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VLBI OBSERVATIONS OF A PULSAR'S SCATTERING DISK

C. R. GWINN,[1] J. M. CORDES,[2] N. BARTEL,[1] A. WOLSZCZAN,[3] AND R. L. MUTEL[4]
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ABSTRACT

VLBI observations of the scattering disk of pulsar 1933+16 agree with predictions for a Kolmogorov spectrum of density fluctuations in the interstellar plasma. A novel least-squares fit technique was used to self-calibrate the data and fit models to the visibilities. Solving for the spectral index of density fluctuations, α, from the visibility as a function of baseline length, we obtain $\alpha_{\rm vis} = 3.52 \pm 0.13$, consistent with $\alpha = 11/3$ for a Kolmogorov spectrum. The scaling of size with frequency agrees with that predicted on the basis of such a spectrum. We place an upper limit of 1:1.7 on the elongation of the scattered image.

Subject headings: interferometry — interstellar: matter — stars: neutron

I. INTRODUCTION

The form of the spectrum of density fluctuations in the interstellar plasma has recently attracted much interest. A power law over some range of length scales, as predicted by the Kolmogorov theory of turbulence and observed in the interplanetary medium (Coles and Harmon 1988), is in agreement with most existing interstellar scattering data (Lee and Jokipii 1975). It has been argued that a power law with an exponent somewhat greater than the Kolmogorov value, or with a small length-scale cutoff at about 10^{11} cm, could explain intensity variations seen in some radio sources by the focusing and defocusing of radiation by large-scale density fluctuations (Rickett, Coles, and Bourgois 1984; Goodman and Narayan 1985; Coles *et al.* 1987).

To study the form of the spectrum of density irregularities in the interstellar plasma, we have observed PSR 1933+16 with intercontinental VLBI networks at 326 and 608 MHz. This pulsar is strong and north of the equator, and so convenient for VLBI observations. It lies at a Galactic latitude of $-2^\circ\!.1$ and at a distance of 6 kpc, as estimated from its dispersion measure (Manchester and Taylor 1981). Its diffractive scattering as measured by single-dish techniques is in the lower range for pulsars with similar dispersion measures (Cordes, Weisberg, and Boriakoff 1985). It was almost completely resolved on our longest baselines at a frequency of 326 MHz; its size therefore matched our (u, v) coverage well. Here we discuss observations of that pulsar's scattering disk, and their interpretation.

Pulsars are excellent probes of interstellar scattering. Alone among known varieties of radio sources, they provide information about the interstellar plasma through pulse dispersion, pulse broadening, and interstellar scintillation. They are pointlike at angular scales probed by VLBI so that any observed structure must be due to interstellar scattering.

II. OBSERVATIONS

We used the Mark II VLBI system with a 2 MHz bandwidth to observe PSR 1933+16. The 326 MHz observations were made on 1986 March 23 with telescopes at Westerbork (phased array), Jodrell Bank (76 m), Arecibo (305 m), Haystack (46 m), Maryland Point (26 m), Green Bank (43 m),[5] Fort Davis (26 m), and Owens Valley (40 m). The 608 MHz observations were made on 1986 October 8 with telescopes at Bonn (100 m), Westerbork (phased array), Jodrell Bank (76 m), Arecibo (305 m), Green Bank (43 m), Iowa (18 m), and Owens Valley (40 m). We recorded left-circular polarization (IEEE convention) at all stations with the exception of Arecibo, which can observe only linear polarization at these frequencies due to feed limitations. The Sun was 69° from the pulsar during the 326 MHz observations, and 101° away during the 608 MHz observations; at these elongations, we expect the solar wind to have negligible effects. During the observations, pulsar pulse phase and period information was recorded onto the cassette at the Green Bank telescope. The NRAO Mark II correlator in Charlottesville, Virginia, used this information to inhibit correlation off-pulse, yielding a factor of 2–5 increase in the signal-to-noise ratio (Gwinn *et al.* 1986). The coherence time is of order 100 s. To eliminate losses we coherently integrated the data for 32 s. The signal-to-noise ratio was less than 5 for all but three scans, with a median of 2.2, and a maximum of 15.

For observations at both 326 and 608 MHz, the (u, v) coverage for PSR 1933+16 was elongated, and would have been almost linear, with a position angle of 80° east of north, except for baselines to Arecibo. Information on the two-dimensional structure of the source, including its elongation, therefore depends on accurate calibration of the Arecibo gain.

The linear polarization recorded at Arecibo presented several problems. Since many pulsars, including PSR 1933+16, produce linearly polarized radiation, the effective gain there could vary with the parallactic angle of the feed and ionospheric Faraday rotation. Moreover, polarization impurities of antenna feeds can lead to nonzero closure phases, even for pointlike sources (Bartel *et al.* 1985). Arecibo presents the worst case, since there the unwanted right-circular polarization is of about the same strength as the desired left-circular polarization, and has the maximum effect on any, presumably small, polarization defects at other stations. Indeed, at 608 MHz, closure phases for some triplets of stations involving Arecibo showed large deviations from 0, with time variations which closely tracked those of the parallactic angle of the feed at Arecibo. We therefore deleted all 608 MHz data from baselines that involved Arecibo. Closure phases for other triplets of

[1] Harvard-Smithsonian Center for Astrophysics.
[2] Department of Astronomy, Cornell University.
[3] Arecibo Observatory, National Astronomy and Ionosphere Center.
[4] Department of Physics and Astronomy, University of Iowa.

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stations at 608 MHz and for all triplets at 326 MHz were consistent with 0.

Several time and frequency scales characterize interstellar scattering of pulsar radiation. A signal emitted from a pulsar at one instant arrives over some time interval $\Delta\tau$ and over a range of angles $\theta \approx (c\,\Delta\tau/D)^{1/2}$, after passage through the scattering medium. Here, D is a typical distance to the scattering material, usually taken as half the pulsar's distance. Interference among the different ray paths produces scintillations with a characteristic bandwidth $\Delta\nu_{\rm ISS} = (2\pi\,\Delta\tau)^{-1}$. Due to motion of the line of sight to the source through the scattering medium at some velocity v, the scintillations appear and disappear with a characteristic time scale $\tau_{\rm ISS} \approx \lambda/(v\theta)$, where λ is the observing wavelength. The time scale for the focusing and defocusing by large-scale plasma fluctuations, known as the refractive time scale, is of order $\tau_{\rm REF} \approx \theta D/v$.

We estimated time and frequency scales for scattering of PSR 1933+16 for our observations by interpolating between decorrelation bandwidth measurements above 1 GHz and pulse smearing measurements below 200 MHz (Cordes, Weisberg, and Boriakoff 1985 and references therein). The decorrelation bandwidth of PSR 1933+16 $\Delta\nu_{\rm ISS}$ is about 100 Hz at 326 MHz and 1.8 kHz at 608 MHz. Its scintillation time scale $\tau_{\rm ISS}$ is 8 s at 326 MHz and 18 s at 608 MHz. The refractive time scale, $\tau_{\rm REF}$, is of order 1.6 yr at 326 MHz and 0.4 yr at 608 MHz. Since our observing bandwidth of 2 MHz was much greater than $\Delta\nu_{\rm ISS}$ and our integration time of 32 s was greater than $\tau_{\rm ISS}$, our observations were averaged over many diffractive scintillation maxima. This averaging smears individual speckles, producing a smooth scattering disk.

III. DATA CALIBRATION AND REDUCTION

To treat telescope gains and source parameters simultaneously and on an equal footing, and to allow for pulsars' large intrinsic flux density variations from pulse to pulse, we wrote a novel least-squares–fit self-calibration program, which fits a model to the correlation coefficients. The model can include station gains and the size and index of the scattering disk as global parameters, and the flux density of the pulsar, the gain at Arecibo, and the station phases, as local parameters, which vary independently for each integration time.

The model correlation coefficient, $C_{AB}(t)$, of signals from stations A and B at time t is given by

$$C_{AB}(t) = \Gamma_A \Gamma_B S(t) \exp\left\{-\frac{1}{2}\left[\frac{\pi}{(2\ln 2)^{1/2}}\,\theta_H B_{AB}\right]^{\alpha_{\rm vis}-2}\right\} \times \exp\left[\imath\phi_A(t) - \imath\phi_B(t)\right]$$

Here, Γ_A and Γ_B are the gains of the stations. The flux density of the pulsar is $S(t)$. The parameter θ_H is a measure of the scattering disk size and is the FWHM of the Gaussian image for $\alpha_{\rm vis} = 4$. Departure of the scattered image from a Gaussian, as predicted by power-law spectra of density irregularities at length scales around the baseline length (Tatarski 1961), is parameterized by $\alpha_{\rm vis}$. The parameters $\phi_A(t)$ and $\phi_B(t)$ model time-dependent phases at each station, as introduced by astronometric errors and changes in ionospheric path length. The length of the projected baseline is B_{AB}, scaled to account for possible elongation of the image by the parameters ϵ and ρ. These parameters scale u and v, the traditional components of the projected interferometer baseline in wavelengths:

$$B_{AB} = \left|\begin{pmatrix}\epsilon\cos\rho & -\epsilon\sin\rho\\ \sin\rho & \cos\rho\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}\right| .$$

The best-fit parameters are determined by minimizing the sum of the squared magnitudes of the differences between the observed and model visibilities.

The technique of sequential least squares (e.g., Kaula 1966; Herring 1984) reduces the order of the matrices of partial derivatives from the total number of parameters, over 2000 for some of our fits, to the number of global and local parameters for one time interval, no more than 21 for our fits. It thus increases the speed and accuracy of the fitting. By this technique, the global parameters are sequentially updated using data from each time interval; their values and variance-covariance matrix are then used to find the local parameters. Since our model is nonlinear, the fit must be iterated. We augment the matrices of partials at each iteration to limit the changes in the parameters, until the fit has converged.

IV. RESULTS AND DISCUSSION

The model we fit to the amplitudes and phases of the correlated flux density of PSR 1933+16 at 326 MHz included parameters θ_H, $\alpha_{\rm vis}$, ϵ, and ρ, as well as station gains and phases. Figure 1 shows the visibility as a function of baseline length after correction for flux density, station gains, and the elongation of the image, as determined by the model fit. Table 1 summarizes the best-fit parameters and the reduced chi-squared. The stated errors are determined from the residuals of the fit. If the Arecibo data are deleted, parameters θ_H and $\alpha_{\rm vis}$ are consistent with those in Table 1, with about the same uncertainties, so their fitted values are not strongly influenced by the Arecibo gain.

Our measured size of 16.4 ± 0.6 mas is significantly less than that of 29 ± 3 mas estimated from previous measurements of decorrelation bandwidth and pulse broadening at other frequencies (Cordes, Weisberg, and Boriakoff 1985 and references therein). The apparent inconsistency could be due to the different weighting given by the two measurement techniques to scattering material along the line of sight. Our observations of PSR 2020+28 and PSR 1919+21 at 326 MHz, when calibrated with the gains determined for PSR 1933+16, yield sizes consistent with zero (Gwinn *et al.* 1988), as expected from their decorrelation bandwidths, increasing our confidence in the fitted gain values.

Our model for the decline in the visibility with baseline length is valid for $\alpha < 4$ and a short length-scale cutoff, a_1, much smaller than 10^{11} cm, in which case refractive effects are small. For $\alpha > 4$ or $a_1 \gtrsim 10^{11}$ cm, theoretical work is not complete, but refractive effects are expected to distort the scattering disk heavily and cause it to wander by of order the scattering disk size over the refractive time scale $\tau_{\rm REF} \approx$ 0.4–1.6 yr. Self-calibration removes wander over time scales longer than the

TABLE 1

FITTED PARAMETERS FOR PULSAR 1933+16[a]

Parameter	Symbol	326 MHz	608 MHz
Size (mas)	θ_H	16.4 ± 0.6	3.99 ± 0.14[b]
Index	$\alpha_{\rm vis}$	3.52 ± 0.13	3.52[c]
Elongation	ϵ	1.55 ± 0.15[d]	...
Position angle (degrees)	ρ	−17 ± 10	...
Reduced chi-squared	χ_ν^2	1.21	1.50

[a] Uncertainties derived from statistical standard errors.
[b] Uncertainty including uncertainty of $\alpha_{\rm vis}$ is 0.31.
[c] Held constant for fit.
[d] Regarded as an upper limit of $\epsilon < 1.70$ (see text).

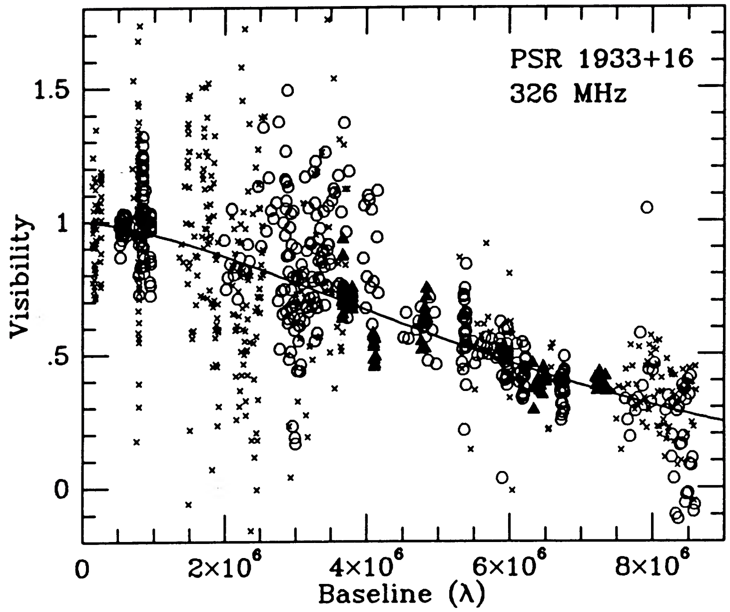

FIG. 1.—Visibility as a function of baseline length, after self-calibration, for PSR 1933+16 at 326 MHz. The data have been integrated for 160 s after fitting, to reduce scatter due to the low signal-to-noise ratio, and projected onto the time-dependent axis defined by the station phases ϕ_A and ϕ_B. Solid triangles indicate baselines involving Arecibo. Crosses indicate baselines involving Maryland Point or Fort Davis, the smallest antennas. Circles indicate other baselines. Baseline lengths have been adjusted to remove effects of the elongation of the source, a small correction except for baselines involving Arecibo. The solid line shows the best-fitting model, summarized in Table 1.

integration time and so underestimates the true angular broadening. It is possible that such a spectrum could, by chance combination of refractive and diffractive effects, produce the observed decline of visibility with baseline length. However, the power-law theory, with $\alpha = 3.52 \pm 0.13$, explains our observations well. Cordes, Weisberg, and Boriakoff (1985) found that the decorrelation bandwidth $\Delta\nu_{\rm ISS}$ scales with frequency ν as $\Delta\nu_{\rm ISS} \propto \nu^{4.22 \pm 0.16}$ for PSR 1933+16. For $\alpha < 4$, one expects $\Delta\nu_{\rm ISS} \propto \nu^{2\alpha/(\alpha-2)}$, yielding $\alpha_{\rm ISS} = 3.80 \pm 0.13$, consistent with our results at the 1.5 σ level.

The apparent elongation of the scattering disk could be caused by either refraction by large-scale density irregularities with sizes of order of the refractive scale, θD, or by elongated small-scale irregularities, as might be produced by large-scale magnetic or velocity fields. If refraction is the cause, subsequent observations, at a time at least $\tau_{\rm REF}$ after those discussed here, should show a different size and elongation. If large-scale magnetic or velocity fields are responsible, one would expect little change in the amount and position angle of elongation. Additional observations, now being processed, may help resolve this issue, and that of wander over times of tens of seconds.

However, the estimated elongation ϵ and position angle ρ are very sensitive to the gain at Arecibo. The time-varying gain at Arecibo as found from the fit tracks the parallactic angle closely, confirming that PSR 1933+16 is linearly polarized and that our model for the correlation coefficient is appropriate. This linearly polarized radiation recorded at Arecibo could corrupt the fitted values of ϵ and ρ, by spurious correlation with signals from feeds with polarization impurities at other antennas, as discussed above, although the closure phases show no evidence for such impurities. We therefore regard the observed elongation as an upper limit of $\epsilon < 1.7$ at present.

At 608 MHz, the data were sufficient to determine only the scattering disk size. We discarded Arecibo data, as noted above, and were unable to fit for source elongation ϵ and position angle ρ. The baselines are too short, given the source size, to find both size θ_H and index $\alpha_{\rm vis}$. The fitted value for θ_H depends on the value assumed for $\alpha_{\rm vis}$. We took $\alpha_{\rm vis} = 3.52$, as given by the 326 MHz data, and found $\theta_H = 3.99 \pm 0.14$ mas. Figure 2 shows the self-calibrated visibilities. Including the uncertainty of $\alpha_{\rm vis}$ determined from our 326 MHz data as another possible source of error in θ_H, we find $\theta_H = 3.99 \pm 0.31$ mas. Assuming that the size scales with frequency ν as $\theta_H \propto \nu^{-\gamma}$, we find $\gamma = 2.28 \pm 0.19$ from θ_H and $\alpha_{\rm vis}$ at 326 MHz and 608 MHz and their standard errors. This value is consistent with the Kolmogorov value of $\gamma = 11/5$ and with our fitted value for $\alpha_{\rm vis}$ at 92 cm, where $\gamma = \alpha/(\alpha - 2)$ for $\alpha < 4$, but is consistent with $\gamma = 2.0$ at the 1.5 σ level as well.

Several groups have observed the scattering disk of the pulsar in the Crab nebula with VLBI, as summarized by Mutel *et al.* (1974). This pulsar is somewhat less scattered than PSR 1933+16 but is much stronger. They conclude that the observed scaling of size, decorrelation bandwidth, and pulse broadening with wavelength is consistent with $3.5 \leq \alpha \leq 4.3$.

Values for $\alpha_{\rm vis}$ from the decline of visibility with baseline length have also been determined for the radio source 2013+370 (Spangler and Cordes 1988), for Cyg X-3 (Wilkinson, Spencer, and Nelson 1988), and for H_2O masers in W49(N) (Gwinn, Moran, and Reid 1988). Spangler and Cordes found $\alpha_{\rm vis} = 3.79 \pm 0.05$ and observed an elongation of 0.70, while Wilkinson *et al.* found $\alpha_{\rm vis} = 3.85 \pm 0.05$ and observed an elongation of 0.95 ± 0.05. Gwinn *et al.* found $\alpha_{\rm vis} = 3.74 \pm 0.05$. Their (u, v) coverage was too nearly linear to determine the elongation. These sources are much more heavily scattered than PSR 1933+16, with values for the strength of scattering C_n^2 of between $10^{0.5}$ and 10^{-1}, as compared with that of $10^{-2.73}$ for the pulsar (Cordes, Weisberg, and Boriakoff 1985). These determinations of $\alpha_{\rm vis}$, and ours, probe the logarithmic

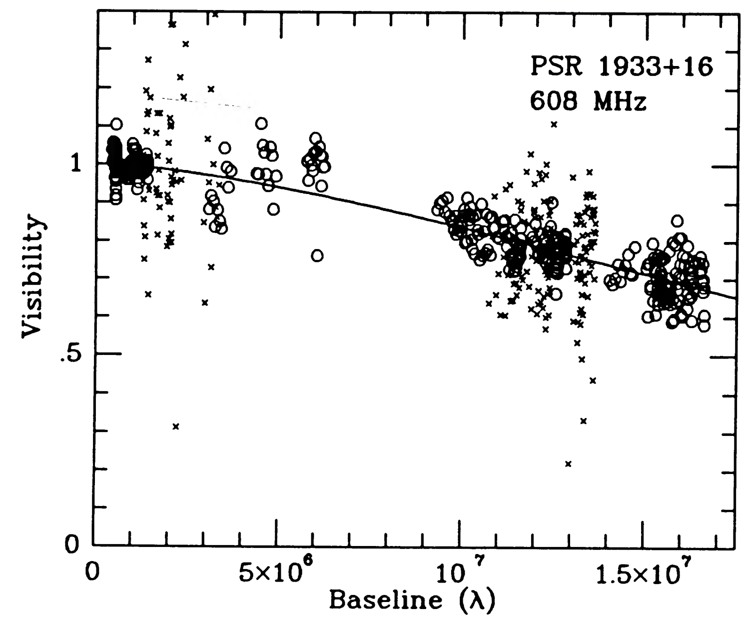

FIG. 2.—Visibility as a function of baseline length at 608 MHz, integrated and projected as for Fig. 1. Crosses indicate baselines involving Iowa, the smallest antenna. Circles indicate other baselines. The solid line shows the best-fitting model, summarized in Table 1.

slope of the spectrum of density variations on scales near the baseline length, or about 10^8–10^9 cm for the observations described here.

V. CONCLUSIONS

For PSR 1933+16, visibility as a function of baseline length and the scaling of size with observing frequency agree with predictions for a spectrum of density fluctuations with an index of about the Kolmogorov value, $\alpha = 11/3$, near our baseline lengths of 10^8–10^9 cm. From the scaling of visibility with baseline length at 326 MHz, we find $\alpha_{\rm vis} = 3.52 \pm 0.13$. This departure of the image from a Gaussian distribution as a function of wavelength indicates that the density irregularity spectrum in the interstellar plasma extends to less than about 10^8 cm. We place an upper limit of 1:1.7 on the elongation of the scattering disk.

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N. BARTEL and C. R. GWINN: Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

J. M. CORDES: Department of Astronomy, Cornell University, Space Sciences Building, Ithaca, NY 14853-6801

R. MUTEL: Department of Physics and Astronomy, University of Iowa, Van Allen Hall, Iowa City, IA 52242

A. WOLSZCZAN: Arecibo Observatory, P.O. Box 995, Arecibo, PR 00613