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ACCRETION ONTO MAIN-SEQUENCE STARS. I. THE TRANSIENT PHASE

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ABSTRACT

We have derived a set of boundary conditions that are suitable for the study of accretion onto main-sequence stars. An important and generally neglected phenomenon is the ram pressure produced by the material accreting onto the stellar surface. However, while important for high rates of mass accretion onto main-sequence stars, it is negligible for accretion onto both white dwarf and neutron stars. We also provide a thermal boundary condition that allows one to include the accretion energy and the accretion luminosity in the evolution. We have used these boundary conditions, together with a Lagrangian, one-dimensional, hydrodynamic stellar evolution code to study mass accreting at high rates onto a main-sequence star with a mass of 1.31 $M_{\odot}$, in an attempt to simulate one of the proposed scenarios for the outburst of the recurrent nova RS Ophiuchi. This study is also relevant to those members of the class of symbiotic variables that are now thought to have main-sequence stars as the mass receiver in the system. We conclude that mass accreting onto an enlarged, massive, main-sequence star cannot be responsible for the outburst of RS Oph.

Subject headings: stars: accretion — stars: individual (RS Oph) — stars: novae

I. INTRODUCTION

Over the past few years it has become clear that a large fraction of stars in our Galaxy gain (or lose) mass during some phases of their evolution. The fraction of stars in binary systems that exchange mass with their companions, at some stage in their lifetime, is a large fraction of the binary population. While a significant amount of work has been done on the response of white dwarf stars, neutron stars, and protostars to mass accretion, very little work has been done on the response of main-sequence stars to an increase in mass. Part of the reason for this lack of interest has been the general belief that an isolated main-sequence star only gains a significant amount of mass during the protostellar phase of its evolution to the main sequence and, in addition, that the accretion phase has little effect on the long-term evolution of the star (except to increase its mass). Of course, while it has long been realized that a main-sequence companion to an evolved star could gain a significant amount of mass, there have been only a few studies of the response of a main-sequence star during this phase of its evolution.

However, in the past few years, Kenyon and his collaborators (Kenyon and Webbink 1984; Kenyon 1986) have shown quite convincingly that at least some of the symbiotic variables must have a main-sequence star as the compact component of the system. In addition, Livio, Truran, and Webbink (1986, hereafter LTW) and Webbink *et al.* (1987) have claimed that a significant fraction of recurrent novae have main-sequence companions. In fact, they except only U Sco and T Pyx. LTW have also proposed that the outburst of the recurrent nova RS Oph is caused by episodic mass transfer onto a bloated main-sequence star when the increased size is a result of episodes of rapid mass transfer. Although the simulations of episodic mass transfer show that such a phenomenon does not result in the ejection of a large amount of mass (Starrfield, Sparks, and Williams 1982; Prialnik and Livio 1985), this scenario is still thought to be a viable alternative to the thermonuclear runaway model for a few selected systems. Therefore, it is important to study the response of a main-sequence star under those conditions which could occur in a symbiotic or recurrent nova system.

In this paper we will derive boundary conditions for and present some numerical simulations of accretion onto a main-sequence star. However, it should be realized that the actual physics of both the accretion process and the interaction between the inner edge of the accretion disk and the stellar surface is not very well known (see Shaviv 1987; Shaviv and Starrfield 1987*b*). While Regev (1984; see also Regev and Hougerat 1988) has made an important first attempt to understand the disk-star boundary, there is still a great deal of work to be done. For example, we still do not, as yet, know what fraction of the accretion energy,

$$\frac{1}{2}\frac{GM}{R}\dot{m}\,, \tag{1}$$

is actually radiated by the boundary layer and what fraction will be present in the material that arrives at the surface of the star. Recently, we have investigated this problem by studying the response of a white dwarf star to accretion and parameterized the amount of accretion energy that was present in the infalling material (Shaviv and Starrfield 1987*a*). We found that by including this energy, *which is actually present in the infalling material*, we caused a significant change in the evolution of a white dwarf to a runaway.

In the next section we briefly review previous work on the treatment of accretion in stellar evolution studies and then go on in § III to discuss in some detail our treatment of the boundary conditions that are necessary to treat accretion. We follow that, in § IV, by a calculation of transient accretion at high rates in an attempt to simulate a proposed model for the symbiotic outbursts. In § V we discuss the implications of our results for close binary systems with a red giant secondary filling its Roche lobe and end with a discussion.

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II. PREVIOUS TREATMENTS OF ACCRETION

In most studies of the accretion of matter onto compact objects it has been assumed both that the material landed "softly" (with no kinetic energy) on the surface of the star, and also that it did not acquire any energy from the infall (Giannone and Weigert 1967; Taam and Faulkner 1975; Webbink 1976; Sparks and Kutter 1980; Prialnik *et al.* 1982; Starrfield, Sparks, and Truran 1985, 1986). This means that, in these studies, an increase in the internal energy of the accreted layers came only from (1) the $P\,dV$ work done on these layers as they were compressed by the continuing infall, (2) the flow of energy into these layers from the interior, and (3) the nuclear reactions that were occurring in the accreted layers.

However, as will be discussed in more detail in the next section, there is a fourth source of internal energy for these layers, which is the fraction, as yet unknown, of the accretion energy that remains within the infalling layers after they have arrived on the surface of the star. This term has generally been neglected in the calculations of accretion onto white dwarf or neutron stars but was included in a study of accretion onto main-sequence stars by Webbink (1976, hereafter W76). We note, however, that Prialnik *et al.* (1982) did assume that the material arrived at the surface with the internal energy of the accretion disk (as determined from standard accretion disk theory; Williams 1980) and obtained smaller accreted masses at the time of runaway than did the other investigators (MacDonald 1983; Starrfield, Sparks, and Truran 1986).

More recently, Prialnik and Livio (1985) tried to dispense with the assumption of "cold" infall by including the (parameterized) internal energy of the accreting material. They expressed the internal energy of the infalling material in the form

$$U_{\rm acc} = \alpha \frac{GM}{R}, \qquad (2)$$

where α represents the fraction of the accretion energy retained in the accreted material. They then evolved a series of evolutionary sequences and varied α in order to determine the effects of α on the final configuration. The initial model for all of the sequences that they studied was a very low mass, completely convective main-sequence star, and they found that its response to accretion was dependent on both the choice of α and the rate of accretion. Since the main-sequence stars in both recurrent novae and symbiotic binary systems are much more massive than the stars that Prialnik and Livio considered, and are not completely convective, we chose to study accretion onto stars with masses exceeding 1.3 $M_\odot$.

III. BOUNDARY CONDITIONS FOR ACCRETION

Shaviv and Starrfield (1987*a*) have demonstrated that it is important to include the accretion energy in the studies of accretion onto white dwarf stars. In this section we apply and extend the techniques developed in that study to accretion onto main-sequence stars and show that there are additional factors of the accretion process that can be even more important for main-sequence stars than for white dwarf stars. We also explore in more detail the effects of accretion on the structure of the star and, in particular, the probable heating of the star from energy produced in the boundary layer between the accretion disk and the stellar surface. We show that under most conditions encountered in accretion both $L_{\rm acc}$ (the total accretion luminosity) and $L_{\rm BL}$ (the luminosity of the boundary layer) are very much greater than the intrinsic luminosity of the star.

Although a number of workers have studied the structure of the boundary layer in some detail (see Shaviv 1987 for a recent review), it is not yet clear how the material from the Keplerian disk actually arrives onto the star. It is not the purpose of this section to continue the discussion but only to point out that the kinetic energy of the infalling material must be released in some fashion in order for the material to arrive, at rest, onto the star (W76; Regev 1984). It is hypothesized that there is a strong shock that thermalizes the infall energy and that the energy radiated by the shock is directed both outward, away from the star, and inward, heating the outer layers of the star. It must also be realized that these outer layers consist of material that has been recently accreted and may still be hot as a result of passage through the standoff shock (Kylafis and Lamb 1979).

If the mass accretion rate is low and the infalling material is optically thin, then the shock lies close to the surface and reaches temperatures exceeding 10^8 K (Pringle and Savonije 1979). Most of the infall energy will be radiated, and what is not radiated will be transported into the star by the material that passes through the shock. For a higher mass accretion rate, the infalling material is optically thick, the standoff shock lies at some distance above the surface, and most of the infall energy is radiated by the shock. However, because it lies at some distance from the surface, the gravitational potential is less and the temperatures are much less: 10^5 K (Pringle 1977). The material that passes through the shock must then radiate and cool, in hydrostatic equilibrium, in order to become part of the surface of the star. In this case, the infall will heat the star both by radiation from the shock and by the energy carried by the infalling material that was not radiated (it is optically thick). Unfortunately, we can only parameterize the amount of energy that arrives in the infalling material and vary our parameters in order to determine the effects of the boundary layer on the evolution.

In order to clarify our discussion of the effects of accretion on the evolution, in this section we will consider the various phenomena as separate and distinct, although we realize that this cannot be true. In addition, we will actually make no physical statement about either the boundary layer or the shock but simply treat them as energy sources to be included in the evolution.

Therefore, we propose that the boundary layer heats the star (and the accreted layers) by two main processes. First, the hot boundary layer radiates into the layers of the star which lie below it; second, the hot infalling material mixes with the outer layers of the star. Since the physical processes which act to dissipate the infall and Keplerian energy of the accreting material and allow the material to reach the surface of the star are unknown, we parameterize the fraction of the accretion energy carried by the infalling material and then treat it under extreme conditions. We note, in addition, that this process is even further complicated by the possibility of shear mixing and shear turbulence (Kutter and Sparks 1987; Sparks and Kutter 1987).

First, let us examine the two time scales that are relevant to this problem. The accretion time scale is

$$\tau_{\rm acc} = \frac{\Delta m_{\rm acc}}{\dot{m}_{\rm acc}}, \qquad (3)$$

where Δm_{acc} is the accreted mass and $\dot{m}_{acc}$ ($M_\odot$ yr^{-1}) is the rate of mass accretion. The second time scale to consider is the thermal relaxation time scale for the envelope (Cox and Giuli 1968; W76):

$$\tau_{th} = \frac{3}{4} \frac{GM \Delta m_{acc} \Delta R}{R^2 L} , \quad (4)$$

where ΔR is the change in the mean radius of the accreted material and M is the total mass of the star (we have assumed a γ of 5/3). Our definitions of these two time scales differ from those of Prialnik and Livio (1985), who used the total mass of the star, M, instead of Δm_{acc}. Consequently, they obtained significantly larger values for the accretion time scale than we find. However, it is obvious that it is the accreted mass that must be used in this equation (W76). Note also that the luminosity used, in the definition of the thermal time scale, should be the luminosity obtained either during or after accretion has begun. The ratio of these two time scales is

$$\frac{\tau_{acc}}{\tau_{th}} = \frac{4}{3} \frac{R^2 L}{G\dot{m}_{acc} \, M \, \Delta R} . \quad (5)$$

This equation implies that, as long as accretion continues, for small L and large ΔR the accreted envelope will never have time to reach thermal equilibrium. In addition, the larger the rate of mass accretion, the less likely that the accreted envelope will be in thermal equilibrium, and we can assume that the material in the optically thick boundary layer will be accreted adiabatically. Once accretion ends, the relaxation of the star to equilibrium may depend on the details of the accretion phase.

a) Pressure Boundary Condition

Since we have to solve both the hydrodynamic and the heat equation, we will need two boundary conditions. We proceed first with the hydrodynamic boundary condition by determining the ram pressure of the infalling material and use that instead of a zero pressure at the boundary (the usual assumption). The equation of momentum conservation is solved in a Lagrangian coordinate system; however, the photosphere is defined by the distance into the star where the optical depth τ is equal to $\frac{2}{3}$. Therefore, we define the atmospheric mass Δm_{atm} to be the amount of material above an optical depth of $\frac{2}{3}$. Obviously, the amount of mass in the atmosphere may change during the evolution. Now, let Δm_{atm} be the last mass shell (the surface) of the model, i.e., the location where the surface boundary pressure is applied. In Lagrangian coordinates,

$$P_{atm} = \frac{GM}{4\pi R_{atm}^4} \left(\frac{\Delta m_{atm}}{2}\right) , \quad (6)$$

where R_{atm} is the radius of this shell. Since we assume that the accretion is both in a steady state and spherical, the ram pressure,

$$P_{ram} = \rho v_{ff}^2 , \quad (7)$$

of the infalling material can be written as

$$P_{ram} = \frac{\dot{m}}{4\pi R^2} \left(\frac{2GM}{R}\right)^{1/2} , \quad (8)$$

where we have assumed the free-fall velocity. The ratio of the ram pressure to the atmospheric pressure is then

$$\frac{P_{ram}}{P_{atm}} = \frac{1.4 \times 10^{-3} \dot{m}(M_\odot \text{ yr}^{-1})(R/R_\odot)^{3/2}}{(M/M_\odot)^{1/2}(\Delta M_{atm}/M_\odot)} . \quad (9)$$

If we use typical values for a 1.0 $M_\odot$ main-sequence star where ΔM_{atm} varies from $\sim 10^{-7}$ $M_\odot$ to $\sim 10^{-8}$ $M_\odot$, then

$$P_{ram}/P_{atm} = 1.4 \times 10^5 \dot{m}(M_\odot \text{ yr}^{-1}) . \quad (10)$$

It is clear that for accretion rates close to Eddington, as will occur in one of the proposed mechanisms for the symbiotic star outburst (LTW; Webbink *et al.* 1987), the *ram pressure will completely dominate the atmospheric pressure*. In contrast, the ratio for typical rates of accretion onto a white dwarf is

$$P_{ram}/P_{atm} = 1.4 \times 10^{-6} \dot{m}(\Delta M_{atm}/M_\odot) . \quad (11)$$

Therefore, for all accretion rates less than Eddington, the ratio is negligible and one can ignore the ram pressure in studies of accretion onto white dwarfs. In addition, since $P_{atm} \ll P_{base}$ (which is about 10^{20} dynes cm^{-2} at the time of runaway; MacDonald 1983), we do not expect the ram pressure to have any effect on the evolution of a thermonuclear runaway and, thus, the nova outburst. It is also obvious that the ram pressure is unimportant for small mass accretion rates onto main-sequence stars. This result was also found by W76.

b) Thermal Boundary Condition

The second boundary condition is needed to treat the heat transport into the star from the infalling material. In standard stellar evolution calculations, in which the mass is used as a Lagrangian independent coordinate, the surface mass shell is treated in a special fashion. While in all other shells the radiative (or convective) flux into and out of the mass shell is given by the diffusion approximation, the flux out of the top of the surface mass shell must be given by an approximation to the theory of stellar atmospheres. In addition, in normal stellar evolution calculations there is no source of energy "above" the surface and one can assume a zero inward flux. Because in this study we must include the heating from the boundary layer "above" the surface, we must treat the surface flux with more care. In Figure 1 we show a sketch of how we "view" the surface structure.

In order to proceed, and also to clarify how we include the boundary-layer heating, we assume that the accretion process occurs in two steps. In the first step we adiabatically (see the time-scale discussion) add material that has been accreted during the time δt with an internal energy, U_{BL}, and mix it with the surface layer that has a mass δm^0 and an internal energy U_S. Next, we assume that the increased mass of the new surface layer (accreted mass plus surface mass) will cause a compression to the appropriate density. As a result there will be work done on both the accreted mass (step 1) and the new surface mass (step 2). In the following paragraphs we write down the various thermodynamic equations that govern these changes but ignore the radiative fluxes until near the end of the discussion.

The mass of the surface shell is δm^0 at $t = t_0$ and increases to $\delta m^0 + \dot{m}\,\delta t$ at time $t_1 = t_0 + \delta t$, where $\dot{m}$ is the mass accretion rate (g s^{-1}). The accreted mass brings into this mass shell an internal energy of $\dot{m}\,\delta t U_{BL}$. Therefore, the total internal energy of the mass shell, which consists of the accreted mass plus the surface mass at time t_0, will be

$$\delta m^0 U_s^0 + \dot{m}\,\delta t U_{BL} . \quad (12)$$

As the newly accreted matter moves from the low-density boundary layer to the high-density star, it is compressed and the work done on it during the compression is

$$\delta w_1 = \dot{m}\,\delta t \int_{\rho_{BL}}^{\rho_s{}^0} P\,dV , \quad (13)$$

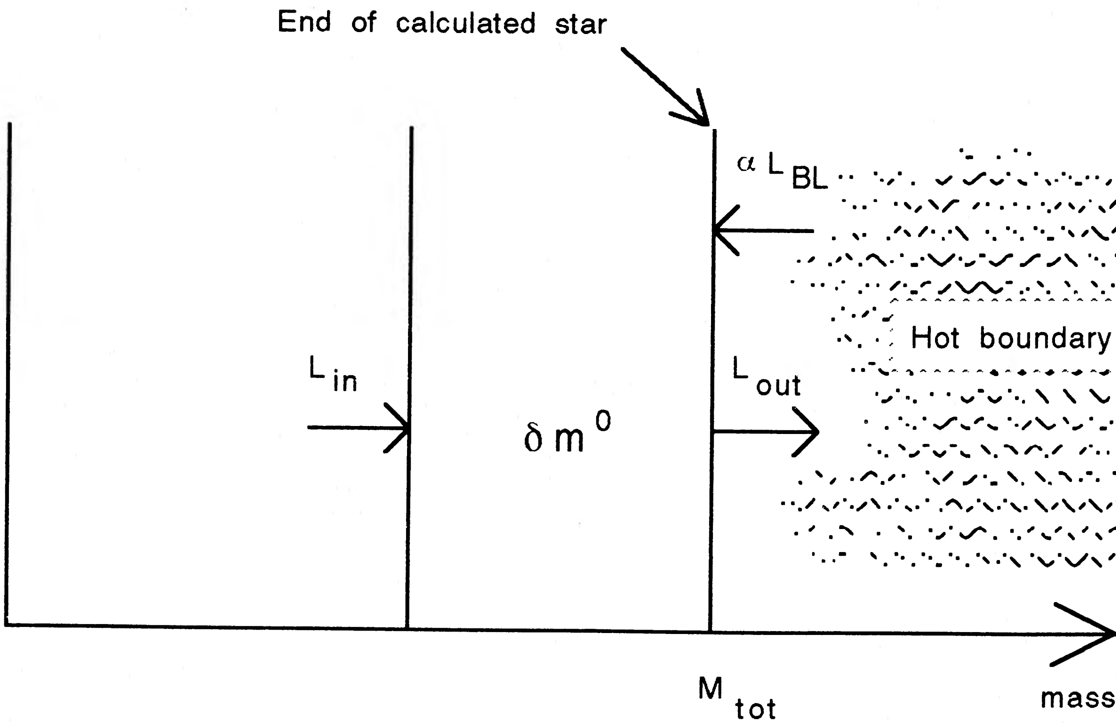

FIG. 1.—Sketch of how we view the surface boundary with the hot boundary layer on top of the surface mass shell

where δw_1 is the total work done on the accreted mass and V is the specific volume. At the same time, the internal energy of the accreted material will change, and this change, δu_1, can be expressed as

$$\delta u_1 = \dot{m}\,\delta t(U_{\rm BL} - U_s^0) \tag{14}$$

(note that we are using lowercase letters to denote the integral quantities, while the uppercase letters denote the values of the quantities per unit mass). Here we have assumed that this compression takes place adiabatically and the change can be obtained from the difference between the internal energy of the accreted and surface layers.

In step 2 we assume that the new surface shell, which now consists of the original surface shell plus the accreted mass, is compressed from its original density to the final density ρ_s^1 at time $t_1 = t_0 + \delta t$. The total work done on this material (in ergs) will be

$$\delta w_2 = (\delta m^0 + \dot{m}\,\delta t)\int_{\rho_s{}^0}^{\rho_s{}^1} P\,dV \,. \tag{15}$$

If, as in step 1, we assume that this layer is compressed adiabatically, then we can write that the total change in internal energy of these layers (the original surface plus the accreted mass) will be equal to the mass times the difference in the internal energy at the beginning and end of the time step:

$$\delta u_2 = (\delta m^0 + \dot{m}\,\delta t)(U_s^0 - U_s^1) \,. \tag{16}$$

We can now write down the equation for conservation of energy if we again assume that the mass accretion rate is large enough that the material is being accreted adiabatically:

$$\delta w_1 + \delta u_1 + \delta w_2 + \delta u_2 = 0 \,, \tag{17}$$

or, explicitly,

$$(\delta m^0 + \dot{m}\,\delta t)\left[(U_s^0 - U_s^1) + \int_{\rho_s{}^0}^{\rho_s{}^1} P\,dv\right] = \dot{m}\,\delta t\left[(U_{\rm BL} - U_s^0) + \int_{\rho_s{}^0}^{\rho_s{}^1} P\,dv\right] . \tag{18}$$

Note that equation (18) is written so that the left-hand side expresses the changes during step 2, while the right-hand side expresses the changes during step 1. Obviously, when there is no accretion, the right-hand side vanishes and the left-hand side is the usual term used in stellar evolution calculations (except for the change in the flux, which will be added in the next equation).

We now assume that the layer is radiative and that convection can be neglected. We express the radiative flux at the bottom as $L_{\rm in}$ and the flux out of the top as $L_{\rm out}$ (see Fig. 1). In a normal stellar evolution code $L_{\rm out}$ is $4\pi R^2 \sigma T_e^4$, where T_e is defined to be the effective temperature of the star. Therefore, if there were no accretion, we would only have to add a term, $(L_{\rm in} - L_{\rm out})\delta t$, to equation (18). In the case of accretion, where we assume that the boundary layer has a luminosity $L_{\rm BL}$, we must add another term $\alpha L_{\rm BL}\,\delta t$ which expresses the fact that only a fraction α of the accreted luminosity will be directed toward the surface of the star (Shaviv and Starrfield 1987*a*). For example, $\alpha = \frac{1}{2}$ is the maximum for an optically thin boundary layer and was the value used by Shaviv and Starrfield (1987*a*) in their study of accretion onto a white dwarf star. We can now add the radiative fluxes to equation (18) and arrive at the following result:

$$(\delta m^0 + \dot{m}\,\delta t)\left[(U_s^0 - U_s^1) + \int_{\rho_s{}^0}^{\rho_s{}^1} P\,dv\right] = \dot{m}\,\delta t\left[(U_s^0 - U_{\rm BL}) + \int_{\rho_{\rm BL}}^{\rho_s{}^0} P\,dv\right] + (L_{\rm in} - L_{\rm out})\delta t + \alpha L_{\rm BL}\,\delta t \tag{19}$$

The radiation of the boundary layer is assumed to be at a temperature different from that of the radiation inside the outer layers of the star where it is absorbed. Usually we expect that it will be much harder than that of the stellar surface. If we define a as the albedo of the surface, then only a fraction $(1 - a)$ of the radiation incident on the stellar surface will be absorbed and the rest will be reflected. Hence, we will have to multiply αL_{BL} by $(1 - a)$. Calculations by King and Watson (1987) show that a should lie between 0.25 and 0.28. We now define a new α $[=(1 - a)\alpha]$ such that αL_{BL} is the actual fraction of the boundary-layer luminosity that is absorbed by the star. We also assume that the last shell is sufficiently massive so that the radiation from the boundary layer is fully absorbed within it.

We now parameterize the problem by introducing two additional parameters β and γ defined by

$$U_{BL} = \beta\left(\frac{1}{2}\right)\frac{GM}{R}, \quad L_{BL} = \left(\frac{1}{2}\right)\dot{m}\frac{GM}{R}; \tag{20}$$

and

$$\gamma = -\int_{\rho BL}^{\rho_s 0} P\,dv\left(\frac{GM}{R}\right)^{-1}. \tag{21}$$

Using these parameters, the right-hand side (RHS) of equation (19) can be written in the form

$$\mathrm{RHS} = (L_{in} - L_{out})\delta t - \dot{m}U_s^0\delta t + \frac{\dot{m}}{2}\frac{GM}{R}\delta t(\alpha + \beta + 2\gamma). \tag{22}$$

The first term is the radiative flux which is present whether there is accretion occurring or not. The second term must be added, because of energy conservation, even if $\alpha = \beta = \gamma = 0$ (the accreted material is assumed to arrive on the surface of the star with no infall energy), and its existence is a consequence of the fact that the mass of the star is changing as a result of accretion. If we were to consider the accretion of a single particle, then $\beta = \gamma = 0$ (there would be no work done on the particle and thus no change in the internal energy). If we were to assume further that $\alpha = 0$ (the particle does not radiate as it accelerates toward the surface of the star), then we would still have to correct the energy equation for the change in mass of the surface mass shell. Prialnik *et al.* (1982) simulated the accretion of hydrogen-rich material onto a white dwarf and then followed the resulting thermonuclear runaway. They assumed $\alpha = 0$, $\beta \ll 1$, and $\gamma = 0$. They arrived at this value of β by assuming that the only available energy was the accretion energy from the material in the disk just above the boundary layer. In this way they ignored the rotational kinetic energy in the accretion disk (W76; Regev 1984). However, most of the internal energy of the boundary layer must come from the thermalization of the rotational kinetic energy. The assumption that $\beta = 0$ was sufficient to decrease significantly the mass of the accreted envelope at the time when the thermonuclear runaway occurred. This is because even a small β implies that the accreted material will be hotter than the surface material. surface material.

In his study of accretion onto main-sequence stars, Webbink (W76) assumed that all of the accretion luminosity was radiated into the surface of the star, but neglected the internal energy of the accreting material. Prialnik and Livio (1985) defined their heat equation so that they were effectively using $\delta = \alpha + \beta + 2\gamma$ as a free parameter. In their highest accretion case, their treatment was equivalent to assuming $\gamma = 0$. So, in reality, the value of δ used by Prialnik and Livio (1985) was less than 0.5, and they probably underestimated the effects of accretion.

As long as the problems of the structure of the boundary layer and its interaction with the star are not solved, we are left with two free parameters. It is of interest to estimate the effect of the accretion luminosity on the star. In the case of a main-sequence star with $L \sim 1\ L_\odot$ accreting at a rate of $\dot{m}\ (M_\odot\ \mathrm{yr}^{-1})$, we have

$$L_{BL} \cong 1.6 \times 10^7 \dot{m}(M_\odot\ \mathrm{yr}^{-1})\ L_\odot. \tag{23}$$

Suppose that the accretion rate is $10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$ or higher, as will be the case if the secondary is a red giant filling its Roche lobe. The accretion luminosity is then $1.6 \times 10^3\ L_\odot$. For δ as low as 0.01, the heating by the hot boundary layer will be sufficient to disturb the radiative balance of the star tremendously. However, we emphasize that there is no *a priori* reason to assume that δ is as small as 0.01.

It is also interesting to check the numbers for a white dwarf in a cataclysmic variable system. Here we have

$$L_{BL} \cong 1.6 \times 10^9 \dot{m}(M_\odot\ \mathrm{yr}^{-1})\ L_\odot. \tag{24}$$

In order to obtain a thermonuclear runaway which results in a nova outburst, the intrinsic luminosity of the white dwarf should be $\sim 10^{-2}$ to $10^{-3}\ L_\odot$ (see Starrfield and Sparks 1987, Starrfield 1987, and Shaviv and Starrfield 1987*b* for recent reviews of the nova outburst). Even for an accretion rate as low as $10^{-10}\ M_\odot\ \mathrm{yr}^{-1}$, the accretion luminosity from the boundary layer will exceed the intrinsic luminosity of the white dwarf by a large factor (Shaviv and Starrfield 1987*a*).

Shara *et al.* (1985) have recently proposed that there is a phase in the inter-outburst evolution of a nova system when the rate of mass accretion decreases to a very low level, and they have named this the hibernation phase. During this time not much happens to the system except that some as yet undetermined process of angular momentum loss causes the two stars to spiral toward each other. When hibernation ceases and the rate of accretion increases to $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, the heating of the star by the accreted matter can be overwhelming and completely change the proposed evolution (Shaviv and Starrfield 1987*a*, *b*).

IV. NUMERICAL CALCULATIONS

Our code uses a few of the ideas introduced in the numerical code of Rakavy, Shaviv, and Zinamon (1967). Additional features plus changes and enhancements are described in the series of papers on the nova outburst (Prialnik, Shara, and Shaviv 1978, 1979; see Shaviv and Starrfield 1987*b* for a review and references). The code includes all of the physical processes necessary to treat either the quasi-static or the hydrodynamic evolution of a star from protostellar collapse through the end of white dwarf cooling. The initial model is a very slightly evolved main-sequence star of solar composition in Z and a total mass of 1.31 $M_\odot$. The star is in radiative and thermal equilibrium at the beginning of the evolution. The process by which we bring a model into equilibrium necessarily involves some evolution, but no further structural changes have occurred in the initial model. However, the hydrogen mass fraction at the center has decreased to 0.67, while the initial value was 0.70.

a) *Accretion Phase*

In this paper we shall study primarily the influence of α, the incident radiation from the boundary layer, on the response of main-sequence stars to accretion. Just as in Prialnik and Livio (1985), we shall assume that $\beta = \gamma = 0$. We evolved two parallel calculations. In model sequence A, which we use as our baseline calculation, we assume that no heating of the surface by the accreting material is occurring ($\alpha = 0$). In model sequence B we assume that half of the boundary-layer luminosity is radiated into the last mass shell ($\alpha = 0.5$). The time evolution of the two sequences is shown in Figure 2, which is a plot of luminosity versus time. The basic difference between the two series of evolutionary calculations is that in model sequence A (no heating) the luminosity drops very quickly after the onset of accretion, while in model sequence B it rises immediately to a plateau. In model A the luminosity increases at later times, while in model B it stays practically constant. The bumps in the light curve for model sequence A are caused by episodes of convection which increase the flow of energy from the interior and then die away. It is to be expected that the luminosity of the models in sequence A would continue to increase to the luminosity reached in sequence B after accretion was stopped (0.64 yr). In sequence B the luminosity quickly reached an almost constant value of about 20% of the boundary-layer luminosity. The difference between the surface and boundary-layer luminosities ($\frac{1}{2}$–$\frac{1}{5}$) was reflected in the net gain in gravitational energy of the outer layers as they expanded. This difference decreases as R increases.

The structures of the initial and final models of each sequence are compared in Figures 3, 4, and 5. Figure 3 shows the variation of entropy density with mass shell as a function of zone number. Note that the abscissae of the figures are the shell number increasing from the center of the star and that for our calculations this is a Lagrangian coordinate system. The surface of the initial model is at mass shell 62, while the surface of the final model extends to mass shell 92, so that shell numbers from 63 to 92 denote the added mass.

In sequence A we found sporadic periods of envelope convection. The presence of the convective region acted to spread the entropy from the infalling material down into the outer layers of the original star. However, the final configuration, at the time that we end the accretion phase of the evolution, was in radiative equilibrium. When no heating from the accreting material is assumed to take place (model sequence A), the accreted matter blocks the outgoing radiative flux from the stellar interior. This caused the outgoing energy from the interior to accumulate in the newly accreted matter and heat it.

However, when the incident radiation from the boundary layer is included in the calculations (sequence B) there is an immediate heating of the accreted matter and, hence, the luminosity reaches the plateau more quickly than for sequence A. We chose our two evolutionary sequences to represent the extremes of accretion heating, and a comparison of these two sequences shows that both models approach the same asymptotic configuration. However, in model sequence A most of the heat comes from the interior and not from the accreting layers. In addition, because there is no accretion energy included, it takes model sequence A much longer to reach the plateau luminosity. We stress the similarity of the two evolutionary sequences. They both appeared to converge to the same final state independent (almost) of boundary conditions.

A comparison of the initial and final densities and radii (Fig. 4) shows that the outer layers were compressed in both sequences. We show the radii of the outer layers in Figure 4 to emphasize the fact that there is virtually no increase in the radius of either sequence as a consequence of the accretion, and, in fact, the original surface layers now have smaller radii. We also point out that the two sequences end with virtually identical surface radii. This means that our conclusions that accretion onto a main-sequence star will not cause it to become "bloated" are unaffected by our assumptions about

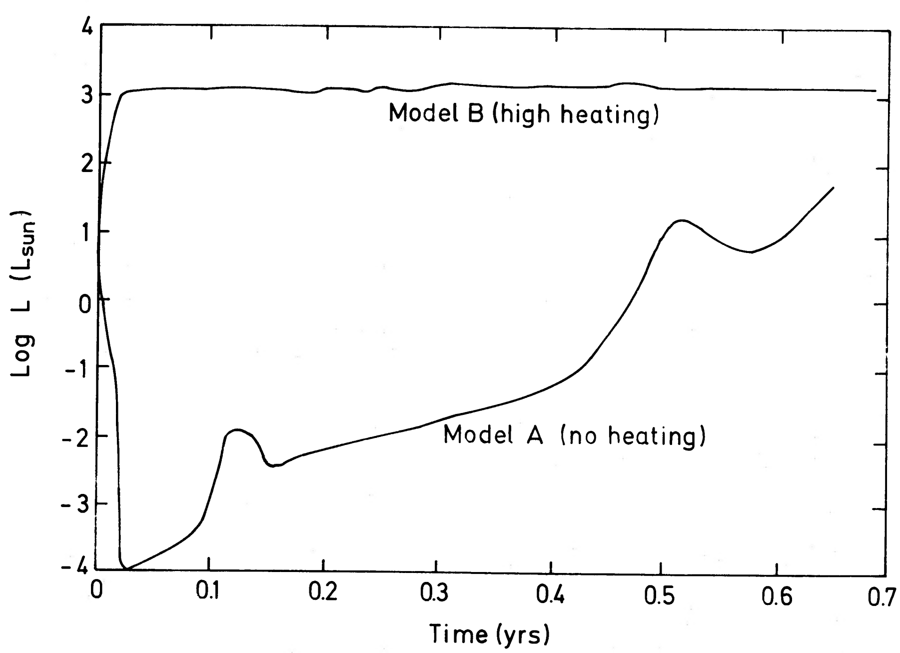

FIG. 2.—Time evolution of the luminosities of the two evolutionary sequences during the accretion phase. The two bumps which occur at times of ~0.12 and ~0.55 yr are caused by episodes of convective instability in the accreted layers.

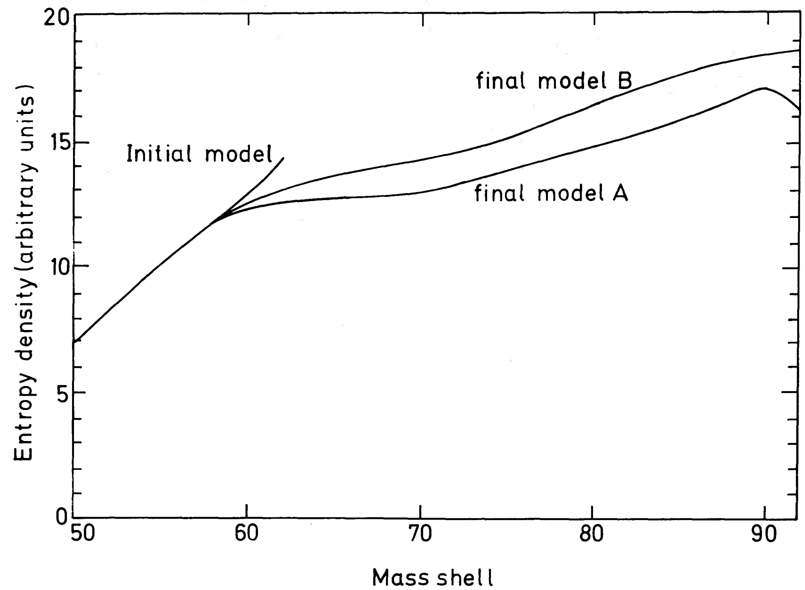

FIG. 3.—Initial and final distributions of the entropy density in sequence A and sequence B. The mass shell number serves as a Lagrangian independent coordinate. The surface zone for the initial model was at shell 62, while the final model extends to zone 92.

how to treat the accretion energy of the infalling material. We stress that the results for the luminosity and radius were obtained for a high accretion rate and a short accretion time. We do not, as yet, know what the results would be if we had accreted enough material to increase the mass of the star significantly. This calculation is beyond the scope of this paper, and work on this problem is in progress.

b) *Postaccretion Relaxation*

The accretion phase was assumed to continue for 0.64 yr, after which the accretion rate declined to zero and the star was allowed to relax. The final luminosity structure of both models is shown in Figure 5. The "bump" in the luminosity of model sequence A between mass shells 60 and 70 was caused by one of the strong episodes of convection in the evolution. We also show Figure 5 in order to emphasize the importance of including the accretion energy in the calculations. The accreted layers in sequence B are significantly more luminous than those in sequence A. This energy must be radiated before the envelope can achieve equilibrium, and it will take longer for sequence B to reach equilibrium than for sequence A, as is seen in Figure 6. The relaxation of both models to equilibrium, after accretion is ended, is shown in Figures 6 and 7. As soon as the accretion diminishes, the ram pressure goes to zero, the very outermost

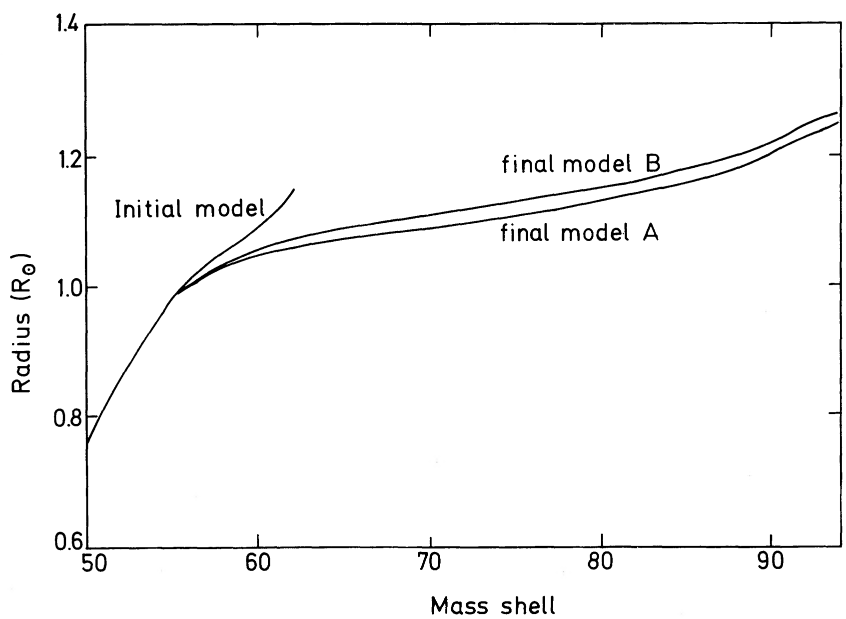

FIG. 4.—Initial and final radius of the outer layers for sequence A and sequence B as a function of the Lagrangian shell number. Note that shell 62, the original radius of the models, has decreased in radius as the two sequences accreted mass. Because the outer layers of the model, with the accretion energy included, are hotter, they have larger radii.

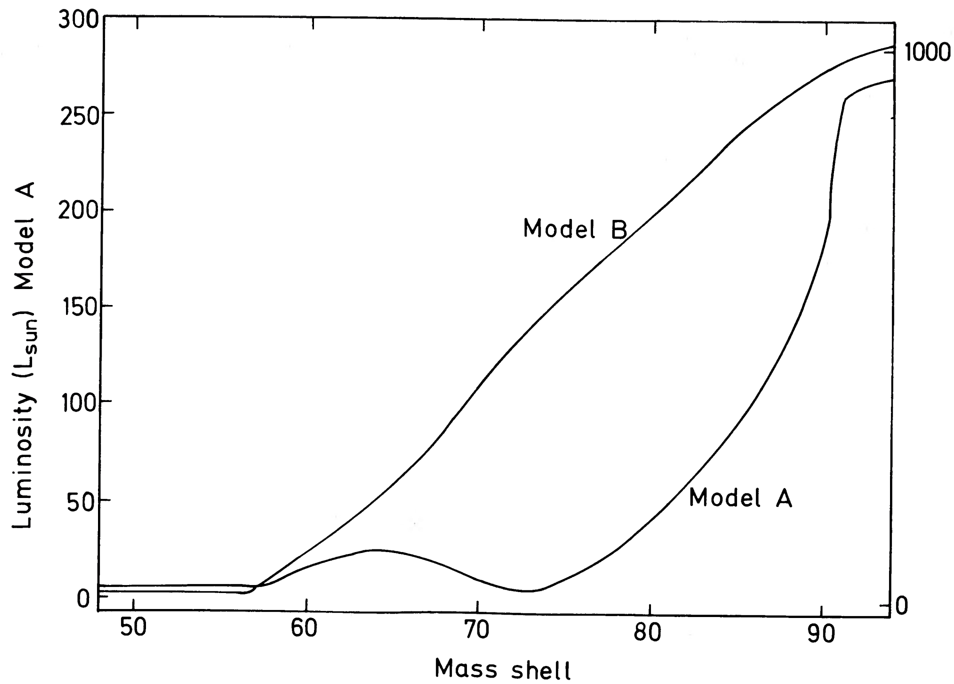

FIG. 5.—Luminosity as a function of mass for the final models in each sequence. This figure emphasizes the effects of including the accretion energy in the evolution.

layers expand (and cool), and the luminosity decreases by almost a factor of 10. after a period of the order of τ_{acc} (the accretion period in this case), the luminosity decreases by another factor of 10, and after a few τ_{acc} the star relaxes completely to its initial configuration. The relaxation time is much shorter than simple analytical estimates (cf. Prialnik and Livio 1985) would suggest because the luminosity of the perturbed star is much higher than the luminosity of the unperturbed star used in their estimates. We also note that no hydrodynamic effects occurred during the evolution.

Of importance and interest to the proposed model for the outburst of RS Oph is the behavior of the effective temperature of the star during the decline as seen in Figure 7. We find that the effective temperature always stays low and even at maximum reaches only about 20,000 K. As soon as accretion stops, the effective temperature declines as rapidly as the

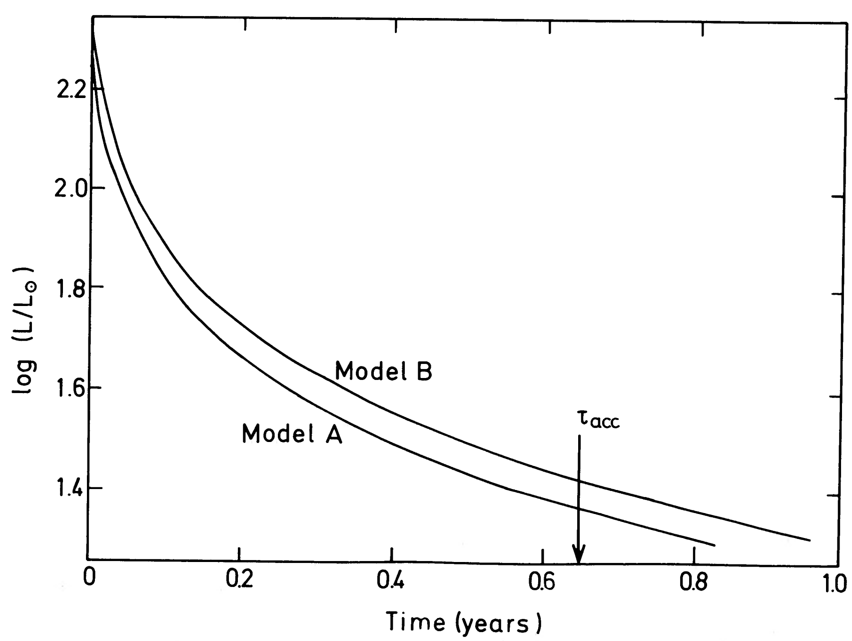

FIG. 6.—Decline of the luminosity as function of time during postaccretion relaxation. Because the outer layers of sequence B are hotter than those of sequence A, it takes sequence B longer to reach equilibrium. However, part of the difference in relaxation time is lessened by the larger luminosity of sequence B.

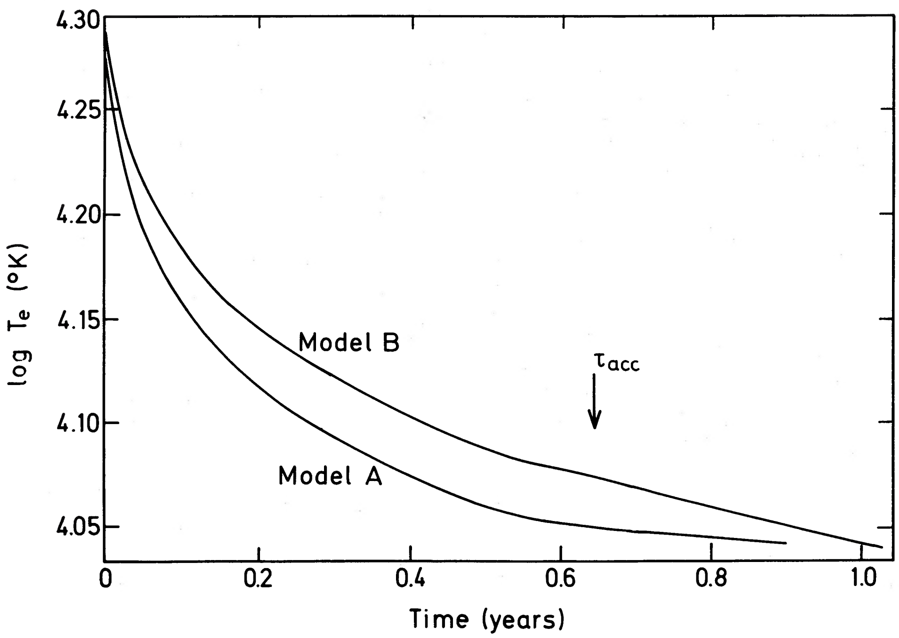

FIG. 7.—Relaxation of the effective temperature during the postaccretion phase. Note that at no time are these layers hot enough to provide a source of high-energy photons to photoionize the ejected shell. Therefore, this evolution does not agree with the observations of RS Oph during outburst.

luminosity. Finally, Figure 8 shows the variation of luminosity and temperature during the phase of relaxation back to quiescence. The (almost) constant line indicates that the changes in radius of the star are very small, in contrast to the speculations of LTW. We note that just at the time that we end accretion, the readjustment of the star to no ram pressure involves a brief period of expansion. However, this phase is very rapid, there is only a small increase in radius, and past this phase the radius hardly changes.

V. SUMMARY AND DISCUSSION

As discussed in § I, Livio, Truran, and Webbink (1986) and Webbink *et al.* (1987) proposed that some of the recurrent nova systems consisted of a red giant filling its Roche lobe and a main-sequence star. For RS Oph they additionally speculated that the main-sequence star had an enlarged radius, for its mass, caused by mass accretion. The outburst is then caused by an instability in the red giant leading to a very high (though

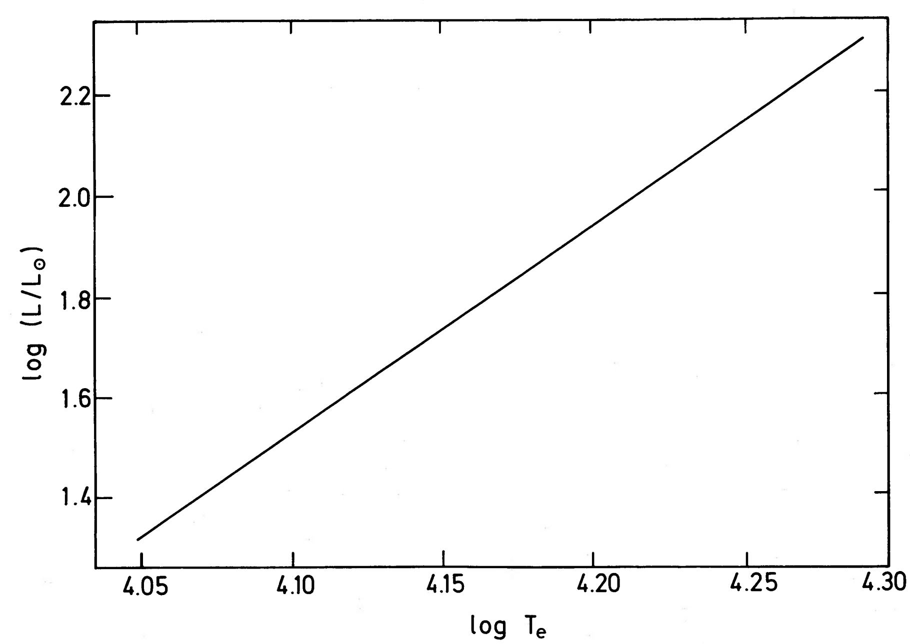

FIG. 8.—Luminosity–effective temperature relation during the relaxation phase. Note that time increases from right to left as the star cools and dims after mass accretion is turned off.

short-lasting) rate of accretion onto the main-sequence star. After the outburst occurs, the accretion rate declines to a quiescent state where there is either no accretion or very low accretion. Clearly, if the period between eruptions is longer than the time scale for the perturbed envelope to relax to quiescence, then the accretor will have ample time to return to a physical state in which it has the normal radius and temperature for its mass. If, however, the time between outbursts is much shorter than the relaxation time of the envelope, then the converse will be true and the star will always have an enlarged envelope. Our calculations how that the relaxation time for the envelope is very short and of the same order as the accretion time. Since the observed outbursts of the recurrent novae seem to last only a few months and the interoutburst times are of the order of many years, it is not possible for the radius of a main-sequence star to become enlarged as a direct result of mass accretion at high rates.

This conclusion is opposite to that reached by LTW, who, however, based their scenario on earlier calculations of accretion onto main-sequence stars (Kippenhahn and Meyer-Hofmeister 1977; Ulrich and Burger 1976). Kippenhahn and Meyer-Hofmeister obtained an enlarged main-sequence star in their calculations but not until it had accreted about 0.25 $M_{\odot}$ at a very high rate. A red giant losing mass at a rate of 10^{-4} $M_{\odot}$ yr^{-1} will need about 1000 yr to lose that much mass (assuming that a red giant losing that much mass will continue to fill its Roche lobe). The model proposed by LTW suggests that, on top of this high mass-loss rate, an instability occurs in the red giant that then drives a *further* increase in the rate of mass loss by a factor of order 10. It is inconceivable to us that immediately after (or during) an episode of high mass loss a red giant can sustain *an even higher rate of mass loss*. In fact, we predict that it will shrink and effectively decrease its mass loss to a very low value, at least for a couple of years, allowing the main-sequence star to relax to the unperturbed state. Finally, we note, for the conditions studied in this paper, that the results are independent of the entropy density of the accreted matter.

Of importance in the connection between our calculations and the proposed model for the outburst of RS Oph is the behavior of the observed level of ionization and excitation in the outgoing matter. Observations of RS Oph show "a trend toward increasing excitation as function of time" (Kenyon 1986; Hjellming *et al.* 1986). This behavior is very characteristic of heating of the expanding material by a hot white dwarf remnant on which a thermonuclear runaway has just occurred and is not that of a main-sequence star which has just accreted a small amount of material. In the study reported in this paper we found that after the "outburst" the star cooled and faded. Therefore, the most straightforward explanation of the observations is that a thinning envelope is uncovering the hot underlying white dwarf.

If the accretor is a main-sequence star, we have two cases: for a short outburst, as shown in the calculation reported here, the effective temperature increases by only a small amount. Over long periods of accretion, Kippenhahn and Meyer-Hofmeister (1977) have shown that the effective temperature decreases for a while as the radius increases and then increases as the radius decreases. However, in no case does the temperature attain very high values. Thus we claim that the measure of the excitation temperature following the eruption is one of the best means to unravel the nature of the underlying star. LTW also estimate that the luminosity of the accretor is $\sim 1000\ L_{\odot}$ while that of the red giant is $\sim 2500\ L_{\odot}$. Following Kippenhahn and Meyer-Hofmeister, this is the luminosity that a 2 $M_{\odot}$ star would have after accreting about 1 $M_{\odot}$. Finally, we point out that such a high rate of mass loss will dramatically affect the mass-losing star, leading to a very peculiar structure, and it would not remain a normal red giant.

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