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AGN BROAD EMISSION LINE AMPLIFICATION FROM GRAVITATIONAL MICROLENSING

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ABSTRACT

The possible gravitational lens amplification of the broad-line region (BLR) of active galactic nuclei (AGNs) by stars in foreground galaxies is studied. For various possible kinematic structures of the BLR, the line shape changes caused by microlensing are generated and classified. Microlensing could significantly enhance the central portion of emission lines emitted from outflow structures and could dramatically amplify the wing region of emission lines that derive from infall structures. Microlensing could also slightly change the redshift of an AGN whose BLR is dominated by Keplerian rotation. Observational consequences of BLR lensing are discussed, including the possible application of this analysis to the quasar pair QSO 1343.4+2640.

Subject headings: galaxies: nuclei — gravitational lenses — quasars

I. INTRODUCTION

Microlensing, the gravitational lensing of distant sources by stars in intervening galaxies, has usually concerned itself with the amplifications of the smallest regions of active galactic nuclei (AGNs). The smallest regions are of interest because they are capable of undergoing the greatest amount of amplification by gravitational lensing. These small regions show themselves in the continuum emission of the optical, the ultraviolet, and the X-ray bands.

No microlensing event has yet been positively identified, although there are several papers that predict the possible appearance of such an event. Paczyński (1986) analysed the effects of many stars acting at once—a regime called high optical depth. Other recent work on the probable appearance of microlensing light curves is presented in Kayser, Refsdal, and Stabell (1986), and Schneider and Weiss (1987). A review of microlensing is given by Nemiroff (1987), along with many of the predicted properties of this phenomenon.

In this paper, a relatively new aspect of microlensing is introduced: lensing of a region outside the continuum. Specifically, lensing of the broad-line region (BLR) is discussed, and the possible amplification and distortion of the main feature of this region: broad spectral emission lines. Microlensing of this region was not studied before because it was thought that the region had too large an angular size. In the following models of BLR lensing, we find that there are regimes not ruled out by current observation where BLR lensing is possible, and show that the very kinematic nature of the BLR lends itself to microlensing very nicely.

The possible effect of foreground stars gravitationally amplifying the light emitted from this region was first introduced by Canizares (1982). Later Weedman (1986) also suggested the possibility; however, neither author discussed the idea in detail, and both suggested that future work in this area might be fruitful.

II. REVIEW OF AGNS AND GRAVITATIONAL LENSING PROPERTIES

There are many theories of the structure of the BLR region, and no general consensus of its physical composition has emerged. Most of these theories involve the motion of many low-mass clouds. The cloud's physical size is hypothesized to be less than 1 AU. With $\sim 10^{-9}\ M_{\odot}$, the whole BLR region is thought to contain between 1 and 100 $M_{\odot}$. For a general review of AGN and BLR properties, see Wiita (1985) or Weedman (1986), and references therein; much of the current discussion is based on these reviews.

The major observational feature which BLR theories strive to explain kinematically is the shape of the BLR spectral emission lines. The lines are seen to have logarithmic shapes away from their peaks, with widths that imply motions of the order of 10^4 km s^{-1}. Most theories use the macroscopic motion of the clouds to explain these velocities. The discrete Doppler shift from each cloud creates a cumulative emission line that is much broader than the hypothesized temperature of the individual clouds would imply.

We wish to discuss the BLR region with regard to the effects that gravitational lenses can have on them. Specifically, we will consider a standard microlensing paradigm: a single stellar lens of 0.01 $M_{\odot}$ in a nearby galactic halo at 10^8 pc acts to amplify a BLR in a quasar much further in the distance than the lens. We pick this mass star as a plausible candidate of the halos of galaxies. Stars much more massive than this populating galactic halos have been ruled out by recent observational studies (Boeshaar and Tyson 1985). Other lensing scenarios, such as those of higher mass lenses in the luminous parts of galaxies, or those of lower mass in the halo, have not been ignored here: our analysis is sufficiently general so that they also can be considered by a rescaling of coordinates.

We will assume distances in the lens plane to be 10 times that of distances in the source plane. The exact distance to the source is then a function of cosmology.

Distances intrinsic to the BLR will be given in two convenient units. The first is that of parsecs (pc) in the source plane, while the second will be the Einstein ring unit (ERU) of the lens. The ERU is angular, defined by

$$\mathrm{ERU} = [2(1 - D/d)R_s/D]^{1/2}\ ,$$

(Liebes 1964). The distance D is the distance from the observer to the lens, d is the distance from the observer to the source, and R_s is the Schwarzschild radius of the point lens. Including ERU in the discussion allows our analysis to be applied to a wider range of lensing scenarios than the specific lensing paradigm cited above. Any combination of R_s, D, and d will define a characteristic ERU, so that different lensing scenarios (like that of a different mass lens) can be considered simply by noting how the ERU scales from one scenario to the other. The conversion factor between the two systems of units in our current paradigm is 230 ERU pc^{-1}.

A stellar gravitational lens will become projected onto a quasar BLR region when a galaxy seen between the observer and the quasar drags the stellar lens across. The galaxy may act as a lens itself, creating more than one image of the quasar, but this is not important here. Multiple macroimages are not necessary for microlensing to take place. Galaxy—quasar alignments are not unusual; for a list of the current known systems of this type, see Monk *et al.* (1986). Discussion and calculations of those systems thought most likely to undergo microlensing are given in Nemiroff (1987).

Because the hypothesized cloud size is small when compared to the angular size of the ERU, the stellar lens will act to amplify each cloud as if were a point source. The amplification by a point lens of a point source is given by

$$A = \frac{r^2 + 2}{r(r^2 + 4)^{1/2}}$$

(Liebes 1964). Here r is the angular distance from the lens to the unlensed source position in ERU. The total amplitude of lensing of the BLR is the sum of the lensing amplitudes of its component clouds divided by the unlensed brightness of the clouds. For a large cloud, we will see that it is possible that the whole BLR region can experience only a small amplitude change, while specific regions at a given Doppler shift can experience a large amplification. Because the Doppler shift maps the BLR region onto a wide range of wavelenths, it is possible to see the amplification of only a specific part of the BLR.

III. BLR MODELS AND LENSING CONSEQUENCES

In the following analysis we will concentrate on five basic models of the movements of the BLR clouds. These models are (*a*) random motions, (*b*) constant acceleration–radial outflow, (*c*) constant velocity–radial outflow, (*d*) gravitational infall, and (*e*) Keplerian disk. We will discuss possible characteristic emission-line changes of the BLR region due to gravitational lensing for each of these scenarios.

All of the following models were calculated numerically. For each model, typically 10^6 clouds were placed randomly. The numerical simulations were run on an IBM 4341 mainframe, with preliminary results obtained on a Leading Edge PC. The graphs presented have been smoothed with polynomial interpolation techniques.

a) *Random Motions*

The first model we will consider is that of random cloud movements. Models involving random cloud movements have been discussed previously by others including Capriotti, Foltz, and Byard (1980, hereafter CFB) and Osterbrock (1978). In generating the BLR spectral lines with this model, several assumptions were made. All clouds were considered to have the same brightness. The position of each was assumed to be random and uniform inside a sphere of radius $r_{max} = 1$ pc. Velocities were also assumed random, following a Gaussian distribution centered on 0 km s^{-1} and having a dispersion velocity of 2000 km s^{-1}.

A typical BLR emission line generated with these assumptions is shown in Figure 1. The ordinate has been normalized to unit luminosity. Its exact maximum value is not important for the present purposes, since it is strictly a function of the number and brightness of the individual clouds. Similarly, the abscissa is scale invariant; the cloud velocities could have been scaled to any dispersion.

The shape of the spectral line was not greatly altered by the placement of a foreground lens anywhere in the field. When r_{max} was picked to be 1 pc, no lensing effects were evident. Even when r_{max} was reduced to 0.1 pc, as depicted by the Figure 1, gravitational distortions of the emission line increased, but were still very small, of the order of 1%. This slight effect is visible as an enhancement of the central peak of the spectral line. Figure 1 shows an emission line generated with the above model. The numerical error in the luminosity was ~ 0.02 normalized luminosity units.

b) *Constant Acceleration Radial Outflow*

Many of the leading theories of BLR kinematics involve radial outflow. Blumenthal and Mathews (1975) considered theories of outflow generated by radiative emission from the continuum region, and showed such a theory can successfully account for the logarithmic shape of the spectral lines away from the peak. Another theory predicts that the outflow is caused by a radial wind of particles emitted by the continuum region (Weymann *et al.* 1982). Both of these models hypothesize the acceleration of the clouds in a radial direction, away from the continuum region. Many dynamical variations of these themes that have been considered (see CFB and references within).

The calculations described below will involve several assumptions which are part of many of the outflow theories. Spherical symmetry is assumed about the origin of the BLR. The luminosity of the continuum region at the center of the BLR is not included in the lensing calculations or in the descriptions of the line profiles. The clouds will be created at a characteristic radius r_{min}.

In the acceleration model used in this section, the clouds start with zero velocity and maintain constant luminosity. They will undergo constant acceleration until they reach r_{max}, the outer radius, where they will abruptly "turn off" or become dark. All of the clouds are assumed identical.

The time of movement for a cloud to go from starting point to finishing point was first computed. The clouds were then placed by assuming a uniformly random time for each in this interval—each time in this interval was equally likely. The angular position of the clouds with respect to the center was also picked in a uniformly random fashion. From this information, it was then possible to calculate the resultant position and velocity for each cloud. We assume the clouds to have a small covering factor, such that even the clouds closest to r_{min}, where the cloud density is greatest, neither have their light blocked by foreground clouds nor block the light from more distant clouds.

We found that the lens can change the shape of the emission line under certain conditions. A typical lens distortion of a BLR spectral emission line is depicted in Figure 2. Here r_{max} was assigned to be 1 pc and r_{min} was equal to 10^{-3} pc (0.23 ERU). It will be typical in our numerical simulations to assume r_{min} is $r_{max}/1000$. A lens of the type described above was superposed on the BLR region; it was placed 1 ERU from the center of the BLR, as seen by the observer.

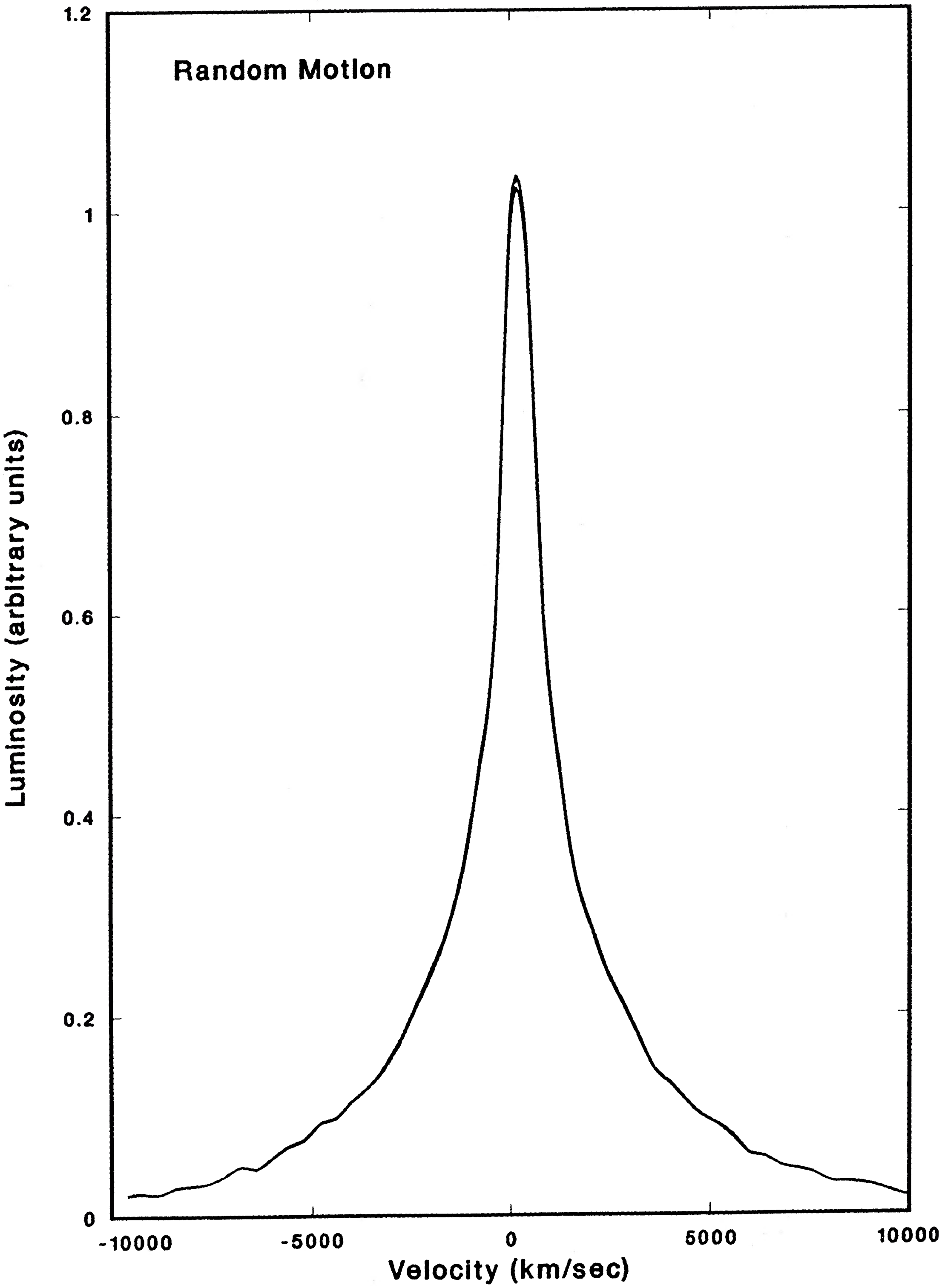

FIG. 1.—The broad emission-line profile generated from random (Gaussian) cloud motions. See the text for a more complete description of the cloud motions and of the microlensing paradigm assumed. The dispersion of the velocity distribution is 2000 km s^{-1}. The effects of a lens 0.1 ERU away from the BLR center do not strongly affect the shape of the line. Only a small (1%) effect of the brightening of the central peak is noticeable. The region was assumed to be 0.1 pc in radius.

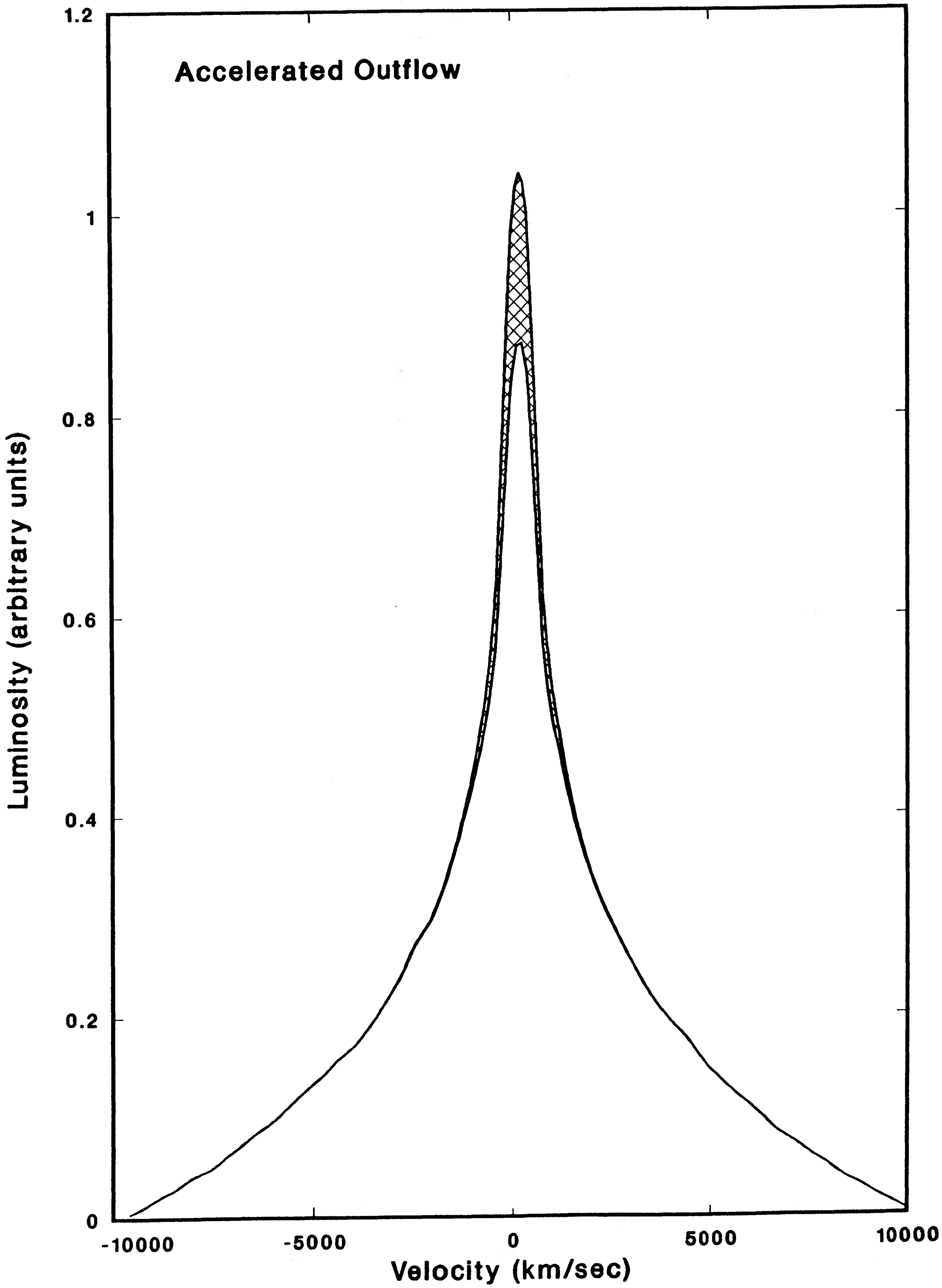

FIG. 2.—Constant radial acceleration outflow model of BLR emission line generation. A lens 1 ERU distant from the BLR center increases the emission from the central peak. The excess emission due to gravitational lensing is denoted on the graph by the hatched area. The parameters that define the size of the BLR region are r_{max} (outer radius) = 1 pc, r_{min} (inner radius) = 10^{-3} pc.

Inspection of Figure 2 shows that the lens noticeably amplified the center of the spectral line, while the rest of the spectral line remained essentially unchanged. This is the typical effect of a single gravitational lens in an accelerated outflow model. The lens affects the center of the spectral line the most strongly because small radial velocities predominantly originate in the central region of the BLR, which also has the highest concentration of source points. Here the numerical error in the luminosity calculation was less than 0.01 normalized luminosity units.

The line distortion effect is extremely sensitive to the value of $r_{\rm min}$. The smaller is $r_{\rm min}$, for a given $r_{\rm max}$, the greater the lens effect. Current theories are not especially specific on the value of $r_{\rm min}$. Wiita (1987) suggests that its size can be as small as several Schwarzschild radii of the hypothesized central black hole at the BLR center. This would place the value of $r_{\rm min}$ at $\sim 10^{-5}$ pc (0.0023 ERU) for a $10^7 M_\odot$ black hole. Weymann *et al.* (1982) consider scenarios with $r_{\rm max}$ between 0.15 and 0.96 pc, while $r_{\rm min}$ is of the order of 0.03 pc (6.8 ERU), but state that these values of $r_{\rm min}$ are only an assumption. If Weymann *et al.* (1982) are correct in this assumption, no lens distortion of the BLR spectral line is to be expected. For BLR line amplification to take place, $r_{\rm min}$ must be of the order of a few ERU or smaller.

The lens effect is also sensitive to the value of $r_{\rm max}$. For a given $r_{\rm min}$, the greatest lens effects arise from the smallest $r_{\rm max}$. The lens must also occupy a region extending from the center of the BLR to within a few ERU outside of the projected $r_{\rm min}$. If the lens is outside this region, it will not disort the emission line shape significantly.

As the simulations model the effect of a single lens, the lens can never act to significantly de-amplify portions of the spectral line, only to amplify it. Also, the requirement of spherical symmetry demands that the lens will always affect both sides of the spectral line equally.

c) Constant Velocity Radial Outflow

The other type of radial outflow which will be considered is that of constant velocity of the clouds (CFB). In this model the clouds are created and given a specific velocity (the typical value used in the simulations was 10^4 km s^{-1}) at $r_{\rm min}$ and allowed them to coast out to $r_{\rm max}$ where they turn off. Here the clouds do not maintain constant brightness as they leave the center, but rather the brightness drops off inversely with distance from the center. This decrease in cloud luminosity is required to create the observed logarithmic shapes of the wings of the spectral lines (CFB).

The lens affects the constant velocity model qualitatively much the same as for the constant acceleration model. This is shown in Figure 3 for the same lens and source parameters that generated Figure 2. As before, the lensing effects are extremely sensitive to the assumed value of $r_{\rm min}$ and the location of the lens with respect to the BLR center. Again the observable distortion of the spectral lines that can be generated is that of an increase in the strength of the central part of the line.

d) Gravitational Infall

Some very interesting gravitational lens effects can arise from the distortion of a BLR region dominated by the gravitational infall of its component clouds. This BLR model, discussed in CFB, involves clouds falling radially from some radius $r_{\rm max}$. This model also resembles one discussed by Kwan and Carroll (1982), with the clouds following parabolic orbits about the central region. We did not allow the clouds to get any closer than $r_{\rm min}$. The clouds, maintaining constant luminosity over their trajectory, were dropped from rest at a radius of $r_{\rm max}$ and allowed to fall to a radius of $r_{\rm min}$, where they abruptly turned off. The mass of the central object was that necessary to cause a velocity of 10^4 km s^{-1} at $r_{\rm min}$.

The gravitational lens effects were typically most noticeable on the wings of the lines, rather than near the center. The lens effects were sensitive to the values of both $r_{\rm max}$ and $r_{\rm min}$, particularly $r_{\rm max}$. For an $r_{\rm max}$ of 1 pc, and $r_{\rm min}$ of 10^{-3} pc, the lens effect could be quite dramatic. For a lens perfectly centered, the maximum lens effect produced wings of the line which were increased by several times their original luminosity, while the center changed much less than this, of the order of 10%. This is shown in Figure 4.

Dramatically different lens effects were generated depending on where the lens was superposed on the BLR region. If the lens was several ERU away from the central region, its effects were weaker but concentrated on the central region, as shown in Figure 5. When the lens was between these two regions, the portion of the line between the wings and the center was most greatly enhanced, as is shown in Figure 6.

Were a lens to move across a BLR region of this character and dimension, the first effects noticeable would be an enhancement of the central part of the emission line. As the lens moved toward the center, the line distortion would increase and move outward from the center of the line. Finally, as the lens approached the center, the wings of the line would be the most enhanced by the gravitational distortion. Then, as the lens exited the BLR region, the whole procedure would reverse. Equivalently, in time order, we would see Figures 5, 6, 4, 6, and back to 5 again.

The time scale for these changes is a strong function of the relative observer, lens, and source velocity. This time scale is of the same order as the time scale for microlensing induced changes in the continuum light. As discussed in Nemiroff (1987), a typical value is expected to be on the order of years.

The emission line generated with analysis does not have a logarithmic profile (CFB). To generate a profile of this type, one must demand that cloud brightness drop off as the square of its distance from the center. Our attempts to model the BLR accurately in this way with the numerical methods outlined above proved too time consuming for the computers being used. We hypothesize that dimming clouds far from the center would only strengthen the effects outlined above, which predominantly result from the lensing of the central region.

The numerical error of the gravitational infall simulation was greater than in all of the other simulations. The error was greatest near the wings of the lines, where there were the fewest clouds, and also behind the lens, where small numerical dispersions in cloud placement were greatly amplified by the lens. The error in lensed luminosity was on the order of the additional lensed luminosity itself. When the lens was far from the center the error was comparatively small, less than 1% of the additional lensed luminosity.

e) Keplerian Disks

There are many theoretical disk models currently considered as possible progenitors of the emission lines generated in the BLR. Some of these theories have been put forward by Osterbrock (1978), CFB, and others. The main problem with the disk models is that they usually fail to reproduce the observed shape of the spectral lines. Without specific assump-

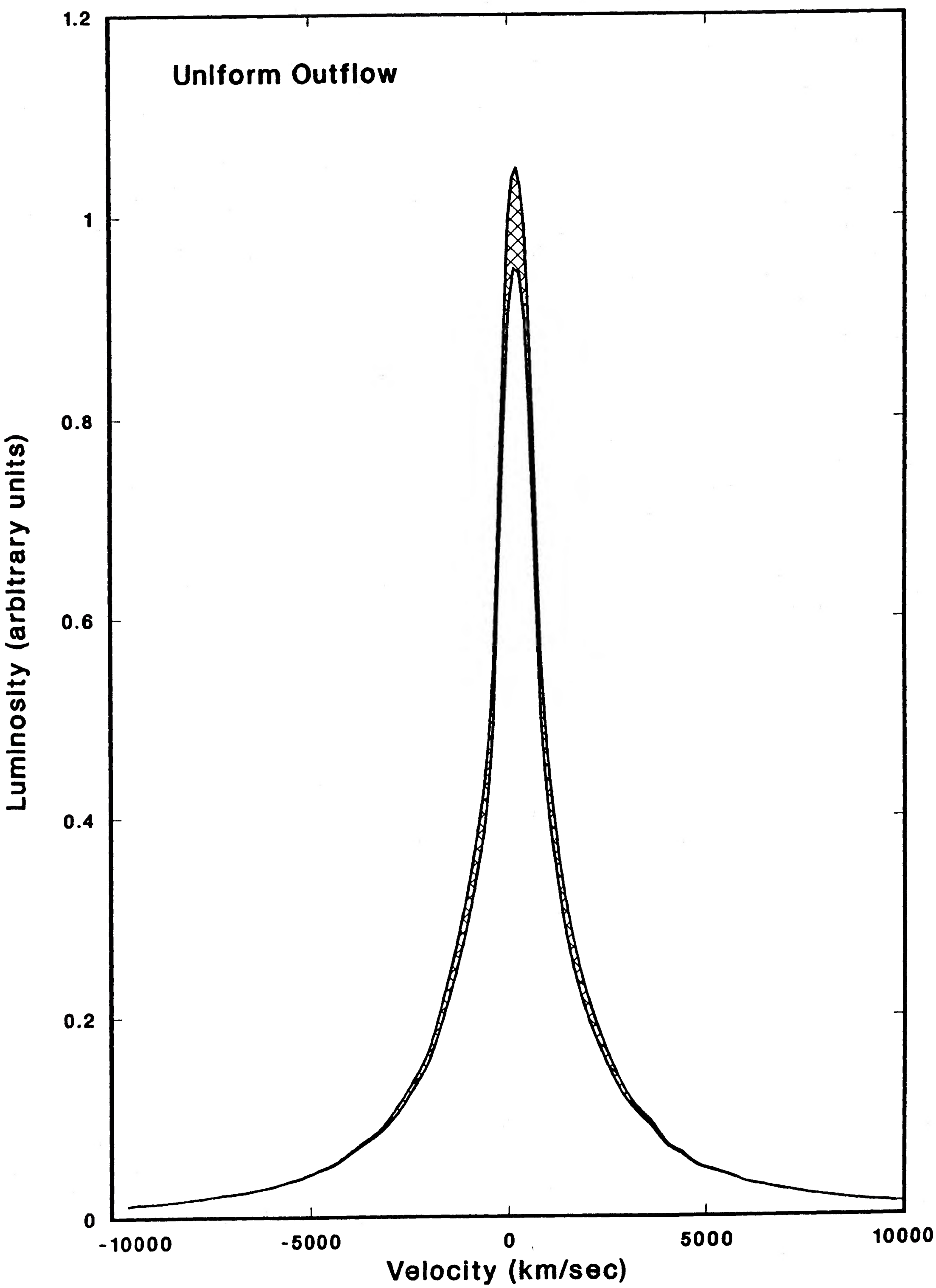

FIG. 3.—Constant radial velocity outflow model of BLR emission line generation. A lens 1 ERU distant from the BLR center increases the emission from the central peak. The size of the BLR region is the same as that in the constant acceleration model above.

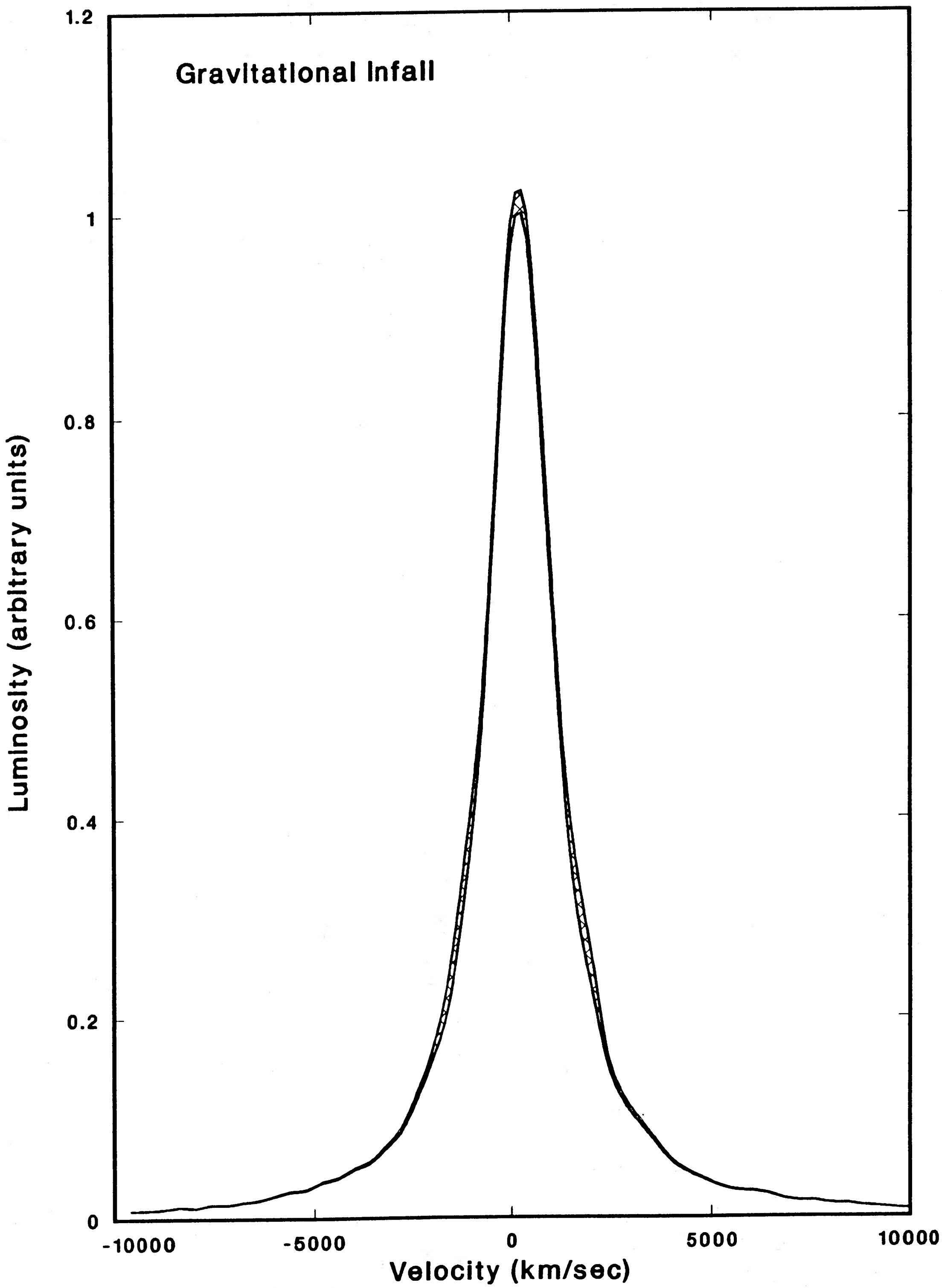

FIG. 4.—Radial gravitational infall model of BLR emission line generation. The region was modeled with r_{max} = 1 pc and r_{min} = 10^{-3} pc. A lens 2.3 ERU distant from the center of the region creates the line distortion shown. Note that the lens primarily affects the peak of the line.

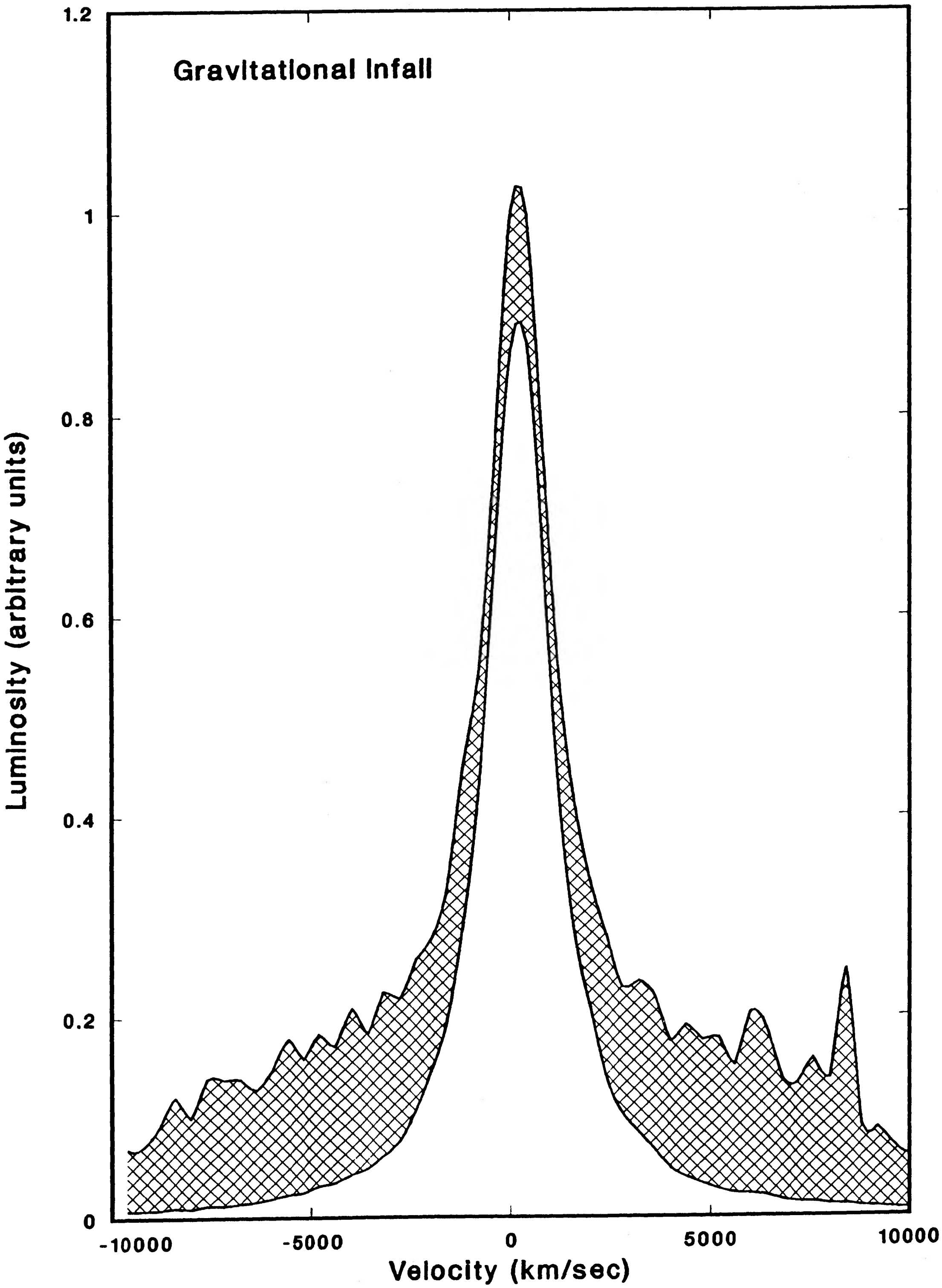

FIG. 5.—Radial gravitational infall model of BLR emission line generation. The BLR parameters are the same as before, but here a lens is 0.11 ERU from the region center. Note that the lens effects are greatest in wings of the line.

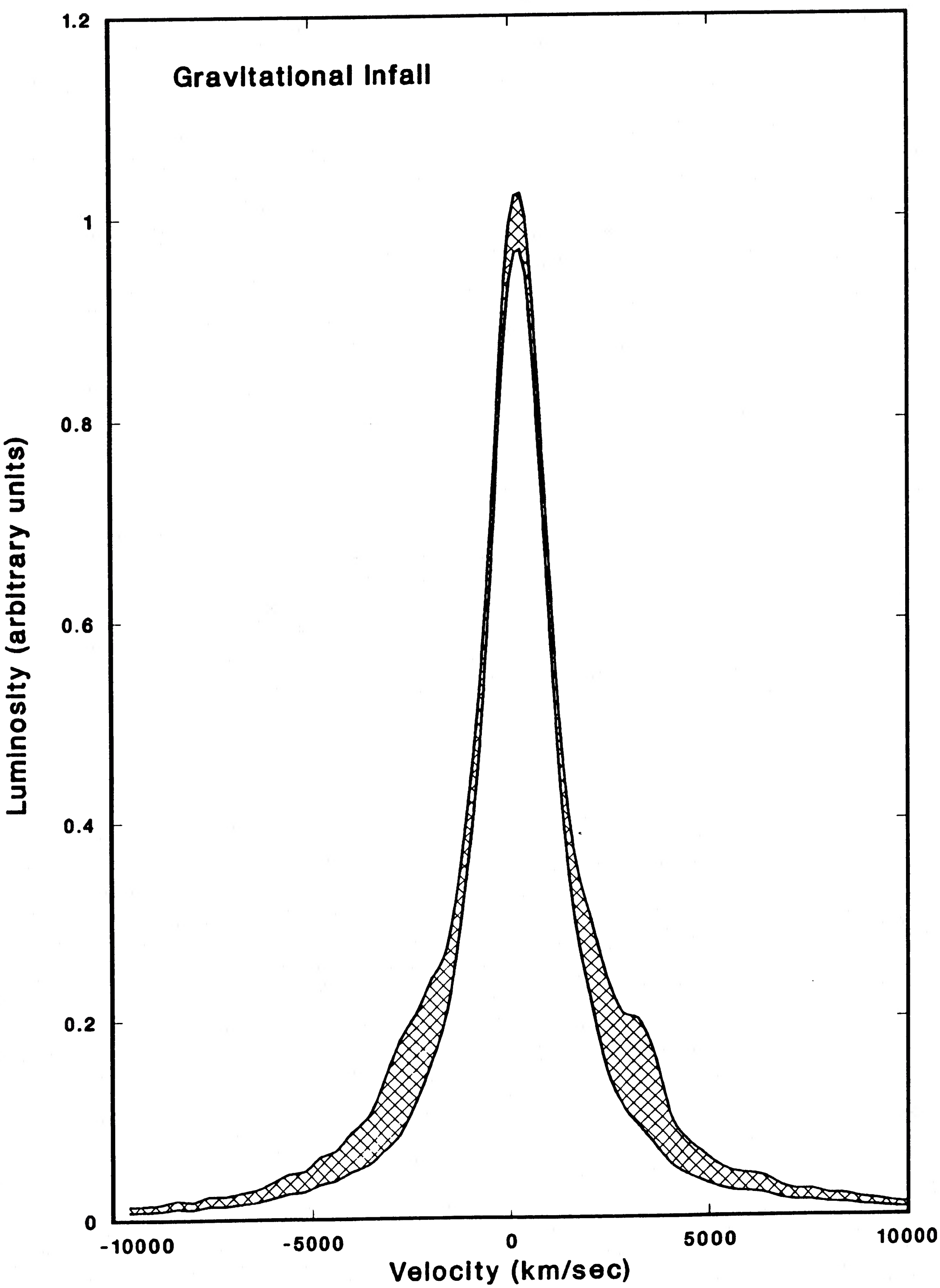

FIG. 6.—Radial gravitational infall model of BLR emission line generation. Again the BLR parameters are the same as before, except that the lens is 0.68 ERU from the region center. Note that the lens effects are most prominent between the peak and the wings.

tions, many of these models would create spectral lines with two peaks or nonlogarithmic wing profiles.

In this section we will concentrate on a model similar to the original one put forth by Osterbrock (1978). We will assume a Keplerian disk with r_{max} (outer radius) of 0.1 pc. Again, r_{min} will be assumed to be $r_{max}/1000$. The exact value of r_{min} is not so important as it was in previous models. We will consider neither the density of clouds nor their brightness to be enhanced in the central region.

The disk will involve Keplerian rotation of clouds around the center in circular orbits, the clouds being too small to block each other (the disk is assumed to be optically thin). The height of the disk will be assumed to be two-fifths the value of r_{max}.

One simulation yielded the spectral line shown in Figure 7. In this simulation, we also made the following assumptions. The disk is seen tilted 45° to our line of sight. The velocity of rotation was 2×10^4 km s^{-1} at r_{min}. The value of r_{max} was 0.2 pc and r_{min} was $r_{max}/1000$.

Following Osterbrock (1978), in addition to the circular motion we added a random motion two-fifths that of the cloud velocities at r_{min}. The random motion was assumed Gaussian in nature.

Line distortion effects are again most evident in the center of the line, but for first time in any of our simulations, the distortions were typically asymmetric. In Figure 7, the lens was superposed 0.7 ERU from the center and on a region away from the projected axes of the projected ellipse. Here the lens affected one side of the spectral line, near the center, more strongly than the other side. These distortions were typically small compared to the velocities in the problem, and showed the most asymmetry when the lens was placed closest to the central region.

The reason for the distortion asymmetry in the lensed emission line derives from the fact that the lens is superposed on a region with net bulk radial motion with respect to the observer. The lens acts to amplify the light from this region more than the rest of the BLR, and hence creates an abnormally large contribution toward the complete spectral line. This abnormal contribution results in the observed line asymmetry.

The lens distortions of Keplerian disks can act to change the central redshift of the BLR line, but only to a small degree. For the simulation described above, the lens could cause the center of the line to "sway" of the order of 500 km s^{-1}. Figure 8 depicts the same scenario described above, but with the exception that the lens was projected onto the opposite side of the BLR region. Inspection of the two graphs shows the redshift change clearly when one notes the comparative width of the hatched areas to either side of the emission line.

The numerical error in the luminosity was on the order of 0.01 normalized luminosity units. The error in the values of the "sway" of the line was on the order of 10 km s^{-1}.

This line shift may be important in identifying QSOs with multiple macroimages. Were microlensing effects strong, they could cause a difference in the redshifts between images by as much as 0.01. One might then not rule out close QSOs with a redshift discrepancy in this range, but consider them candidates for both macrolensing and microlensing. This prediction is testable in that one would expect the redshifts of the AGN to change back to congruence within a few years.

IV. DISCUSSION

The analysis of this paper is aimed at opening a new window through which gravitational lensing may be detected. The gravitational distortion of spectral lines in the BLR may give us a tool to explore many aspects of quasars and gravitational lenses. Information on the size, make-up, geometry, and dynamics of the BLR region may be recoverable. Similarly, BLR lensing has the potential to tell us about the mass, number density, and dynamics of the lenses involved.

The analysis invokes several assumptions. The weakest assumptions probably involve the estimation of the angular sizes of the lenses' ERU compared to the QSOs BLR region. A change in any of a number of parameters will result in a change of the relative sizes of the regions. These changes could involve parameters such as the mass of the lens, the distances to the lens and the source, the actual size of the BLR region, and the cosmology of the universe. Were the masses of the lenses considered larger, the lenses assumed closer, or the BLR regions assumed smaller, a larger amplification effect would be expected. The reverse conditions would result in a smaller lens affect.

Another assumption that is important to the accuracy of our analysis is that of the spherical symmetry of the BLR region. Any large geometric asymmetries or asymmetries in the brightness distribution may end up as irregularities in the shape of the spectral line. These asymmetries are sure to affect the way a lens would distort a BLR line.

It is important to realize that for the microlensing paradigm given above, these line distortion effects are *not* improbable to measure, relative to continuum enhancement effects. The probability of spectral line distortion is at least equal to the probability of amplification of the inner continuum region, and in some cases can even slightly exceed it. That the BLR region maps spatial regions onto a spectral line can *help* in distinguishing regions that are gravitationally amplified from those that are not.

We speculate that spectral distortions due to critical lines, common at higher optical depths (see, for example, Chang and Refsdal 1979), would not be characteristically different than the single star distortions investigated here, at low optical depths. The amplification effects should be greater, as the lines amplify to a greater extent, but the distortion of the emission lines is more a function of the kinematics of the BLR region than the type of amplifying lens.

The above analysis is predictive in several aspects. First, one can compare those QSOs that are candidates for gravitational lensing with those which are not, looking for comparative differences in the shapes of their broad emission lines. One might investigate the shapes of the transient emission features of the current BL Lac objects that are thought to be lensed (Nottale 1986; Ostriker and Vietri 1985). One might also investigate those currently known macrolensed systems for anomalous line shapes and comparative differences in line shapes, as they are also prime candidates for microlensing.

One might also look for time variability of the broad-line phenomenon to see if it conforms to any of the models outlined above. One would expect not only changes, but a specific pattern in the changes to characterize specific microlensing scenarios.

The current analysis may be immediately relevant to the recent observations of Crampton and Cowley (1987). They have discovered a QSO pair, 1343.4+2640, with identical redshifts but different emission line shapes. This could be explained if one of the images was undergoing a microlensing event, while the other image was not. Alternatively, if the density of microlenses is high, on the order of one lens per

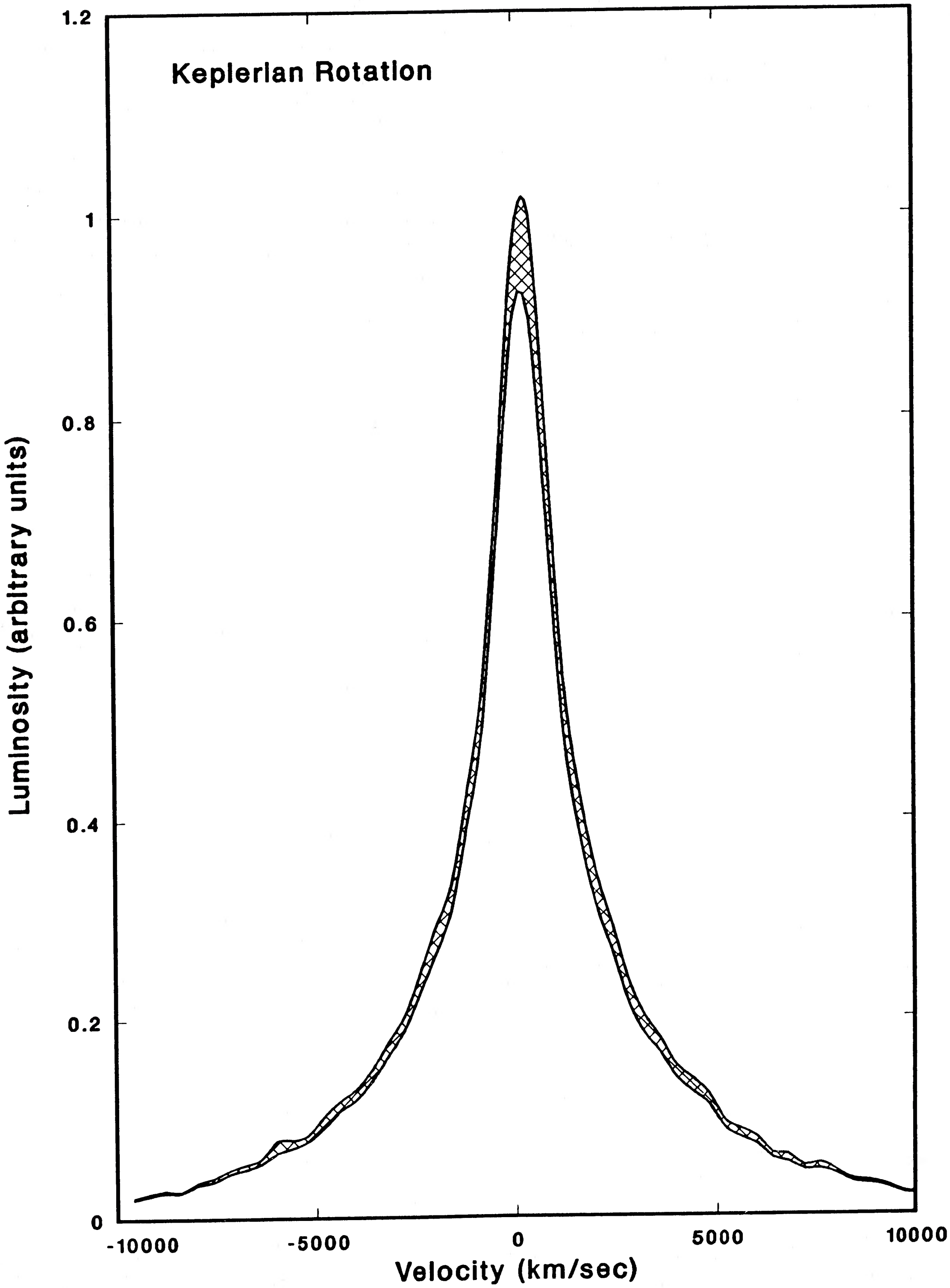

FIG. 7.—Keplerian disk model of BLR emission line generation. Here the disk is tilted 45° to the line of sight, and has $r_{max} = 0.1$ pc and $r_{min} = 10^{-4}$ pc, and a height of 0.04 pc. Part of the cloud motion is turbulent: a random (Gaussian) distribution with a dispersion velocity of 2000 km s^{-1}. The rest of the cloud motion is ordered: a circular Keplerian orbit about the BLR center, with $v = 2 \times 10^4$ km s^{-1} at r_{min}. Here the lens was placed 0.7 ERU from the center and situated so that it would affect one side of the line differently from the other. This effect is not large, but a slight bias of the line toward smaller redshifts is discernable.

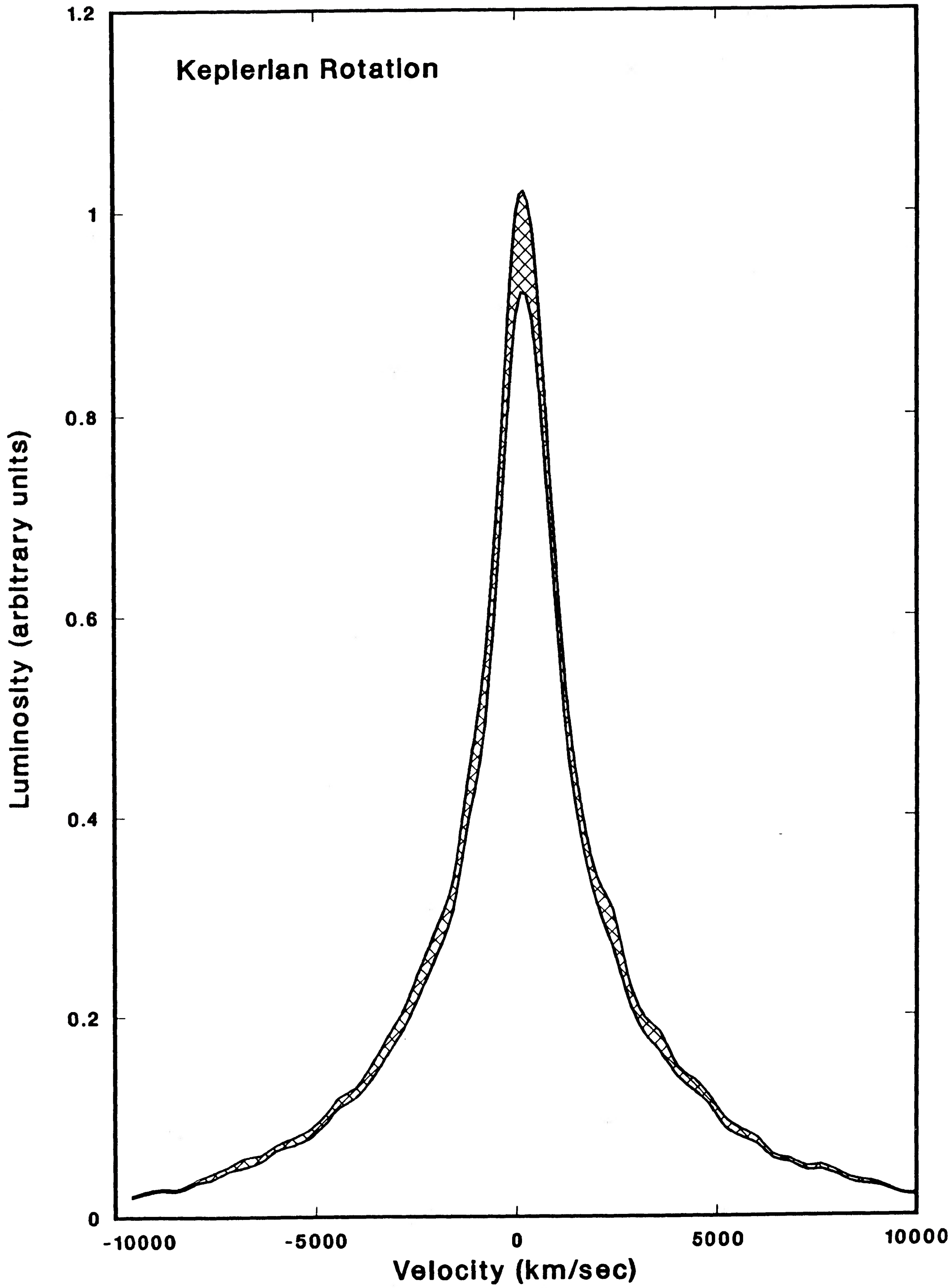

FIG. 8.—Keplerian disk model of BLR emission line generation. The BLR model is the same as in the one used to generate Fig. 7, except that the lens is superposed on the opposite side of the BLR disk region. This causes the line to tilt in the opposite direction from the previous line, resulting in a redshift difference between the two of the order of hundreds of km s^{-1}

Einstein ring or more, both images could be undergoing a different microlensing distortion.

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