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ON THE POSSIBILITY OF DETECTING WEAK MAGNETIC FIELDS IN VARIABLE WHITE DWARFS

PHILIP W. JONES
Department of Astrophysical, Planetary, and Atmospheric Sciences and Joint Institute for Laboratory Astrophysics, University of Colorado
W. DEAN PESNELL
Department of Astronomy, New Mexico State University
CARL J. HANSEN
Department of Astrophysical, Planetary, and Atmospheric Sciences and Joint Institute for Laboratory Astrophysics, University of Colorado
AND
STEVEN D. KAWALER
Center for Solar and Space Research, Yale University
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ABSTRACT

We suggest that "weak" magnetic fields of strengths less than 10^6 G may be detectable in some variable white dwarfs. Weak fields can cause subtle changes in the Fourier power spectra of these stars in the form of "splitting" in frequency of otherwise degenerate signals. Present-day observational and analysis techniques are capable of detecting these changes. We suggest further, by listing some well-studied candidate stars, that perhaps the magnetic signature of splitting has already been observed in at least one object and that the difficult task of intensive measurements of weak fields should now be undertaken of those candidates.

Subject headings: stars: magnetic — stars: pulsation — stars: white dwarfs

I. INTRODUCTION

In present usage, the term *magnetic* white dwarf seems to be reserved for those white dwarfs exhibiting surface magnetic field strengths in excess of 10^6 G. This numerical limit appears, however, to be forced upon us by present-day observational constraints on Zeeman and continuum circular polarization measurements, and spectropolarimetry. Measured field strengths of below 10^6 G usually have quoted errors comparable to the field strengths themselves (as, for example, in the tabulations of Angel, Borra, and Landstreet 1981). Thus both "firm" and null determinations are uncertain.

On the other hand, there is no reason to suspect that *weak* magnetic fields with $B < 10^6$ G should not be present in white dwarfs; the origins and evolution of magnetic fields in stars are still sufficiently uncertain that weak fields cannot be ruled out and, indeed, their discovery and measurement would help elucidate questions of origin and evolution. (For a brief review of such questions associated with white dwarfs see Wendell, Van Horn, and Sargent 1987.) The techniques used in this study are not applicable to strong fields and we shall discuss them no further; see the review of Schmidt (1988, and references therein) for a full discussion of the two dozen known strong magnetic field white dwarfs.

The purpose of this communication is to suggest that weak magnetic fields may, in principle, be detectable in variable white dwarfs through observation and frequency analysis of their light variations. If successful, such a program might be called "white dwarf magnetoseismology." The next section discusses what are the possible signatures of weak magnetic fields in the variables, and we include some illustrative numerical calculations. We conclude with a discussion of the observational problems of detection, what steps are necessary to resolve them, and we list some candidate stars.

II. EFFECTS OF WEAK MAGNETIC FIELDS ON PULSATION

The frequency spectrum of low-amplitude pulsations in a variable white dwarf may be characterized, in the simplest instance, by sets of three indices—l, m, and k—and a mode type. The first two indices specify the order of spherical harmonic, $Y_{l,m}(\theta, \phi)$, which describe the angular form of fluid displacements. The k index is roughly a count of the number of nodes in the displacement in the radial direction. It is thought that most, if not all, variable white dwarfs are gravity (g)-mode pulsators with $l > 0$ in which buoyancy is the restoring force for fluid motions. (For a review of this and other matters, see Winget 1987, 1988. General references to nonradial pulsation theory are Unno *et al.* 1979 or Cox 1980.)

If the star is spherically symmetric, not rotating, and contains no magnetic fields, then the frequency (or frequencies) of pulsation, σ, depends only on k and l but not on m; the $2l + 1$ possible values of $m = 0, \pm 1, \pm 2, \ldots, \pm l$ give rise to the same degenerate frequency σ_{kl}. However, either rotation or embedded magnetic fields can destroy underlying spherical symmetries and, at the same time, remove the degeneracy in m so that frequency depends on m also; that is, $\sigma = \sigma_{klm}$. A particular mode is then said to be "split" with respect to m. We shall see that magnetic and rotational splitting may compete in an observed white dwarf variable and, for that reason, we now discuss rotational splitting. Needless to say, the frequency spectra of some variable white dwarfs show strong evidence for splitting and the issue will be the precise cause.

a) Splitting by Rotation

The most extensively analyzed example of potential splitting of modes in variable white dwarfs has been for the case of slow, solid body rotation. By "slow" we mean that the angular frequency of rotation, Ω_0, is small compared to the pulsation frequency. In the limit of adiabatic pulsations, the degree of splitting (as observed in an inertial frame) due to Coriolis forces in this situation is given by

$$\sigma_{klm} = \sigma_{kl} - m\Omega_0[1 - C(k, l)] , \quad (1)$$

where $C(k, l)$ is a simple ratio of integrals of the eigenfunctions (linearized displacements) of the corresponding nonrotating star which has a mode frequency of σ_{kl}. This is the result quoted by Ledoux and Walraven (1958, § 82), and used by Brickhill (1975), Hansen, Cox, and Van Horn (1977), and Pesnell (1985) (among others). In the limit of large k (high overtone) for g-modes, C approaches $[l(l + 1)]^{-1}$. The important feature of equation (1) is that a given frequency in the absence of rotation is split into $(2l + 1)$ equally spaced signals by rotation and the splitting between these signals is of the order of the rotation frequency. (Which, if any, of these sublevels may be excited in a real star to observable amplitude is a largely unexplored question.) However, as is the case for magnetic fields, the rotation properties of the interiors of stars is unknown—with the possible exception of the Sun. Hansen, Cox, and Van Horn (1977) investigated the splitting of modes in cooling white dwarfs for a particular "law" of axially symmetric differential rotation and found that equation (1) was modified to read

$$\sigma_{klm} = \sigma_{kl} - m\Omega_0[1 - C(k, l) - C_1(k, l, |m|)] , \quad (2)$$

where Ω_0 is the rotation frequency along the rotation axis and $C_1(k, l, |m|)$ again involves integrals of eigenfunctions but depends only on the magnitude of m. There are still $(2l + 1)$ components of σ_{klm} but, although the splitting is symmetric about $m = 0$, it need not be equally spaced throughout. Similar deviations from equation (1) may occur even with solid body rotation if higher order corrections in Ω_0 are included (as in Chlebowski 1978).

b) Splitting due to Magnetic Fields

We assume that conductivities in the white dwarf stellar plasma are sufficiently large that the "MHD" approximation is valid; that is, the temporal evolution of the magnetic field, B, is governed by

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) , \quad (3)$$

where $\boldsymbol{v}$ is the instantaneous fluid velocity (Jackson 1975, § 10.3). Also implicit in this equation is the assumption that displacement currents may be ignored. This is justified in the present context because pulsation amplitudes are assumed to be small and observed pulsation periods in variable white dwarfs are long (around 10 minutes). To completely describe the system, again in the adiabatic approximation where we examine only mechanical effects, we must include Lorentz forces in the equation of motion and then see how the system responds to periodic low amplitude fluid displacements. The details of this procedure are given in Unno *et al.* (1979, § 18) and we will only summarize the results here.

If $\boldsymbol{\xi}(\boldsymbol{r})$ is the amplitude of the local displacement field describing how a parcel of fluid is shifted about during a pulsation of the variable star, and $\boldsymbol{B}_0(\boldsymbol{r})$ is the magnetic field resident in the unperturbed (not pulsating) star, then the perturbed magnetic field is given by

$$\boldsymbol{B}'(\boldsymbol{r}) = \nabla \times (\boldsymbol{\xi} \times \boldsymbol{B}_0) , \quad (4)$$

where a temporal dependence of the form $e^{i\sigma t}$ has already been assumed. At this juncture Unno *et al.* (1979) make the important assumption that the unperturbed field is force free and satisfies

$$(\nabla \times \boldsymbol{B}_0) \times \boldsymbol{B}_0 = 0 . \quad (5)$$

We shall also make this assumption because without it any equilibrium models to be considered would have to include the effects of the field on the structure of those models. This is a task we are not going to undertake in this preliminary study where we are concerned only with order of magnitude effects. We also make the additional assumption (as do Unno *et al.* in their final analysis) that $\boldsymbol{B}_0$ is a potential field satisfying $\nabla \times \boldsymbol{B}_0 = 0$.

The effect of magnetic fields in pulsation is to modify displacements and pulsation frequencies by means of tensions in the field lines resisting fluid motions. However, if the fields are weak such that they change frequencies by only a small amount, then a (Rayleigh) variational principle may be invoked to find those small frequency changes. This principle uses only the eigenfunctions, $\boldsymbol{\xi}$, for a mode in which no magnetic effects are present. This is directly analogous to how the results of splitting due to rotation were obtained. (If fields are strong, then both frequencies and eigenfunctions may be strongly modified and the problem is nearly intractable—see, for example, the simplified calculations of Carroll *et al.* 1986 in a neutron star context.)

If we let $\sigma_{kl}(\boldsymbol{B}_0 = 0)$ be the pulsation frequency of the nonmagnetic star and denote σ_{klm}' as the correction due to small magnetic effects, then the final frequency of the magnetic star is given by

$$\sigma_{klm}(\boldsymbol{B}_0) = \sigma_{kl}(\boldsymbol{B}_0 = 0) + \sigma_{klm}' , \quad (6)$$

where the variational analysis yields

$$\sigma_{klm}' = \frac{1}{8\pi\sigma_{kl}(\boldsymbol{B}_0 = 0)} \times \frac{\int_0^M \rho^{-1} |\boldsymbol{B}'|^2 \, dm_r - \int_S [(\xi^* \times \boldsymbol{B}_0) \times \boldsymbol{B}'] \cdot \hat{\boldsymbol{n}} \, ds}{\int_M |\xi|^2 \, dm_r} . \quad (7)$$

Here ξ is the eigenfunction of a given mode for the field-free star, and the integrals are either over the entire stellar mass or, as in the rightmost integral, over the stellar surface. An asterisk means complex conjugate and $\hat{\boldsymbol{n}}$ is an outwardly directed unit normal to the surface S. Note that σ_{klm}' is proportional to $|\boldsymbol{B}_0|^2$, and we include the surface integral in our calculations although it was assumed to be negligible by Unno *et al.* The latter term usually only affects our result at 10% level.

While the simplicity of equation (7) is appealing, it can be misleading. We have assumed total reflection of pulsation kinetic energy by the stellar surface when finding $\boldsymbol{\xi}$. If the initial magnetic field is such that $\boldsymbol{B}_0 \cdot \hat{\boldsymbol{n}} \neq 0$ at the surface, then this kinetic energy is not totally reflected but rather propagates outward into the external field. This situation requires a brutal piece of analysis (as in Biront, Goosens, and Mestel 1982) and is, hopefully, not necessary for obtaining the sort of estimates we seek. A more relevant difficulty is the mixing of spherical harmonics by this process. Each normal mode would have several Y_{lm} components in its angular dependence which could confuse the observed power spectrum and severely test both theory and observation.

The next phase of the analysis is indeed tedious and only a sketch will be given here. The eigenfunctions $\boldsymbol{\xi}$ may be decomposed into two independent radial functions multiplied by spherical harmonics and their angular derivatives in θ and ϕ. Once a sample magnetic field is chosen, then the angular integrations implicit in equation (7) can be performed usually with difficulty. What remains is a radial (or mass) integration over the star.

We have chosen two sample fields for our calculations: a constant field with $\boldsymbol{B}_0 = B_0(\cos\theta e_r' - \sin\theta e_\theta')$ with B_0 a constant, and a dipole field

$$\boldsymbol{B}_0 = \frac{B_0}{r^3}(2\cos\theta e_r' + \sin\theta e_\theta') . \quad (8)$$

The first field is certainly unrealistic whereas the second may be a reasonable representation for the global field of a white dwarf. However, our final results are relatively insensitive to which of these two fields we choose.

Once the symmetry axis of the magnetic field is chosen, then the angular eigenfunctions may be found using degenerate perturbation theory. With our choices of $\boldsymbol{B}_0$, the correction term σ_{klm}' will depend only on m^2 and the angular dependence will consist of standing wave spherical harmonics. The remaining degeneracy may be removed by adding some rotation (if we so desired). Combinations of rotation and magnetic fields in the case of unaligned symmetry axes are discussed in Pesnell (1988) for Ap stars.

The splitting for the constant magnetic field example has been solved by Unno *et al.* (without the surface integral term) and our final expressions agree with those they obtain (with the correction of a minor typographical error—the left-hand side of their eq. [18.56] should contain a factor of r^2). Our expression for the integrands in equation (7) typically contain some 20 terms and will not be given here but are available upon request.

The result that σ_{klm}' depends only on m^2 and not on m directly implies that the frequency of a given g-mode in a white dwarf will be split by our magnetic fields into $l + 1$ levels and not $2l + 1$ as is the case for rotation (as discussed, for example, by Goosens 1976). This is then one signature of the weak magnetic field. Note too that, unlike the case for rotation, the mode with $m = 0$ is also shifted in frequency. It remains to be shown whether such signatures can be seen in variable white dwarfs for interesting field strengths.

c) Models and Numerical Examples

We have explored the splitting due to weak constant and dipole magnetic fields in three white dwarf models chosen from a pure carbon evolutionary sequence of mass 0.6 $M_\odot$. Some characteristics of these models are listed in Table 1 and more information about the evolutionary sequence may be found in Kawaler, Hansen, and Winget (1985) and Kawaler *et al.* (1986).

The models were chosen to represent the effective temperatures and luminosities of the three known kinds of white dwarf (or pre-white dwarf) variables. These are, in order of increasing model number and age, the PG1159 (GW Vir) variables at around 10^5 K, the DB (helium atmosphere) variables with effective temperatures a bit below 30,000 K, and the cool DA (hydrogen) ZZ Ceti variables which are near 13,000 K. The pure carbon models cannot reproduce the latter variables very well in the surface layers because of the composition dissimilarities, but the overall structures as well as pulsation periods and eigenfunctions are similar.

TABLE 1

EVOLUTIONARY MODEL CHARACTERISTICS

Model	$\log L/L_\odot$	$\log R$ (cm)	$\log T_{\rm eff}$
1.........	2.00	9.203	5.084
2.........	−0.815	8.958	4.501
3.........	−2.13	8.936	4.184

We have applied the magnetic field splitting formalism of § II*b* to all three models using both constant and dipole field structures. All modes with frequencies corresponding to pulsation periods in the range 100 s to 1000 s and for $l = 1, 2, 3$ were computed for the nonmagnetic case, and then the corrections σ_{klm}' were found. The numerical technique used is similar to that reported in Kawaler, Hansen, and Winget (1985); the full adiabatic nonradial pulsation equations (as an eigenvalue problem) were integrated along with equation (7) using a high-order integrator with spline interpolation.

Some results are shown in Figures 1 and 2. What is given for the abscissae of these figures is the difference in frequency, $f = \sigma/2\pi$ in Hz, between a signal of a given m and that of $m = 0$ after a magnetic dipole field of $B_0 = 10^5$ G has been applied to modes of $l = 1$, and 2 in model 2. The ordinate shows the period of the unperturbed mode with no field. For example, in Figure 2 an unperturbed mode with a period of 214 s would be split into three modes $(l + 1)$ with spacings of 1.2×10^{-5} Hz (between m of 0 and 1) and 4.8×10^{-5} Hz (between m of 0 and 2). By our assumption of small changes in frequency due to magnetic effects, the period of all three signals should be close to the orignal 214 s.

The ordering of frequency in the split mode does not seem to depend on the orginal model structure or value of l in any obvious way. For $l = 2$ in model 2, the frequencies decrease with m whereas for $l = 1$ the order is reversed except for the lowest order (shortest period) modes. The overall effect of the field is to increase frequency because magnetic fields "stiffen" the stellar fluid against motions; that is, σ_{klm}' is always positive. Furthermore, it should be evident that because the splitting varies as m^2 then, for example, $(f_{m=0} - f_{m=2}) = 4(f_{m=0} - f_{m=1})$. This too should be a signature of splitting due to simple weak fields for $l > 1$—namely, a splitting in frequency which is quadratic in the integers $1, 2, \ldots, l$.

The results shown in the figures for the longer period (higher order k) modes are a bit deceptive because the amount of splitting tends to be comparable to the periods of the unperturbed modes. This violates one of our original assumptions. A smaller field would, of course, reduce that splitting (and this may easily be effected in figures by scaling with $|\boldsymbol{B}_0|^2/10^{10}$). The explanation for why the splitting increases with mode order follows from the properties of g-mode fluid displacements in variable white dwarfs and the nature of the integrands in equation (7). As order increases, the amplitudes of horizontal and radial displacements near the stellar surface increase relative to displacements in the deeper interior. This means that, for a fixed normalization of one of the eigenfunctions at the model surface, the overall amplitude of fluid displacement in the core decreases with increasing k. Thus, with other considerations set aside, the contribution to splitting from the first most important integral in the numerator is concentrated toward the surface and becomes more concentrated as mode order increases. Note also the factor of ρ^{-1} in the numerator; it effectively changes the implied integral of the magnetic field perturbations over mass to one over volume. This is not true of the integral in the denominator (which describes the kinetic

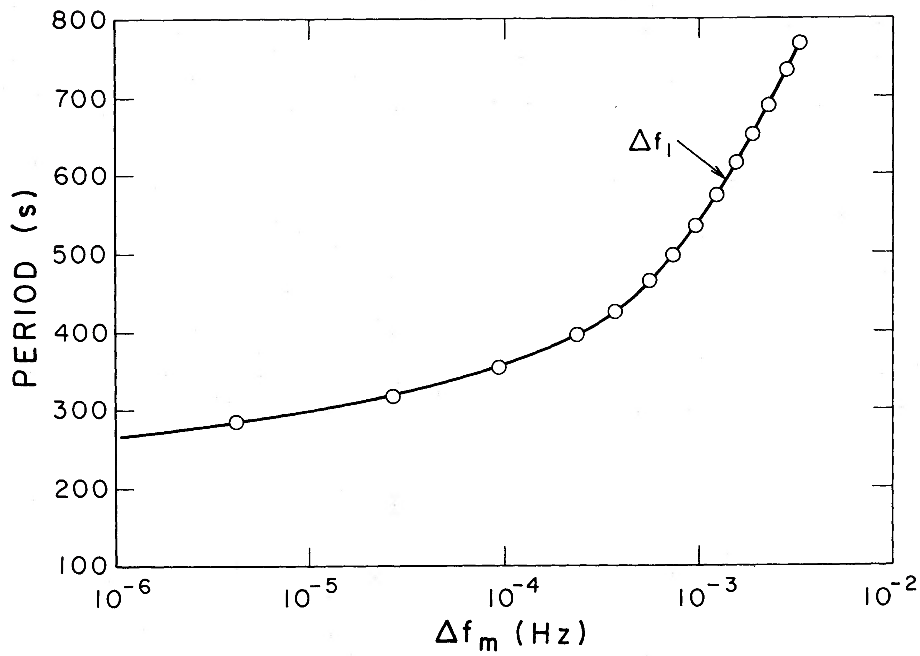

FIG. 1.—The relative frequency splitting $\Delta f_1 = f_0 - f_1$ (in Hz) between $m = 0$ and $m = 1$ modes for $l = 1$ in model 2. The assumed field is dipole with $B_0 = 10^5$ G. The period on the ordinate is that for the unmagnetized star. Each point represents a particular (fixed k) mode in the unperturbed model.

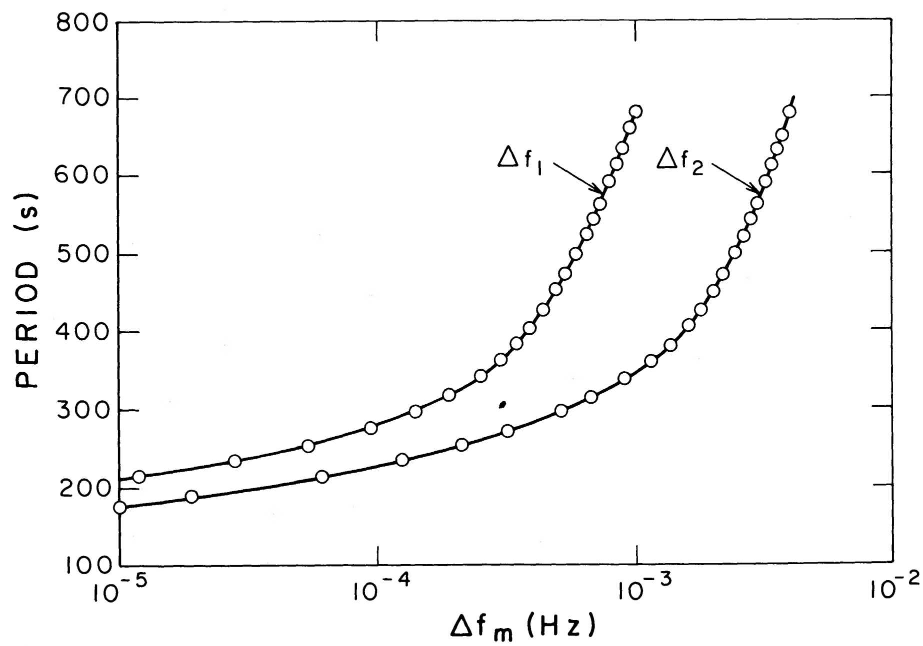

FIG. 2.—The relative frequency splitting $f_0 - f_1$ and $\Delta f_2 = f_0 - f_2$ for $l = 2$ in model 2. The field is again a dipole with $B_0 = 10^5$ G as in Fig. 1.

energy distribution) where the low-density material toward the surface has little weight. The net effect is that the numerator decreases less rapidly with period than does the denominator. A similar situation does not apply to splitting due to rotation because the integrals in equation (1) which are analogous to those in equation (7) are all over mass.

The frequency splitting results for the constant magnetic field are similar to those obtained for the dipole field. In addition, the splittings as a function of unperturbed periods for all models with both fields and l ranging from 1 through 3 are similar, give or take a factor of 3. In summary, a field of $B_0 \sim 10^5$ G yields values of $(f_{m=0} - f_m)$ greater than (or much greater than) 10^{-5} Hz for periods greater than about 200 s. Is this observable or has it already been observed?

III. FREQUENCY SPLITTING IN VARIABLE WHITE DWARFS

Of the almost 30 known variable white dwarfs, all show some degree of complexity in both their light curves and amplitude versus frequency spectra from, say, Fourier transforms; it appears that not one of these objects shows only one pure frequency signal. However, there is a trend for the ZZ Ceti variables in that those with longer periods (of, say, a few hundred to a thousand seconds) show greater complexity whereas those with shorter periods (100–300 s) are relatively simple and have just a few modes present in their power spectra (Winget and Fontaine 1982). With a few important exceptions (such as PG 1159−035) only the "simple pulsators" have been observed over long enough times and in sufficient detail that their power spectra have been resolved to high precision. It is from among these latter that we will seek candidates for magnetic field splitting.

We first consider an object which probably does not contain a magnetic field of any appreciable strength. The DA (ZZ Ceti) variable white dwarf G226-29 (WD 1647+591) shows evidence of only three signals, down to the noise level, at frequencies (semiamplitudes) 9.134721 mHz (3.1 mmag), 9.150865 mHz (1.2 mmag), and 9.167009 mHz (3.2 mmag) with a mean period of 109.3 s. A complete description of these results is reported in Kepler, Robinson, and Nather (1983). Because the triplet is evenly spaced in frequency by $\Delta f = 1.614 \times 10^{-5}$ Hz within observational error (and the amplitudes show a pleasing symmetry), it is very probable that slow rotation has split an $l = 1$ mode and that the rotation period is $P_{\rm rot} \sim (\Delta f)^{-1} \sim 1$ day. However, Angel, Borra and Landstreet (1981) report a magnetic field of $B = (1.7 \pm 0.8) \times 10^5$ G. Our calculations indicate that if this were a dipole field, we would then expect uneven splitting of only around 10^{-7} Hz for an $l = 2$ mode of period near 109 s in model 3 (which has an effective temperature near that of G226-29). A magnetic field does not seem to be the cause of the splitting. To reinforce this conclusion, J. R. P. Angel (in a private communication) informs us that the field result quoted could just as well be a null determination if the errors were reinterpreted slightly. It is a tough business.

Our most promising candidate star for a weak magnetic field is R548 (WD 0133−116) which is ZZ Ceti itself. Stover *et al.* (1980) have reported on the results of 8 yrs of observations of this star and they find two doublets with periods 212.768427 s, 213.132605 s for one doublet and 274.250814 s, 274.774562 s for the other. The corresponding frequency separations are, respectively, $\Delta f = 8.03 \times 10^{-6}$ Hz and 6.95×10^{-6} Hz. Angel, Borra, and Landstreet (1981) report a null detection of $B = (-1.3 \pm 2.3) \times 10^5$ G. If each doublet is a magnetically split $l = 1$ mode, then our results for model 3 indicate that a dipole field of slightly less than 10^5 G should reproduce the observed splitting for modes with periods close to the above. A closely spaced doublet is very hard to understand in the context of rotational splitting (for any l) unless some members of a multiplet are not excited to observable amplitude for some reason or inclination effects produce the same result (Pesnell 1985). We thus urge that observations be made of this star to determine whether magnetic fields are indeed present. However, we also caution the observers to be prepared for the nasty situation of dealing with a star that is varying in light output in four ways at once.

Another simple pulsator is GD 385 (WD 1950+250) which has been described in detail by Kepler (1984). The power spectrum contains only a singlet with a period of 128.15 s and one doublet with a mean period of 256.23 s where the splitting in frequency of the latter is 3.1×10^{-6} Hz. We have no good idea why only one doublet is seen whereas the singlet appears to be untouched. We hesitate to promote GD 385 as a candidate magnetic star but it should be kept in mind.

Just as no two planets in the solar system are alike, so all the well-studied white dwarf variables differ in important respects. The ZZ Ceti variable G117−B15A (WD 0921+354) contains two widely spaced triplets with some curious, and as yet unexplained, numerical relationships between frequencies (enumerated by Kepler *et al.* 1982). We note here that the frequencies in each triplet are spaced in the ratio 1:4 to almost within the quoted observational errors. This is consistent with the spacing ratio expected from a triplet with $l = 2$. However, the spacing itself is on the order of 10^{-3} Hz and, if due to magnetic effects, would imply a rather strong field. We suspect that magnetic fields are not at work here but this star may bear further scrutiny in the future.

We cannot say very much about the more complex pulsators except to suggest that in some cases the complexity may in part be due to magnetic fields. Kawaler (1988) has proposed that at least part of the power spectrum of PG 1159−035, a hot variable white dwarf, is due to sequence of modes spaced equally in period—a property of modes of a given l but with large k. Further observations may confirm this suggestion but would not guarantee that other complex objects conform as nicely. Knowledge of what kinds of modes exist in the hot variables is important because these stars are the fastest evolving of the white dwarfs and secular time changes in the frequency of a given mode reflect these evolutionary time scales (Winget *et al.* 1985; Kawaler, Hansen, and Winget 1985). In order to make a firm connection between the observations, pulsation analyses, and evolutionary studies, the role of magnetic fields (and rotation) must be clarified.

If, indeed, magnetic fields are measured in some of the variable white dwarfs, then the theoretical problem of modeling the observations must be done with much more care than we have taken. Primary among the problems is the consideration of more realistic magnetic fields and, furthermore, the competing mechanism of rotation must also be faced. An additional problem is that of the strengths of even the "weak" fields considered here. A major assumption of our calculations has been that field strengths are small enough that the perturbation techniques of § II are valid. This is certainly true in the deep interior of the star, but in the low density and pressure regions near the surface, such assumptions may fail. If $(B^2/8\pi)$ is the magnetic pressure at some level in the star, then it should be less than the local gas pressure in order that structural effects and perturbations of the pulsation eigenfunctions due to fields may be considered small. For our model 3, the level of equality of these two pressures is only 0.01% in radius below the surface for a field of 10^5 G. Thus, in the higher surface layers, magnetic pressures exceed gas pressures and we expect fluid displacements to be effected in perhaps a gross way. To incorporate these effects is no easy matter because the pulsation problem must then be solved in an entirely consistent fashion (as in Carroll *et al.* 1986 for a simplified geometry). Additional analysis must also be done if the fields turn out not to be force-free and derivable from a potential—as we have assumed—or if the fields are asymmetric in some way. Rotation naturally complicates matters both from the pulsation standpoint and because the possibility of differential rotation implies complex fields.

Despite these concerns, a program designed to treat realistic fields is worthwhile if weak fields are discovered. Besides the benefit of telling us more about the evolution of magnetic fields in very old stars where stellar cores have been exposed to view, such a program should yield vital information on what really goes on in a variable white dwarf. For example, if it does turn out the R548 has a measurable field of roughly the strength our calculations would suggest, then our assignment of $l = 1$ to both of the doublets observed in that star would appear to be firm. This, in conjunction with the observed periods, would then fix not only l but also the mode order k of both modes. The importance of this lies in the observation that not one mode in any variable white dwarf has been unambiguously assigned *any* of the indices k, l, or m. Determining any of these quantities would be a benchmark for future pulsation analyses.

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Carl J. Hansen and Philip W. Jones: Joint Institute for Laboratory Astrophysics, University of Colorado, Box 440, Boulder, CO 80309

Steven D. Kawaler: Center for Solar and Space Research, P.O. Box 6666, New Haven, CT 06511

W. Dean Pesnell: Department of Astronomy, New Mexico State University, Box 4500, Las Cruces, NM 88003.