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RADIAL VELOCITY STUDIES AND ABSOLUTE PARAMETERS OF CONTACT BINARIES. I. AB ANDROMEDAE

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ABSTRACT

New radial velocity curves have been obtained for the contact binary AB And, using the cross-correlation technique. A mass ratio of 0.479 is determined, which is revised to 0.491 when the velocities are corrected for proximity effects using a light curve model. These values differ by less than ten percent from the photometric mass ratio.

An analysis of the symmetric *B* and *V* light curves reported by Rigterink in 1973 using the spectroscopic mass ratio yields a consistent set of light and velocity curve elements. These also produce a reasonably good fit to the infrared *J* and *K* light curves reported by Jameson and Akinci in 1979. Absolute elements are determined, and these indicate that both components have a main-sequence internal structure. These absolute parameters, together with the Galactic kinematics, suggest an age for the system similar to or greater than that of the Sun.

Subject headings: radial velocities — stars: eclipsing binaries — stars: individual (AB And) — stars: W Ursae Majoris

I. INTRODUCTION

AB Andromedae (BD +36°5017, SAO 073069) was discovered to be a variable by Guthnick and Prager (1927). Photoelectric visual-band light curves have been published by Binnendijk (1959), Landolt (1969), Rigterink (1973), Rovithis-Livaniou and Rovithis (1981), and Bell, Hilditch, and King (1984), among others. Infrared light curves at *J* (1.2 μm) and *K* (2.2 μm) bands have been published by Jameson and Akinci (1979).

The spectrum of the system has been classified as G5 by Struve *et al.* (1950) and G5n by Hill *et al.* (1975). Strömgren indices of AB And obtained by Rucinski and Kaluzny (1981; see also Rucinski 1983) are similar to those of a K0 main-sequence star, when corrected for extinction. The only previous radial velocity study is by Struve *et al.* (1950), who obtained a mass ratio of 0.62 based upon seven spectra, only four of which were measured as double-lined. Thus their solution must be regarded as very preliminary.

AB And has not been observed to date in the satellite ultraviolet with *IUE* (*IUE Merged Log*, through 1987 May 31), nor has it been observed in the X-ray region with the *Einstein* (Seward and Macdonald 1983) or *EXOSAT* (Sternberg *et al.* 1986) satellites. In the longer wavelength regime, it is not included as a detection in the *IRAS Point Source Catalog*, nor have radio observations been published.

Several of the photoelectric light curves have been analyzed using modern techniques. Early synthetic light curve analyses were performed by Lucy (1973) of the *B* and *V* light curves of Binnendijk (1959), by Rigterink (1973) and by Berthier (1975) of Rigterink's *B* and *V* light curves. The results of more recent synthetic light curve analyses have been published by Mochnacki (1981), complied from the light curve analyses of L. W. Twigg (unpublished) and Bethier (1975), and by Bell, Hilditch, and King (1984) of their analysis of their intermediate-band blue light curves. A Fourier technique analysis has been published by Rucinski (1974) of Binnendijk's (1959) light curves, and Rovithis-Livaniou and Rovithis (1986) have used Kopal's frequency domain technique to analyze their *B* and *V* light curves (Rovithis-Livaniou and Rovithis 1981). Some of these analyses adopted the mass ratio of 0.62 determined spectroscopically by Struve *et al.* (1950), while others permitted it to adjust in their solutions, obtaining values ranging from 0.52 to 0.68. Most found the system to be in contact or overcontact, with the others finding one of the components to be slightly detached.

Although contact binaries are generally regarded as the most common type of interacting binary, until recently there existed a paucity of radial velocity studies of these systems. *The Seventh Catalogue of the Orbital Elements of Spectroscopic Binary Systems* (Batten, Fletcher, and Mann 1978) lists only some two dozen contact systems, almost all of which are classified as poor (d–e) in quality. The spectra of contact binaries are broad-lined and blended, which make them hard to measure. In addition, exposure times should be kept short to minimize phase smearing.

The use of the cross-correlation technique, by which the spectrum of a similar spectral type standard star is cross-correlated with that of the binary to derive the velocities of the two components, has been successfully applied to contact binaries in the past half-dozen years, beginning with the work of McLean (1981, 1983) and McLean and Hilditch (1983). By using an image intensifier tube, we have been able to obtain spectra of 10th and 11th mag stars at a dispersion of 15 Å mm^{-1} in 10 to 15 minutes or less with the 1.8 m telescope at the Dominion Astrophysical Observatory (DAO). Thus exposure times can be kept short.

In this paper, we present and discuss the results of a new radial velocity study of AB And. Then we use these results together with a solution of the light curve to obtain the absolute parameters of the system. As in previous papers, we use the cross-correlation technique to derive the component velocities (Hrivnak *et al.* 1984; Milone, Hrivnak, and Fisher 1985). We have initiated a program to study a number of contact binaries in this fashion, using primarily new Reticon data, and this is the first publication in this series. Our goal is to build up a new reliable and consistent data base of absolute parameters of these systems to aid in the understanding of their physical properties and evolution. Preliminary results for AB And have been presented by Hrivnak and Milone (1988).

II. SPECTROSCOPIC OBSERVATIONS

AB And was observed on eight nights in 1984 and 1985 with the 1.8 m telescope at DAO. A total of 52 spectra were obtained. The use of an image intensifier tube kept the exposure times short; the exposure time averaged 5 minutes or $0.01P$. Two-thirds of the spectra were recorded with the 1872 element Reticon detector, and the rest were recorded on IIa-O plates. The spectra were obtained at a reciprocal dispersion of 15 Å mm^{-1}, with a resolution of about 0.8 Å.

Table 1 contains a listing of the observations. The heliocentric Julian dates refer to the midpoints of the exposures. The phases were computed using an epoch based upon the times of minima of Bell, Hilditch, and King (1984) and the period of Landolt (1969). The ephemeris used was

$$\text{Hel. Pri. Min.} = 2{,}444{,}913.3534 + 0.3318922\,E\,.$$

The type of detector used, either Reticon (R) or photographic plate (P), is also listed in Table 1.

A number of standard velocity stars were observed in this program with the same equipment, and are used in the subsequent reduction. An example of an unrectified Reticon spectrum of AB And and of the velocity standard star HD 217014 is shown in Figure 1.

The G5 spectral type previously assigned to the system appears appropriate, with the spectra clearly appearing a little earlier in spectral type when recorded near quadrature than when closer to eclipse. This slight phase dependence in the spectral type is in the sense that one would expect from limb and gravity darkening. No Ca II emission was obvious in these spectra of AB And, which included in all cases at least the Ca II H line. We made no attempt, however, to measure possible infilling by emission of the Ca II lines.

III. SPECTROSCOPIC REDUCTION AND ANALYSIS

We have previously discussed the general reduction procedures used for obtaining the component velocities with the cross-correlation technique (Hrivnak *et al.* 1984; Hrivnak and Milone 1986). Here we will outline the procedures discussed previously; the application to Reticon data requires only slight modification. The photographic plates were first oversampled in steps of approximately 0.1 Å with the PDS at DAO using the program SCANN (Fisher, Morris, and Hoffman 1983). The data were then wavelength calibrated, rectified in intensity units using an approximate calibration, and linearized in ln λ using the computer program REDUCE (Hill, Fisher, and Poeckert 1982). The Reticon data were likewise wavelength calibrated, rectified, and linearized in ln λ.

The program spectra were then cross-correlated with spectra of comparison standard stars reduced in the same way. The program VCROSS (Hill 1982) was used to cross correlate the spectra. The wavelength interval used in the cross correlation was 4000 to 4270 Å, with the Hδ line excluded. This wavelength range is illustrated in Figure 1. The lower wavelength limit serves to exclude the broad Ca II H line and the upper limit excludes the *G* band and the λ4290 Sr I line which is blended with it. Tests showed that it was important to use care in setting the bounds, for at the large component velocities encountered in contact binaries, the broadened lines are shifted by several angstroms.

Following the cross correlation of the comparison spectrum with the program spectrum, a cross correlation function (c.c.f.)

TABLE 1

OBSERVATIONS OF AB ANDROMEDAE

JD$_\odot$ −2,440,000	Phase	Detector	V_P (km s^{-1})	V_S (km s^{-1})
5886.8607	0.203	R	79	−246
5886.8666	0.221	R	73	−246
5886.8721	0.238	R	78	−253
5886.8770	0.252	R	84	−245
5886.8815	0.266	R	71	−248
5886.8867	0.282	R	75	−243
5886.8926	0.299	R	67	−245
5886.8971	0.313	R	65	−231
5886.9023	0.329	R	73	−227
5886.9075	0.344	R	54:	−214
5913.9192	0.731	P	−125	214
5913.9254	0.750	P	−116	213
5913.9310	0.767	P	−124	213
5914.7672	0.286	P	83	−241
5914.7741	0.307	P	70	−243
5914.7828	0.333	P	76	−230
5915.9057	0.717	R	−125	206
5915.9099	0.729	R	−124	208
5915.9137	0.741	R	−133	201
5915.9175	0.752	R	−134	205
5915.9214	0.764	R	−131	202
5915.9255	0.776	R	−136	202
6272.8035	0.059	R	44	−154[a]
6272.8125	0.086	R	49	−178[a]
6272.8191	0.106	R	54	−193[a]
6272.8281	0.133	R	59	−214
6272.8365	0.159	R	76	−230
6272.8441	0.182	R	80	−238
6273.8685	0.268	R	94	−243
6273.8727	0.281	R	85	−242
6273.8782	0.297	R	73	−243
6273.8824	0.310	R	90	−232
6273.8880	0.327	R	72	−228
6273.8928	0.341	R	66	−220
6273.8970	0.354	R	67	−208
6273.9015	0.368	R	81	−181
6273.9053	0.379	R	43	−197
6273.9109	0.396	R	42[a]	−182
6273.9154	0.409	R	43[a]	−159
6273.9210	0.426	R	32:[a]	−151:
6283.9597	0.673	P	−134	188
6283.9622	0.681	P	−116	188
6283.9650	0.689	P	−151	189
6283.9705	0.706	P	−128	205
6283.9740	0.716	P	−144	198
6283.9768	0.725	P	−154	199
6283.9795	0.733	P	−138	198
6283.9823	0.741	P	−147	217
6283.9851	0.750	P	−138	209
6285.9602	0.701	P	−144	186
6285.9640	0.712	P	−129	196
6285.9689	0.727	P	−140	208

NOTE.—The velocity measurements indicated with colons were given half-weight in the analysis.

[a] Not used in the velocity curve solutions, since component partially in eclipse.

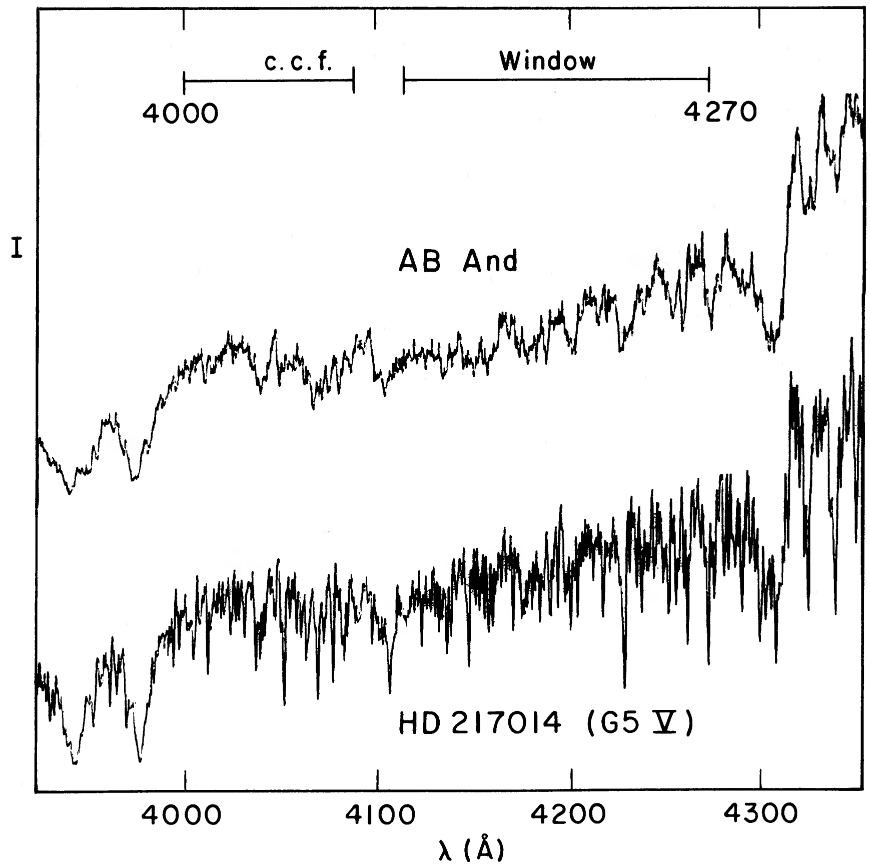

FIG. 1.—Examples of the spectra of AB And (phase = 0.282) and the standard star HD 217014, produced from unrectified Reticon data. The wavelength window used in the cross-correlation reduction is indicated.

profile is plotted as a function of ln λ or velocity shift. Two prominent peaks are seen in the profiles, which represent the velocities of the two components. We have fitted these simultaneously with two Gaussian functions with zero baseline, taking the centers of the Gaussian functions to represent the component velocities.

Examples of the c.c.f. profiles obtained at the two quadratures are displayed in Figure 2, for spectra obtained with both the Reticon and photographic plates. The spectra were all clearly resolved as two peaks except the two spectra between phases 0.40 and 0.45. Many of the spectra obtained near the first quadrature on both years show an asymmetry in the red-shifted (primary) component in the sense that the side of the profile closer to the other component appears somewhat distorted. This effect is not seen in the c.c.f. profiles of the spectra obtained at the other quadrature. This suggests a celestial source. The exposure times were of short enough duration to make skylight contributions negligible. The origin may be due to a significantly asymmetric light distribution on the photosphere of this (primary) component, or phase-locked circumbinary material.

The velocities obtained from the cross-correlation analysis are listed in Table 1. In one case a component possessed a large asymmetry in its c.c.f. profile and in another case, near eclipse, the components were very blended. These uncertain velocities are marked with a colon and were given only half-weight in the analysis. The other observations near eclipse also possessed some blending in their c.c.f. profiles and could have been assigned some intermediate weight, but were analyzed with full weight. These velocities were all derived from the correlation of standard velocity comparison star spectra with the binary's spectra. The standard stars observed in this program are listed in Table 2, where N is the number of individual spectra obtained for the star and σ is either the m.s.e. (mean standard error) among the individual spectral measurements or among the lines in a single spectrum. The velocities were measured from the digitized spectra with the program VELMEAS (Hill *et al.* 1982), using lines appropriate for F and G stars (Batten *et al.* 1971). The velocities are very close to standard values, and within the precision expected for sharp-lined, late-type stars observed with an image-intensifier tube, which increases the noise in the spectra. The standard templates used in this study of AB And were composed of several spectra of HD 217014, co-added to reduce noise. The template for the Reticon spectra was composed of the four individual Reticon spectra of HD 217014, and that for the plates of the three photographic spectra. A test of several spectra using a different standard star yielded velocities which differed by not more than a few km s^{-1} from those listed in Table 1.

We tested the accuracy of our c.c.f. technique on synthetic binary spectra. These were created by adding together, at a variety of specified velocity differences, two different spectra of our standard star linearized in ln λ. Using our standard template, we found that at velocity separations ranging from 150 to 400 km s^{-1} and light ratios ranging from 1.0 to 0.6, all measured velocities were within 5 km s^{-1} of the specified

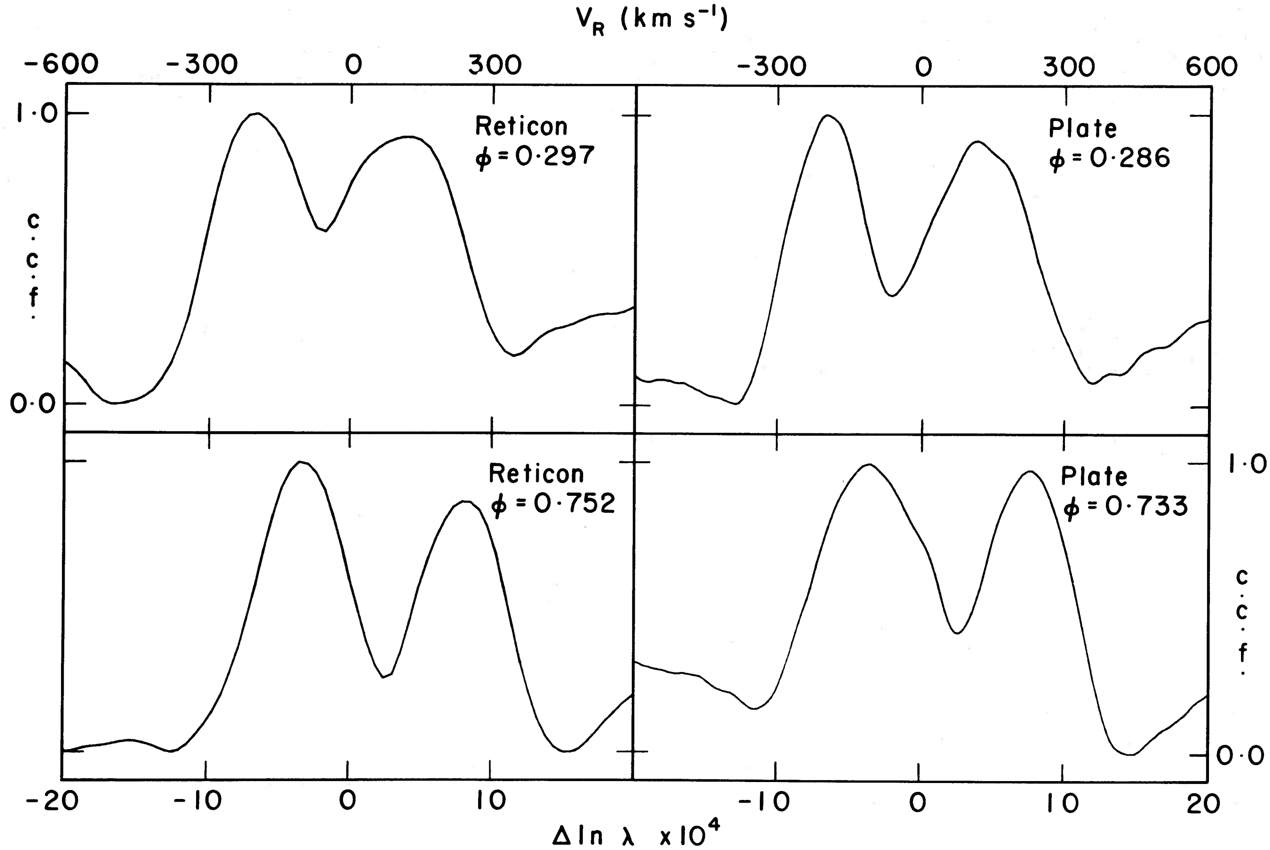

FIG. 2.—Examples of the cross-correlation function (c.c.f.) profiles obtained at the two quadratures of the orbit, from spectra recorded with both the Reticon and photographic plates. The exact phases are indicated.

values, with a mean of less than 2 km s^{-1}. It would have been more realistic to perform these tests using broad-line spectra, had they been available, rather than the narrow-lined spectra of the standard star. We estimate that a realistic uncertainty in the velocities derived for the contact binaries is less than or on the order of 10 km s^{-1}. This value compares favorably with the velocity residuals derived from the velocity curve analysis, as seen below.

It was noted earlier that care was taken to exclude the broad *G* band from the range of wavelengths used in the cross-correlation analysis. This is because the inclusion of this broad feature degrades the resolution of the cross-correlation profiles. This can be seen in Figure 3, where we display for illustrative purposes the effect on the resolution of including or excluding the *G* band in the cross-correlation. This would particularly effect the accuracy of the results when the components are not well separated in velocity.

The radial velocity curves were solved assuming a circular orbit using the spectroscopic orbit program of C. L. Morbey. As noted earlier, the components indicated with a colon were given half-weight in the analysis. The two velocity measurements of the secondary component between phases 0.05 and 0.09 produced unusually large negative residuals, which is what would be expected due to the rotation effect during a

TABLE 2

STANDARD STAR VELOCITIES

Star	SpT	Detector	V_{obs} (km s^{-1})	σ (km s^{-1})	*N*	V_{std} (km s^{-1})	ΔV (km s^{-1})
HD 217014..........	G5 V	R	−32.9	1.3	4	−33.0[a]	+0.1
		P	−32.7	1.6	3	−33.0[a]	+0.3
HD 187691..........	F8 V	R	−1.9	1.4	3	+0.1[b]	−2.0
		P	−1.1	1.6	4	+0.1[b]	−1.2
HD 222368..........	F7 V	P	3.2	0.3	2	+5.3[b]	−2.1
HD 182572..........	G8 IV	R	−100.3	1.3	4	−100.5[b]	+0.2
		P	−101.1	0.4	2	−100.5[b]	−0.6
HD 154417..........	G0 IV	R	−17.0	0.1	2	−17.4[b]	+0.4
		P	−19.4	1.4[c]	1	−17.4[b]	−2.0

[a] Beavers *et al.* 1979; Wilson 1953 lists −31.2 km s^{-1}.
[b] Pearce 1957.
[c] m.s.e. in the lines of a single spectrum.

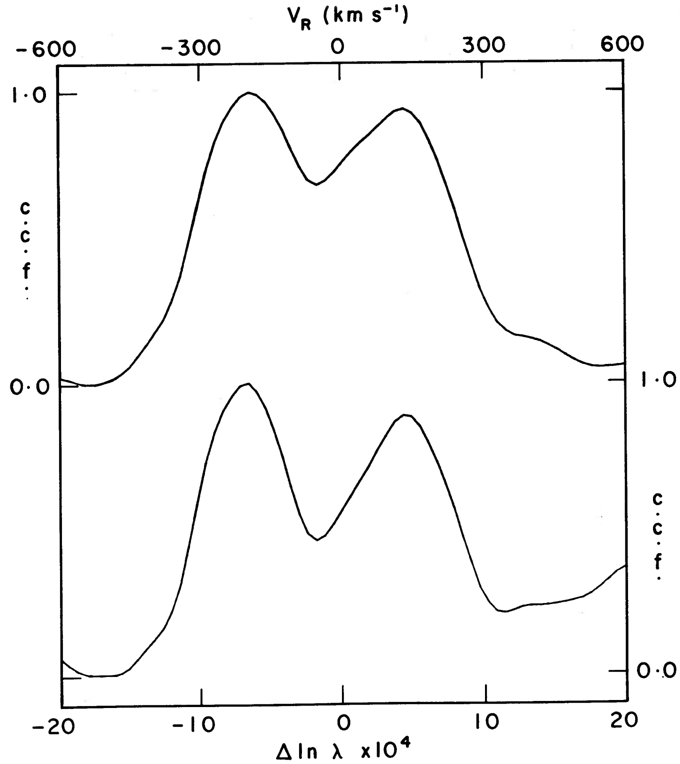

FIG. 3.—An example of the degraded resolution obtained when the broad G band is included in the cross-correlation analysis. The same spectrum of AB And is analyzed with (*upper panel*) and without (*lower panel*) the inclusion of the G band in the wavelength window.

partial eclipse. We thus decided to omit from the analysis all velocity measures within $0.11P$ of minimum for the component undergoing eclipse. This removed three velocity measurements for each component from the analysis. The velocity curve of each component was first analyzed separately, and very good agreement was found between the systemic velocity of each component. Thus there is no indication of gas streams or other large-scale complications in the system producing a systematic difference in these velocities. We note that a similarly good agreement between the systemic velocity of each component is found in our study of OO Aql (Hrivnak 1988), in contrast to the significant differences found in U Peg (Lu 1985) and BV and BW Dra (Batten and Lu 1986). A combined analysis was then made of the two components with the constraint that they have exactly the same V_0. The results are listed in Table 3, in columns (2)–(4). The resulting radial velocity curves are displayed in Figure 4. The fit is very good for a broad-lined contact system. The mean standard deviations of the fits for the two curves is 9.5 km s^{-1}, with that of the primary being 3 km s^{-1} larger than that of the secondary. The system falls in the W-type classification of W UMa systems, with the less-massive (secondary) component eclipsed at primary minimum. We note that the values of the mass ratio derived independently by several investigators for their light curve analyses are slightly (10%) larger than this radial velocity value of 0.479.

These values can be compared with the preliminary results of the radial velocity study by Struve *et al.* (1950). This latter study was based upon seven spectra, only four of which were measured as double-lined. The values derived in our study differ significantly from the values of Struve *et al.* and correct the anomalously large mass values ($M_1 \sin^3 i = 1.69\ M_\odot$, $M_2 \sin^3 i = 1.05\ M_\odot$) which result from their published orbital elements. (The system is not actually 2.5 mag underluminous for its mass as noted by Rucinski 1974; rather its luminosity agrees quite well with its mass, as discussed in § V.) The orbital elements published by Struve *et al.*, assuming a circular orbit, are $V_0 = -45$ km s^{-1}, $K_1 = 165$ km s^{-1}, and $K_2 = 265$ km s^{-1}, which yields $q = 0.62$. Our reanalysis of the same data, assuming equal weights for each observation, yields different values: $V_0 = -38$ km s^{-1}, $K_1 = 130$ km s^{-1}, and $K_2 = 285$ km s^{-1}, with large uncertainties, and a mass ratio of $q = 0.46$. This suggests that they either made a misprint in the published data, used a more complicated weighting scheme, or made a computational error. Their measurements themselves are suspect; it is hard to imagine that they were able to resolve both components within $0.02P$ of secondary minimum, and that the large velocity difference of 217 km s^{-1} that they measured is real. Correcting the phases of the observations of Struve *et al.* by $-0.030P$, based upon the use of a more appropriate emphemeris derived from times of minimum by Oosterhoff (1950) close in time to the spectroscopic observations, results in $q = 0.42$. We conclude that their study suffers from few observations, phase smearing (they began a new spectrum

TABLE 3

ORBITAL ELEMENTS OF AB ANDROMEDAE

	Proximity Effects			
	Not Included			Included
Element (1)	Primary (2)	Secondary (3)	Joint (4)	Joint (5)
V_0 (km s^{-1})	−24.9 ± 1.5	−24.0 ± 1.1	−24.4 ± 0.8	−24.6 ± 0.9
K_P (km s^{-1})	110.8 ± 1.7	...	110.7 ± 1.7	115.7 ± 0.7
K_S (km s^{-1})	...	231.1 ± 1.2	231.0 ± 1.2	235.7 ± 1.5
σ_P (km s^{-1})	11.1	...	11.2	10.7
σ_S (km s^{-1})	...	7.9	8.0	6.9
$a_P \sin i (R_\odot)$	...	...	0.726 ± 0.011	0.759 ± 0.010
$a_S \sin i (R_\odot)$	...	...	1.515 ± 0.008	1.546 ± 0.010
$M_P \sin^3 i (M_\odot)$	...	...	0.926 ± 0.017	1.002 ± 0.017
$M_S \sin^3 i (M_\odot)$	...	...	0.443 ± 0.009	0.491 ± 0.009
$q(=M_S/M_P)$	...	...	0.479 ± 0.008	0.491 ± 0.005

NOTE.—Errors listed are standard errors.

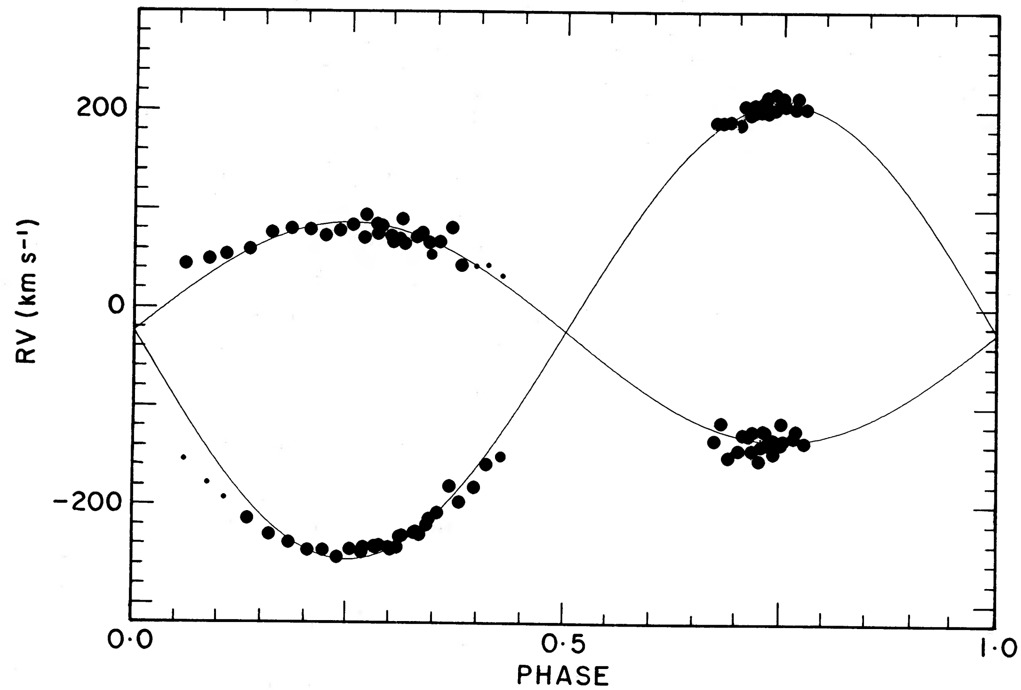

FIG. 4.—Radial velocity curves of AB And. The solid dots represent the observed velocities, and the solid lines the velocity curve solution, without the inclusion of proximity effects. The smaller dots indicate phases when the particular components were within partial eclipse, and were these were not used in the analysis.

every 52 minutes on average), and the complications of measuring broad-lined, blended spectra.

This analysis of our new observations omits the corrections due to proximity effects. These require a model of the light distribution of the system. We discuss such an analysis of the velocity curves including tidal and eclipse effects, together with the light curves, in § IV.

It is possible in principle to obtain the value of the luminosity ratio of the two components from the c.c.f. profiles. For spectra obtained near orbital quadratures, the ratio of the areas of the c.c.f. profile of each component is approximately equal to the light ratio. This was demonstrated in tests made with synthetic binary spectra and is discussed in more detail elsewhere (Hrivnak 1988). At light ratios of 0.8–1.0, the deviations were only a few percent. For the contract binary OO Aql, excellent agreement was found between the light ratio determined from the ratio of the c.c.f. profiles and the more precise light ratio determined from the light curve analysis (Hrivnak 1988). For AB And, we calculate a light ratio (secondary to primary) of 0.86 ± 0.02 (m.s.e.) from the average ratio of the areas of the c.c.f. profiles of the two components. This is based upon 40 spectra obtained within a phase interval of ± 0.10 of the quadratures. The ratio was lower for the spectra obtained in 1985 than in 1984, and lower at the second quadrature than the first. This may represent variations in the surface brightness distribution. The light curve is known to vary in brightness at maximum light, which coincides with the quadratures. A lower value for the light ratio of 0.74 is calculated from the more direct light curve analysis discussed in § IV.

IV. CONSISTENT VELOCITY AND LIGHT CURVE ANALYSIS

To include the proximity effects, such as tidal distortion, the so-called reflection effect, and eclipses, requires a model of the shape and of the light distribution of the binary system. This can be obtained from an analysis of the light curve. Such an analysis was performed previously for XY Leo by Hrivnak *et al.* (1984), and used by Hrivnak (1985) to determine a consistent set of light and velocity curve elements, and then to obtain the absolute parameters of that system.

As noted in the introduction, several photoelectric light curves have been obtained for AB And, and many of these have been previously analyzed to obtain light curve elements. The light curves show an asymmetry in maximum light, with the maximum following primary minimum sometimes the brighter and sometimes the fainter of the two (see Bell, Hilditch, and King 1984). This is a common phenomenon among the cool, W-type contact binaries, and is often attributed to variable starspot activity.

For this analysis, we have chosen to use the B and V light curves of Rigterink (1973). These possess dense coverage obtained in a reasonably short interval of time, 57 days, and display little difference in the heights of maximum light, with the $|B_1|$ term of the outside-eclipse Fourier analysis equal to 0.002. The data were converted to intensity ratios and normalized to 1.000 at maximum light, then gathered into average points in phase intervals of $0.01P$, centered on phase 0.000. The number of individual observations which compose each average point ranges from five to 19, averaging 11, and was used as a relative weight for each average point. These light curves suggest a short interval of totality at primary minimum, although this is not evident in some other light curves of the system.

The light curves were analyzed using the synthetic light curve and differential corrections program of Wilson and Devinney (1971), modified by Wilson (1979) to also include radial velocity observations.

The initial values were set as follows. The mass ratio was initially set at the spectroscopic value. We departed from the photometric practice of calling the star eclipsed at primary minimum star 1; rather we maintained subscripts to denote the more massive primary (P) and less massive secondary (S) components. We then followed the "nonstandard trick" used by Van Hamme and Wilson (1985) of reversing the usual photometric indices and introducing a phase shift of one-half a cycle, which they assert leads to improved solution convergence when the star eclipsed at the deeper minimum is the less-massive one. However, we use this "trick" notation only internally, and list the published results according to the subscripted P and S notation so as to avoid confusion. Note that it is the less massive component, star S which is eclipsed at the deeper minimum. The average temperature of the primary star, T_P, was fixed at 5450 K. This is a compromise between the spectral type and the color, with the recognition that the temperature of the secondary component in a W-type contact system typically appears to be 200–400 K higher than that of the primary. The gravity exponents g and the bolometric albedos A were set at theoretical values of 0.32 and 0.50, respectively, appropriate for stars with convective atmospheres (Lucy 1967 and Rucinski 1969, respectively). Values of the wavelength-dependent limb darkening, x_V and x_B, were derived from the tabulation of Al-Naimiy (1978). These values remained fixed in the subsequent analysis. The Carbon-Gingerich model atmosphere subroutine was used to represent the surface flux distribution. The scatter in the data was assumed to be due to shot noise, and thus the relative weighting of the light curve average points was inversely proportional to the light level. The value of third light, l_3, was set at 0.000.

The values which were permitted to adjust to fit the light curve were the inclination of the orbit i, the temperature of the secondary star T_S, the modified equipotentials Ω_P and Ω_S (which are equal for a contact or overcontact system), and the wavelength-dependent luminosity of the primary star $L_{P,V}$ and $L_{P,B}$. Initial values of these parameters were derived in two different ways: (1) trial-and-error fits to the light curves, starting from a detached configuration (mode 2), and (2) starting from the solution for $\beta = 0.03$ of Bell, Hilditch, and King (1984; where $4\beta = g$), which is a contact (overcontact) configuration (mode 3). The detached configuration immediately adjusted to a contact configuration, and converged to the solution of the initially contact configuration ([2], above). The remainder of the analysis was done with a contact configuration (mode 3). The light curve parameters very quickly converged to a solution in which the differential corrections to the parameters were all less than their probable errors.

The procedure which we use to achieve a consistent set of parameters for the velocity and light curves is to iterate between solutions of the two separately, holding either the light curve or velocity curve parameters fixed while the others adjust. The mass ratio can, however, be determined from both the velocity curve and the light curve, although in the latter case it is coupled with the sizes of the stars through the equipotential surface. One could have chosen to solve the light and velocity curve parameters simultaneously, but then one must give special attention to the proper relative weighting between the light curve and the velocity curve, as the later has many fewer points. We chose to derive the mass ratio strictly from the velocity observations, but taking account of the proximity effects which are derived from the light curve parameters. Thus we hold the mass ratio fixed in the light curve solution. We later investigate the effect of permitting the mass ratio to adjust in the light curve solution.

With this light curve solution, we solved the velocity curve taking into account the proximity effects. The parameters which were permitted to adjust were the mass ratio q ($= M_S/M_P$), the semimajor axis of the relative orbit in solar radii a, and V_0. These adjusted only a slightly. These new values were put back into the light curve solution, and slightly adjusted light curve parameters derived. Iterating between the light curve solution and the velocity curve solution quickly yielded a consistent set of light and velocity curve parameters, with the differential corrections all less than the probable errors in the parameters. The parameters derived from the consistent light curve and velocity curve solutions are listed in Table 4, along with their standard errors. Since the light and velocity curve parameters are actually coupled through q, the errors listed were those derived from a light curve solution including q as an adjustable parameter, which takes this correlation between parameters into account. The solution indicates a secondary component 370 K, or 6.8%, hotter than the primary. The value of the degree of overcontact is 15%.

A comparison of the light curves derived from the parameters of the solution with the observed average points is displayed in Figure 5. The fit to the light curves is good except for the very bottom of the two minima, where the synthetic light curve is not quite deep enough. The light curve solution, in minimizing the residuals, results in a synthetic light curve with too long an interval of totality. As we discuss later, the fits in the bottom of the minima are significantly improved by allowing q to adjust. The other deviations are attributed to light curve complications evident in the first maximum and particularly in the phase interval 0.35 to 0.40 in the V light curve. Color curves were formed by the ratio of the normalized B and V intensity ratios. The fit to the color curve is reasonably good, indicating that the temperature difference determined is accurate. The slight predicted rise during primary minimum is due to the total occultation of the darkened limb of the secondary component, which had contributed to the redness of the light. This last effect is displayed and discussed for W UMa by Linnell (1987), who has emphasized the importance color curves can play in constraining source temperatures in these types of binaries.

The theoretical velocity curves including the proximity effects are displayed in Figure 6, along with the observed velocity points. The curves are calculated from the projected light centers of each component as viewed by the observer. The result of the explicit inclusion of eclipse and tidal effects is to produce more flattened extrema in the curves. As in the purely spectroscopic solution, the three points in each curve where the components are undergoing eclipse were not used in the solution. The resulting theoretical curve predicts values which are very close to the observed ones for the secondary component at phase 0.059 to 0.106. At these phases the secondary component is undergoing a partial eclipse, and the resulting rotation effect is very successfully predicted by the theoretical curve even though these observed points were not used in the solution. On the other hand, the predicted eclipse effect in the primary component is not seen in the observed velocities at phases 0.396 to 0.426, and the sine-curve representation without the inclusion of the proximity effects is a better fit. On the whole, the observed velocity points are better represented by the velocity curve solution including the proximity effects. The spectroscopic elements as revised by the proximity effects are also listed in Table 3 in column (5). The standard deviations of the fits are smaller than those of the fit to the purely spectroscopic solution without proximity effects. The mass

TABLE 4

LIGHT CURVE AND VELOCITY CURVE PARAMETERS FOR AB ANDROMEDAE

PARAMETER	CONSISTENT SOLUTION[a] Light Curve	CONSISTENT SOLUTION[a] Velocity Curve	LIGHT CURVE ONLY LC (q adjusted)
$a\,(R_\odot)$	...	2.308 (13)	...
V_0 (km s^{-1})	...	−24.6 (9)	...
q	...	0.491 (5)	0.524 (5)
i	86.8 (3)	...	85.8 (3)
$g_P = g_S$	0.32[b]	...	0.32[b]
T_P (K)	5450[b]	...	5450[b]
T_S (K)	5821 (6)	...	5798 (6)
$A_P = A_S$	0.50[b]	...	0.50[b]
$\Omega_P = \Omega_S$	2.816 (9)	...	2.880 (9)
$L_P/(L_P + L_S)_V$	0.589 (2)	...	0.580 (2)
$L_P/(L_P + L_S)_B$	0.578 (2)	...	0.569 (2)
$x_{P,V} = x_{S,V}$	0.65[b]	...	0.65[b]
$x_{P,B} = x_{S,B}$	0.80[b]	...	0.80[b]
l_3	0.000[b]	...	0.000[b]
$r_{P,\,\mathrm{pole}}$	0.423 (1)	...	0.417 (1)
$r_{P,\,\mathrm{side}}$	0.451 (2)	...	0.444 (2)
$r_{P,\,\mathrm{back}}$	0.482 (3)	...	0.475 (3)
$r_{S,\,\mathrm{pole}}$	0.306 (1)	...	0.310 (1)
$r_{S,\,\mathrm{side}}$	0.320 (2)	...	0.325 (2)
$r_{S,\,\mathrm{back}}$	0.358 (3)	...	0.362 (3)
f (% overcontact)	15 (3)	...	13 (3)
$\Sigma\,\mathrm{wt} \times (O - C)^2$	0.0229	...	0.0175

NOTE.—Standard errors in the last digits are listed in parentheses, not probable errors as are usually listed with Wilson-Devinney analyses.

[a] Errors listed are those appropriate to a light curve solution which includes q as an adjustable parameter.

[b] Not adjusted in the solution.

center semiamplitudes are slightly larger than the light center semiamplitudes, and thus the calculated masses are accordingly larger. As a consequence of the correction for proximity effects, the calculated values of the masses are larger by approximately 10%.

We also determined a solution to the light curves with the mass ratio permitted to adjust. Initial values for the parameters were taken from the consistent light curve and velocity curve solution listed in Table 4. After a few iterations, the light curve parameters converged to a solution with $q = 0.524$. The parameters derived from this solution are listed in the rightmost column of Table 4. This increase in mass ratio results in a solution with lower inclination and somewhat smaller temperature difference. The fit to the observed light curve is definitely better in the minimum, which results in a reduction in the weighted sum of the residuals squared by 23%. Part of this can be expected due to the inclusion of one additional free parameter in the solution. The color curve fit is similar to that displayed in Figure 5, which was obtained using the spectroscopic mass ratio. Thus the photometric mass ratio is 7% larger than the corrected spectroscopic mass ratio. We regard this as reasonably good agreement and favor the spectroscopically determined mass ratio.

Our light curve solutions are rather similar to those of Berthier (1975) using Rigterink's light curves and of Bell, Hilditch, and King (1984) from the study of their light curves using Rucinski's light curve synthesis program. The average values determined by Bethier are $i = 85\overset{\circ}{.}9$, $q = 0.53$, $f = 0.08$, and $\Delta T/T_P = 0.054$, and the average values determined by Bell, Hilditch, and King are $i = 86\overset{\circ}{.}6$, $q = 0.56$, $f = 0.24$, and $\Delta T/T_P = 0.073$. A more complete comparison of previous analyses is listed by Bell, Hilditch, and King (1984). The compilation by Mochnacki (1981) lists $f = 0.45$, which is a much larger degree of overcontact than found in any of the other solutions. The detached solution of Rigterink (1973) is clearly in error, perhaps influenced somewhat by the large mass ratio determined.

Infrared J (1.2 um) and K (2.2 um) light curves have been published for AB And by Jameson and Akinci (1979). They were obtained on 1977 September 27 and 29. In these infrared light curves, the two minima are of nearly equal depth. However, the two maxima in each curve have a relatively large difference in brightness, about 0.05 mag in each curve with that in J slightly larger. This is a large difference in maximum light, larger than has been recorded in any of the published visible-band light curves of AB And. Asymmetries in the heights of maximum light are common in the W-type W Ursae Majoris binaries and are frequently attributed to (variable) starspot activity. However, one would expect such an effect to be reduced in amplitude in the infrared. The data do show changes in the heights of the two maxima on the two different nights of observation, which may be due to short-term intrinsic variation in the light of the system, and perhaps even some small, unaccounted for atmospheric or instrumental effects.

As noted above, the two minima are of nearly equal depths in the infrared. The minimum at phase 0.00 remains the deeper, but the difference in depths decreases monotonically with increasing wavelength from the visible to the infrared. This is a common effect seen in the infrared light curves of W UMa binaries (Jameson and Akinci 1979; Zhukov 1984). This observed effect might initially suggest that the two components actually have equal temperatures, thus producing the nearly equal depths in the infrared light curves. The observed differences in depths in the visible could then be attributed to some wavelength-dependent complication, such as the effect of cool spots on the photosphere of the more massive component.

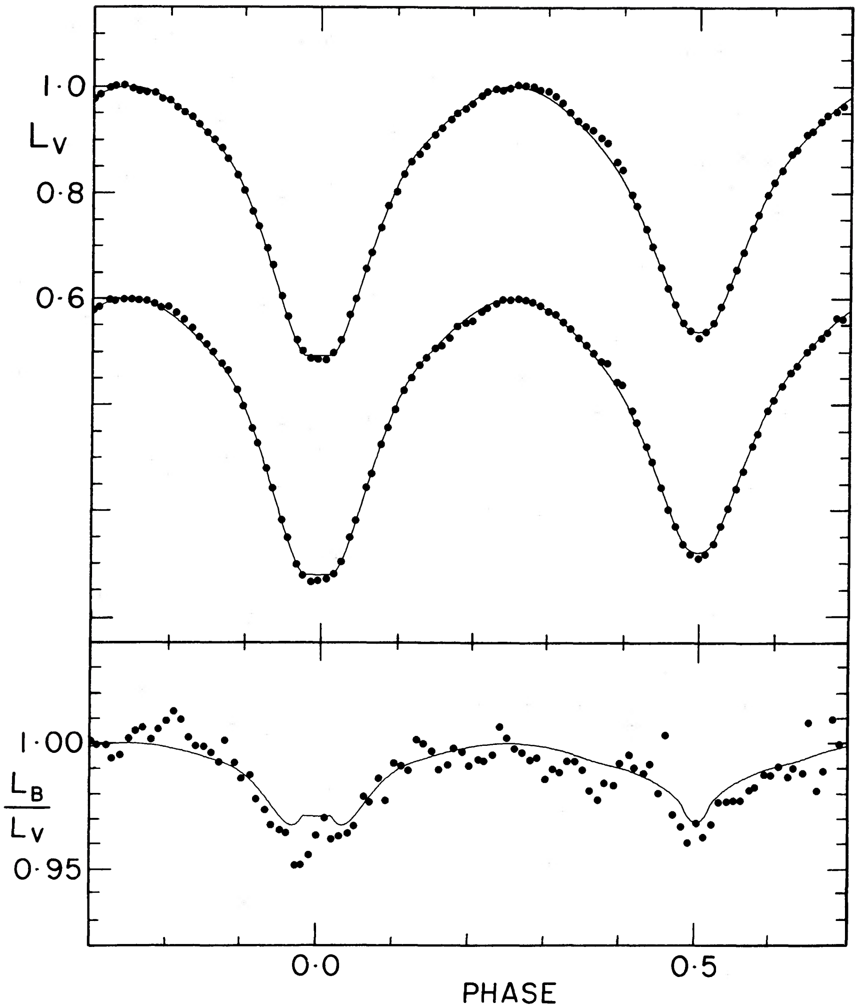

FIG. 5.—Light and color curves of AB And. The solid dots represent the observed average points by Rigterink (1973), and the solid lines the results of the consistent light and velocity curve solution.

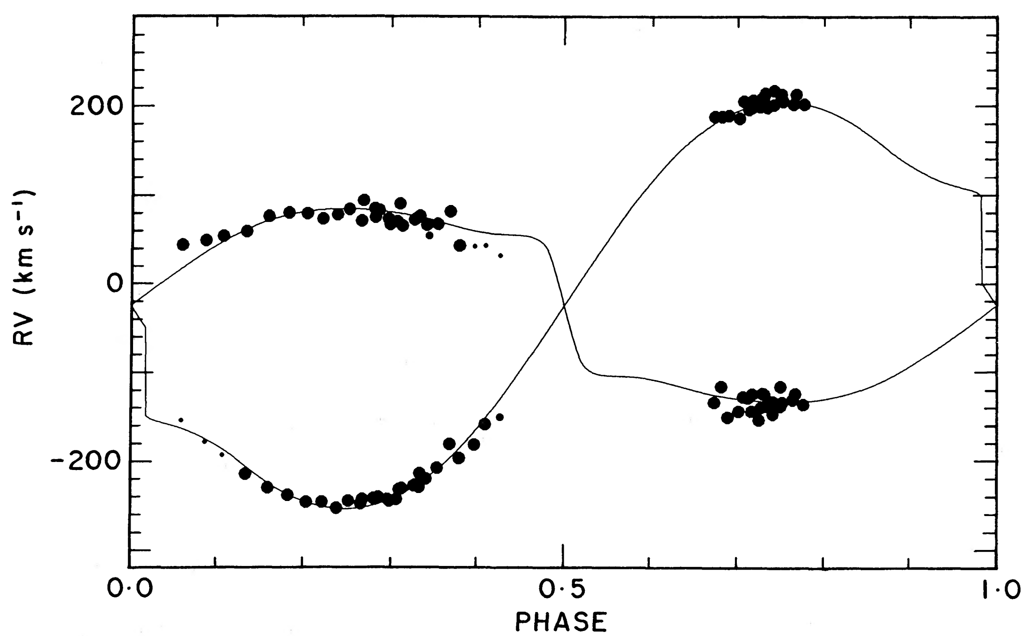

FIG. 6.—Radial velocity curves of AB And, modified to include proximity effects such as tidal distortion and eclipses. The symbols have the same meaning as in Fig. 4.

Such cool spots would help to explain the W-type phenomenon found in the cooler W UMa–type binaries, the fact that the more massive components are eclipsed at secondary minimum and thus appear to have the lower temperatures. However, equal temperatures are not a necessary consequence of equal depths. Limb darkening and to a lesser extent gravity darkening affect the relative depths of the two minima. At the transit eclipse, when the smaller star is in front, the observed light curve includes the effects of limb darkening on both stars, while at the occultation minimum it is mainly the limb darkening of the larger star which is included. To produce minima of equal depths in the light curve thus requires that the larger, more massive component have a lower temperature.

To see how well the light curve model derived from the B and V light curves fit the infrared observations, we produced synthetic J and K light curves using the Wilson-Devinney program. Limb-darkening coefficients were taken from Al-Naimiy (1978). For the purposes of this study, we simply removed the asymmetry in the maximum light by a subtraction of the first-order and second-order sine terms from the light curves. The sine coefficients were derived from a Fourier analysis of the outside eclipse light curves.

The parameters were fixed at the consistent light curve and velocity curve solution, and only L_1 was permitted to adjust. The resulting light curves are plotted together with the sine-corrected observations in Figure 7. The fit to these infrared light curves is acceptable in general, although the synthetic light curves are not deep enough in the minima. The fractional light of the primary is 0.624 and 0.637 in J and K, respectively. The parameters of the "Light Curve only" solution, with $q = 0.524$, again produce a significantly better fit in the two minima. If we let T_2 adjust, only a slight additional improvement in the fit results, and T_2 converges to a value approximately 60 K lower. This still would leave the secondary 300 K hotter than the primary. Thus the hot secondary model produces a good fit to the infrared light curves also.

V. SUMMARY AND DISCUSSION

Using these new radial velocity data, we have determined a consistent set of light and velocity curve parameters for AB And. These provide a good fit to the B and V light curves and color curve, and also a reasonably good fit to the infrared J and K light curves. The absolute parameters for AB And can now be determined. These are listed in Table 5, along with the standard error of the last figure in each parameter, which is listed in parentheses. The radius is the mean volume radius. It and the mean surface gravity were calculated using the tables of Mochnacki (1984), which are based upon the Roche geometry. For the average temperature of each component, we have assumed a standard error of 300 K. The luminosity is based upon the temperature and surface area of each component. Following Popper (1980), we have adopted for solar values $T_{\text{eff}} = 5780$ K, $M_{\text{bol}} = 4.69$, and BC $= -0.14$. The bolometric correction used for each component is from Popper's (1980) tabulation. The calculated visual magnitude difference between the two components agrees exactly with the light ratio determined from the light curve analysis. For reddening, we have adopted a value of 0.03 mag, which is the average of the values suggested by Eggen (1967) and Rucinski (1983), and for the observed V magnitude we used the value of 9.49 derived from the data of Rucinski and Kaluzny (1981) at maximum light.

The mass and radius of the primary component of AB And have values very similar to those of the Sun. For XY Leo, we (Hrivnak 1985) compared the mass and luminosity of the two components with those of detached, lower main-sequence stars as listed by Popper (1980) and plotted by VandenBerg *et al.* (1983). The secondary component of XY Leo was highly over-

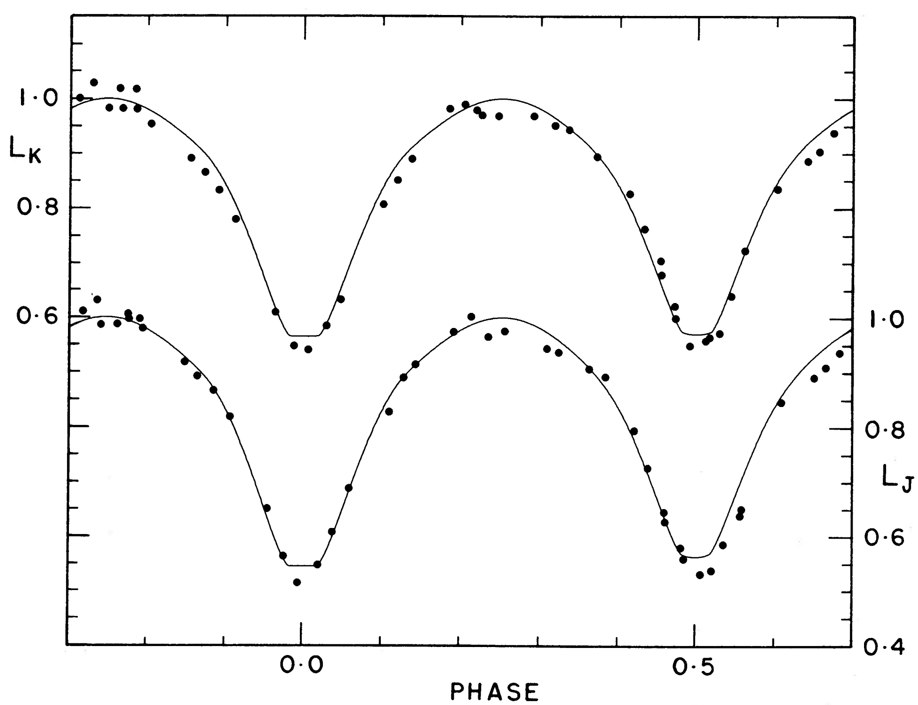

FIG. 7.—Near-infrared light curves of AB And. The solid dots represent the observed data by Jameson and Akinci (1979), and the solid lines the results of the light curve fit using the solution obtained for the visible light and velocity curves.

TABLE 5

ABSOLUTE PARAMETERS OF AB ANDROMEDAE

Parameter	Primary	Secondary
Mass ($M_{\odot}$)	1.01 (2)	0.49 (1)
Radius ($R_{\odot}$)	1.05 (1)	0.76 (1)
log g (cgs)	4.36 (1)	4.34 (1)
log T (K)	3.74 (2)	3.77 (2)
log $L/L_{\odot}$	−0.06 (10)	−0.23 (9)
$M_{\rm bol}$ (mag)	4.85 (24)	5.26 (22)
M_V (mag)	5.04 (24)	5.39 (22)
Distance[a] (pc)	98 (15)	

NOTE.—Standard errors in the last digits are given in parentheses.

[a] Assumed reddening of 0.03 mag.

TABLE 6

GALACTIC MOTION OF AB ANDROMEDAE

Element	Value
u_{α} cos δ (arcsec yr^{-1})	+0.110 ± 0.012
u_{δ} (arcsec yr^{-1})	−0.059 ± 0.010
U (km s^{-1})	27 ± 8
V (km s^{-1})	−46 ± 4
W (km s^{-1})	−33 ± 8
S (km s^{-1})	63 ± 6
U' (km s^{-1})[a]	18 ± 8
V' (km s^{-1})[a]	−34 ± 4
W' (km s^{-1})[a]	−26 ± 8
S' (km s^{-1})[a]	46 ± 6

NOTE.—Errors listed are standard errors.

[a] Primed quantities are refered to the LSR.

luminous. We find a similar situation for AB And. The secondary is overluminous by 2.5–3.0 mag. As with XY Leo, we can test to see if the total luminosity of the system is consistent with that of two main-sequence components with energy transfer from the primary to the secondary causing the overluminosity of the secondary. Following Mochnacki (1981) and assuming an average mass-luminosity relationship of the form L(internal) $\propto M^{4.4}$, we find corrected bolometric magnitudes for the primary and secondary of 4.3 and 7.7, respectively. This places both components among the locus of detached, lower main-sequence stars on the mass-luminosity plane. Thus the total luminosity of the system, when compared with the mass of the two stars, is consistent with both components still in the main-sequence phase of their evolution.

We compared the absolute parameters of the primary component with the results of stellar evolution models by VandenBerg (1985). The bolometric magnitude and the temperature were corrected for energy transfer following the procedure of Mochnacki (1981). This resulted in a primary component with mass and bolometric magnitude which agreed very well with evolutionary models for ($Z = 0.0169$ [solar], age = 10.0 Gyr), (0.0100, 6.0 Gyr), and (0.0060, 4.0 Gyr). The $Z = 0.0100$, age = 6.0 Gyr model had exactly the same temperature as the primary of AB And, when the primary's temperature had been corrected for energy transfer from the primary to the secondary. The other two models had temperatures which were within one standard error of this value. These corrected magnitude and temperature values are only approximate, and depend upon the assumptions of the energy transfer corrections in addition to the uncertainties in the absolute parameters. They do, however, indicate an age for the primary on the order of or slightly greater than that of the Sun, with perhaps a slight underabundance in metals when compared with the Sun.

The space motion can be calculated for AB And, and also indicates an old age. Proper motion components are listed in the *SAO Catalogue*. These were combined with the systemic velocity, calculated distance, and positional coordinates to yield the kinematics of the system. These are listed in Table 6. The quantities U, V, W, and S refer to the motion relative to the Sun, U outward in the Galaxy, V in the direction of Galactic rotation, and W perpendicular to the plane. S is the space motion relative to the Sun. The primed quantities refer to motion relative to the local standard of rest (LSR). The space motion S' of 46 km s^{-1} is relatively large and is consistent with the system belonging to the intermediate to old disk population. This suggests an age on the order of that of the Sun or older, which appears to be a general result found for W UMa binaries (Guinan, Bradstreet, and Robinson 1987). In fact, Eggen and Iben (1988) have recently claimed that AB And is a member of the old disk group HR 1614.

Thus for AB And, we found a good fit to the visible and infrared light curves using a synthetic light curve model with a hot secondary. Good agreement (within 10%) is found between the spectroscopic and photometric mass ratios. This is a general result which is emerging from the comparison of photometric mass ratios determined from synthetic light curve solutions with the new cross-correlation spectroscopic mass ratios (see Hrivnak and Milone 1986). With the spectroscopic elements, the absolute parameters can now be determined accurately. The binary system appears to consist of main-sequence components and has an age comparable to that of the Sun or older.

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