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MEASURING H_0 AND q_0 WITH X-RAY LINES FROM GALAXY CLUSTERS

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ABSTRACT

A new method of measuring angular diameter distances, and hence H_0 and q_0, by means of X-ray observations of galaxy clusters is proposed. It resembles the method which combines maps of X-ray continuum emission with maps of the Sunyaev-Zel'dovich effect, but substitutes measurements of X-ray absorption lines in the spectra of background quasars for the Sunyaev-Zel'dovich effect. When the high resolution permitted by the AXAF microcalorimeter becomes available, this method should yield results with significantly smaller uncertainty.

Subject headings: cosmology — galaxies: clustering — X-rays: general

Ten years ago a new method to measure H_0 and q_0 using measurements of the angular diameter distance to galaxy clusters was simultaneously proposed by a number of people (Gunn 1978; Silk and White 1978; Cavaliere, Danese, and De Zotti 1979). The heart of the method was to combine one observation interpretable as a column density with another interpretable as an emission measure in order to arrive at an absolute lengthscale, and then to compare it to the measured angular diameter. Unfortunately, closer examination of this method showed that several features in its practical implementation made reliable results very difficult to obtain (Boynton *et al.* 1982). In this *Letter* we propose a modification of this method which will remove the principal difficulty. To make clear the advantages of the new method, we will begin by setting out the old method in schematic form. It was as follows:

1. Find a galaxy cluster containing a substantial quantity of hot gas.

2. Measure its temperature-weighted electron column density along a number of lines of sight via the Sunyaev-Zel'dovich effect (Sunyaev and Zel'dovich 1972). Experimentally, this means mapping the fractional depression of the Rayleigh-Jeans tail of the cosmological microwave background

$$\frac{\Delta I}{I} = \int dl n_e \sigma_{\rm T} \frac{kT}{m_e c^2}\,, \qquad (1)$$

where l is pathlength through the cluster, n_e is the electron density, $\sigma_{\rm T}$ is the Thomson cross section, and T is the gas temperature. It is expected that $\Delta I/I \sim 10^{-4}$.

3. Find the emission measure along the same lines of sight by mapping the X-ray continuum surface brightness

$$S_x = \frac{1}{4\pi(1+z_c)^4} \int dl n_e^2 \Lambda_b g(T) T^{1/2}\,. \qquad (2)$$

Here z_c is the cluster redshift, Λ_b is the bremsstrahlung emission coefficient (which is only dependent on atomic physics provided the heavy elements have no more than $\sim 10\times$ their solar abundances), and $g(T)$ describes what fraction of the total emitted flux is captured in the detector's bandpass. If the hardest X-ray photons to which the detector responds have energy $\epsilon_{\rm max}$, $g(T) \sim \epsilon_{\rm max}/(kT)$ for $\epsilon_{\rm max} < kT$ and $g(T) \simeq 1$ for $\epsilon_{\rm max} > kT$.

4. Measure the transverse angular scale θ_t of the gas from the map constructed in step 3.

5. Estimate T from the X-ray spectrum.

6. Given the cluster redshift z_c, and assuming that the radial length scale l_r is the same as the transverse scale $l_t = \theta_t D_A$ [D_A is the angular diameter distance, which is $(1+z_c)^{-2}$ times the luminosity distance D_L], the three measured quantities $\Delta I/I$, S_x, and θ_t can be combined to determine the three unknown quantities n_e, $l = l_r = l_t$, and D_A. If z_c is not too large, H_0 can be found directly from D_A and z_c with only a weak dependence on q_0. The same procedure applied to clusters at larger redshift then gives q_0.

Unfortunately, this method has not yet been successfully implemented. The principal sticking point is the difficulty of step 2. The value of $\Delta I/I$ is so small that many sources of noise, including most notably sources of radio emission inside the galaxy cluster, can easily mask the effect. Consequently, despite many years of effort, to date there has been but one reported detection (Birkinshaw, Gull, and Hardebeck 1984), and even that is a bit shaky. In this *Letter* we propose a variant of this method, which, using instrumentation of the coming decade, substitutes for $\Delta I/I$ a quantity with essentially the same information content but which can be measured accurately with much greater ease.

To be precise, we propose applying this metod to galaxy clusters which lie in front of X-ray–bright background quasars. To replace the map of $\Delta I/I$, we suggest using a single line-of-sight measurement of several X-ray absorption line equivalent widths W_i. For later convenience, we define W_i here in terms of the frequency-dependent optical depth $\tau_i(q)$:

$$W_i = m_e c^2 \int dq 1 - \exp\left[-\tau_i(q)\right]\,, \qquad (3)$$

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with

$$\tau_i(q) = \int dl n_{\rm H} X_j X_i(T) 2\pi^{3/2}\alpha\lambda_c^2 \frac{f_i}{q_i(\Delta v_j/c)} \times \exp\left[-\left(\frac{1-q/q_i}{\Delta v_j/c}\right)^2\right], \quad (4)$$

where $n_{\rm H}$ is the hydrogen nucleus density; α is the fine-structure constant; λ_c is the electron Compton wavelength; f_i is line i's oscillator strength; q is the photon energy, and q_i the resonance energy in units of $m_e c^2$; Δv_j is element j's velocity dispersion; X_j is the abundance of element j; and X_i is the fractional abundance of the ionization state responsible for line i. X_i is, in these conditions, a function solely of temperature. As we will show in § II, there are a number of X-ray lines for which we expect $\tau_i(q_i) \sim 1$. Such optical depths are, of course, the most desirable, for extracting a column density from the equivalent width is easiest when $\tau_i(q_i) \lesssim 1$.

Relabeling the measurement of W_i as step 2, we then introduce two subsidiary steps:

2a. If they are resolved, obtain Δv_j from emission lines associated with the same element off the quasar line of sight.

3a. Measure the strength of off-target emission lines as an alternate gauge of the emission measure and elemental abundances.

Because there are a number of lines which can be used, multiple constraints can be obtained, and combining them will reduce the overall uncertainty.

One essential instrumental ingredient is required to implement this program: a high-resolution X-ray spectrometer. The expected equivalent widths of these absorption lines are 0.5–3 eV if the gas is quiescent, increasing to as much as 4–25 eV if the gas has transonic random velocities. The first instrument capable of measuring such small equivalent widths in the X-ray band is the microcalorimeter being developed to fly on AXAF, whose resolution is expected to be better than 10 eV (Holt 1987; McCammon *et al.* 1987). Thus, the success of this method hinges crucially on the performance of that and subsequent similar instruments. Mapping the emission line surface brightness (step 3b) should be easily accomplished with the AXAF CCD imaging spectrometer (Nousek *et al.* 1987).

In the next section of this *Letter*, we will show which lines should be useful for this purpose: in § III we will discuss how to find quasars aligned properly behind galaxy clusters; and in § IV we will summarize by explicitly setting out the details of the method, and estimating the likely final error level in distances obtained by it.

II. SPECIFIC LINES

To demonstrate the feasibility of this method, we have calculated the expected line center optical depths for a number of X-ray lines. Only a few elements need be considered as candidates, for, when gas is hot enough to radiate an X-ray continuum, it is hot enough to strip nearly every cosmically abundant species. The most common elements which retain any electrons are Si, S, Ca, Ar, and Fe, but each of these possesses at least one line between 1 and 7 keV.

Different clusters have different temperatures and column densities of gas, as well as somewhat different elemental abundances, of course. Optical depths are easily adjusted for different columns, so we will present results only for a single, fairly typical (Sarazin 1986) column: $N_{\rm H} = 10^{22}$ cm^{-2}. Observed temperatures in galaxy clusters range from 2 to 10×10^7 K (Sarazin 1986), and the relative line strengths depend on temperature, so we tabulate the τ_i for a variety of values of T in this range. Heavy element abundances also vary, but are generally found to be roughly 0.1–1 times solar (Sarazin 1986); like column densities, the calculated optical depths can be easily adjusted to the abundance of the moment, so we will quote only a single example: all elements heavier than He have half their solar abundance (Rothenflug and Arnaud 1985). Finally, we assume in all cases that the velocity dispersion of the atoms is due solely to thermal motion.

The optical depths of nine X-ray lines calculated under these assumptions are compiled in Table 1 (cf. Gil'fanov, Sunyaev, and Churazov 1987 for the Fe lines). We remind the reader that when $\tau_i = 1$, $W_i = 1.29 q_i m_e c^2 \Delta v_j/c = 1.5(q_i m_e c^2/2\ {\rm keV})(T/5 \times 10^7\ {\rm K})^{1/2}(A/28)^{-1/2}$ eV. There are several more Fe K lines near 7 keV which we do not tabulate, even though their optical depths can be sizable, because the sensitivity of any likely focusing instrument falls off so rapidly above $\simeq 5$ keV that finding quasars bright enough to provide adequate photon statistics will be difficult. We quote the Fe XXV line as an indicator of these lines' optical depths. It is important to remember, however, that if the cluster redshift is $\simeq 0.3$ or larger, these lines become much more useful. Below 3×10^7 K, there are always at least eight lines whose line center depths are greater than 0.3, and hence should produce measurable equiva-

TABLE 1

STRONG ABSORPTION FEATURES IN HOT PLASMAS

log T	Si XIII 1.865	Si XIV 2.006	Si XIV 2.337	S XV 2.46	S XVI 2.621	Ar XVII 3.140	Ca XIX 5.039	Fe XXIII 1.129	Fe XXIV 1.165	Fe XXIV 1.538	Fe XXV 6.70
7.0	14.6	4.69	0.75	8.48	0.50	3.78	0.73	5.90	2.93	0.57	0.36
7.1	7.91	5.14	0.82	6.39	0.90	3.29	0.70	8.65	8.58	1.67	2.19
7.2	3.30	4.05	0.65	4.01	1.25	2.63	0.62	5.23	9.79	1.90	4.48
7.3	1.22	2.56	0.41	2.07	1.26	1.86	0.52	2.38	7.63	1.48	5.79
7.4	0.44	1.42	0.23	0.89	0.99	1.12	0.40	1.03	5.34	1.04	6.19
7.5	0.16	0.84	0.13	0.36	0.67	0.57	0.26	0.43	3.68	0.72	5.98
7.6	0.06	0.51	0.08	0.14	0.42	0.25	0.15	0.19	2.31	0.45	5.33
7.7	...	0.32	0.05	0.06	0.26	0.12	0.07	0.08	1.44	0.28	4.52
7.8	...	0.20	...	...	0.16	0.05	...	...	0.81	0.15	3.42
7.9	...	0.13	...	...	0.10	...	...	...	0.42	0.08	2.37
8.0	...	...	...	...	...	...	...	...	0.19	0.04	1.46

NOTE.—Each column is headed by the ion and a value of E, measured in keV. The ellipsis dots correspond to predicted optical depths less than 0.05. Note also that Si XIV 2.006 keV, S XVI 2.621 keV, Fe XXIV 1.165 keV, and Fe XXIV 1.538 keV are all doublets.

lent widths; at higher temperatures there are still generally at least two lines. Even at 10^8 K there is still one strong line remaining. Because the higher temperature lines are at higher energy, their equivalent widths tend to be larger as well.

III. FINDING ALIGNED QUASARS

To perform this experiment, there must be a background quasar whose projected position lies within the cluster and which is bright enough for good signal-to-noise ratio to be achieved. If we define the minimum flux as that which permits a $\pm 10\%$ measurement at 2 keV with 3×10^5 s integration, $F_{\rm min} = 2.5 \times 10^{-13}$ ergs cm^{-2} s^{-1} keV^{-1} for the nominal effective area of the AXAF microcalorimeter (Holt 1987). From the results of the *Einstein Observatory* Medium Sensitivity Survey (Stocke *et al.* 1983), we estimate that there are $\sim 10^3$ quasars in the sky brighter than this. Roughly half have redshifts greater than $\simeq 0.4$.

On the other hand, there are $\simeq 2400$ clusters in the Abell catalog, with an average angular size $\simeq 0.05$ deg^2. This catalog covers roughly only one-quarter of the sky when incompleteness in surveyed areas is combined with completely unsurveyed regions, and its median redshift is $\simeq 0.2$. If we assume that in all alignments the quasar lies behind the cluster, the probability per cluster of a background quasar is $\simeq 1.2 \times 10^{-3}$, so that roughly a dozen Abell-class clusters should have aligned quasars bright enough to obtain good measurements of absorption lines. Of course, if the catalog were extended to high enough redshift that its median z were 0.4, the number of alignments would rise by roughly a factor of 4.

A few examples are already known. One such is GQ Comae, a relatively bright ($m_V = 15.5$) quasar which is near the edge of A1455, and whose redshift (0.165) is consistent with its being either in the background, or possibly even inside the cluster. Another particularly prominent example, though not strictly speaking a quasar, is the combination M87 and the Virgo Cluster. In this case, a bright continuum X-ray source is found right in the middle of hot diffuse gas within a rich cluster. Because the X-ray–emitting gas is believed to be primarily bound to M87 itself, it is reasonable to suppose that the gas is distributed in a spherically symmetric fashion centered on the galaxy.

The *ROSAT* all-sky survey, scheduled for 1990, will be the most efficient way of finding the best clusters on which to perform these measurements. Its flux limit will be well below $F_{\rm min}$ and should provide both a complete list of all X-ray quasars bright enough to use, and also all clusters with detectable X-ray–emitting gas. Searching the two catalogs for positional coincidences will immediately pick out the best clusters to observe.

IV. EXPLICIT PROGRAM AND ERROR ANALYSIS

The algebraic result of combining the measurements described in § I can be written in either of two forms:

$$D_A^{(1)} = \frac{1}{4\pi(1+z_c)^4} \frac{N_i^2(\boldsymbol{p})}{\theta_t} \frac{S_x(\boldsymbol{p}')}{S_k^2(\boldsymbol{p}')} \frac{I_{2k}^2(\boldsymbol{p}')}{I_1(\boldsymbol{p}')I_3^2(\boldsymbol{p})} \quad (5)$$

or

$$D_A^{(2)} = \frac{1}{4\pi(1+z_c)^4} \frac{N_i^2(\boldsymbol{p})}{\theta_t} \frac{S_x(\boldsymbol{p}')}{S_i(\boldsymbol{p}')S_k(\boldsymbol{p}')} \frac{I_{2i}(\boldsymbol{p}')I_{2k}(\boldsymbol{p}')}{I_1(\boldsymbol{p}')I_3^2(\boldsymbol{p})} . \quad (6)$$

In these expressions, N_i is the column density in line i and S_k is the surface brightness of a subsidiary or forbidden line from the same element appearing in emission. It is the comparison with these latter lines which eliminates explicit dependence on the elemental abundance. The line of sight to the background quasar is labelled by $\boldsymbol{p}$; the lines of sight contributing to emission features are labelled by $\boldsymbol{p}'$.

The functions I_1 and I_2 are integrals with the dimensions of emission coefficients, while I_3 is dimensionless. All express different averages of the density, temperature, and ionization fractions along the line of sight:

$$I_1(\boldsymbol{p}') = \int dy\, d^2(\boldsymbol{p}', y) x_i \Lambda_b g[T(\boldsymbol{p}', y)] T^{1/2}(\boldsymbol{p}', y) , \quad (7a)$$

$$I_2(\boldsymbol{p}') = \int dy\, d^2(\boldsymbol{p}', y) X_i[T(\boldsymbol{p}', y)] \Lambda_i[T(\boldsymbol{p}, y)] , \quad (7b)$$

and

$$I_3(\boldsymbol{p}) = \int dy\, d(\boldsymbol{p}, y) X_i[T(\boldsymbol{p}, y)] . \quad (7c)$$

Here $d(\boldsymbol{p}, y)$ is the hydrogen nucleus density at the position $(\boldsymbol{p}, y)$ normalized to some characteristic density n_0, x_i ($= 1.1$) is the number of ions per hydrogen nucleus, y is the coordinate along the line of sight normalized to the cluster characteristic length scale l, and Λ_i is the emissivity of line i. It can be found from the expression

$$\Lambda_i(T) = 8.63 \times 10^{-6} \frac{E_i \Omega_i(T)}{\omega_i T^{1/2}} \exp\left(-\frac{E_i}{kT}\right) \text{ergs cm}^3 \text{ s}^{-1} , \quad (8)$$

where Ω_i, the collision strength, is only weakly dependent on T, and ω_i is the statistical weight of the upper level of the transition. Note that when $E_i \lesssim kT$, Λ_i is particularly insensitive to T.

There are a variety of ways to evaluate I_1, I_2, and I_3. For example, if spherical symmetry, constant temperature, and hydrostatic equilibrium are good assumptions, there are analytic expressions for these integrals. The only additional effort required to evaluate them is a measurement of the galactic velocity dispersion in the cluster. Other techniques to deproject the observed surface brightness to an emissivity, and thence to runs of density and temperature, are also in common use (e.g., Fabian *et al.* 1981). These latter techniques may be particularly useful because they require gathering only enough galaxy redshifts to find the *mean* cluster redshifts, not the much larger number required to find its dispersion (Boynton *et al.* 1982 emphasize the difficulties involved in the latter task).

To complete the computation of D_A, two more quantities must be defined in terms of observables: T and θ_t (X_i is determined by calculation as a function of T). If the bandpass of the X-ray detectors goes to high enough energy to see the curvature above photon energies $\sim kT$, the temperature is best measured by fitting to the shape of the continuum. If the bandpass is too soft for the temperature to be measured by this simple method, the ratios of fluxes from emission lines reflecting different ionization states in the same element can be used to bound T. Unfortunately, this method introduces extra error due to uncertainties in ionization and recombination coefficients. The value of θ_t can be measured by fitting the continuum surface brightness map to a functional form chosen to be consistent with that derived from the deprojection discussed in the previous paragraph.

The relation between column density N_i and equivalent width W_i depends on whether the line is in the linear or the flat part of the curve of growth. In the former case the relation is very simple:

$$N_i = \frac{W_i/m_e c^2}{3.4\pi^{3/2}\alpha\lambda_c{}^2 f_i}\,. \tag{9}$$

In the latter case, the relation is more complicated, with W_i depending much more strongly on Δv_j than on N_i. The attendant uncertainty in N_i is then much greater. To determine which case applies, it is necessary to either resolve the line or find a measure of the velocity dispersion. If the line can be resolved, then the depth of the line at line center immediately answers the question. If it cannot be resolved, a less direct approach must be taken. If other emission lines are resolved, it may be possible to use their dispersions to determine whether Δv_j is primarily thermal or primarily bulk motion. Even if there are no resolved lines, it is at least possible to assume thermal dispersion and check for self-consistency if several lines from the same species are present.

Before arriving at D_A, one more subtlety must be dealt with: strictly speaking, the surface brightness in an emission line only directly reflects the emission measure on that line of sight in the limit of small optical depth. Scattering of emission-line photons smooths their surface brightness over the entire optically thick region of the cluster (Gil'fanov, Sunyaev, and Churazov 1987). Within the context of the cluster model already derived for the integral computation, this effect can be corrected for, but the uncertainty involved when the optical depth is much more than one gives another incentive to relying primarily on lines on the linear part of the curve of growth.

When all this has been done, one must evaluate the uncertainty in the result. The measurements themselves are, of course, subject to the usual random error, but there are also three sources of systematic error: the assumption of spherical symmetry, the deconvolution required to evaluate the integrals, and the atomic data. One of the advantages of this method is that by adroit intercomparisons, many of these sources of error can be limited.

The most obvious way in which these comparisons limit the final error is simply by providing independent measurements of the same quantities. If the error in each independent distance determination is ϵ, then in a cluster with N_Q aligned quasars, each of which shows N_l lines, the error in the mean of these distances is $\epsilon/(2N_Q N_l)^{1/2}$, where the factor 2 enters because of the two different methods defined by equations (5) and (6).

However, there are other ways to reduce the final uncertainty beyond simple averaging. If there are multiple lines arising from the same ion, then it is very likely that at least one of them will be on the linear part of the curve of growth (cf. the Fe XXIV lines in Table 1). If that is so, the uncertainty in N_i is reduced by a considerably larger factor than the square root of the number of lines from that ion. Moreover, if the density and temperature are slowly varying, and the line k occurs in the same ionization stage as line i, the ratio $(I_2/I_3)^2$ which appears in both $D_A{}^{(1)}$ and $D_A{}^{(2)}$ eliminates dependence on the atomic data with the greatest uncertainty, the ionization and recombination coefficients. These coefficients are typically uncertain by 20%–30%, although it is important to note that when, as is often the case, $X_i \simeq 1$, errors in the atomic data have little effect on X_i.

By comparison, the line-specific atomic data are fairly well known. Oscillator strengths for these simple ions are generally good to within a few percent. The uncertainties in the collision strengths are probably somewhat larger, but they are also difficult to estimate because the Ω_i are in general calculated rather than measured. Distorted Wave or Close Coupling calculations (e.g., those of Hayes 1979 for Fe XXIV 1.165 keV, Pradhan, Nordcross, and Hummer 1981 and Pradhan 1985 for Si XIII 1.865 keV and S XV 2.46 keV, and Calloway 1983 for Si XIV 2.006 keV, S XVI 2.621 keV, and Fe XXVI 6.9 keV) are generally thought to be good to $\simeq 20\%$ at the present; they will probably improve by the time the X-ray measurements are available.

In sum, we have proposed a new way to measure angular diameter distances to distant galaxy clusters. When the instruments on which it depends become available, it should present many advantages, both by skirting difficulties inherent in other methods, and by providing multiple independent measurements of the same distance.

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