\left(\frac{1}{2\pi i} - s \mid z\right) . \tag{36}$$

Given the observed luminosity function and the optical depths and redshifts of the absorbers detected in a magnitude limited sample of quasars, the problem of determining the mean and variance of the optical depth along random lines of sight is completely specified by equations (28) through (33) and equation (36).

Koo and Kron (1988) have combined the data from several optical surveys, including their own, to estimate the luminosity function of quasars at different redshifts in the range $1 \le z \le 3$. We have converted the results, originally expressed in J magnitudes, into B luminosities using the relations $B - J = 0.1$ and $\phi_o(L, z) = \phi_{KK}(M, z) \mid \partial M/\partial L \mid$. The observed luminosity function is plotted as the solid curves in Figure 5 for $q_0 = 0.5$ and $H_0 = 50$ km s^{-1} Mpc^{-1}. We have fitted to this a Schechter function with a redshift-dependent knee:

$$\phi_o(L, z) = \phi_*(L/L_*)^{-\beta} \exp [-(1 + z)^{-\gamma}L/L_*] . \tag{37}$$

Minimizing χ^2, we obtain

$$\beta = 1.74 \pm 0.05 , \quad \gamma = 3.9 \pm 0.4 , \tag{38a}$$

$$\log (\phi_*/\text{Gpc}^{-3} L_\odot^{-1}) = -6.9 \pm 0.6 , \quad \log (L_*/L_\odot) = 11.0 \pm 0.2 . \tag{38b}$$

Our model, shown as the solid lines in Figure 5, provides an excellent fit to the data, even at fairly low redshifts. The Fourier transform of $\phi_o(L, z)$, again on the variable ln L, is

$$\tilde{\phi}_o(s, z) = \phi_* L_*^{\beta}[(1 + z)^{\gamma}L_*]^{2\pi i s - \beta}\Gamma(2\pi i s - \beta) , \tag{39}$$

where Γ denoted the usual gamma function.

We now consider the optical depths τ_a that appear in the expressions for $\bar{\tau}(z)$ and $\langle\Delta\tau^2(z)\rangle$. To obtain unbiased estimates, we must include all the candidates for damped Lyα systems in the WTSC survey, not just those confirmed to be damped by intermediate-resolution spectroscopy. Most of the features with rest frame equivalent widths in the range 5 Å $\le W_a <$ 10 Å are in fact blends of Lyα forest lines on the flat part of the curve of growth (Wolfe, Turnshek, and Smith 1989). The column densities of neutral hydrogen in these systems are smaller than in damped Lyα systems with the same equivalent widths by amounts that depend sensitively on the

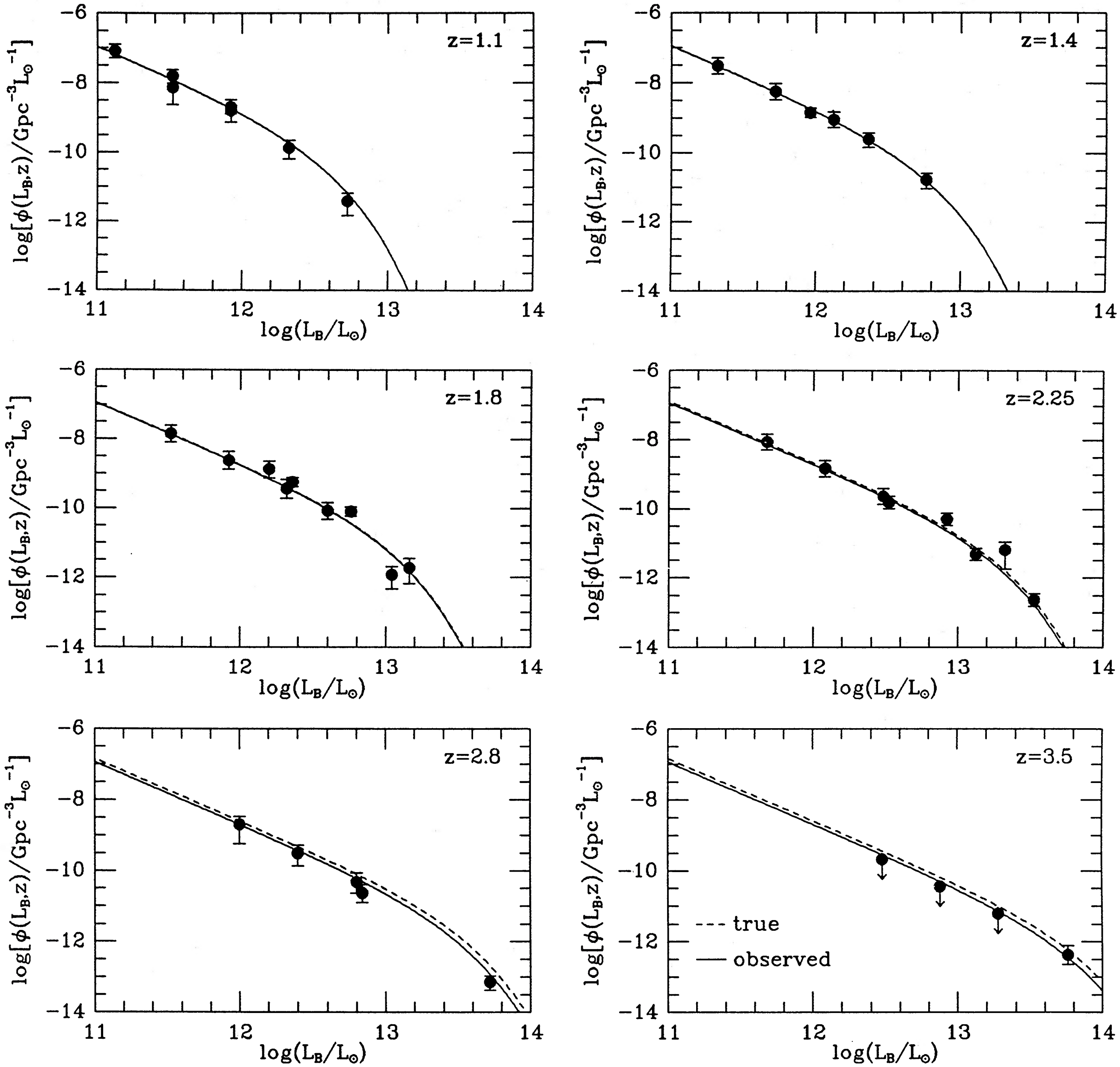

FIG. 5.—Luminosity function of quasars at different redshifts for $H_0 = 50$ km s^{-1} Mpc^{-1} and $q_0 = 0.5$. The data points with error bars are from Koo and Kron (1988). The solid curves represent the observed luminosity function in the form of eq. (37), with parameters determined by minimizing χ^2. The dashed curves represent the true luminosity function computed from eqs. (36) and (39) and the approximation $\bar{p}(s \mid z) \simeq \bar{p}(s \mid 1.63, z)$. All luminosities are in the B band.

unknown Doppler broadenings. Thus, from equations (4) through (6), the optical depth in the B band of the observer, for either damped or saturated lines, must satisfy the relation

$$\tau_a \leq 0.19k(W_a/10\ \text{Å})^2 \xi[4400\ \text{Å}/(1 + z_a)] . \tag{40}$$

Once again, $\xi(\lambda)$ is the nondimensional extinction curve (GAL, LMC, or SMC), and k is the nondimensional dust-to-gas ratio. To set limits on the mean and variance of the optical depth, we use the 95% confidence limits on k derived in the previous section, the column densities from Wolfe (1987) for the confirmed damped Lyα lines, and the equivalent widths from WTSC for the other features. We perform two sets of calculations, one that includes the 48 features with $W_a \geq 5$ Å and the other that includes the 16 features with $W_a \geq 10$ Å. The first set provides generous upper limits because all the features, including blends of Lyα forest lines, are treated as if they were single damped lines. The second set of calculations is probably more realistic but may not provide strict upper limits because a few of the damped lines are omitted.

Lyα systems are detected in the WTSC survey only at wavelengths longer than 3200 Å and therefore with redshifts greater than $z_l = 1.63$. As a result of this restriction, the procedure outlined above requires slight modification. We define $p(\tau \mid z_l, z)d\tau$ as the probability that the total optical depth of all absorbers with redshifts between z_l and z lies in the interval $(\tau, \tau + d\tau)$. It is easy to show that the Fourier transform of $p(\tau \mid z_l, z)$ is given by the right-hand side of equation (28). One can also show, using the convolution theorem in the form $\tilde{p}(s \mid 0, z) = \tilde{p}(s \mid 0, z_l)\tilde{p}(s \mid z_l, z)$, that the left-hand sides of equations (29) and (30) must be replaced by $\bar{\tau}(z) - \bar{\tau}(z_l)$ and $\langle \Delta\tau^2(z) \rangle - \langle \Delta\tau^2(z_l) \rangle$. The relation between $\phi_t(L, z)$ and $\phi_o(L, z)$ depends, of course, on all absorbers, including those with redshifts less than z_l. Most of the obscuration will occur at higher redshifts because the density of absorbers increases with redshift and the extinction curves decrease with wavelength. Thus, to a good approximation, we have $p(\tau \mid z) \simeq p(\tau \mid z_l, z)$ for $z > z_l$. The normalization factors $Q(z_a)$ are first computed from equation (31) with all $w(z_e) = 1$. Equation (28) then gives an approximation to $\tilde{p}(s \mid z)$ that can be used in equation (36) to compute a first approximation to $\phi_t(L, z)$. This is used in equation (32) to compute a better approximation to the $w(z_e)$ and hence $\tilde{p}(s \mid z)$. The procedure is repeated until all quantities have converged, and the final values of $Q(z_a)$ are then used to compute $\bar{\tau}(z) - \bar{\tau}(z_l)$ and $\langle \Delta\tau^2(z) \rangle - \langle \Delta\tau^2(z_l) \rangle$.

The results of our calculations are shown in Figures 5–8. We have plotted the true luminosity function of quasars, computed with the Galactic extinction curve and all Lyα systems with $W_a \geq 5$ Å, as the dashed curves in Figure 5. The small difference between $\phi_t(L, z)$ and $\phi_o(L, z)$ indicates that neglecting absorbers with $z < 1.63$ introduces only small errors in the normalization factors $Q(z_a)$. In fact, in the first iteration, with all $w(z_e) = 1$, and therefore no dependence on the luminosity functions, the $Q(z_a)$ are only 25%–33% larger than their final values. Figure 6 shows $p(\tau \mid 1.63, z)$, again computed with the Galactic extinction curve and all Lyα systems with $W_a \geq 5$ Å. The upper limits on $\bar{\tau}_B(z) - \bar{\tau}_B(1.63)$ and $[\langle \Delta\tau_B{}^2(z) \rangle - \langle \Delta\tau_B{}^2(1.63) \rangle]^{1/2}$ are plotted against redshift in Figures 7 and 8

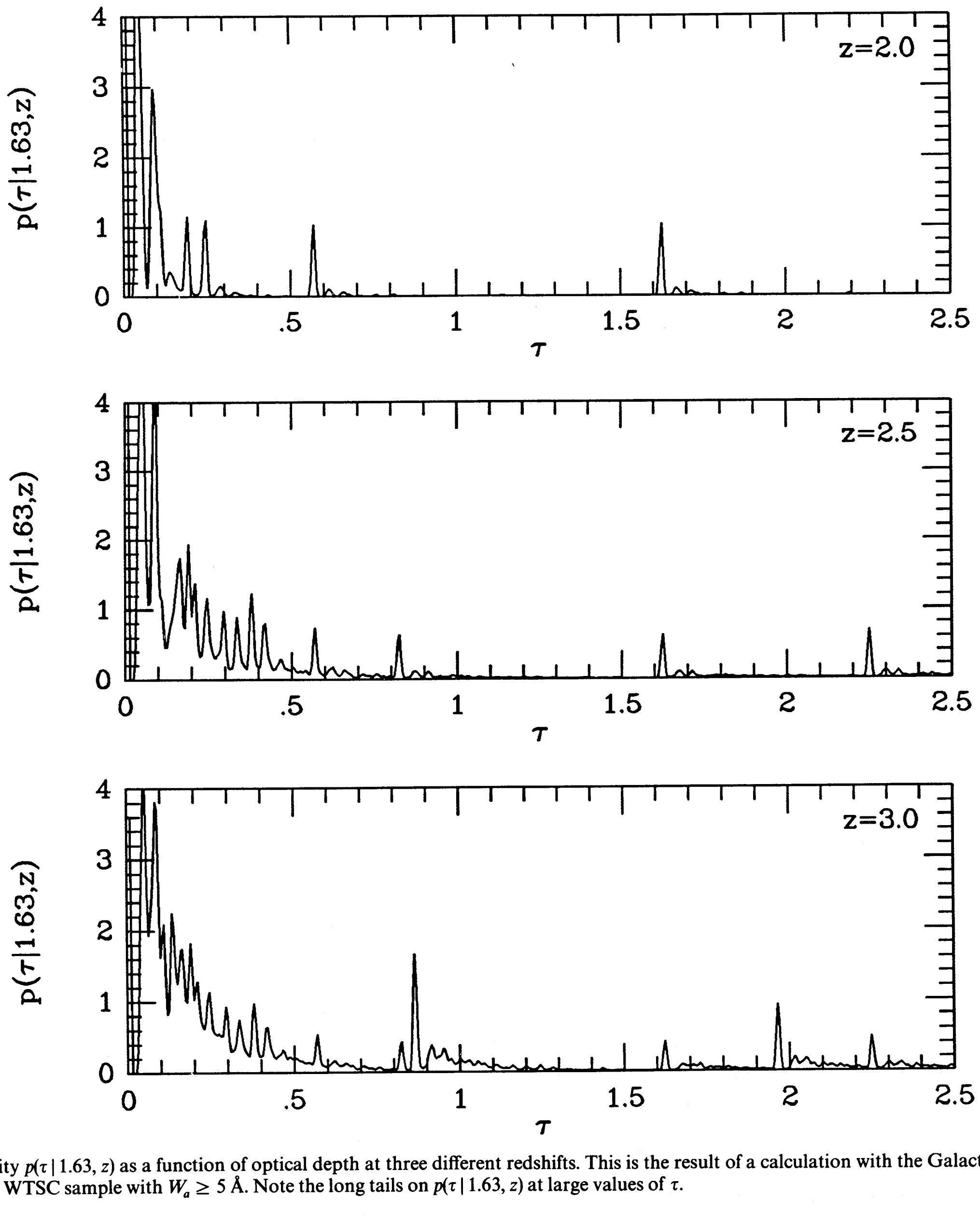

FIG. 6.—Probability $p(\tau \mid 1.63, z)$ as a function of optical depth at three different redshifts. This is the result of a calculation with the Galactic extinction curve and all Lyα systems in the WTSC sample with $W_a \geq 5$ Å. Note the long tails on $p(\tau \mid 1.63, z)$ at large values of τ.

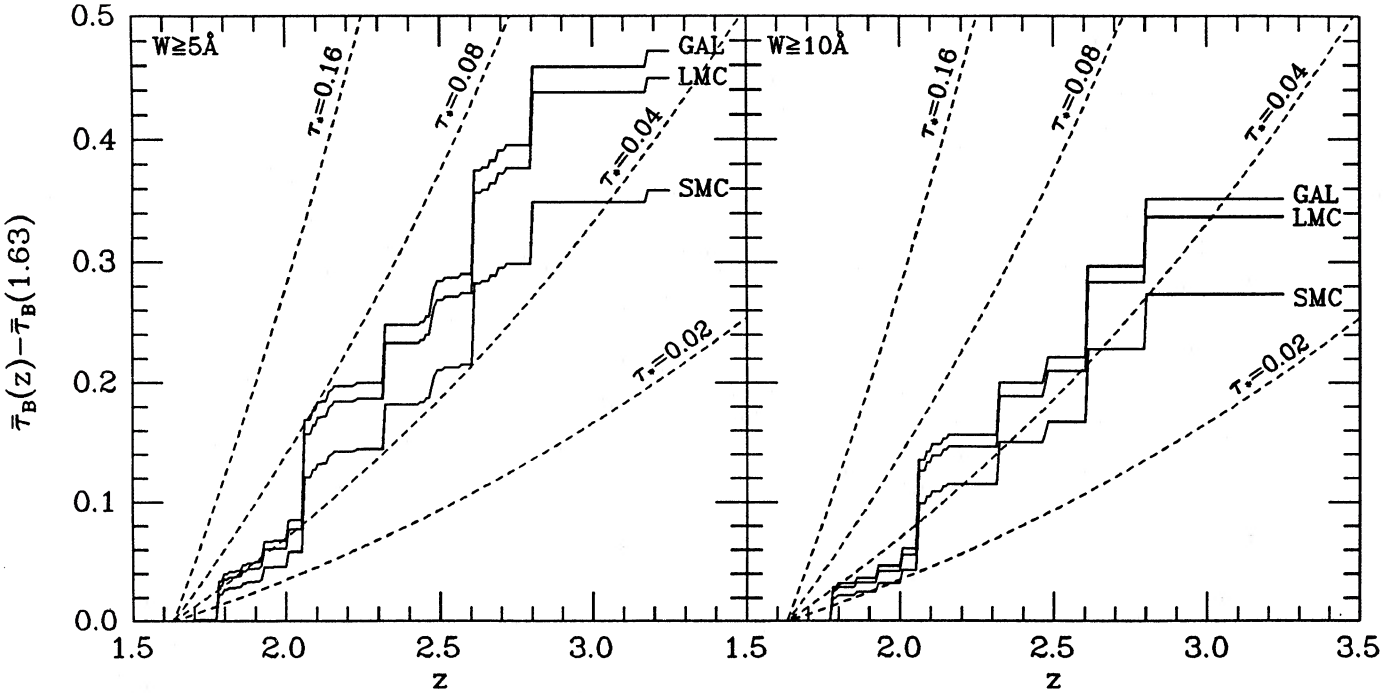

FIG. 7.—Upper limits on the mean optical depth along random lines of sight as a function of redshift. Results are shown for the Galactic, LMC, and SMC extinction curves. The left-hand and right-hand panels are based, respectively, on calculations that include all Lyα systems in the WTSC sample with $W_a \geq 5$ Å and $W_a \geq 10$ Å. The dashed curves are from eq. (41) with the indicated values of τ_*.

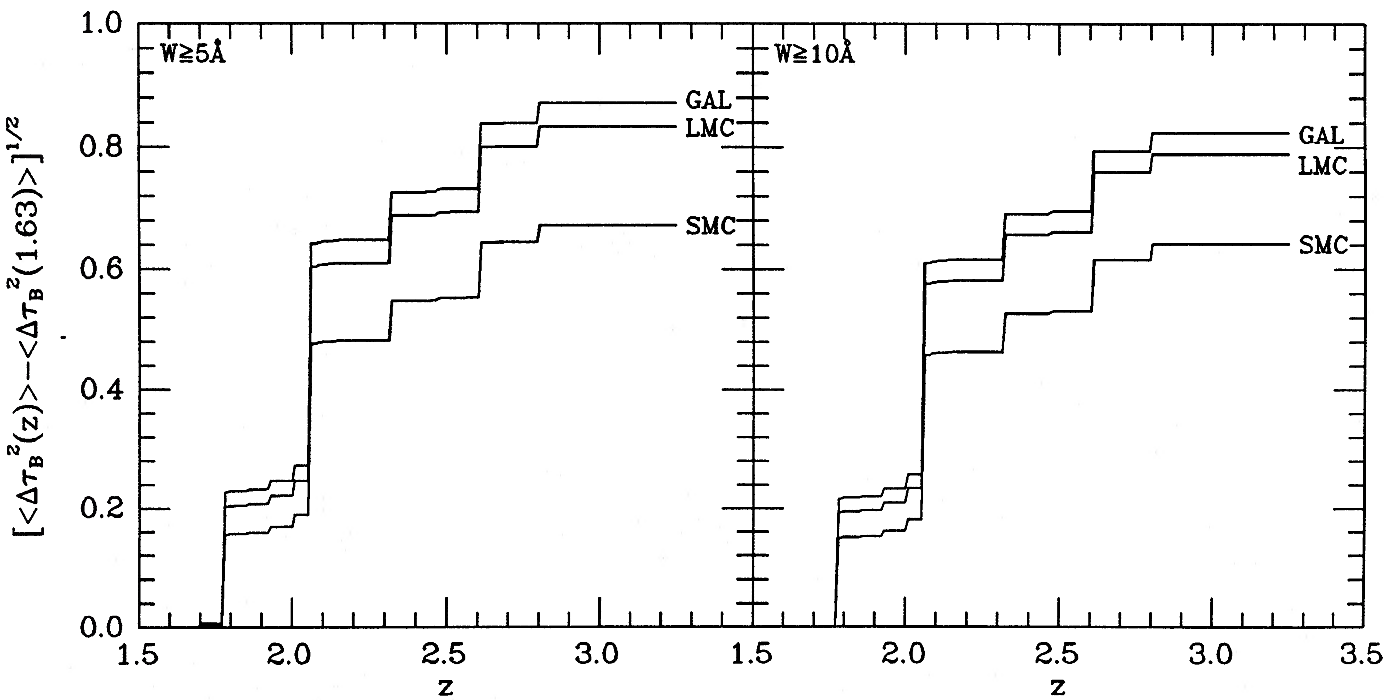

FIG. 8.—Upper limits on the dispersion about the mean optical depth along random lines of sight as a function of redshift. Results are shown for the Galactic, LMC, and SMC extinction curves. The left-hand and right-hand panels are based, respectively, on calculations that include all Lyα systems in the WTSC sample with $W_a \geq 5$ Å and $W_a \geq 10$ Å. Note that $[\langle\Delta\tau_B{}^2(z)\rangle - \langle\Delta\tau_B{}^2(1.63)\rangle]^{1/2}$ is comparable to or larger than $\bar{\tau}_B(z) - \bar{\tau}_B(1.63)$.

for the three different extinction curves and the two limits on W_a. As one might expect from the long tail on $p(\tau \mid 1.63, z)$, the dispersion about the mean is comparable to the mean itself. For the calculations with $W_a \geq 5$ Å, the upper limits on $\bar{\tau}_B(z) - \bar{\tau}_B(1.63)$ lie between 0.35 and 0.46 at $z = 3$, and the upper limits on $[\langle \Delta\tau_B{}^2(z) \rangle - \langle \Delta\tau_B{}^2(1.63) \rangle]^{1/2}$ lie between 0.67 and 0.87 at $z = 3$; in both cases, the largest values are for the Galactic extinction curve and the smallest values are for the SMC extinction curve. The means are reduced by 23% and the dispersions are reduced by 5% when the calculations include only the absorbers with $W_a \geq 10$ Å. This indicates that most of the contributions are from the damped Lyα systems rather than from the more numerous Lyα forest systems.

In the models proposed by Ostriker and Heisler (1984) and Heisler and Ostriker (1988), the mean optical depth along random lines of sight is given by the simple formula

$$\bar{\tau}_B(z) = 0.4\tau_*[(1 + z)^{5/2} - 1] . \tag{41}$$

The 1984 models have $\tau_* = 0.4$ or 0.8, and the 1988 models have $\tau_* = 0.16$; the first is based on a power-law extinction curve, and the second is based on the Galactic extinction curve. To compare our results with the predictions by Ostriker and Heisler, we have plotted as the dashed curves in Figure 7 the difference $\bar{\tau}_B(z) - \bar{\tau}_B(1.63)$ given by equation (41) for several values of τ_*. Our calculations with $W_a \geq 5$ Å imply the following limits: $\tau_* \leq 0.07$ (GAL), $\tau_* \leq 0.06$ (LMC), and $\tau_* \leq 0.05$ (SMC). These can be interpretated as 95% confidence limits because they are based on similar probabilities for the dust-to-gas ratios and because the observational errors in the column densities cause negligible uncertainties in $\bar{\tau}_B(z)$. A comparison of the dispersions about the mean optical depths, which requires some lengthy calculations, gives similar results. We therefore conclude that all the Ostriker-Heisler models predict too much obscuration, the 1984 models by factors of 6–16 and the 1988 models by factors of 2–3. Finally, we note that our limits on τ_* can be re-expressed in terms of the mean optical depth to a redshift of 3: $\bar{\tau}_B(3) \leq 0.9$ (GAL), $\bar{\tau}_B(3) \leq 0.7$ (LMC), and $\bar{\tau}_B(3) \leq 0.6$ (SMC). These are twice as restrictive as the 95% confidence limit inferred from the weak correlation between the spectral indices and the redshifts of quasars in the Richstone-Schmidt (1980) survey.

IV. CONCLUSIONS

We have introduced a new method to determine or set limits on the dust-to-gas ratio in the damped Lyα systems and any obscuration they cause along random lines of sight. Our method is based on two assumptions: that the emitted spectra of the background quasars are independent of the presence or absence of damped Lyα in the observed spectra, and that the dust-to-gas ratio is the same in all the damped Lyα systems. The first assumption seems almost self-evident, while the second must be considered as an approximation. We have repeated the entire analysis described above assuming that all Lyα systems in the WTSC survey with $W_a \geq 10$ Å have the same optical depth (in their rest frames) rather than the same dust-to-gas ratio. In this case, the upper limits on $\bar{\tau}_B(z)$, the mean optical depth along random lines of sight, increase by only 20%. We have also made a series of calculations in which the logarithms of the dust-to-gas ratios are assumed to have a Gaussian distribution with a dispersion $\sigma(\ln k)$ about a mean value $\langle \ln k \rangle$. For almost any dispersion, the upper limits on $\langle \ln k \rangle$ are within 0.25 of those obtained for $\sigma(\ln k) = 0$ (i.e., a constant dust-to-gas ratio). Moreover, for $\sigma(\ln k) \leq 0.5$, the upper limits on the mean optical depth along random lines of sight are within 30% of the ones derived in the previous section. We therefore conclude that none of our limits depends critically on the assumption that all damped Lyα systems have the same dust-to-gas ratio.

The damped Lyα systems contain virtually all the neutral hydrogen in the universe at redshifts between 1.6 and 3.0. Thus, whether or not they are galactic disks of the kind envisaged by WTSC, one might suppose that they also contain most of the dust. The abundances of heavy elements in the damped Lyα systems are poorly known, but there is some evidence that they are below the solar value (Bergeron and Stasinska 1986; Wolfe 1986). This suggests that the extinction curve for the SMC may be most appropriate and that any limits on the dust-to-gas ratio based on the absence of broad absorption features near 2200 Å in the rest frames of the absorbers should be treated with caution. A low abundance of heavy elements in the damped Lyα systems might also lead one to expect that the dust-to-gas ratio should be smaller than that in the local interstellar medium of the Milky Way. Our limits, including those derived with the extinction curves for the Magellanic Clouds, are entirely consistent with this expectation. It is remarkable that such limits can be obtained from a survey that was not designed in any special way to search for dust in the damped Lyα systems. A better determination of the spectral indices of the quasars would lead to even stronger limits or an actual measurement of the dust-to-gas ratio. This will require spectra with accurate flux calibration to the red of Lyα emission for a large fraction of the quasars in the WTSC survey, including the control sample in which damped Lyα is absent.

The method presented here has several advantages over previous searches for dust on cosmological scales. In particular, we make no assumptions about the degree to which the optical depths of the absorbers or the spectral indices of the quasars evolve with redshift. Our upper limits on the mean optical depth along random lines of sight are comparable to the weak detection claimed by Wright (1981), an order of magnitude smaller than the predictions by Ostriker and Heisler (1984) and several times smaller than the predictions by Heisler and Ostriker (1988). We include corrections for the quasars that would be too obscured to appear in the WTSC survey but ignore the possible obscuration by any dust not associated with the damped Lyα systems. A by-product of our study is a comparison between the true and observed luminosity functions of quasars, which depends on the amount of dust along the line of sight. We find that the luminosity functions differ by less than 60% for $z < 3$ (see Fig. 5 above). Since all information about the counts of quasars, either as a function of magnitude or redshift, is also contained in the luminosity function, we conclude that any dust in the damped Lyα systems cannot be responsible for a cutoff in the counts at redshifts below 3. It is possible that dust at higher redshifts would produce an apparent cutoff, but this seems unlikely on the basis of a smooth extrapolation of our limits on the mean optical depth.

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