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THE BAADE-WESSELINK METHOD AND THE DISTANCES TO RR LYRAE STARS. VI. THE FIELD STARS RS BOOTIS, TW HERCULIS, VY SERPENTIS, AND UU VIRGINIS, AND THE ABSOLUTE MAGNITUDES OF RR LYRAE STARS

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ABSTRACT

We utilize simultaneous *BVK* photometry and radial velocities from high-resolution spectroscopy to derive distances and absolute magnitudes for three metal-rich ([Fe/H] ~ -0.5) RR Lyrae stars: RS Boo, TW Her, and UU Vir, using the surface brightness version of the Baade-Wesselink method. We also employ new *K* photometry along with previously published optical photometry and radial velocities to perform the same analysis upon the extreme velocity metal-poor ([Fe/H] ~ -1.75) RR Lyrae variable VY Ser. We find from our analysis of seven stars, including the three we analyzed previously, that there is little if any dependence of $\langle M_V \rangle$ upon metallicity, but that such a trend may exist for $\langle M_{\rm bol} \rangle$, consistent with theory. We critically reassess the determination of the mean absolute magnitude $\langle M_V \rangle_{\rm RR}$ using statistical parallaxes, the moving cluster method, and globular cluster main-sequence fitting, and find that $\langle M_V \rangle_{\rm RR}$ is closer to 0.85 mag than to 0.60 mag. Our results also support the recent proposal that there may be a simple period-$\langle M_K \rangle$ relationship that is independent of metallicity and temperature effects. Such a relationship would have important applications, such as an improved estimate for the distance to the Galactic center or for any high-reddening region. Finally, we estimate the age of an RR Lyrae–rich globular cluster, M5, for which our field star calibrations are most applicable. Using the CCD data of Richer and Fahlman, the *turnoff luminosity alone* implies an age of $(18 \pm 3) \times 10^9$ yr.

Subject headings: clusters: globular — stars: luminosities — stars: pulsation — stars: RR Lyrae

I. INTRODUCTION

In previous papers in this series (Carney and Latham 1984; Jones *et al.* 1987*a*, *b*; Jones 1988; Jones, Carney, and Latham 1988; hereafter Papers I–V, respectively), we have discussed the application of the surface brightness version of the Baade-Wesselink method to field RR Lyrae stars using simultaneous photometry and spectroscopy. In these papers, we showed that the results depend upon the color index employed in the analysis, a consequence of an apparent redistribution of flux during the expansion phase of the pulsation cycle, and that only the *VK* data consistently yielded radii in phase agreement with those derived from spectroscopy. We further determined that there is apparently little if any dependence of $\langle M_V \rangle_{\rm RR}$ upon [Fe/H], although no definite conclusions could be drawn, because of the unknown evolutionary status of our three program stars. Finally, our results, and those of the most recent statistical parallax analyses (Hawley *et al.* 1986; Barnes and Hawley 1986; Strugnell, Reid, and Murray 1986, hereafter SRM), indicated that RR Lyrae stars may be less luminous than were previously believed.

In this paper we will extend the analysis to four more field RRab–type variables: three metal-rich ([Fe/H] ~ -0.5) stars (RS Boo, TW Her, and UU Vir) and one metal-poor ([Fe/H] ~ -1.75) variable (VY Ser), in order to test further the dependence of $\langle M_V \rangle_{\rm RR}$ upon [Fe/H]. We will also critically reevaluate the results of other determinations of $\langle M_V \rangle_{\rm RR}$, including those summarized in Stothers (1983) and also more recent values, to see whether RR Lyrae stars are indeed less luminous than 0.6 mag. Finally, we will again compare our results with the predictions of Sandage period-luminosity-amplitude (*P-L-A*) relations (Sandage, Katem, and Sandage 1981; Sandage 1981, 1982), and also test the idea of Fernley, Longmore, and Jameson (1986), Longmore, Fernley, and Jameson (1986), and Fernley *et al.* (1987) that there is a simple relationship between $\langle M_K \rangle$ and the period that is independent of metallicity and temperature effects.

II. OBSERVED CHARACTERISTICS OF THE PROGRAM STARS

a) Photometry and Spectroscopy

BVK photometry of RS Boo, TW Her, and UU Vir, and *K* photometry of VY Ser were obtained by B. W. C. and R. V. J. during 1987 March 23–29 and 1987 May 29–June 3 using the SIMULPHOT system on the KPNO 1.3 m reflector, while *K* measures of VY Ser were also obtained by B. W. C. on 1986 February 28 during the SW Dra observing run (Paper III). Optical photometry and radial velocities of VY Ser were obtained earlier and were presented in Paper I. The SIMULPHOT system consists of both an optical and an infrared photometer mounted in such a way that light from an object can be alternated from one photometer to the other using a (now) motor-driven mirror. The infrared detector "Otto" was used during the March run, while "Hermann" was employed for the second. The procedure followed in the reduction of the photometry is the same as that discussed in the previous

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papers. The nights of 1987 March 23/24, 26/27, and 28/29 and the period 1987 May 31–June 3 were fully photometric, while the conditions on the nights of 1987 March 25/26 and 27/28 and 1987 May 28/29 permitted differential photometry, mainly in the infrared. Repeatability of two nearby comparison stars for each variable and the baseline on the chart recorder allowed us to monitor the observing conditions carefully. The observed magnitudes and colors of the nearby comparison stars are given in Table 1 and were used to determine the extinction coefficients during the photometric nights. Transformations to the standard systems were performed using observations of standard stars from the lists of Landolt (1983) for the BV photometry and Elias *et al.* (1982) for the K photometry, so that the latter results are on the "CIT" system. The photometric results for the four program stars are presented in Tables 2–6 and depicted in Figures 1–4.

Radial velocities of RS Boo, TW Her, and UU Vir were obtained on various nights between 1987 March and July by observers at the CfA using an echelle spectrograph and photon-counting Reticon on both the 1.55 m Wyeth reflector at the Oak Ridge Observatory near Harvard, Massachusetts, and the 1.5 m Tillinghast reflector at the Whipple Observatory on Mount Hopkins near Tucson, Arizona. Details of the observing procedure can also be found in the previous papers in this series, as well as in Latham (1985) and Wyatt (1985). These observations are listed in Tables 7–9 and plotted in Figures 5–8.

It should be noted here that RS Boo possesses the Blazhko effect, with a secondary period of 537 days (Oosterhoff 1946; Szeidl 1976) or 533 days (Kanyo 1980) and a minor variation of the secondary period with a period of between 58 and 62 days (Kanyo 1980). Spectroscopic observations of this star were obtained at different times during the observing period to monitor possible variations in the radial velocity curve due to these effects; the results are shown in Figure 6. As can be seen, there is a marked difference between the observations obtained in March and those obtained in June and July, particularly in the amplitude, the rise time, and the strength of the "dip" at about phase 0.7.

Finally, it was discovered during the photometric observations of TW Her that the nearby F2 star BD +29°3132 is in fact a variable. BVK observations of this star are presented in Table 10 and in Figure 9. It is apparent from the amplitude, the period, and the correlation between the V and the $B-V$ light curves that this star is probably a δ Scuti or a dwarf Cepheid pulsating variable, and it appears from Figure 9 that it is multiperiodic. Further observation of this star must be undertaken to determine its behavior more precisely.

b) Corrections to Previously Published Observations

As noted earlier, the spectroscopic and optical photometric observations of VY Ser were presented in Paper I. However, in that paper, the JD values of the observations were not converted into HJD. A heliocentric correction of +0.0055 days was applied to all of the JD values in Tables 1–4 of that paper to account for this, and also the phasing was redefined so that phase 0.0 occurred at maximum V light. This resulted in a shift of +0.0224 in the phases of the observations.

It was discovered during a reevaluation of the observations of X Ari (Paper II) and SW Dra (Paper III) that a sign error occurred during the application of the heliocentric correction to the JD values. This is a minor problem for SW Dra, since the heliocentric correction was essentially constant during the time of observations and the phases were computed with respect to a time of maximum V light obtained during the observing period, so that the phases were not affected by this error. The HJD of all the observations, as well as that of the zero point of the ephemeris, should be increased by 0.0048 (twice the heliocentric correction) to account for this error. This error is more serious for X Ari, since the observations were obtained over a longer period of time. The HJD of the observations obtained in 1985 September and also of the zero point of the ephemeris should be increased by 0.0068 (optical data) and 0.0078 (infrared data), while the HJD of all the other observations should be increased by 0.0112 and the associated phases by 0.0067.

It was also discovered that the JD of the radial velocity observations listed in Papers I–III were the times of the end of the exposure, not those of midexposure, as was assumed. Although variable exposure lengths were used during the acquisition of the spectra, so that each time had to be corrected individually, generally the exposure times for a given star were roughly the same. On the average, the HJD of the radial velocities of VY Ser should be shifted by +0.0022 and the phases by +0.0176, while for X Ari the corresponding shifts are +0.0052 and −0.0022. Finally, the corresponding shifts for SW Dra are +0.0010 and −0.0072. It should be noted that all of these shifts also include the effects described in the preceding paragraphs.

c) Metallicities and Reddenings

For VY Ser, the adopted metallicity, [Fe/H] = −1.77, is the value derived by Carney and Jones (1983) from an analysis of a high-dispersion spectrogram. Butler (1975) obtained values of [Fe/H] = −0.47 and −0.49 for TW Her and UU Vir, respectively. We note, however, that the spectra of the stars that he used in his analysis were obtained during the phases of

TABLE 1

COMPARISON STAR PHOTOMETRY

Variable	Comparison	V	n	$B-V$	K	n
RS Boo	BD +32°2487	10.644 ± 0.003	15	0.520 ± 0.002	...	...
	HD 127664	8.551 ± 0.03	15	1.178 ± 0.001	...	...
	HR 5447[a]	...	...	...	3.486 ± 0.001	14
TW Her	BD +30°3073	9.762 ± 0.001	14	0.389 ± 0.001	8.809 ± 0.003	15
	BD +30°3083	9.087 ± 0.001	17	1.409 ± 0.001	5.856 ± 0.001	17
VY Ser	HD 138041	...	...	...	7.753 ± 0.005	15
UU Vir	HD 105390	9.273 ± 0.005	29	0.433 ± 0.004	8.181 ± 0.006	19

[a] Infrared standard (K = 3.485 mag).

TABLE 2

SIMULPHOT PHOTOMETRY OF RS BOOTIS

HJD[a]	Phase	V	$B-V$	K	HJD[a]	Phase	V	$B-V$	K
8.6441	0.7991	...	...	9.631	8.8438	0.3284	...	...	9.397
8.6470	0.8072	...	...	9.627	8.8504	0.3460	10.474	0.375	...
8.6530	0.8228	...	...	9.643	8.8561	0.3611	10.492	0.388	...
8.6561	0.8309	...	...	9.657	8.8589	0.3686	...	...	9.394
8.6592	0.8390	...	...	9.657	8.8620	0.3768	...	...	9.396
8.6634	0.8503	10.872	0.447	...	8.8739	0.4082	10.563	0.410	...
8.6645	0.8531	10.877	0.441	...	8.8769	0.4162	...	...	9.407
8.6705	0.8692	10.842	0.423	...	8.8800	0.4244	...	...	9.411
8.6734	0.8767	...	...	9.705	8.8855	0.4389	10.600	0.429	...
8.6764	0.8847	...	...	9.679	8.8906	0.4525	10.619	0.426	...
8.6845	0.9062	10.683	0.355	...	8.8965	0.4681	...	...	9.439
8.6903	0.9216	10.518	0.295	...	8.8995	0.4760	...	...	9.434
8.6932	0.9293	...	...	9.569	8.9082	0.4992	10.660	0.438	...
8.6959	0.9365	...	...	9.537	8.9142	0.5151	10.665	0.443	...
8.7009	0.9496	10.078	0.175	...	8.9202	0.5310	...	...	9.477
8.7059	0.9628	9.873	0.131	...	8.9232	0.5389	...	...	9.479
8.7084	0.9695	...	...	9.397	9.6703	0.5188	10.678	0.432	...
8.7111	0.9767	...	...	9.369	9.6769	0.5364	10.682	0.431	...
8.7140	0.9843	9.673	0.068	...	9.6823	0.5507	...	...	9.463
8.7198	0.0000	9.625	0.051	...	9.6853	0.5587	...	...	9.464
8.7224	0.0068	...	...	9.315	9.6899	0.5709	10.693	0.430	...
8.7270	0.0190	9.645	0.061	...	9.6947	0.5835	...	...	9.481
8.7282	0.0221	9.659	0.056	...	9.6976	0.5913	...	...	9.475
8.7312	0.0300	...	...	9.332	9.7005	0.5989	10.717	0.431	...
8.7339	0.0373	...	...	9.318	9.7064	0.6144	10.730	0.427	...
8.7369	0.0452	9.700	0.113	...	9.7112	0.6271	...	...	9.508
8.7417	0.0580	...	...	9.330	9.7140	0.6347	...	...	9.523
8.7466	0.0710	9.806	0.107	...	9.7174	0.6436	10.739	0.426	...
8.7518	0.0846	9.853	0.128	...	9.7231	0.6587	10.744	0.420	...
8.7546	0.0922	...	...	9.340	9.7280	0.6719	...	...	9.531
8.7610	0.1092	9.927	0.148	...	9.7308	0.6793	...	...	9.544
8.7635	0.1157	...	...	9.333	9.7350	0.6902	10.763	0.435	...
8.7663	0.1231	9.973	0.169	...	9.7413	0.7071	10.782	0.423	...
8.7687	0.1294	...	...	9.356	9.7467	0.7217	...	...	9.562
8.7715	0.1370	10.010	0.185	...	9.7499	0.7297	...	...	9.562
8.7742	0.1439	...	...	9.344	9.7528	0.7376	10.805	0.433	...
8.7777	0.1533	10.057	0.210	...	9.7578	0.7506	10.818	0.436	...
8.7807	0.1612	...	...	9.339	9.7627	0.7638	...	...	9.591
8.7831	0.1677	10.119	0.232	...	9.7658	0.7719	...	...	9.593
8.7914	0.1897	10.161	0.250	...	9.7685	0.7791	10.836	0.438	...
8.7947	0.1984	...	...	9.367	9.7755	0.7976	10.863	0.437	...
8.8057	0.2274	10.265	0.285	...	9.7815	0.8135	...	...	9.617
8.8087	0.2356	...	...	9.366	9.7849	0.8225	...	...	9.631
8.8123	0.2450	10.299	0.311	...	9.7878	0.8302	10.873	0.445	...
8.8285	0.2879	10.381	0.335	...	9.7931	0.8444	10.875	0.441	...
8.8307	0.2938	...	...	9.395	9.7990	0.8599	...	...	9.655
8.8345	0.3039	10.410	0.351	...	9.8020	0.8678	...	...	9.660
8.8376	0.3120	...	...	9.386	9.8079	0.8834	10.802	0.409	...
8.8407	0.3202	10.429	0.368	...	9.8125	0.8957	10.754	0.382	...

[a] HJD −2446940.

maximum V light, when the stellar atmospheric parameters are poorly defined by static models, so that his values may not be very accurate. According to Preston (1959, $\Delta S = 2$ for RS Boo, yielding [Fe/H] $= -0.55$ using the calibration of Butler (1975), while McDonald (1977) derived $\Delta S = 0.9$ for this star, which corresponds to [Fe/H] $= -0.37$. Preston (1959) also obtained $\Delta S = 2$ for both TW Her and UU Vir, while Butler (1975) derived $\Delta S = 0.7$ for UU Vir, which yields [Fe/H] $= -0.34$ for that star. Finally, Lub (1979) has determined that the value of his line-blanketing estimator $\Delta[B-L]$ is 0.50 for UU Vir, which corresponds to a ΔS value ~ 3, using his conversion, and a value of [Fe/H] ~ -0.7. It will be shown in § III that this last value may be the most appropriate one for UU Vir. Therefore, we adopt metallicities of [Fe/H] $= -0.5$ for RS Boo and TW Her, -0.7 for UU Vir, and -1.77 for VY Ser.

Previously, we have adopted the reddening scale of Lub (1979, hereafter Lub) in our Baade-Wesselink analysis. This reddening scale was chosen over that of Sturch (1966) because Lub employed more accurate photometry and also used more advanced model atmospheres to account for line-blanketing effects than did Sturch (1966). SRM have shown that Lub's

TABLE 3

SIMULPHOT PHOTOMETRY OF TW HERCULIS

HJD[a]	Phase	V	$B-V$	K	HJD[a]	Phase	V	$B-V$	K
4.7102	0.2464	...	...	10.155	6.9101	0.7516	...	...	10.346
4.7182	0.2666	...	...	10.145	6.9138	0.7609	11.649	0.459	...
4.7227	0.2777	11.201	0.339	...	6.9284	9,7974	...	...	10.365
4.7400	0.3211	11.263	0.379	...	6.9318	0.8060	...	...	10.408
4.7469	0.3382	11.300	0.388	...	6.9353	0.8148	...	...	10.390
4.7607	0.3729	11.381	0.421	...	6.9306	0.8232	11.742	0.477	...
4.7701	0.3963	...	...	10.197	6.9478	0.8460	11.770	0.468	...
4.7731	0.4040	...	...	10.186	7.7503	0.8544	11.771	0.480	...
4.7763	0.4118	...	...	10.190	7.7572	0.8717	11.774	0.488	...
4.7795	0.4200	11.445	0.444	...	7.7597	0.8780	...	...	10.458
4.7855	0.4350	11.472	0.445	...	7.7629	0.8858	...	...	10.481
6.7055	0.2398	11.131	0.328	...	7.7659	0.8934	...	...	10.476
6.7128	0.2580	...	...	10.173	7.7801	0.9290	11.575	0.398	...
6.7159	0.2657	...	...	10.155	7.7871	0.9463	11.306	0.313	...
6.7190	0.2735	...	...	10.163	7.7897	0.9530	...	...	10.335
6.7228	0.2831	11.230	0.346	...	7.7927	0.9603	...	...	10.291
6.7342	0.3115	...	...	10.145	7.8049	0.9909	10.499	0.094	...
6.7373	0.3193	...	...	10.184	7.8122	0.0093	10.481	0.082	...
6.7404	0.3271	...	...	10.156	7.8177	0.0229	...	...	10.138
6.7544	0.3620	...	...	10.182	7.8206	0.0303	...	...	10.136
6.7576	0.3700	...	...	10.182	7.8285	0.0499	10.594	0.101	...
6.7606	0.3777	...	...	10.171	7.8363	0.0694	10.656	0.122	...
6.7641	0.3864	11.409	0.426	...	7.8396	0.0777	...	...	10.138
6.7786	0.4226	...	...	10.200	7.8427	0.0856	...	...	10.156
6.7816	0.4392	...	...	10.204	7.8458	0.0933	...	...	10.169
6.7847	0.4378	...	...	10.185	7.8590	0.1263	10.833	0.194	...
6.7882	0.4467	11.503	0.443	...	7.8660	0.1439	10.889	0.212	...
6.7979	0.4710	...	...	10.169	7.8685	0.1501	...	...	10.158
6.8010	0.4787	...	...	10.206	7.8715	0.1577	...	...	10.138
6.8041	0.4864	...	...	10.208	7.8745	0.1652	...	...	10.162
6.8085	0.4974	...	...	10.208	7.8830	0.1865	11.014	0.258	...
6.8122	0.5066	11.552	0.455	...	7.8901	0.2042	11.057	0.275	...
6.8277	0.5455	...	...	10.218	7.8930	0.2143	...	...	10.148
6.8309	0.5534	...	...	10.243	7.8960	0.2190	...	...	10.130
6.8340	0.5612	...	...	10.212	7.8996	0.2278	...	...	10.151
6.8371	0.5691	11.552	0.458	...	7.9068	0.2460	11.147	0.322	...
6.8499	0.6012	...	...	10.240	7.9142	0.2645	11.178	0.339	...
6.8530	0.6088	...	...	10.237	7.9183	0.2748	...	...	10.125
6.8560	0.6164	...	...	10.247	7.9226	0.2855	...	...	10.145
6.8604	0.6273	11.578	0.461	...	7.9264	0.2950	...	...	10.157
6.8736	0.6603	...	...	10.289	7.9349	0.3163	11.278	0.379	...
6.8766	0.6679	...	...	10.278	7.9426	0.3357	11.321	0.386	...
6.8855	0.6902	11.598	0.473	...	7.9553	0.3674	...	...	10.154
6.9034	0.7348	...	...	10.319	7.9585	0.3754	...	...	10.148
6.9067	0.7431	...	...	10.321	7.9615	0.3829	...	...	10.160

[a] HJD −2446940.

reddening scale is essentially the same as that of Burstein and Heiles (1982), although Lub's values are slightly larger on the average. Both scales yield smaller reddening values than does that of Sturch (1966). In order to utilize Lub's reddenings, it is necessary to transform his Walraven $E(V-B)$ values into Johnson $E(B-V)$. We accomplished this by using equations (A1b) and (A2b) in the appendix of Lub (1979) to derive $E(B-V) = 2.175E(V-B)$. For UU Vir, the value $E(B-V) = 0.028$ mag was obtained using this relationship, while a value of 0.037 mag was derived for VY Ser. We have instead chosen to adopt the value of 0.030 mag for the latter star, which is the value derived by Carney and Jones (1983) and adopted in Paper I. The resultant differences in our analyses are negligible.

A drawback to the valuable work of Lub is that he did not extend it to the northern hemisphere variables, so that RS Boo, SW Dra, and TW Her have not been analyzed. In Paper III we adopted $E(B-V) = 0.00$ mag for SW Dra for convenience; however, it is better to find a way to derive reddenings directly based on Lub's scale. Lub derived a relationship between the value of the Walraven $V-B$ index at minimum temperature ($0.5 \le \phi \le 0.8$, where ϕ is the phase), corrected for line-blanketing effects, of an RR Lyrae star and its period. It is expected, then, that RR Lyraes which possess the same metallicity and the same period should have the same color at minimum temperature. Therefore, we searched the atlas of Lub (1977) for stars with metallicities (from ΔS values) and periods similar to those values of RS Boo ($\Delta S = 0.9$ or 2, period

TABLE 4

BV PHOTOMETRY OF UU VIRGINIS

HJD[a]	Phase	V	B−V	HJD[a]	Phase	V	B−V
78.6959	0.7344	10.899	0.425	78.9069	0.1780	10.338	0.261
78.7146	0.7736	10.944	0.439	78.9398	0.2471	10.483	0.325
78.7158	0.7780	10.945	0.438	78.9412	0.2501	10.495	0.324
78.7258	0.7971	10.986	0.435	78.9424	0.2526	10.493	0.335
78.7270	0.7976	10.990	0.431	78.9570	0.2833	10.552	0.354
78.7421	0.8314	11.035	0.437	78.9582	0.2859	10.568	0.344
78.7433	0.8339	11.036	0.449	78.9594	0.2883	10.560	0.351
78.7588	0.8666	11.050	0.469	81.8022	0.2655	10.516	0.342
78.7601	0.8693	11.059	0.450	81.8027	0.2666	10.516	0.347
78.7983	0.9497	10.372	0.250	81.8580	0.3829	10.709	0.400
78.7995	0.9521	10.349	0.244	81.8590	0.3850	10.708	0.410
78.8007	0.9547	10.328	0.238	81.8765	0.4217	10.751	0.424
78.8023	0.9580	10.301	0.220	81.8770	0.4228	10.752	0.423
78.8034	0.9603	10.275	0.210	81.9036	0.4787	10.803	0.433
78.8045	0.9626	10.249	0.195	81.9046	0.4809	10.810	0.429
78.8118	0.9779	10.022	0.148	81.9383	0.5518	10.845	0.431
78.8129	0.9802	9.998	0.137	81.9393	0.5539	10.841	0.434
78.8140	0.9826	9.977	0.125	81.9614	0.6002	10.864	0.437
78.8155	0.9858	9.955	0.121	81.9624	0.6023	10.865	0.446
78.8167	0.9882	9.936	0.120	82.9032	0.5805	10.853	0.436
78.8179	0.9907	9.922	0.119	82.9043	0.5827	10.841	0.441
78.8251	0.0061	9.905	0.112	83.7111	0.2791	10.555	0.338
78.8263	0.0085	9.904	0.117	83.7121	0.2812	10.555	0.343
78.8275	0.0110	9.910	0.110	83.7362	0.3319	10.630	0.380
78.8289	0.0139	9.915	0.109	83.7372	0.3340	10.636	0.376
78.8300	0.0162	9.915	0.119	83.7536	0.3686	10.684	0.404
78.8311	0.0186	9.927	0.116	83.7546	0.3706	10.686	0.407
78.8446	0.0469	10.006	0.135	83.8535	0.5785	10.852	0.442
78.8457	0.0493	10.014	0.139	83.8545	0.5807	10.851	0.443
78.8469	0.0518	10.021	0.142	83.8745	0.6227	10.876	0.442
78.8634	0.0865	10.114	0.175	83.8754	0.6247	10.874	0.451
78.8645	0.0889	10.128	0.170	83.8996	0.6756	10.897	0.440
78.8657	0.0913	10.130	0.175	83.9007	0.6777	10.897	0.442
78.8847	0.1312	10.225	0.219	83.9232	0.7251	10.889	0.433
78.8859	0.1338	10.230	0.220	83.9242	0.7273	10.890	0.443
78.8870	0.1361	10.238	0.215	83.9459	0.7729	10.942	0.430
78.9057	0.1755	10.333	0.256	83.9469	0.7750	10.948	0.437

[a] HJD −2446800.

$P = 0.377$ days), SW Dra ($\Delta S = 3.5$, $P = 0.570$ days), and TW Her ($\Delta S = 2$, $P = 0.399$ days). This search yielded AA Aql ($\Delta S = 0$ [Preston 1959; Lub 1977] or 1.0 [Butler 1975], $P = 0.362$ days) and RW TrA ($\Delta S = 1$ [Lub 1977], $P = 0.374$ days) as stars similar to RS Boo, V445 Oph ($\Delta S = 1$ [Preston 1959; Lub 1977], $P = 0.397$ days) and HH Pup ($\Delta S = 2$ [Lub 1977]; $P = 0.391$ days) as stars comparable to TW Her, and V341 Aql ($\Delta S = 3$ [Preston 1959; Lub 1977] or 4.0 [Butler 1975], $P = 0.578$ days) as a star similar to SW Dra. Since Lub's photometry of these stars is on the Walraven system, it must first be transformed to the standard *BV* system. This was performed using equations (A1) and (A2) in the appendix of Lub (1979) to derive

$$(B-V)_0 = [(V-B) - E(V-B) + 0.003]/0.4 . \qquad (1)$$

The values of $(B-V)_0$ at minimum temperature of the five stars listed above were derived from this equation and the values published in Lub (1977, 1979), yielding values of $(B-V)_{\min,0}$ of 0.393 and 0.422 mag for AA Aql and RW TrA (average value 0.408 mag), 0.435 and 0.383 mag for V445 Oph and HH Pup (average value 0.409 mag), and 0.418 mag for V341 Aql. These values can be directly compared with the values for RS Boo (0.432 mag), TW Her (0.461 mag), and SW Dra (0.451 mag) to yield reddenings of 0.024, 0.052, and 0.033 mag, respectively. Burstein (1987) quotes values of 0.000, 0.056, and 0.014 mag, respectively, for these stars, and he also lists values of 0.004 and 0.021 mag for UU Vir and VY Ser. It can be seen that the values on Lub's scale are moderately larger, particularly for the stars at high Galactic latitude. This difference is probably due to the fact that Lub adopted a small nonzero reddening at the Galactic poles. It does appear, however, that the reddenings for stars away from the Galactic poles are in better agreement, since the values for TW Her on the two systems are virtually identical, as are the values for DH Peg (0.080 mag from Lub; 0.072 mag from Burstein 1987). We will continue to use values based on Lub's reddening scale, but note that we may be slightly overestimating the reddening values of the stars near the Galactic poles.

In Paper II we adopted the reddening of Manduca *et al.* (1981), $E(B-V) = 0.153$ mag, for X Ari. Lub derived a value of $E(V-B) = 0.074$ mag, which transforms to $E(B-V) = 0.161$ mag, by averaging the value obtained for this star from his

TABLE 5

K PHOTOMETRY OF UU VIRGINIS

HJD[a]	Phase	*K*	HJD[a]	Phase	*K*	HJD[a]	Phase	*K*
78.7006	0.7443	9.624	78.9609	0.2917	9.416	81.8744	0.4176	9.473
78.7017	0.7466	9.588	78.9620	0.2940	9.373	81.8942	0.4591	9.484
78.7029	0.7489	9.620	78.9631	0.2962	9.423	81.8952	0.4612	9.464
78.7187	0.7824	9.618	78.9641	0.2984	9.419	81.8961	0.4631	9.452
78.7200	0.7852	9.618	80.7171	0.9843	9.472	81.8970	0.4650	9.452
78.7211	0.7874	9.670	80.7181	0.9863	9.473	81.9295	0.5333	9.501
78.7223	0.7899	9.661	80.7193	0.9887	9.458	81.9303	0.5351	9.479
78.7355	0.8178	9.706	80.7203	0.9909	9.438	81.9312	0.5370	9.514
78.7366	0.8200	9.647	80.7213	0.9931	9.453	81.9321	0.5388	9.467
78.7388	0.8247	9.695	80.7223	0.9952	9.484	81.9679	0.6136	9.520
78.7455	0.8387	9.687	80.7265	0.0039	9.459	81.9687	0.6158	9.521
78.7466	0.8409	9.696	80.7275	0.0061	9.443	81.9696	0.6177	9.519
78.7481	0.8442	9.700	80.7286	0.0084	9.430	81.9705	0.6195	9.532
78.7640	0.8776	9.798	80.7297	0.0106	9.438	82.8928	0.5588	9.493
78.7652	0.8802	9.754	80.7228	0.1013	9.448	82.8937	0.5607	9.495
78.7664	0.8828	9.745	80.7738	0.1034	9.441	82.8947	0.5628	9.528
78.7678	0.8856	9.828	80.7748	0.1055	9.440	82.8957	0.5649	9.503
78.7881	0.9283	9.658	80.7759	0.1077	9.417	82.9248	0.6260	9.532
78.7892	0.9307	9.664	80.7796	0.1156	9.422	82.9257	0.6280	9.521
78.7959	0.9447	9.597	80.7808	0.1182	9.437	82.9266	0.6299	9.531
78.8063	0.9666	9.556	80.7819	0.1205	9.423	82.9276	0.6319	9.543
78.8073	0.9688	9.509	80.7829	0.1227	9.418	83.7189	0.2958	9.424
78.8084	0.9710	9.498	80.8841	0.3353	9.448	83.7204	0.2988	9.423
78.8094	0.9731	9.508	80.8851	0.3373	9.437	83.7220	0.3021	9.406
78.8195	0.9943	9.438	80.8870	0.3414	9.436	83.7263	0.3112	9.426
78.8205	0.9964	9.421	81.6341	0.9123	9.739	83.7278	0.3144	9.428
78.8215	0.9985	9.458	81.6351	0.9143	9.670	83.7294	0.3178	9.431
78.8225	0.0001	9.413	81.6360	0.9162	9.740	83.7313	0.3219	9.421
78.8382	0.0337	9.444	81.6370	0.9183	9.724	83.7433	0.3470	9.464
78.8393	0.0360	9.455	81.6402	0.9251	9.671	83.7448	0.3501	9.432
78.8407	0.0389	9.430	81.6413	0.9274	9.705	83.7462	0.3532	9.435
78.8417	0.0411	9.452	81.6423	0.9294	9.678	83.7477	0.3562	9.435
78.8581	0.0754	9.468	81.6432	0.9313	9.645	83.7587	0.3794	9.428
78.8590	0.0774	9.455	81.6508	0.9473	9.542	83.7615	0.3854	9.451
78.8600	0.0795	9.442	81.6519	0.9496	9.587	83.7634	0.3884	9.442
78.8610	0.0815	9.420	81.6529	0.9518	9.559	83.7644	0.3914	9.447
78.8674	0.0951	9.446	81.6539	0.9540	9.600	83.8600	0.5924	9.523
78.8686	0.0976	9.446	81.6588	0.9643	9.585	83.8614	0.5953	9.533
78.8698	0.1000	9.406	81.6597	0.9662	9.519	83.8628	0.5982	9.531
78.8708	0.1023	9.426	81.6607	0.9683	9.538	83.8808	0.6361	9.550
78.8886	0.1396	9.446	81.6617	0.9703	9.506	83.8822	0.6391	9.558
78.8896	0.1417	9.431	81.8102	0.2825	9.434	83.8836	0.6420	9.561
78.8906	0.1438	9.428	81.8111	0.2844	9.435	83.9057	0.6886	9.570
78.8916	0.1460	9.427	81.8121	0.2864	9.413	83.9071	0.6914	9.592
78.9084	0.1813	9.401	81.8130	0.2884	9.418	83.9084	0.6942	9.557
78.9094	0.1834	9.447	81.8495	0.3650	9.425	83.9296	0.7387	9.630
78.9105	0.1856	9.417	81.8504	0.3670	9.447	83.9310	0.7417	9.606
78.9115	0.1877	9.432	81.8513	0.3690	9.438	83.9324	0.7446	9.605
78.9447	0.2576	9.425	81.8523	0.3710	9.446	83.9338	0.7476	9.601
78.9457	0.2597	9.401	81.8717	0.4118	9.444	83.9522	0.7863	9.630
78.9467	0.2618	9.376	81.8726	0.4138	9.463	83.9536	0.7892	9.650
78.9477	0.2639	9.450	81.8736	0.4157	9.423	83.9549	0.7920	9.638

[a] HJD −2446800.

period–color(minimum temperature) relation, 0.067 mag, and that derived using its mean value from β photometry ($\langle\beta\rangle$), 0.082 mag. These reddenings do not agree that well with each other; according to Lub, if X Ari is as reddened as its $\langle\beta\rangle$ value indicates, it would be roughly 200 K hotter than similar RR Lyrae stars such as UY Boo, SS Leo, and V675 Sgr. Since we have essentially utilized the period–color(minimum temperature) relation to derive the reddenings for the other stars, we should adopt the value from this relation for X Ari, which transforms to $E(B-V) = 0.146$ mag, for consistency. It is possible, however, that for some reason X Ari is anomalously bluer than other RR Lyraes with a similar period, so that a higher reddening may be more appropriate for it. In support of this is the value from Burstein (1987), 0.169 mag. Since it is not clear which of Lub's values to adopt, 0.146 mag or 0.161 mag, we will continue to use the 0.153 mag value, which fortuitously

TABLE 6

K PHOTOMETRY OF VY SERPENTIS

HJD[a]	Phase	*K*	HJD[a]	Phase	*K*	HJD[a]	Phase	*K*
489.8748	0.0815	8.744	882.8430	0.3848	8.717	883.0096	0.6181	8.798
489.9032	0.1212	8.726	882.8439	0.3860	8.720	883.0137	0.6238	8.804
489.9255	0.1524	8.735	882.8479	0.3916	8.729	883.0152	0.6259	8.806
489.9463	0.1816	8.707	882.8489	0.3930	8.717	883.0212	0.6343	8.816
489.9717	0.2171	8.721	882.8500	0.3956	8.730	883.0227	0.6364	8.816
489.9941	0.2485	8.705	882.8639	0.4140	8.722	883.0276	0.6433	8.814
490.0184	0.2825	8.706	882.8649	0.4154	8.734	883.0290	0.6442	8.831
490.0451	0.3149	8.712	882.8658	0.4167	8.736	883.7950	0.7179	8.863
878.8914	0.8510	8.969	882.8813	0.4384	8.732	883.7964	0.7199	8.844
878.8985	0.8610	8.978	882.8823	0.4398	8.735	883.7977	0.7217	8.864
879.0106	0.0180	8.776	882.8832	0.4411	8.744	883.8113	0.7407	8.889
879.0117	0.0195	8.774	882.8843	0.4426	8.720	883.8127	0.7427	8.889
879.0181	0.0285	8.775	882.9128	0.4825	8.750	883.8140	0.7445	8.899
880.8593	0.6068	8.805	882.9139	0.4840	8.736	883.8262	0.7616	8.907
880.8603	0.6082	8.800	882.9150	0.4866	8.741	883.8276	0.7636	8.900
881.8650	0.0152	8.775	882.9160	0.4870	8.758	883.8289	0.7654	8.911
881.8658	0.0163	8.792	882.9507	0.5356	8.767	883.8438	0.7863	8.916
881.8876	0.0468	8.773	882.9518	0.5371	8.761	883.8452	0.7882	8.914
881.8886	0.0482	8.760	882.9578	0.5455	8.759	883.8465	0.7900	8.928
881.9230	0.0964	8.739	882.9606	0.5494	8.752	883.8667	0.8183	8.938
881.9239	0.0977	8.768	882.9701	0.5627	8.758	883.8681	0.8203	8.932
881.9248	0.0989	8.745	882.9711	0.5641	8.780	883.8899	0.8508	8.949
881.9517	0.1366	8.739	882.9721	0.5655	8.779	883.8915	0.8531	8.949
881.9526	0.1379	8.737	882.9731	0.5669	8.781	883.9123	0.8822	8.934
881.9739	0.1677	8.737	882.9830	0.5808	8.778	883.9137	0.8841	8.920
881.9747	0.1688	8.733	882.9841	0.5824	8.803	883.9364	0.9159	8.882
882.0108	0.2194	8.722	882.9854	0.5842	8.785	883.9377	0.9178	8.877
882.0117	0.2206	8.698	882.9947	0.5972	8.799	883.9593	0.9480	8.868
882.0126	0.2219	8.692	882.9957	0.5986	8.779	883.9607	0.9500	8.857
882.0247	0.2388	8.716	882.9966	0.5999	8.788	884.0143	0.0250	8.780
882.0257	0.2402	8.708	882.9976	0.6013	8.777	884.0157	0.0270	8.778
882.0333	0.2509	8.703	882.9986	0.6027	8.769	884.0170	0.0288	8.773
882.0341	0.2520	8.706	883.0046	0.6111	8.790	947.7745	0.3132	8.710
882.0383	0.2579	8.697	883.0055	0.6123	8.802	947.7764	0.3159	8.699
882.0392	0.2591	8.701	883.0065	0.6137	8.805	947.7989	0.3474	8.714
882.8296	0.3660	8.713	883.0085	0.6165	8.820	947.8008	0.3501	8.704
882.8328	0.3705	8.704						

[a] HJD −2446000.

falls nearly halfway in between, but note that the reddening of X Ari may be a little more uncertain than those of the others.

d) Observed Characteristics of the Program Stars

Table 11 presents the mean observed quantities, including the dereddened magnitudes and colors and also the systemic velocities, γ, of the program stars, while Table 12 lists the amplitudes and the rise times, $\Delta\phi_{\rm rise}[\equiv \phi(\text{maximum}) - \phi(\text{previous minimum})]$. The mean magnitudes were computed both by averaging the magnitudes over phase (magnitude average) and by first converting the magnitudes into intensities, averaging these values over phase, and then converting the mean intensity into a magnitude (intensity average). The quantities of stars analyzed in previous papers in this series have been included in these tables for convenience and also because some of these values, notably the adopted reddening and the observed dereddened quantities of SW Dra, are slightly different from the values adopted previously.

Several items of interest in these tables will be discussed in more detail here. First, the amplitude of variation of the radial velocity curve is obviously not the same for all RR Lyraes (error in amplitude should be $\sim$2 km s^{-1}), as assumed by Woolley and Aly (1966) and to some extent by McDonald (1977), but appears to be correlated with the amplitudes of the light curves. In particular, if a linear relationship is assumed to exist between the radial velocity amplitude, $A_{\rm vrad}$, and that of the V light curve, A_V, then an unweighted least-squares fit to the data of Table 12 (excluding the c-type variable DH Peg, which does not follow the relation) yields

$$A_{\rm vrad} = 35(\pm 2)A_V + 28(\pm 2)\ . \qquad (2)$$

A simple relation between $A_{\rm vrad}$ and the period may also exist for stars of similar metallicity, which is expected if A_B and $A_{\rm vrad}$ are correlated, as seems likely from Table 12, and if a relation exists between A_B and P, as shown by Sandage, Katem, and Sandage (1981) and Sandage (1981, 1982). It is also expected that the zero point of a period-$A_{\rm vrad}$ relation should vary with metallicity (the Sandage period shift), and it is also possible that the slope of the relation may vary as well (see § IV*b*).

Another feature of interest seen in Table 12, in Figures 1–4, and in Papers II, III, and V is the variation in the shape of the K light curve between the program stars. In particular, the

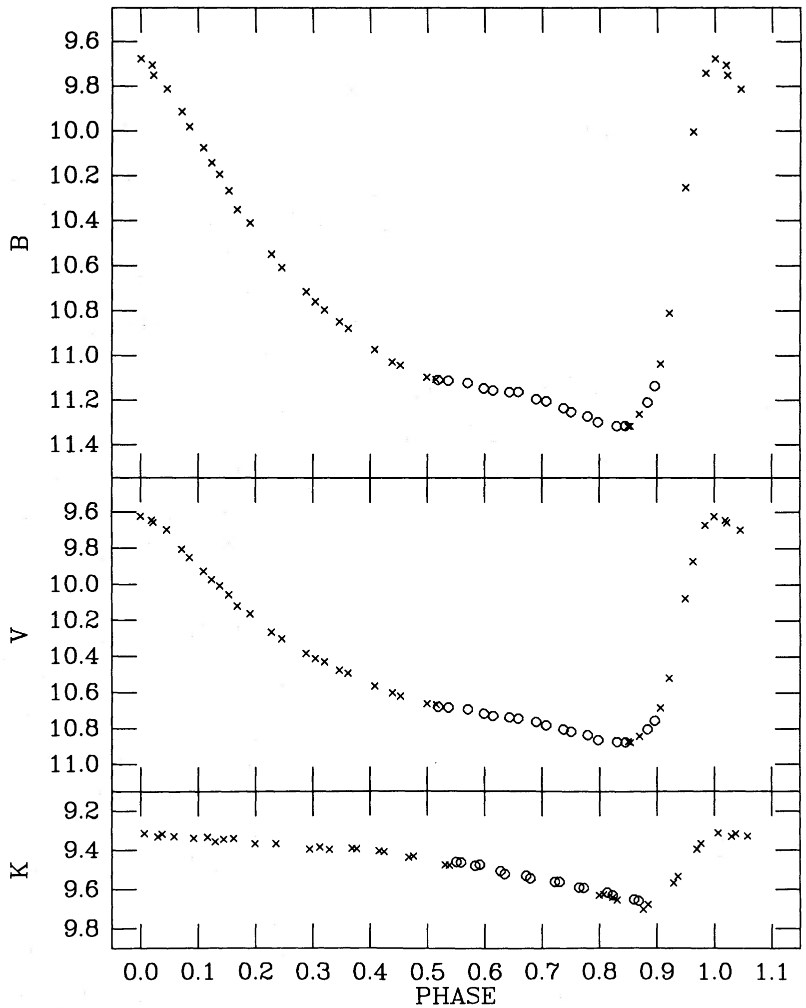

FIG. 1

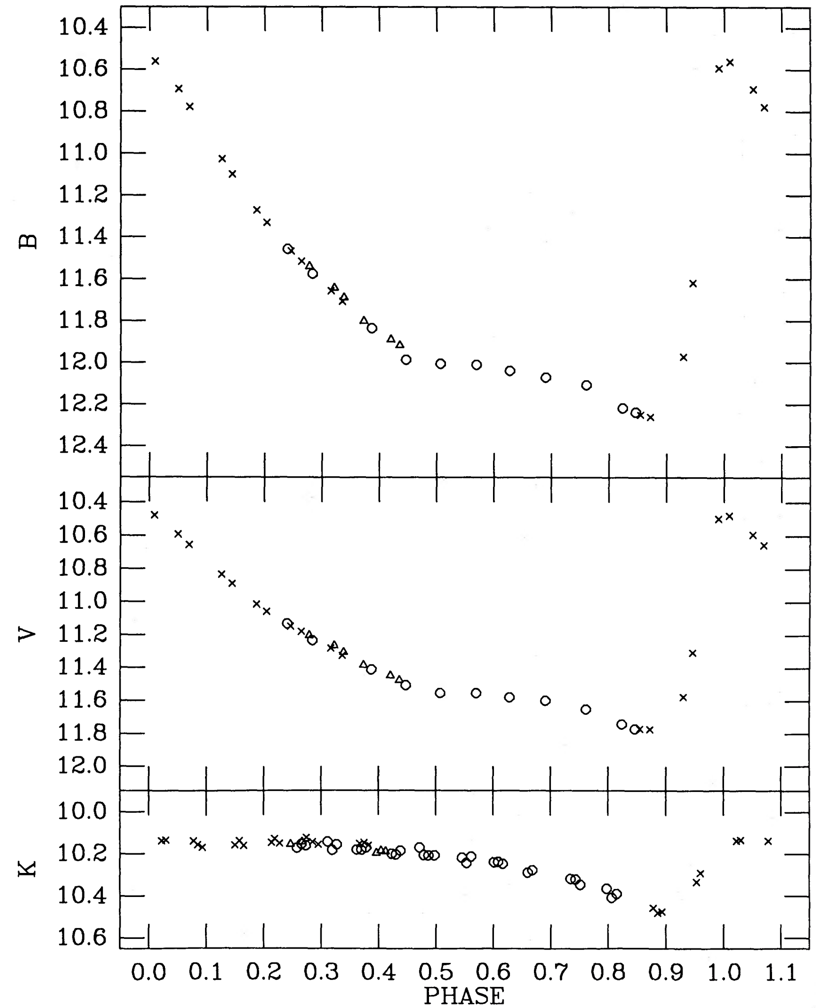

FIG. 2

FIG. 1.—*B*, *V*, and *K* magnitudes (*top to bottom*) of RS Boo plotted against phase. The symbols refer to data obtained on the following nights: 1987 June 1/2 (*crosses*) and 1987 June 2/3 (*open circles*).
FIG. 2.—Same as Fig. 1, for TW Her. Data were obtained on 1987 May 28/29 (*open triangles*), 1987 May 30/31 (*open circles*), and 1987 May 31/June 1 (*crosses*).

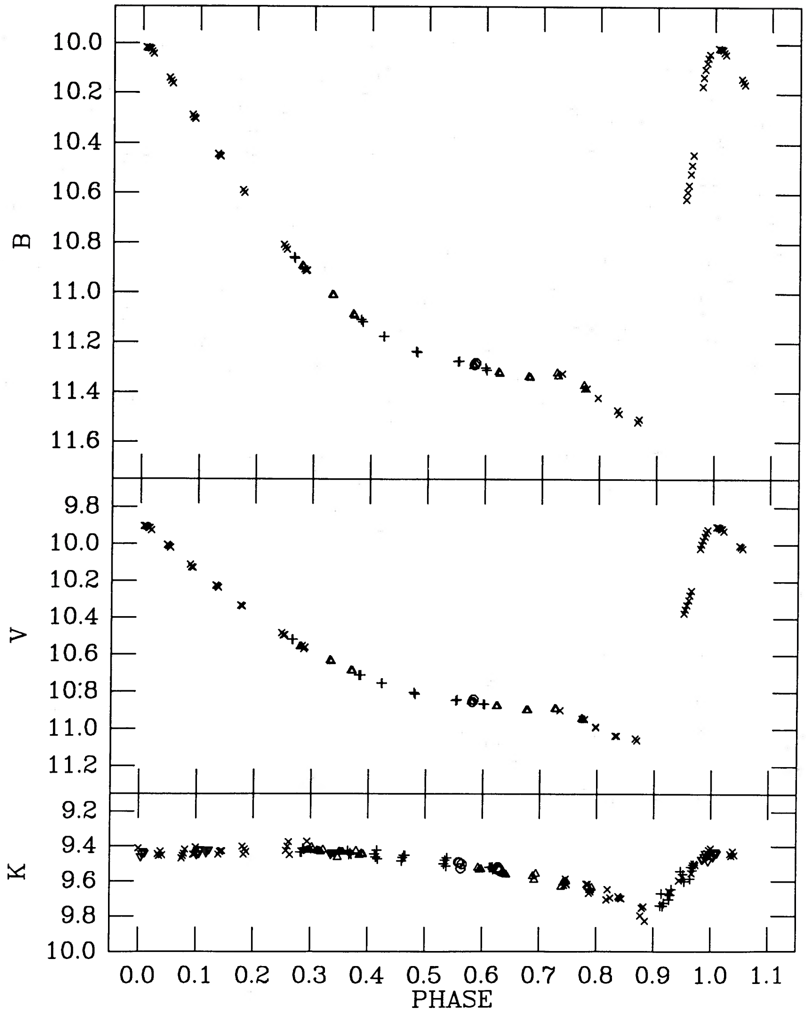

FIG. 3.—Same as Fig. 1, for UU Vir. Data were obtained on 1987 March 23/24 (*crosses*), 1987 March 25/26 (*inverted open triangles*), 1987 March 26/27 (*plus signs*), 1987 March 27/28 (*open circles*), and 1987 March 28/29 (*open triangles*).

strength of the "bump" which appears at about phase 0.0 in these stars appears to be correlated with the amplitudes of the optical light curves, indicating that this feature is a temperature effect as well. The strength of this bump can be determined from the quantity ΔK_{tr}, listed in Table 12, which is defined as

$$\Delta K_{tr} = K_{max,t} - K_{max,r} , \quad (3)$$

where the quantities $K_{max,t}$ and $K_{max,r}$ refer to the maximum values of K at about phase 0.0, which is probably due to temperature effects, and at about phase 0.3 (0.1 for DH Peg), which arises from radius effects. Two stars (DH Peg and VY Ser) do not appear to possess a bump on their K light curves, so the value of $K_{max,t}$ was taken to be the value at phase 0.0 for these stars. At the other extreme, the K light curve of RS Boo is so dominated by the bump that the maximum due to the radius appears only as a shoulder at about phase 0.35. It is not clear why the bump is so much stronger in this star than in the others, particularly TW Her, which has about the same amplitude; perhaps it is associated with the Blazhko effect. A detailed study of the behavior of the K light curve over the course of a secondary period is recommended; such a study may be most feasible with RR Lyrae itself as the target. Finally, it should be noted that for the two stars with ΔK_{tr} values less than zero, RS Boo and TW Her, the value of A_K is the difference between $K_{max,t}$ and the preceding minimum, not between $K_{max,r}$ and K_{min} as for the others.

The phases for the program stars were generated using the ephemerides listed in Table 13. For X Ari and DH Peg, the ephemerides listed are the ones that were derived in Papers II and V, respectively (the zero point for X Ari has been corrected in the manner discussed earlier). The ephemerides for the other stars were derived by using the periods from Kukarkin *et al.* (1970) and setting the zero points from the times of maximum V light that we observed. These zero points differed slightly from those of Kukarkin *et al.* (1970), but only by about 0.01 in phase. No attempt was made to derive new periods for these stars, however, since not enough maxima were observed, so that these ephemerides may not accurately predict the past or future behavior of these stars. Finally, the phases computed from the listed ephemeris of RS Boo are not strictly accurate, since the variations due to the Blazhko effect have not been included.

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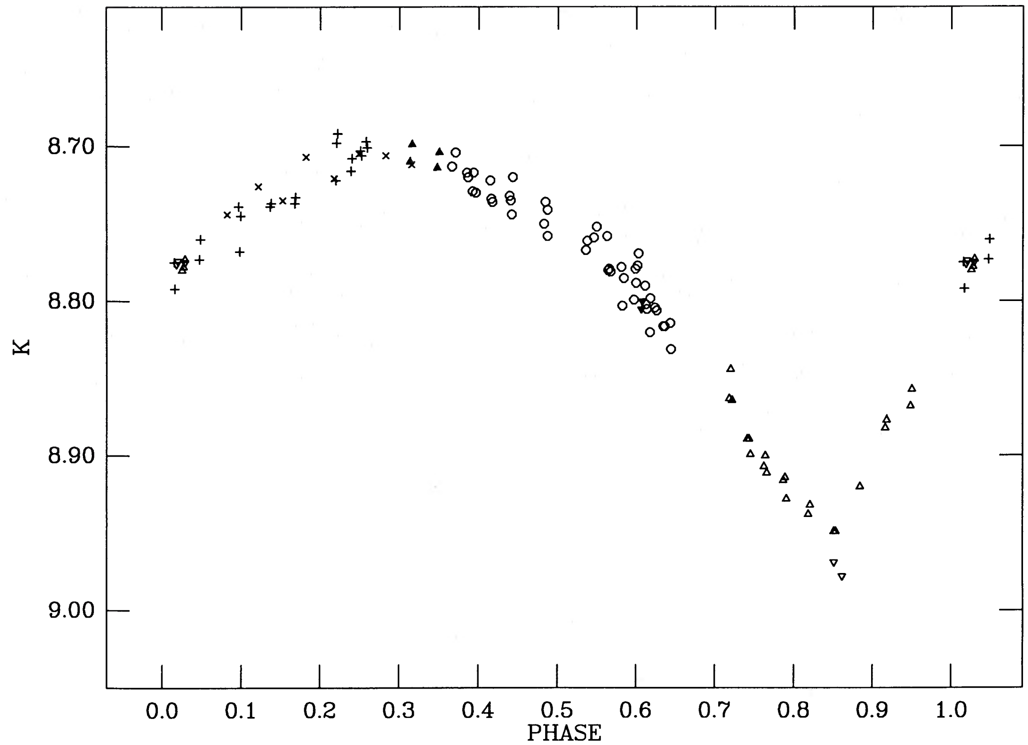

FIG. 4.—*K* magnitudes of VY Ser plotted against phase. Data were obtained on 1986 February 27/28 (*crosses*), 1987 March 23/24 (*inverted open triangles*), 1987 March 25/26 (*inverted filled triangles*), 1987 March 26/27 (*plus signs*), 1987 March 27/28 (*open circles*), 1987 March 28/29 (*open triangles*), and 1987 May 31/June 1 (*filled triangles*).

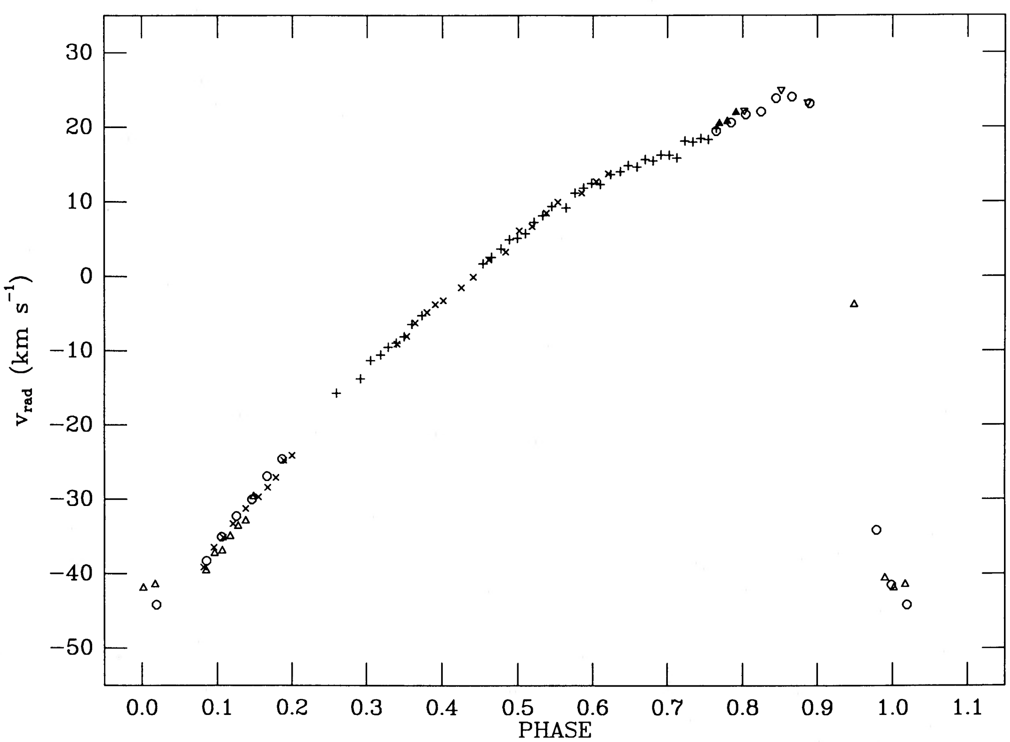

FIG. 5.—Radial velocities of RS Boo versus phase. Symbols refer to data obtained on the following nights: 1987 June 4/5 (*filled triangles*), 1987 June 6/7 (*crosses*), 1987, June 9/10 (*inverted open triangles*), 1987 June 10/11 (*open circles*), 1987 June 11/12 (*plus signs*), and 1987 June 12/13 (*open triangles*).

TABLE 7

RADIAL VELOCITIES OF RS BOOTIS

HJD[a]	Phase	v_{rad}	HJD[a]	Phase	v_{rad}	HJD[a]	Phase	v_{rad}	HJD[a]	Phase	v_{rad}
879.5664	0.7245	17.03	879.8775	0.5489	10.49	957.7753	0.9984	-41.52	959.6633	0.0019	-41.95
879.5745	0.7460	19.56	879.8843	0.5671	12.16	957.7832	0.0194	-44.14	959.6691	0.0173	-41.43
879.5827	0.7677	19.59	879.8912	0.5853	12.47	957.8080	0.0851	-38.29	959.6946	0.0849	-39.61
879.5908	0.7892	22.41	879.8981	0.6036	12.85	957.8155	0.1050	-35.07	959.6987	0.0958	-37.30
879.5988	0.8102	22.98	879.9048	0.6214	12.94	957.8231	0.1251	-32.24	959.7026	0.1061	-36.95
879.6064	0.8306	23.54	879.9108	0.6373	13.65	957.8309	0.1458	-30.06	959.7065	0.1164	-35.01
879.6143	0.8515	24.03	951.6514	0.7692	20.41	957.8385	0.1659	-26.90	959.7105	0.1270	-33.64
879.6217	0.8710	18.70	951.6555	0.7800	20.69	957.8461	0.1861	-24.58	959.7144	0.1374	-32.94
879.6287	0.8896	14.44	951.6597	0.7912	21.90	957.8738	0.2595	-15.72	959.7184	0.1480	-29.65
879.6367	0.9108	-9.59	953.6560	0.0817	-39.11	958.6405	0.2914	-13.77	985.6378	0.8383	23.10
879.6428	0.9270	-21.41	953.6610	0.0949	-36.50	958.6456	0.3049	-11.35	985.6417	0.8486	24.10
879.6489	0.9432	-34.58	953.6657	0.1074	-35.13	958.6506	0.3181	-10.58	985.6457	0.8592	24.34
879.6546	0.9582	-38.28	953.6705	0.1201	-33.32	958.6545	0.3285	-9.55	985.6496	0.8696	23.58
879.6600	0.9726	-41.08	953.6770	0.1373	-31.26	958.6585	0.3391	-8.95	985.6537	0.8804	23.14
879.6653	0.9867	-38.30	953.6835	0.1546	-29.69	958.6625	0.3497	-8.10	985.6577	0.8911	20.38
879.6707	0.0009	-40.45	953.6881	0.1667	-28.43	958.6664	0.3600	-6.47	985.6616	0.9014	17.69
879.6762	0.0155	-40.10	953.6923	0.1779	-27.11	958.6713	0.3730	-5.26	985.6656	0.9120	13.00
879.6816	0.0299	-39.59	953.6963	0.1885	-24.77	958.7018	0.4538	1.71	985.6696	0.9226	8.10
879.6870	0.0441	-38.83	953.7005	0.1996	-24.11	958.7061	0.4652	2.56	985.6742	0.9348	-2.42
879.6923	0.0582	-36.74	953.7535	0.3401	-9.11	958.7107	0.4774	3.69	985.6781	0.9451	-7.72
879.6978	0.0727	-34.51	953.7584	0.3531	-8.08	958.7150	0.4888	4.93	985.6821	0.9557	-18.62
879.7033	0.0873	-32.99	953.7626	0.3642	-6.27	958.7190	0.4994	5.10	985.6860	0.9660	-25.30
879.7091	0.1027	-32.16	953.7685	0.3798	-4.85	958.7230	0.5100	5.73	985.6900	0.9766	-32.04
879.7156	0.1200	-28.58	953.7727	0.3909	-3.79	958.7273	0.5214	7.22	985.6939	0.9870	-36.51
879.7218	0.1363	-25.87	953.7767	0.4016	-3.26	958.7316	0.5328	8.14	985.6979	0.9976	-41.63
879.7276	0.1517	-25.29	953.7857	0.4254	-1.53	958.7362	0.5450	9.35	985.7018	0.0079	-42.15
879.7338	0.1682	-23.09	953.7917	0.4413	-0.08	958.7433	0.5638	9.19	985.7057	0.0183	-42.38
879.7401	0.1847	-20.34	953.7993	0.4614	2.23	958.7479	0.5760	11.18	985.7097	0.0289	-41.63
879.7462	0.2008	-18.64	953.8077	0 4837	3.25	958.7523	0.5876	11.82	985.7136	0.0392	-41.93
879.7523	0.2172	-17.08	953.8145	0.5017	6.10	958.7562	0.5980	12.45	985.7176	0.0498	-41.97
879.7584	0.2334	-15.41	953.8209	0.5187	6.69	958.7606	0.6096	12.33	985.7215	0.0601	-41.39
879.7662	0.2541	-13.09	953.8280	0.5375	8.47	958.7657	0.6232	13.64	985.7254	0.0705	-40.00
879.7720	0.2694	-11.67	953.8338	0.5529	9.96	958.7707	0.6364	14.03	985.7293	0.0808	-38.65
879.7775	0.2840	-10.92	953.8458	0.5847	11.15	958.7747	0.6470	14.81	985.7339	0.0930	-38.14
879.7834	0.2997	-8.64	953.8531	0.6040	12.68	958.7791	0.6587	14.65	985.7418	0.1139	-35.13
879.7897	0.3163	-7.03	953.8592	0.6202	13.74	958.7834	0.6701	15.61	985.7457	0.1243	-33.29
879.7961	0.3333	-5.63	956.5691	0.8018	22.17	958.7874	0.6807	15.42	985.7496	0.1346	-31.94
879.8038	0.3537	-3.72	956.5876	0.8509	24.96	958.7914	0.6913	16.24	985.7536	0.1452	-31.02
879.8121	0.3757	-1.42	956.6011	0.8866	23.25	958.7955	0.7021	16.19	985.7575	0.1555	-29.47
879.8196	0.3956	-1.06	957.6870	0.7644	19.41	958.7994	0.7125	15.83	985.7614	0.1659	-28.38
879.8265	0.4139	1.07	957.6945	0.7843	20.57	958.8034	0.7231	18.11	985.7654	0.1765	-27.09
879.8334	0.4322	2.15	957.7019	0.8039	21.66	958.8074	0.7337	17.96	985.7693	0.1868	-25.77
879.8423	0.4558	4.31	957.7095	0.8241	22.04	958.8114	0.7443	18.42	985.7733	0.1974	-24.11
879.8471	0.4684	5.23	957.7171	0.8442	23.81	958.8153	0.7546	18.30	985.7772	0.2077	-22.26
879.8542	0.4873	6.79	957.7250	0.8651	24.00	958.8193	0.7652	19.76	985.7811	0.2181	-21.30
879.8612	0.5058	7.41	957.7340	0.8890	23.09	959.6433	0.9489	-3.85	985.7850	0.2284	-21.00
879.8683	0.5247	9.51	957.7678	0.9786	-34.18	959.6587	0.9897	-40.62	985.7890	0.2390	-19.54

[a] HJD − 2446000.

TABLE 8

RADIAL VELOCITIES OF TW HERCULIS

HJD[a]	Phase	v_{rad}	HJD[a]	Phase	v_{rad}
3.8997	0.1683	-29.13	5.8425	0.0302	-45.42
3.9076	0.1881	-27.92	5.8439	0.0337	-44.18
3.9154	0.2076	-24.61	5.8464	0.0399	-44.79
3.9246	0.2306	-22.39	5.8504	0.0500	-44.44
3.9325	0.2504	-20.49	5.8545	0.0602	-43.36
3.9402	0.2697	-16.25	5.8585	0.0702	-42.62
3.9481	0.2894	-16.68	5.8625	0.0802	-41.18
4.8147	0.4581	1.10	5.8665	0.0902	-39.10
4.8221	0.4766	2.73	5.8705	0.1003	-38.73
4.8299	0.4961	4.18	5.8745	0.1103	-37.28
4.8376	0.5154	5.20	5.8785	0.1203	-36.32
4.8454	0.5349	6.72	5.8825	0.1303	-35.61
4.8571	0.5642	9.05	5.8865	0.1403	-33.46
4.8658	0.5860	10.68	5.8905	0.1503	-33.15
4.8732	0.6045	11.57	5.8945	0.1603	-31.27
4.8807	0.6233	14.04	5.9083	0.1948	-26.62
4.8881	0.6418	14.95	5.9123	0.2049	-25.53
4.8956	0.6606	15.22	5.9163	0.2149	-24.53
4.9030	0.6791	15.17	5.9203	0.2249	-23.40
4.9181	0.7169	16.20	5.9250	0.2366	-20.91
4.9255	0.7354	16.22	5.9308	0.2512	-20.07
4.9330	0.7542	17.06	5.9360	0.2642	-18.35
4.9406	0.7732	17.65	5.9399	0.2739	-17.55
4.9496	0.7957	19.52	5.9439	0.2839	-16.01
4.9575	0.8155	21.43	5.9479	0.2939	-15.59
4.9654	0.8352	22.02	5.9518	0.3037	-13.61
4.9736	0.8558	24.93	5.9558	0.3137	-13.19
4.9819	0.8765	25.34	5.9598	0.3237	-11.91
5.7646	0.8352	21.37	5.9638	0.3337	-10.89
5.7741	0.8590	22.75	5.9678	0.3437	-9.18
5.7846	0.8853	23.73	5.9718	0.3538	-8.89
5.7952	0.9118	27.06	5.9757	0.3635	-8.45
5.8064	0.9398	16.21	5.9798	0.3738	-7.44
5.8165	0.9651	-21.83	5.9838	0.3838	-5.52
5.8266	0.9904	-38.43	5.9878	0.3938	-4.84
5.8345	0.0102	-46.21	5.9919	0.4041	-3.89
5.8357	0.0132	-47.12	5.9959	0.4141	-1.99
5.8384	0.0199	-46.60			

[a] HJD −2446920.

TABLE 9

RADIAL VELOCITIES OF UU VIRGINIS

HJD[a]	Phase	v_{rad}	HJD[a]	Phase	v_{rad}	HJD[a]	Phase	v_{rad}
29.7195	0.0156	-47.51	30.7943	0.2754	-19.42	33.6941	0.3725	-10.34
29.7234	0.0239	-46.03	30.7983	0.2838	-19.33	33.6983	0.3814	-9.27
29.7277	0.0328	-46.47	31.7408	0.2655	-19.94	33.7025	0.3901	-8.25
29.7317	0.0412	-46.66	31.7447	0.2737	-19.76	33.7070	0.3996	-6.30
29.7358	0.0498	-42.05	31.7489	0.2826	-18.42	33.7112	0.4084	-5.42
29.7397	0.0581	-44.18	31.7529	0.2909	-17.76	33.7151	0.4168	-4.35
29.7437	0.0664	-42.58	31.7568	0.2991	-17.33	33.7193	0.4255	-4.51
29.7476	0.0746	-40.88	31.7608	0.3076	-16.34	33.7238	0.4349	-3.37
29.7516	0.0831	-41.16	31.7648	0.3160	-15.17	33.7278	0.4434	-2.29
29.7561	0.0926	-40.28	31.7688	0.3244	-13.89	33.7317	0.4516	-1.79
29.7601	0.1009	-38.69	31.7727	0.3326	-13.98	33.7359	0.4604	-1.44
30.5601	0.7830	15.88	31.7768	0.3412	-12.80	33.7401	0.4691	-0.24
30.5713	0.8066	17.64	31.7814	0.3509	-12.49	33.7441	0.4776	0.36
30.5825	0.8301	18.82	31.7854	0.3593	-11.68	33.7481	0.4861	0.82
30.5939	0.8541	19.19	31.7894	0.3677	-9.75	33.7523	0.4948	1.24
30.6066	0.8808	21.34	31.7935	0.3763	-9.80	33.7562	0.5030	1.83
30.6182	0.9051	20.06	31.7975	0.3848	-7.98	33.7607	0.5125	3.29
30.6280	0.9258	18.58	32.7287	0.3427	-12.67	33.7646	0.5207	3.10
30.6371	0.9449	-11.61	32.7327	0.3510	-11.35	33.7686	0.5291	4.02
30.6647	0.0029	-43.09	32.7367	0.3595	-11.04	33.7729	0.5382	5.12
30.6977	0.0724	-43.30	32.7411	0.3688	-10.05	33.7768	0.5464	6.11
30.7018	0.0810	-41.81	32.7450	0.3770	-8.78	33.7808	0.5547	6.82
30.7061	0.0899	-39.87	32.7501	0.3876	-8.11	33.7858	0.5653	7.63
30.7100	0.0981	-39.17	32.7544	0.3967	-7.83	33.7900	0.5741	7.30
30.7141	0.1067	-38.47	32.7586	0.4056	-6.52	37.5589	0.4986	2.01
30.7181	0.1152	-38.35	32.7629	0.4146	-5.35	37.5677	0.5171	2.53
30.7221	0.1237	-34.97	32.7669	0.4230	-4.40	37.5772	0.5371	7.00
30.7261	0.1320	-34.76	32.7709	0.4314	-3.91	37.5879	0.5595	8.52
30.7301	0.1405	-35.45	32.7751	0.4402	-2.81	37.5981	0.5810	10.64
30.7341	0.1488	-32.87	32.7791	0.4486	-1.61	37.6094	0.6047	9.47
30.7380	0.1570	-32.94	32.7830	0.4568	-0.72	37.6216	0.6304	11.41
30.7420	0.1655	-31.30	32.7870	0.4653	-0.83	37.6340	0.6564	12.35
30.7458	0.1736	-31.79	32.7911	0.4739	-0.13	37.6449	0.6794	12.75
30.7507	0.1837	-28.77	32.7950	0.4821	0.33	37.6561	0.7029	12.52
30.7548	0.1923	-27.46	32.7993	0.4911	1.15	37.6678	0.7275	12.67
30.7587	0.2006	-26.82	33.6727	0.3275	-14.28	37.6797	0.7527	14.08
30.7638	0.2113	-26.67	33.6770	0.3366	-13.58	37.6938	0.7822	14.63
30.7678	0.2197	-25.74	33.6813	0.3456	-12.41	37.7086	0.8132	15.97
30.7718	0.2281	-24.82	33.6856	0.3547	-11.85	37.7241	0.8458	18.13
30.7760	0.2370	-23.86	33.6898	0.3634	-11.23			

[a] HJD −2446900.

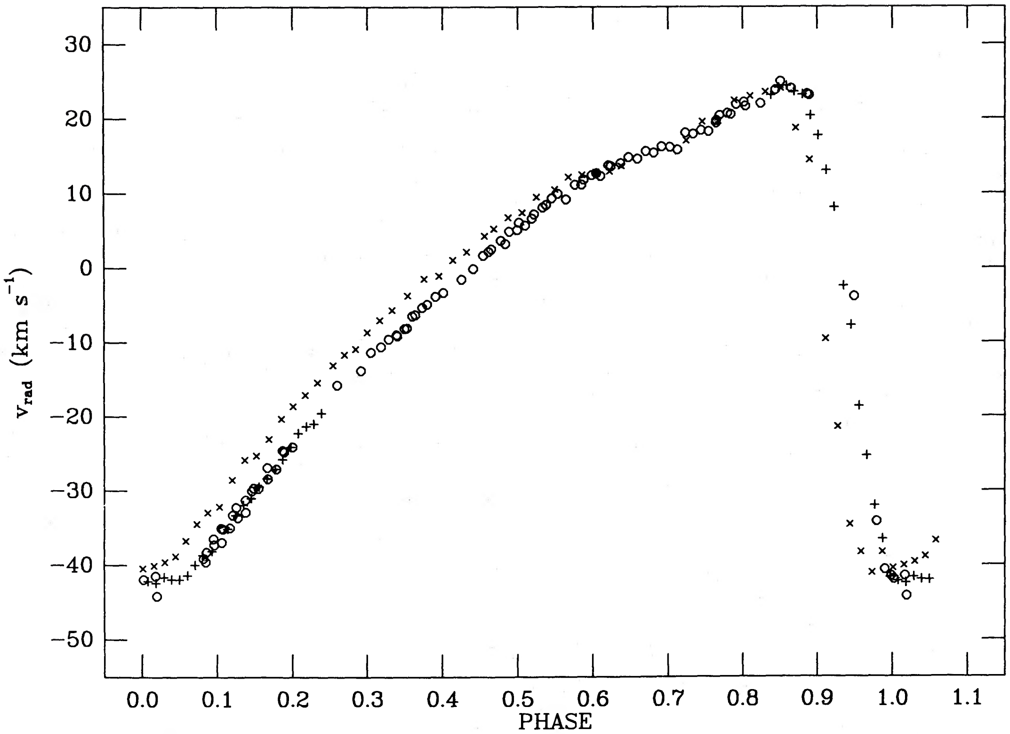

FIG. 6.—Same as Fig. 5. Data were obtained on 1987 March 24/25 (*crosses*), 1987 June (*open circles*, plotted also in Fig. 5), and 1987 July 8/9 (*plus signs*).

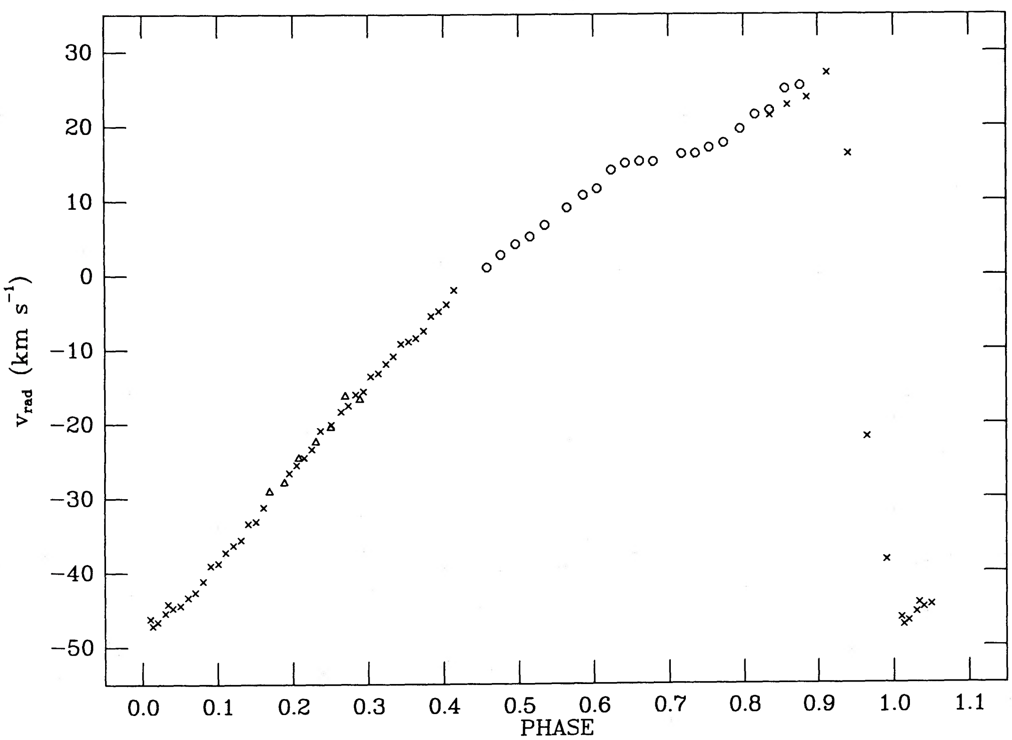

FIG. 7.—Same as Fig. 5, for TW Her. Data were obtained on 1987 May 7/8 (*open triangles*), 1987 May 8/9 (*open circles*), and 1987 May 9/10 (*crosses*).

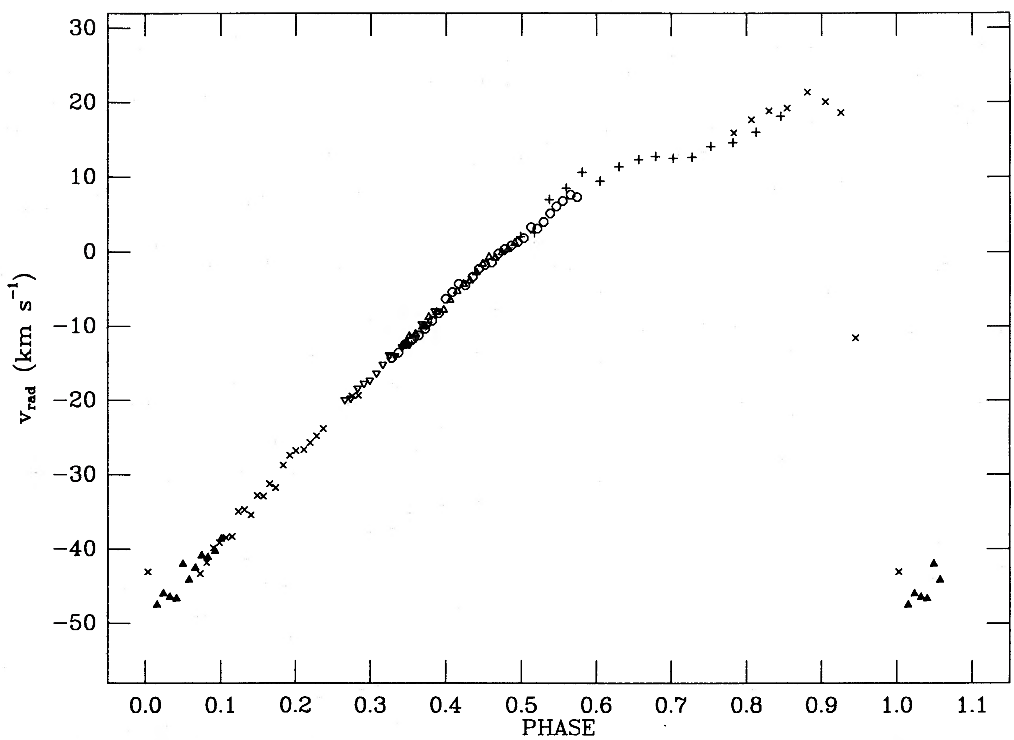

FIG. 8.—Same as Fig. 5, for UU Vir. Data were obtained on 1987 May 13/14 (*filled triangles*), 1987 May 14/15 (*crosses*), 1987 May 15/16 (*inverted open triangles*), 1987 May 16/17 (*open triangles*), 1987 May 17/18 (*open circles*), and 1987 May 21/22 (*plus signs*).

III. ANALYSIS AND RESULTS

Our application of the surface brightness version of the Baade-Wesselink method basically consists of matching the variation of the photometric angular diameter, θ_{phot}, with phase with that of the spectroscopic angular diameter, θ_{spect}. These quantities are defined as

$$\theta_{phot}(\phi) = \text{dex}\,[0.2(42.160 - m_{bol} - 10 \log T_{eff})] , \quad (4)$$

where m_{bol} is the apparent bolometric magnitude and θ_{phot} is in milliarcseconds, and

$$\theta_{spect} = \frac{2R(\phi)}{d} = \frac{2[R(\phi_0) + \Delta R(\phi, \phi_0)]}{d} = \frac{2}{d}\left[R(\phi_0) - \int_{\phi_0}^{\phi} p(v_{rad} - \gamma)P\,d\phi\right] , \quad (5)$$

where d is the distance in parsecs and p converts the radial velocities into pulsational velocities. Details of the conversion of the observed magnitudes and colors into bolometric magnitudes and temperatures can be found in the previous papers of this series; the models of R. Kurucz were again employed in this conversion.

Since there could be systematic effects in either the θ_{phot} or the θ_{spect} curves or both, we devised a method to match the curves that minimizes the influence of one upon the other. The procedure is as follows: define the difference between the θ_{spect} values of two phases, ϕ_1 and ϕ_2, as

$$\Delta\theta_{spect}(\phi_2, \phi_1) = \theta_{spect}(\phi_2) - \theta_{spect}(\phi_1) = \frac{2\Delta R(\phi_2, \phi_1)}{d} , \quad (6)$$

where ΔR is determined from the integration of the radial velocity curve. The value of $\Delta\theta_{spect}$ can be adjusted by changing the distance d until $\Delta\theta_{spect}(\phi_2, \phi_1) = \Delta\theta_{phot}(\phi_2, \phi_1)$, where the $\Delta\theta_{phot}$ values are derived from equation (4). The phases ϕ_1 and ϕ_2 should be chosen so that the $\Delta\theta$ values are as large as possible in order to minimize the effects of relative uncertainties in the values due both to observational uncertainties and to computational roundoff errors. Ideally, the phases would be chosen to be the phases of minimum and maximum radius; however, the θ_{phot} values around minimum radius are not accurately defined because of the rapid changes in the stellar atmosphere, caused by strong accelerations and shock waves, which cannot be modeled by our static model atmospheres. In previous papers, the $\Delta\theta$ values were defined using values from phases around maximum radius and those in the phase interval $0.1 \le \phi \le 0.15$, which yielded distance moduli accurate to roughly 0.05 mag (Paper V). This process has been improved so that the $\Delta\theta$ values are now summed. Define two phase intervals, $\phi_j \le \phi \le \phi_k$ and $\phi_l \le \phi \le \phi_m$, where the first phase interval is the region containing (or close to) maximum radius and the other interval can be in either the expansion or the contraction regions of the pulsational cycle. The sum is performed in such a way that

$$\sum(\text{spect}) = \Delta\theta_{spect}(\phi_l, \phi_j) + \Delta\theta_{spect}(\phi_l, \phi_{j+1}) + \cdots + \Delta\theta_{spect}(\phi_l, \phi_k) + \Delta\theta_{spect}(\phi_{l+1}, \phi_j) + \cdots + \Delta\theta_{spect}(\phi_m, \phi_k) \quad (7)$$

or

$$\sum(\text{spect}) = 2[\Delta R(\phi_l, \phi_j) + \cdots + \Delta R(\phi_m, \phi_k)]/d . \quad (8)$$

Sums are computed for various values of d and then compared

TABLE 10

SIMULPHOT PHOTOMETRY OF BD +29°3132

HJD[a]	V	$B-V$	K
6.7010	9.242	0.413	...
6.7092	...	...	8.380
6.7249	9.205	0.385	...
6.7319	...	...	8.325
6.7470	9.222	0.377	...
6.7519	...	...	8.354
6.7661	9.263	0.387	...
6.7754	...	...	8.337
6.7903	9.300	0.417	...
6.7953	...	...	8.353
6.8145	9.326	0.427	...
6.8246	...	...	8.348
6.8254	...	...	8.349
6.8394	9.332	0.419	...
6.8476	...	...	8.341
6.8620	9.295	0.417	...
6.8672	9.280	0.407	...
6.8690	...	...	8.335
6.8872	9.225	0.378	...
6.8923	9.216	0.376	...
6.8974	...	...	8.311
6.8983	...	...	8.314
6.9155	9.209	0.374	...
6.9230	...	...	8.310
6.9406	9.253	0.387	...
6.9460	9.263	0.392	...
6.9528	...	...	8.334
6.9537	...	...	8.343
7.7523	9.277	0.408	...
7.7682	...	...	8.326
7.7819	9.259	0.390	...
7.8068	9.253	0.394	...
7.8230	...	...	8.346
7.8303	9.260	0.393	...
7.8482	...	...	8.330
7.8606	9.276	0.405	...
7.8769	...	...	8.331
7.8848	9.295	0.409	...
7.9019	...	...	8.341
7.9087	9.297	0.412	...
7.9289	...	...	8.330
7.9369	9.280	0.410	...
7.9638	...	...	8.325

[a] HJD −2446940.

with the sum of the corresponding $\Delta\theta_{\text{phot}}$ values to determine the appropriate distance. The value of $\bar{R}(\phi_0)$ is then adjusted so that the θ_{spect} curve corresponding to this distance overlies that of θ_{phot}.

In practice, the determination of θ_{phot} from an observed color and magnitude is dependent upon the effective gravity, so that a value of the mass of the star must be assumed in order to convert the gravities into radii and vice versa. We assumed a value of 0.6 $M_\odot$ for each of the stars in the analysis. Since the actual masses of these stars may not be exactly 0.6 $M_\odot$, we repeated the analysis using values of 0.5 and 0.7 $M_\odot$ in order to estimate the uncertainty in our results caused by our assumed mass. It can be seen in Table 14, which presents the results of the matching process for VY Ser, that the results derived using the different masses are essentially the same, owing to the relative insensitivity of the $V-K$ color to gravity effects and the small range of masses possessed by RR Lyrae stars. However, the effects of an incorrect assumed mass may be larger for other types of variables with a larger range of masses, especially if other color indices are used to compute the temperatures.

Figures 10–14 present the angular diameter versus phase curves for RS Boo, TW Her, VY Ser, and UU Vir, respectively. It can be seen in all cases that the K, $V-K$ combination yields θ_{phot} values that are more in phase agreement with the θ_{spect} values than are those derived using optical colors and magnitudes; it is also apparent from the small θ_{phot} values derived from the optical colors during the expansion phase that the flux redistribution described in Paper IV occurs in these stars as well. It has been shown in previous papers that the phasing problem illustrated for the $B-V$ and the $b-y$ colors in these figures also occurred when other optical color indices, such as $V-R$ and $V-I$, are used, and also that this problem persisted even when an empirical color-temperature relation, that of Burki and Meylan (1986), is employed (Paper III). A possible explanation of this effect may be found from the fact that the continuum forms at different physical depths at different wavelengths. If the temperature structure of an RR Lyrae atmosphere is different from that of a static star, the continuum flux distribution may be different as well. This idea is illustrated in Table 15, which presents the depths of formation [≡ log (column density)] at different wavelengths of an infinitesimally weak ($\log gf < -5$) high-excitation Fe II line. Since the equivalent width of this line is less than 1 mÅ, these depths can be regarded as the depths of the continuum at different wavelengths. These depths were derived using Kurucz's WIDTH6 program and two of his model atmospheres: one with $T_{\text{eff}} = 6500$ K, $\log g = 2.5$, and $[\text{m/H}] = -2$, corresponding to the conditions in X Ari at phase 0.15, and one with $T_{\text{eff}} = 6000$ K, and the same gravity and metallicity as before, which represents the behavior of X Ari at minimum temperature. In the first case, the continua at the wavelengths corresponding to the central wavelengths of the V and K filters form at essentially the same depth, so that this index may be unaffected by the different temperature structure, which may explain why this index does not possess the phasing problem. At minimum temperature the continua of the two filters no longer form at the same depth; however, the stellar atmosphere apparently does not differ from a static one during these phases, so that this difference is not important. It can also be seen from Table 15 that, for the model representing X Ari at phase 0.15, the continuum forms deeper at the wavelength of the B filter and forms higher in the atmosphere for the R and I filters than it does at the wavelengths of the V and K filters, which may explain the behavior of this star at that phase as discussed in Paper IV.

There are other features of interest in Figures 10–14 that are worth mentioning. First of all, it is apparent in Figure 11 that the large "bump" seen on the optical light curves of TW Her in Figure 2 from about phase 0.45 to minimum light has a definite effect upon the θ_{phot} values from both the $V-K$ and the $B-V$ color indices, since it is probably the culprit behind the bumps on the angular diameter curves during these phases. These phases were therefore excluded during the matching process for this star. Also, the behavior during the expansion phase of the θ_{phot} curve derived from the $V-K$ index of RS Boo in Figure 10 is probably caused by the apparent excess of K flux during this phase region (Fig. 1), which shows that temperature effects can still affect the results from the $V-K$ color. However, since this star exhibits a very shallow secondary "bump" on its optical light curves and an even smaller

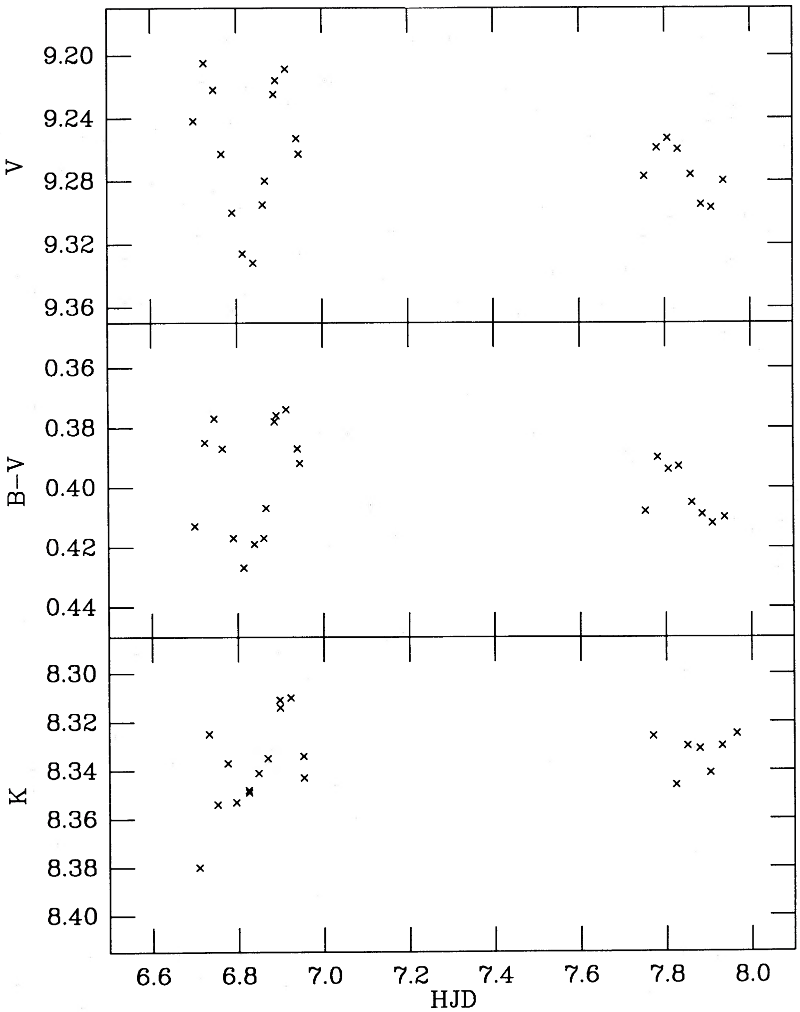

FIG. 9.—V, $B-V$, and K (*top to bottom*) of BD +29°3132 plotted against Heliocentric Julian Date

TABLE 11

MEAN OBSERVED DEREDDENED QUANTITIES: PROGRAM STARS

Quantity[a]	X Ari	RS Boo	SW Dra	TW Her	DH Peg	VY Ser	UU Vir
Fe/H]	−2.2	−0.5	−0.8	−0.5	−0.8	−1.8	−0.7
$E(B-V)$	0.153	0.024	0.033	0.052	0.080	0.030	0.028
$\langle V \rangle_m$	9.119	10.372	10.424	11.160	9.301	10.085	10.548
$\langle V \rangle_i$	9.078	10.302	10.389	11.091	9.287	10.069	10.494
$\langle B \rangle_m$	9.447	10.679	10.764	11.473	9.498	...	10.873
$\langle B \rangle_i$	9.374	10.552	10.699	11.344	9.475	...	10.776
$\langle K \rangle_m$	7.898	9.450	9.330	10.222	8.587	8.783	9.506
$\langle K \rangle_i$	7.894	9.445	9.326	10.217	8.587	8.780	9.501
$\langle m_{\rm bol} \rangle_m$	9.018	10.359	10.391	11.145	9.314	9.982	10.517
$\langle m_{\rm bol} \rangle_i$	8.987	10.292	10.363	11.078	9.301	9.970	10.471
$\langle B-V \rangle_m$	0.328	0.307	0.340	0.313	0.197	...	0.325
$\langle B \rangle_i - \langle V \rangle_i$	0.296	0.250	0.310	0.253	0.187	...	0.282
$\langle V-K \rangle_m$	1.221	0.922	1.094	0.938	0.714	1.302	1.041
$\langle V \rangle_i - \langle K \rangle_i$	1.184	0.858	1.066	0.874	0.701	1.289	0.993
γ(km s^{-1})	−36.8	−3.7	−29.2	−4.5	−71.0	−146.5	−7.1

[a] Subscript m refers to magnitude average, subscript i to intensity average.

TABLE 12

AMPLITUDES AND RISE TIMES: PROGRAM STARS

Quantity	X Ari	RS Boo	SW Dra	TW Her	DH Peg	VY Ser	UU Vir
Amplitude:							
B (mag)	1.26[a]	1.65	1.22	1.69	0.64[b]	0.76[c]	1.50
V (mag)	1.01	1.25	0.93	1.29	0.51	0.63	1.15
K (mag)	0.31	0.36	0.31	0.34	0.12	0.26	0.38
m_{bol} (mag)	0.89	1.24	0.83	1.29	0.49	0.54	1.09
v_{rad} (km s^{-1})	63	70[d]	64	75	26	49	68
v_p (km s^{-1})[e]	82	91[d]	83	98	34	64	88
Radius[e,f]	0.90	0.56	0.80	0.62	0.15	0.79	0.72
ΔK_{tr} (mag)	0.04	−0.07	0.04	−0.01	0.01	0.09	0.03
Rise time, $\Delta\phi_{rise}$:							
B	0.15[a]	0.16	0.16	0.13	0.44[b]	...	0.13
V	0.16	0.16	0.16	0.14	0.44	0.21	0.13
v_{rad}	0.14	0.14[d]	0.15	0.12	0.36	0.19	0.12

[a] From Preston and Paczyński 1964.
[b] From Tifft 1964.
[c] From Varsavsky 1960.
[d] From June data. For March, $A_{vrad} = 66$ km s^{-1}, $A_{vp} = 86$ km s^{-1}, $\Delta\phi_{rise}(v_{rad}) = 0.16$.
[e] Assuming $p = 1.30$.
[f] In solar units.

TABLE 13

EPHEMERIDES[a] OF PROGRAM STARS

Star	HJD(ZP)	P_0 (days)	β
X Ari	2,446,325.8888	0.6511571	1.66×10^{-9}
RS Boo[b]	2,446,948.7198	0.37733691	0
SW Dra	2,446,495.7541	0.56966993	0
TW Her	2,446,947.8085	0.399600104	0
DH Peg	2,446,684.737	0.25551037	0
VY Ser	2,444,738.8585	0.71409384	0
UU Vir	2,446,878.8222	0.47560623	0

[a] HJD (max V light) = HJD(ZP) + $P_0(1 + \frac{1}{2}\beta E)E$; ZP = zero point.
[b] Possesses Blazhko effect (not accounted for here).

"dip" on its 1987 June radial velocity curve, there should not be much distortion of the θ_{phot} curve in the phase region following maximum radius, so this region was utilized in the matching process. Finally, Figures 13 and 14 illustrate the effect of the metallicity upon the derived angular diameters. As can be seen, the θ_{phot} values derived from the $V-K$ index are virtually the same for both metallicities, while there is a noticeable difference between the values derived from the $B-V$ index. Since it is expected from the results of previous papers that the photometric angular diameters should agree during the minimum-temperature phases, it appears that [Fe/H] = −0.7 is more appropriate for this star than is −0.5. Also, the possibility exists that the metallicities of RS Boo and TW Her have been underestimated, since the θ_{phot} values from the $B-V$ color exceed those of the $V-K$ index during the minimum-temperature phases. However, the curves for both stars are afflicted by bumps, so that it is difficult to determine this for certain.

Table 16 lists the adopted absolute quantities and their uncertainties for all of the program stars. It should be noted that the quantities for X Ari and SW Dra are different from those derived in Papers II and III, for reasons which were discussed earlier. The uncertainties are essentially those discussed in Paper V, with a few changes. First of all, it is clear from Table 14 that the uncertainty in the derived distance moduli due to the matching process might be as small as 0.01 mag, instead of the value 0.05 mag adopted in Paper V, because of the implementation of the summing process. However, there are systematic distortions in the θ_{phot} curves, such as the bumps on the curves of UU Vir at about phase 0.3 and of VY Ser at about phase 0.55, which undoubtedly arise from gaps in the light curves, as well as deviations produced by secondary bumps, which may affect the results. An uncertainty of 0.06 mag will therefore be adopted for all stars except DH

TABLE 14

RESULTS OF MATCHING PROCESS FOR VY SERPENTIS

ANGULAR DIAMETER	SUM[a] Case 1	Case 2	Case 3
Photometric[b]	994.6	993.7	993.9
Spectroscopic ($m - M$):			
9.13	1012.8	1012.0	1012.2
9.15	1003.5	1002.7	1002.9
9.17	994.3	993.6	993.8
9.19	985.2	984.5	984.7
9.21	976.1	975.4	975.6

[a] Sum process described in text (sums are in units of 10^{-6} arcsec). Phase intervals: $0.0 \leq \phi \leq 0.3$; $0.3 \leq \phi \leq 0.4$ (maximum radius). Case 1: $E(B-V) = 0.030$ mag, assumed mass = 0.6 $M_\odot$; case 2: $E(B-V) = 0.030$ mag, assumed mass = 0.5 $M_\odot$; case 3: $E(B-V) = 0.050$ mag, assumed mass = 0.6 $M_\odot$.
[b] Obtained using the K, $V-K$ magnitude-color combination.

TABLE 15

DEPTHS OF FORMATION[a] OF CONTINUUM AT VARIOUS WAVELENGTHS

WAVELENGTH (Å)	FILTER	DEPTH[b] T_{eff} = 6500 K	T_{eff} = 6000 K
3500	U	1.227	1.530
4400	B	1.276	1.550
5500	V	1.264	1.538
6400	R_c	1.255	1.531
7900	I_c	1.246	1.523
22000	K	1.261	1.552

[a] Depth = log (column density).
[b] log g = 2.5; [m/H] = −2.

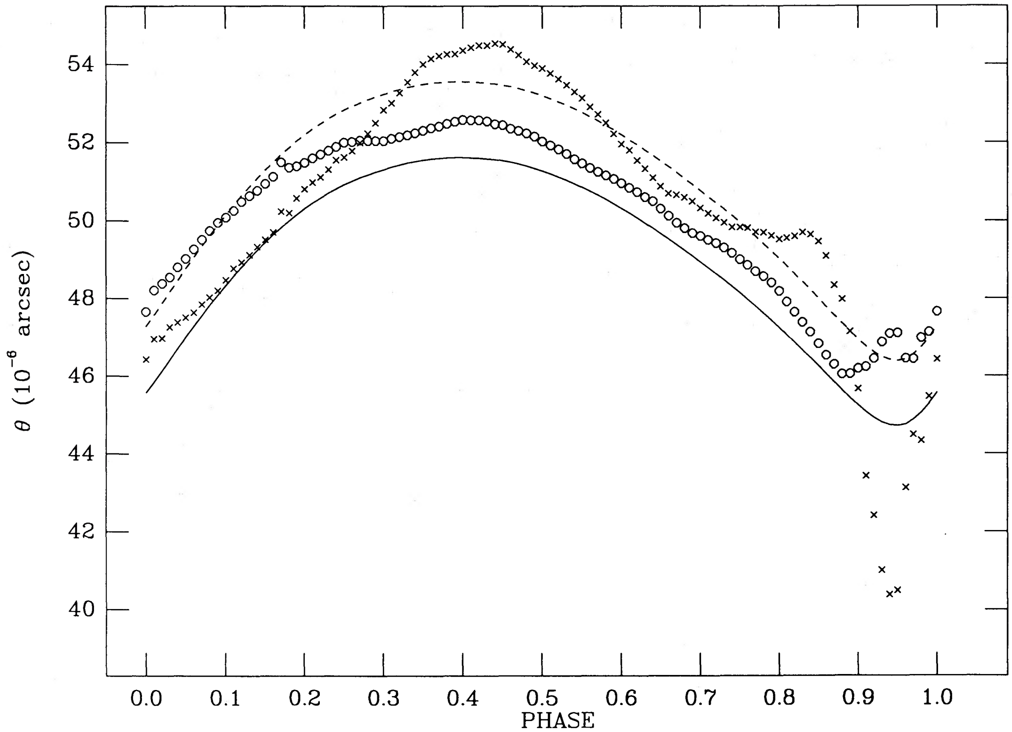

FIG. 10.—Angular diameters of RS Boo. Symbols are photometric diameters derived from synthetic colors with [m/H] = -0.5 using the V magnitude and $B-V$ color (*crosses*) and the K magnitude and $V-K$ color (*open circles*). Lines represent spectroscopic diameters obtained from the radial velocities of Fig. 5 with $m-M = 9.31$ (*dashed line*) and 9.39 (*solid line*); $\log g_{\mathrm{eff}}(\phi = 0.0) = 3.261$.

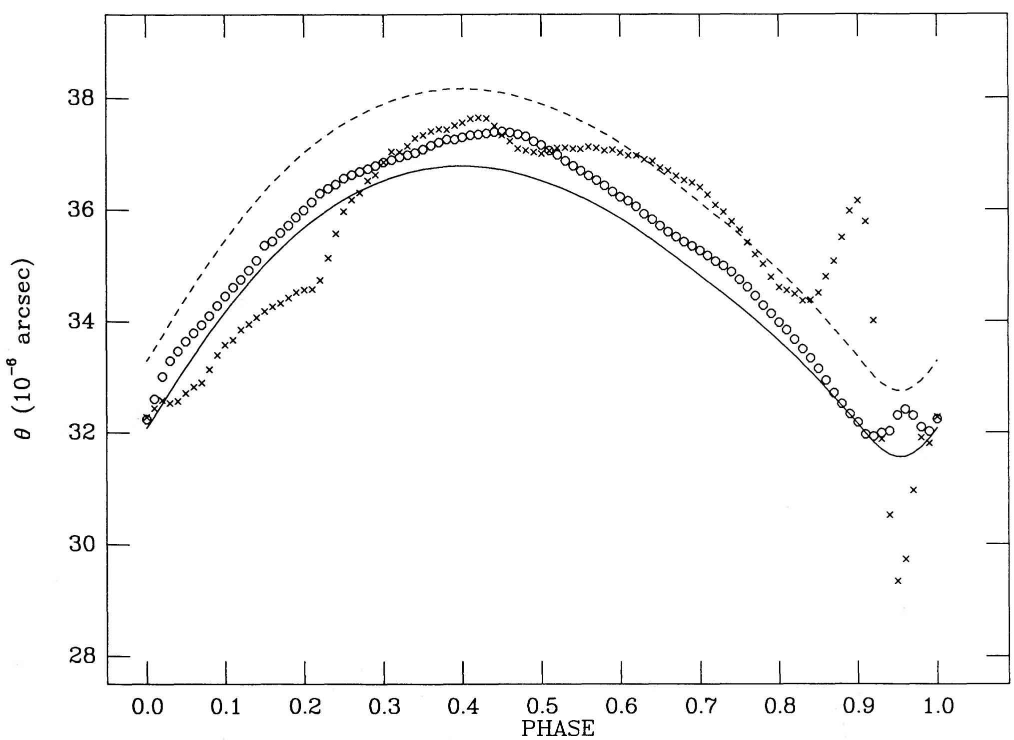

FIG. 11.—Same as Fig. 10, for TW Her. Spectroscopic diameters were obtained with $m-M = 10.15$ (*dashed line*) and 10.23 (*solid line*); $\log g_{\mathrm{eff}}(\phi = 0.0) = 3.424$.

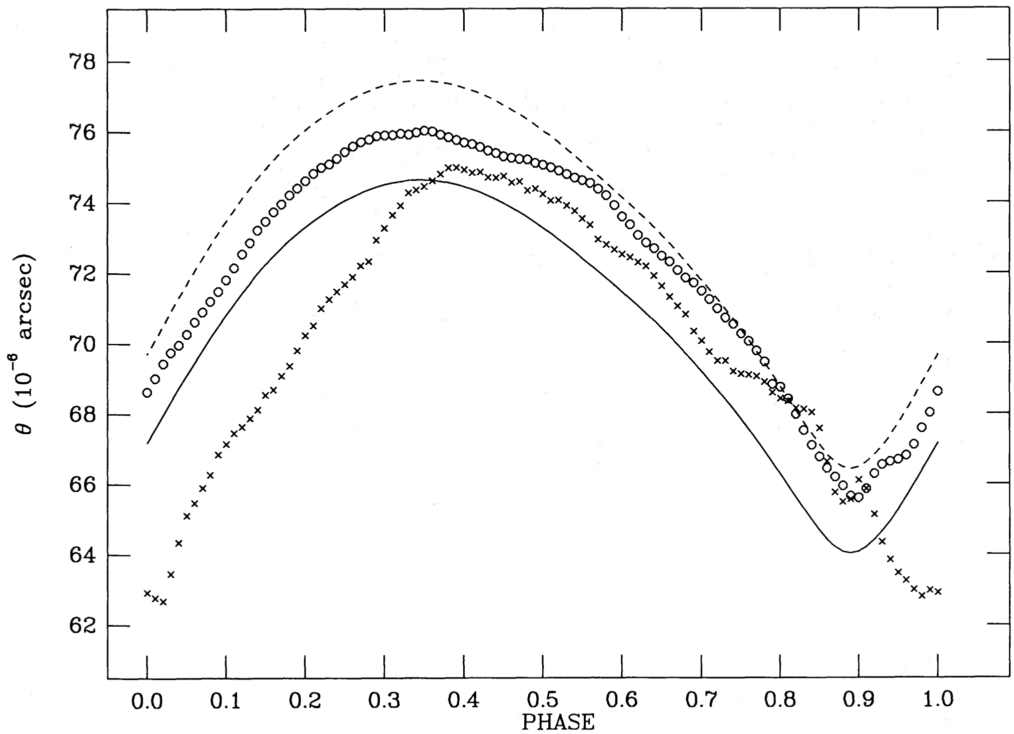

FIG. 12.—Same as Fig. 10, for VY Ser, except [m/H] = −1.75 and crosses represent $b-y$ color. Spectroscopic diameters represent $m-M = 9.13$ (*dashed line*) and 9.21 (*solid line*); $\log g_{\text{eff}}(\phi = 0.0) = 2.820$.

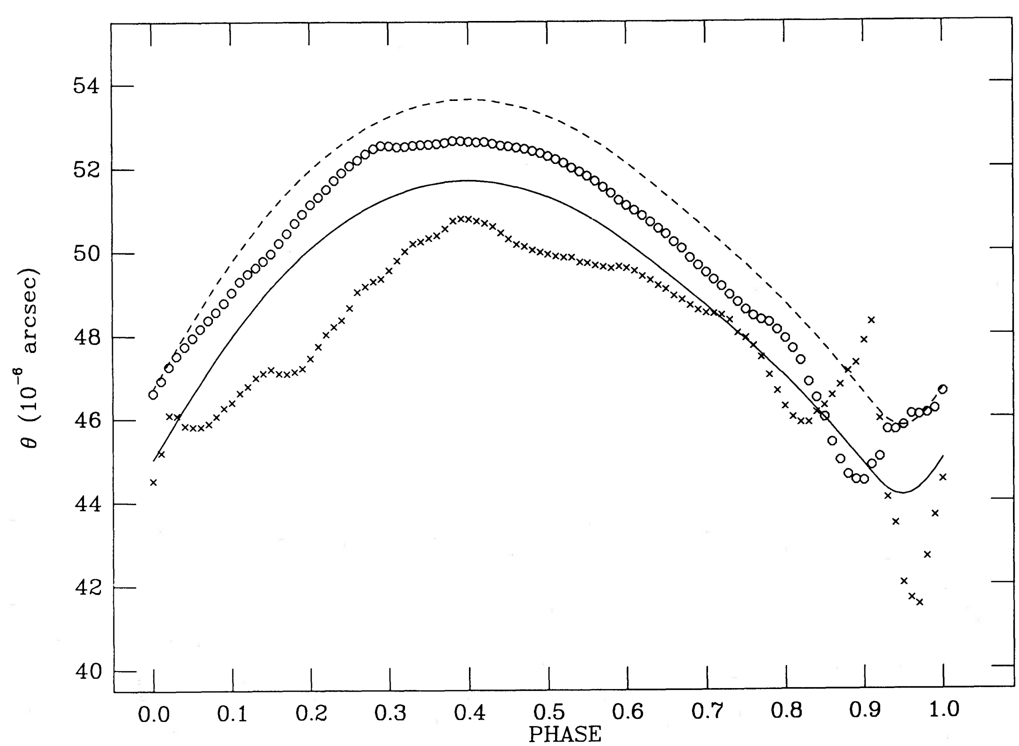

FIG. 13.—Same as Fig. 10, for UU Vir. Spectroscopic diameters derived from $m-M = 9.63$ (*dashed line*) and 9.71 (*solid line*); $\log g_{\text{eff}}(\phi = 0.0) = 3.279$, [m/H] = −0.5.

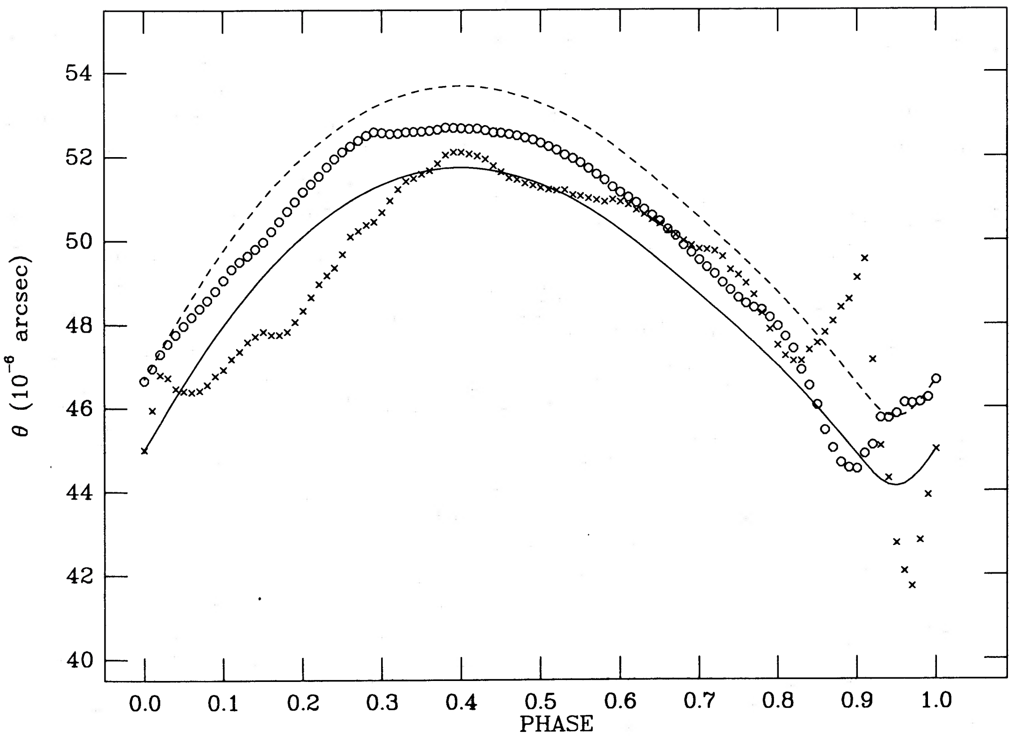

FIG. 14.—Same as Fig. 13, except [m/H] = −0.7

TABLE 16

MEAN ABSOLUTE QUANTITIES AND ESTIMATED UNCERTAINTIES OF PROGRAM STARS

Quantity	X Ari	RS Boo	SW Dra	TW HER	DH Peg	VY Ser	UU Vir
$(m-M)_0$	8.34	9.35	9.61	10.19	8.40	9.17	9.67
Random	0.06	0.08	0.06	0.06	0.12	0.06	0.06
Total	0.14	0.15	0.14	0.14	0.18	0.14	0.14
d (pc)	466	741	836	1091	479	682	859
Random	6	12	10	13	11	8	10
Total	13	22	23	31	17	19	24
$\langle M_V \rangle_i$	0.74	0.95	0.78	0.90	0.89	0.90	0.82
Random	0.08	0.09	0.08	0.08	0.13	0.08	0.08
Total	0.15	0.16	0.15	0.15	0.18	0.15	0.15
$\langle M_B \rangle_i$	1.03	1.20	1.09	1.15	1.08	...	1.11
Random	0.10	0.11	0.10	0.10	0.15	...	0.10
Total	0.16	0.17	0.16	0.16	0.19	...	0.16
$\langle M_K \rangle_i$	−0.45	+0.10	−0.28	+0.03	+0.19	−0.39	−0.17
Random	0.06	0.08	0.06	0.06	0.12	0.06	0.06
Total	0.14	0.15	0.14	0.14	0.18	0.14	0.14
$\langle M_{\rm bol} \rangle_i$	0.65	0.94	0.75	0.89	0.90	0.80	0.80
Random	0.08	0.09	0.08	0.08	0.13	0.08	0.08
Total	0.15	0.16	0.15	0.15	0.18	0.14	0.14
log L^a	1.64	1.52	1.60	1.54	1.54	1.58	1.58
Random	0.03	0.04	0.03	0.03	0.05	0.03	0.03
Total	0.06	0.06	0.06	0.06	0.07	0.06	0.06
$\langle R \rangle^a$	5.30	3.98	4.89	4.16	3.73	5.26	4.60
Random	0.16	0.21	0.16	0.16	0.32	0.16	0.16
Total	0.37	0.39	0.37	0.37	0.46	0.37	0.37
Mass[a]	0.52	0.56	0.51	0.58	0.55	0.45	0.57
Random	0.04	0.07	0.04	0.06	0.12	0.03	0.05
Total	0.09	0.14	0.10	0.13	0.17	0.08	0.12
$\langle T_{\rm eff} \rangle^b$	6315	6795	6460	6770	7165	6160	6570

[a] In solar units.
[b] Phase average of values from the $V-K$ color index.

Peg as the uncertainty arising from the combination of curve-fitting errors and the matching process. The uncertainties in Paper V will be retained for DH Peg because of the many bumps and dips in its angular diameter curve.

Table 14 also shows the effects of increasing the reddening by 0.02 mag. As can be seen, the derived distance modulus remains virtually unchanged by this. This means that the uncertainty in the mean absolute magnitude in a given filter arising from an uncertainty in the reddening is due mainly to the uncertainty in the magnitude absorption in that filter. For out intensity-averaged values, $\langle M_V \rangle_i$ and $\langle M_K \rangle_i$, these absorptions are 3.1 and 0.38 times the uncertainty in the adopted $E(B-V)$ value (Savage and Mathis 1979), so a change in the reddening of 0.02 mag yields changes of 0.063 mag in $\langle M_V \rangle_i$ (and also in $\langle V \rangle_i$) and 0.008 mag in $\langle M_K \rangle_i$. According to Lub (1987), the uncertainty in the reddening values should be $\sim$0.01 mag. As noted earlier, the uncertainty may be greater for variables near the Galactic poles. We will therefore continue to adopt an uncertainty of 0.05 mag in $\langle M_V \rangle_i$ owing to the uncertainty in the reddening, and we will also adopt uncertainties of 0.08, 0.01, and 0.01 mag for the absolute magnitudes in the B and K filters and the distance modulus, respectively. We note that even if the uncertainty in $E(B-V)$ is as large as 0.1 mag, the uncertainty in $\langle M_K \rangle_i$ and in the distance modulus will still be less than 0.05 mag, which means that the relative distances between globular clusters may be more precisely determined than the relative $\langle M_V \rangle_i$ values of their RR Lyrae stars.

Finally, there is an extra source of uncertainty in RS Boo arising from the fact that it possesses the Blazhko effect and that the photometry and spectroscopy were not obtained simultaneously. We believe this uncertainty to be small, since the radial velocity curve employed in the analysis was obtained within 11 days of the photometric observations and RS Boo has a long secondary period of 533 or 537 days (§ II*c*). Using the elements of the secondary period of Kanyo (1980), we determined that our photometric observations were taken 184–185 days after the occurrence of the lowest maximum of the light curve, while the 1987 March and June radial velocities were obtained 115 and 187–195 days after this time of smallest magnitude amplitude. Since the secondary variation of RS Boo is essentially sinusoidal (Oosterhoff 1946), the radial velocity curve that is appropriate to the time period of the photometry should lie between those of the March and June curves, so that the maximum possible uncertainty arising from the Blazhko effect should be determined by reanalyzing the star using the March curve. This analysis resulted in a value of $\langle M_V \rangle$ that is 0.17 mag brighter than our adopted value. We do not believe the uncertainty to be as large as this, since the photometry was obtained nearly simultaneously with the data of Figure 5. Another effect must be taken into account, however: the "minor variation" of the secondary period, which Kanyo (1980) reports to possess a period between 58 and 62 days. This variation is much smaller in amplitude than the long secondary variation, and is most prominent during the maximum of the long cycle. Since the velocities of Figure 5, which were taken over a period of eight days, agree well with each other and also with those from 1987 July (Fig. 6), which were obtained a month later, it is clear that the minor variation has only a small effect upon our results. We will therefore adopt an uncertainty of 0.05 mag for RS Boo due to the Blazhko effect, but note that this estimate is likely to be a conservative one.

Figures 15 and 16 depict $\langle M_V \rangle_i$ and $\langle M_{\mathrm{bol}} \rangle_i$, respectively, versus [Fe/H] for the program stars. As can be seen, particu-

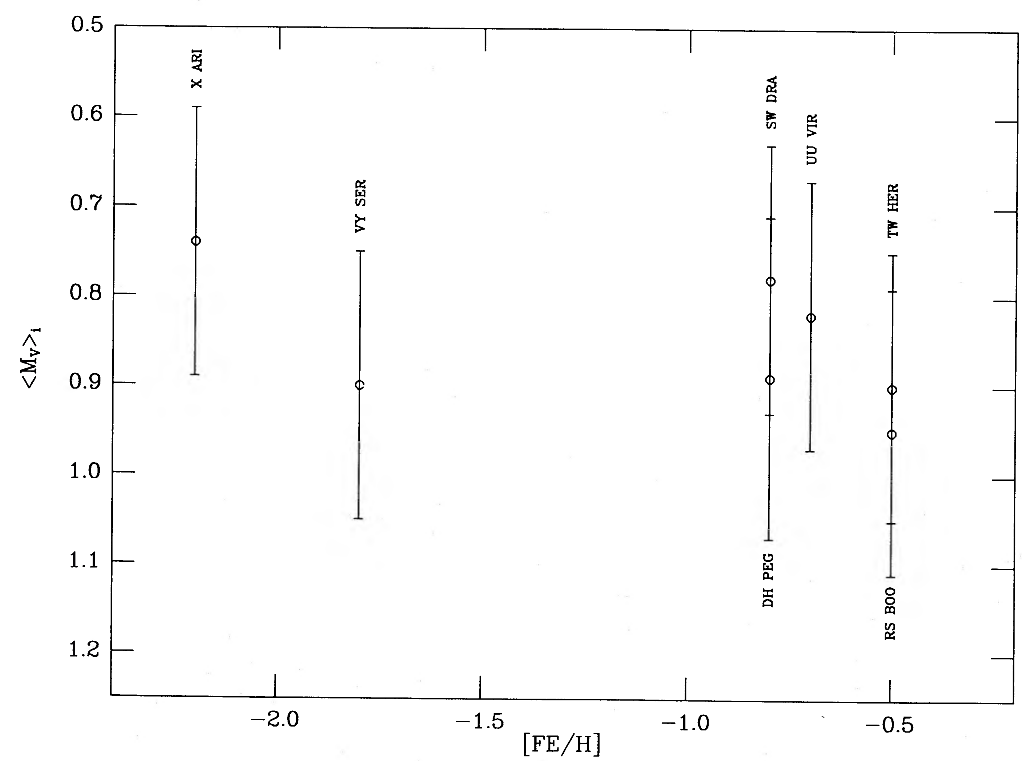

FIG. 15.—Intensity-averaged $\langle M_V \rangle$versus [Fe/H] for the program stars. Error bars represent the estimated total uncertainty, including random and systematic sources; error bars for random uncertainties only are the same as in Fig. 16.

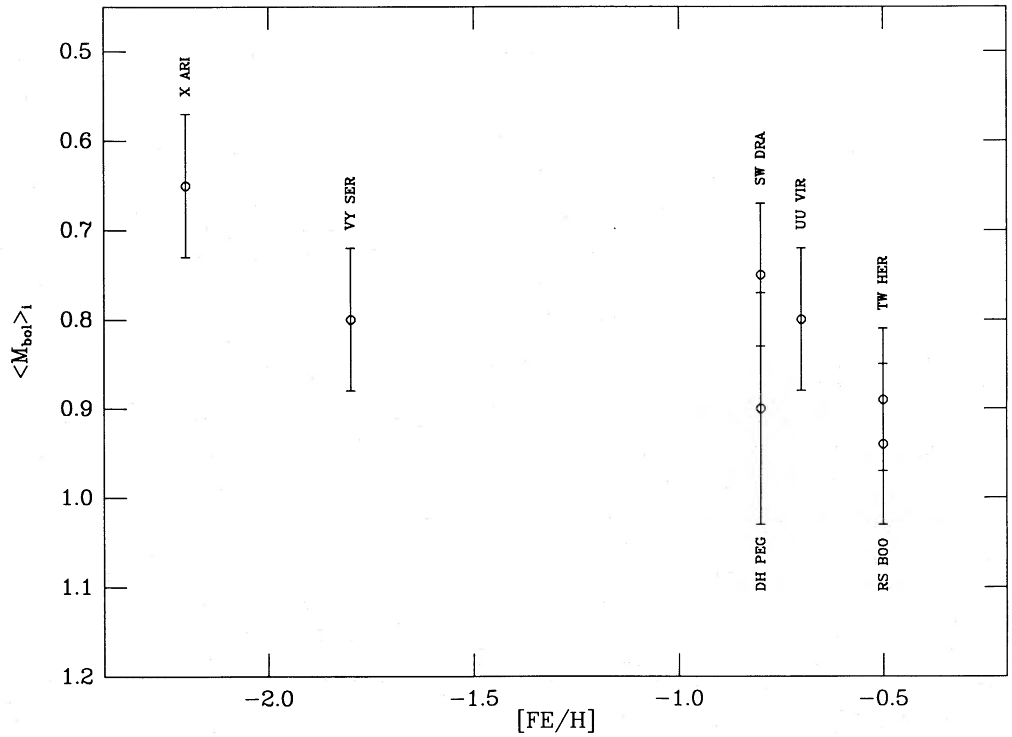

FIG. 16.—Intensity-averaged $\langle M_{\rm Bol} \rangle$ versus [Fe/H]. Error bars represent the random uncertainties only; error bars for total uncertainty are the same as in Fig. 15.

larly in Figure 16, there may be a slight dependence of these values upon the metallicity such that the metal-poor stars are brighter. If the program stars are divided into two groups, one consisting of the metal-poor stars X Ari and VY Ser and the other containing the remaining five stars, then the mean $\langle M_V \rangle_i$ values for the two groups are 0.82 and 0.87 mag, respectively, while for $\langle M_{\rm bol} \rangle_i$, the values are 0.73 and 0.86 mag. A trend probably exists for the bolometric magnitudes, although, as will be discussed later, it does not seem to be as large as that predicted by Sandage (1982). There is essentially no difference between the mean visual absolute magnitudes, and an average of all the values, weighted inversely as the square of the random uncertainties, yields $\langle M_V \rangle_{i,\,\rm RR} = 0.85 \pm 0.12$ mag for these stars (the uncertainty quoted includes the systematic errors). If the program stars are typical RR Lyrae variables, this value indicates that these stars may not be as bright as 0.6 mag as previously thought. This will be considered in more detail in the next section.

IV. DISCUSSION

a) Comparison with Other $\langle M_V \rangle_{\rm RR}$ Values

Stothers (1983) has conveniently summarized the estimates of $\langle M_V \rangle_{\rm RR}$ from various techniques and derived an unweighted mean of 0.61 mag from these determinations. Since several of the results used by Stothers (1983) are now known to be affected by serious systematic biases, it is worthwhile to examine these results again in detail.

i) *Baade-Wesselink Results*

The results presented in Table 3 of Stothers (1983) were all derived using optical color indices to determine the effective temperatures needed in the Baade-Wesselink method, and only Oke and collaborators (Oke and Bonsack 1960; Oke, Giver, and Searle 1962; Oke 1966) employed simultaneous photometry and spectroscopy. We have argued in Papers I–V and in Jones (1987), as have Cohen and Gordon (1987), that all of the optical colors, particularly the blue ones, overestimate the temperatures during the expansion phase of the pulsation cycle, leading to a phasing problem between the photometric and spectroscopic angular diameters, and that the use of the V magnitude enhances this effect. Paper III also argued that the results obtained from the use of empirical color-temperature conversions instead of synthetic colors from static model atmospheres were not immune to this effect. Since some analyses have employed phase shifts to remove any phasing problems, which they assumed to arise from an inaccurate ephemeris, the appropriate corrections to be applied to previous results to account for this effect are not easily determined. To avoid such problems, we will therefore exclude all Baade-Wesselink results, including our own, derived using optical color indices. Only when the effects of the flux redistribution problem are accounted for fully, using dynamic model atmospheres or by restricting analyses to phase intervals when the problem can be minimized, will we recommend the use of results based upon optical data alone.

Other Baade-Wesselink results using the $V-K$ index that probably do not suffer from this phasing problem are those of Longmore *et al.* (1985). Cohen and Gordon (1987) have derived $\langle M_V \rangle = 1.05$ $(+0.15,\ -0.25)$ mag from four variables in the globular cluster M5 using a restricted phase interval to avoid the effects of the flux redistribution problem. Longmore *et al.* (1985) derived values of $\langle M_V \rangle = 0.63 \pm 0.12$ mag and $\langle R \rangle = 6.13 \pm 0.14\ R_\odot$ for VY Ser by employing the technique of Balona (1977) and several magnitude-color combinations. However, as pointed out in Paper II, there appears to be a systematic effect in their results (their Table 3) such that, for all magnitudes, the derived $\langle R \rangle$ values are larger for the bluer colors. This effect was most pronounced when the V magnitude was employed: the result from the V, $V-J$ combination, for example, is 24% larger than that of the K, $V-K$ combination. Since the flux redistribution problem manifests itself as a systematic effect that is largest for the bluer colors and is also enhanced by the use of the V magnitude, these two systematic effects may be related, although there could be other effects, such as gravity, present as well. We will consider only the result from the K, $V-K$ combination, $\langle R \rangle = 5.45 \pm 0.08\ R_{\odot}$, in which case the absolute magnitude of VY Ser would be

$$\begin{aligned} \langle M_V \rangle_{\rm new} &= \langle M_V \rangle_{\rm old} + \Delta M_V \\ &= 0.63 - 5 \log (5.45/6.13) = 0.89 \text{ mag} , \qquad (9) \end{aligned}$$

using $L \propto R^2$, which is virtually the same as the value we derive. More recently, Longmore, Fernley, and Jameson (1986) and Fernley *et al.* (1987) quote $\langle M_V \rangle$ values of 0.55, 0.30, and 1.08 mag for VY Ser, X Ari, and V445 Oph, respectively, while Jameson (1986) lists these values as 0.60, 0.52, and 1.02 mag. A detailed comparison of these results must await publication of the results for the individual magnitude/color index combinations.

In conclusion, the latest Baade-Wesselink results that are clearly free of the phasing problem indicate that $\langle M_V \rangle_{\rm RR}$ may be somewhere between 0.8 and 1.1 mag, instead of the canonical 0.6 mag. The main limitation of this method at present seems to be the need for the use of either static model atmospheres or empirical relations to convert the colors into temperatures. Ideally, the angular diameters of the variables over time would be measured directly, or, more realistically, be obtained from dynamic models, but this is still not possible.

ii) *The Statistical Parallax Method*

Hawley *et al.* (1986) provide a detailed description of the many uncertainties and systematic biases present in the statistical parallax analyses that preceded their work. These uncertainties often led to results from one analysis that were markedly different from those of other analyses despite the use of the same data set; also, the results of Heck and Lakaye (1978) and Clube and Dawe (1978) showed a strong correlation of $\langle M_V \rangle_{\rm RR}$ with [Fe/H] in the sense that the metal-rich stars are more luminous, contrary to theoretical predictions.

The more recent analyses of Hawley *et al.* (1986; corrected by Barnes and Hawley 1986) and SRM, using a more advanced statistical technique based upon the Principle of Maximum Likelihood model of Murray (1983) and the proper motions of Wan, Mao, and Ji (1981), yielded similar results of $\langle M_V \rangle_i = 0.68 \pm 0.14$ and 0.75 ± 0.2 mag, respectively. It should be noted, however, that the result of Hawley *et al.* (1986) was derived using the reddening scale of Sturch (1966), and, as we have discussed earlier, this scale has been superseded by that of Lub, which yields smaller reddenings more in agreement with those of Burstein and Heiles (1982). To place the two sets of results on the same reddening scale and roughly the one that we have chosen as well, we correct the result of Hawley and collaborators to 0.80 ± 0.14 mag, as recommended by SRM. It can be seen that these latest statistical parallax results are in excellent agreement with each other, although they are not entirely independent, and also seem to indicate that $\langle M_V \rangle_{\rm RR}$ is closer to 0.8 than to 0.6 mag.

iii) *The Moving Cluster Method*

Eggen and Sandage (1959) obtained a value of $\langle M_V \rangle = 0.6$ mag from five stars which they considered to be members of moving groups. They stressed that their results should be considered preliminary because of large uncertainties in the proper-motion and radial velocity data of the RR Lyrae stars as well as uncertainties in the mean magnitudes and interstellar absorption effects adopted in their work. In view of new data obtained since then, it is worth reconsidering the method and its results. Perhaps the most significant improvement is in the proper motions, as summarized by Hawley *et al.* (1986). We will discuss the five stars of Eggen and Sandage (1959) in turn.

Even with the improved proper motions, W CVn remains a poor calibrator, for the errors in its proper motions amount to 20% of the proper motions themselves. This would lead to a 20% error in the derived distance even if the convergent point were precisely defined, which it is not.

TU Uma must also be set aside, for its purported group partner, BD +17°4708, is not a good calibrator. Carney and Latham (1987) reported it to be a probable single-lined spectroscopic binary, and Latham *et al.* (1988) have recently determined an orbit for it with a period of about 219 days. Further, Lu *et al.* (1987) have employed speckel techniques to show that the star is also an astrometric binary (CHARA 119), with a separation of 0".205 and an estimated orbital period of 30 yr. These results suggest that the proper motions for BD +17°4708 might give a misleading result for any convergent point solution. Finally, the metallicity of TU UMa does not agree with that of BD +17°4708, as we might expect for common group membership. For the latter, Peterson (1981) found [Fe/H] = −1.95, whereas the ΔS value of 6 indicates [Fe/H] = −1.2 for TU UMa, using the calibration of Butler (1975).

Both X Ari and SU Dra are now claimed by Eggen (1977) to be members of the same group, which also includes ST Leo and U Lep. The convergent point is defined by Kapteyn's star (HD 33793), which has a very well-defined parallax, 0".251 ± 0".007. We find it difficult to accept these group memberships, however, because the four RR Lyraes differ somewhat in metallicity. For X Ari, SU Dra, ST Leo, and U Lep, Butler (1975) gives ΔS = 11.7, 9.5, 7.3, and 9.4, which lead to [Fe/H] = −2.1, −1.75, −1.4, and −1.7, respectively. Much worse, however, is the fact that the calibrating star, HD 33793, has [Fe/H] = −0.55 (Mould 1976). Although the M_V values derived by Eggen for this proposed group's RR Lyraes average 0.8 mag, thereby supporting the Baade-Wesselink and statistical parallax results, we regard the agreement as accidental.

Finally, we are left with the original and best-defined RR Lyrae–subdwarf pair, comprising RR Lyrae itself and HD 103095 (also known as HR 4550 and Groombridge 1830). Since the original work of Eggen and Sandage (1959), two more high-precision parallax/proper motion determinations have been published for HD 103095 (Beardsley, Gatewood, and Kamper 1974; Heintz 1984). The new results, when merged with those obtained previously, result in $\pi = 0''.117 \pm 0''.003$, $\mu = 7''.0597 \pm 0''.0004$ yr^{-1}, and $\theta = 145^\circ.339 \pm 0^\circ.002$. The convergent point derived from these new results ($\alpha = 16^{\rm h}19^{\rm m}43^{\rm s}$, $\delta = -54°22'$ [1950.0]) differs negligibly from that quoted by Eggen and Sandage (1959). Their original result for RR Lyrae, $\langle M_V \rangle = 0.8$ mag, must be altered, however, because the data for this star have changed somewhat. Using the new proper motions of Hawley *et al.* (1986), $\mu = 0''.2229$

$\pm$ 0″.0039 yr^{-1} and $\theta = 209°.4 \pm 0°.25$, $E(B-V) = 0.016$ mag (McNamara, Helm, and Wilcken 1970), $\langle v_{\rm rad}\rangle = -76$ km s^{-1} (Sanford 1949), and $\langle V\rangle = 7.76$ mag (Manduca *et al.* 1981) leads to $\langle M_{v,0}\rangle = 0.52$ mag. However, we question again whether the variable shares the convergent point with the subdwarf. Both stars have well-determined proper motions, and we can therefore derive the two stars' convergent point. The result then implies a parallax of 0″.093 $\pm$ 0″.003 for HD 103095, which is inconsistent with the observed trigonometric parallax at a very high level of significance.

We conclude, therefore, that the convergent point method cannot be used at present to improve the accuracy of our absolute magnitude estimates for field RR Lyraes.

iv) *Globular Cluster Main-Sequence Fitting*

In principle, the distance to a globular cluster can be determined directly by fitting its main sequence to one obtained from nearby halo dwarfs possessing the same metallicity as the cluster. This is not feasible at present, owing to the small number (8) of metal-poor subdwarfs with trigonometric parallaxes accurate enough to define such a sequence. Sandage (1982) performed direct fits of the main sequences of five globular clusters to a main sequence defined by these eight metal-poor subdwarfs and obtained results indicating that $\langle M_V\rangle$ is $\sim$0.87 mag, and that the most metal-poor RR Lyraes are brighter, in agreement with theoretical predictions. It is not clear from his paper, however, whether or not he corrected the cluster main sequences for differential line blanketing with respect to the subdwarf main sequence; if not, then the fit would tend to overestimate the luminosity of the most metal-poor stars and underestimate that of the more metal-rich variables.

Another approach is to account for line-blanketing effects by utilizing subdwarfs with well-defined trigonometric parallaxes and metallicities [as indicated by the normalized ultraviolet excess, $\delta(U-B)_{0.6}$ (Sandage 1969), which will hereafter be abbreviated as δ] to derive the values of the magnitude difference, ΔM_V, from a standard main sequence as a function of metallicity. The distance to a globular cluster possessing a given value of δ can then be determined by applying the appropriate ΔM_V value to the standard main sequence. The derived distances are independent of the location of the standard main sequence in the M_V-color diagram, since changing the location of the standard main sequence also changes the derived ΔM_V values; essentially, the cluster main sequences are fitted to nearby subdwarfs of the same metallicity. Sandage (1970) determined values of $\langle M_V\rangle$ for RR Lyraes in the globular clusters M3, M13, M15, and M92 by fitting the main sequence of these clusters to a fiducial main sequence, assuming that all of the main sequences were of the same form, so that the ΔM_V values depended only upon δ. His results indicated that the mean $\langle M_V\rangle$ values for these clusters is 0.6 mag when systematic biases are taken into account; however, his results also indicated that the more metal-rich clusters contained the more luminous RR Lyrae stars, contrary to theoretical predictions. Hanson (1979) reanalyzed the Sandage (1970) data, employing more advanced corrections to account for systematic errors in the determination of luminosities from trigonometric parallaxes, and derived $\langle M_V\rangle$ values for the four clusters that were substantially brighter than those of Sandage (1970), but which showed the same trend with metallicity. In his derivation of the ΔM_V values, Hanson (1979) assumed that there was a linear relationship between these corrections and δ and performed an unweighted least-squares fit upon the values obtained from nearby subdwarfs grouped according to δ. However, it should be noted that an unweighted fit is influenced disproportionately by the groups with the fewest stars and the least accurate ΔM_V values, namely, the most metal-poor groups. Also, the use of a linear relationship may underestimate the corrections for the most metal-poor stars, and hence overestimate their luminosities, if in fact the relationship is nonlinear as assumed by Sandage (1970). It is possible, therefore, that the $\langle M_V\rangle$ values from Hanson (1979) are systematically too bright. Laird, Carney, and Latham (1988, hereafter LCL) have derived new relations between M_V and $B-V$ for the main sequences defined by the Hyades (with an assumed distance modulus of 3.30 mag; Hanson 1980) and by the eight metal-poor subdwarfs. The main sequence of the latter was defined by using the models of VandenBerg and Bell (1985) to correct the values of the subdwarfs to the same metallicity ($\delta = 0.25$). Their results indicated that the form of the main sequence depends upon the metallicity, since the Hyades main sequence had a steeper slope than that of the subdwarfs, so their derived magnitude differences, $\Delta M_V^{\rm H}$, defined using the Hyades as the standard main sequence, depended upon $B-V$ as well as upon δ for $0.0 \leq \delta \leq 0.25$. For $\delta > 0.25$, LCL's magnitude differences were derived from the model isochrones of VandenBerg and Bell (1985) and were independent of color. We will use their results here to rederive distances to some globular clusters in order to see what effect this color dependence has on the derived distances.

We have restricted our analysis to globular clusters with $E(B-V) < 0.05$ mag in order to minimize reddening effects. We further restrict our analysis to stars in the interval $B-V = 0.70$–0.80 mag, which has the dual advantages that such stars are essentially unevolved in the clusters, so age differences do not affect our results, and that the calibrating halo dwarfs in this color range have the best-determined parallaxes. The disadvantage is that such stars are faint in the clusters, so accurate color indices are hard to measure. For this reason, we have restricted our analysis to clusters with high-precision CCD data only. Also, we have utilized the metallicities of Zinn and West (1984) and the calibration of Carney (1979) to derive the δ-values for the clusters, so that these values are on the same scale.

The results for five globular clusters are presented in Table 17. The mean implied value for the five clusters' RR Lyraes is $\langle M_V\rangle_{\rm RR} = 0.55 \pm 0.07$ mag (standard error of the mean). Also listed in this table are the M_V(RR) values derived in the sources of the CCD photometry. These values were determined by fitting the cluster main sequences to the eight metal-poor halo dwarfs, using the models of VandenBerg and Bell (1985) to account for differential line blanketing. Except for the value of M5 of Richer and Fahlman (1987), the corrections of Lutz and Kelker (1973) had been applied; these corrections were not applied to our results. It is not clear that these corrections should be applied, or that they are as large as estimated by Lutz and Kelker (see Richer and Fahlman 1987 for a discussion of this). If they were applied, our results would be 0.1 mag brighter. It can be seen that there is good agreement between our results and those of the models for all clusters except 47 Tuc. LCL also determined that the model magnitude differences do not agree with those derived empirically in this metallicity region, which they attributed to an incorrect treatment of line blanketing in the models. The horizontal-branch $\langle M_V\rangle$ results of Table 17 are, except for that of M5, brighter

TABLE 17

RESULTS OF MAIN-SEQUENCE FITTING OF FIVE GLOBULAR CLUSTERS

Quantity	$(B-V)_0$	47 TUC	M5	NGC 6752	M30	M92
[Fe/H][a]	...	−071	−1.40	−1.54	−2.13	−2.24
δ[a]	...	0.146	0.217	0.230	0.277	0.285
No. (RR)	...	1?	90	0	3	12
$E(B-V)$		0.04	0.02	0.04	0.02	0.02
M_V(HB)[b]	...	0.7	0.81	0.39	0.40	0.45
Source[c]	...	1	2	3	4	5
ΔM_V^{H}	0.7	0.72	1.35	1.46	1.81	1.86
	0.8	0.68	1.26	1.37	1.69	1.74
M_V[d]	0.7	5.78	6.41	6.52	6.73	6.74
	0.8	6.30	6.88	6.99	7.19	7.20
V_0(MS)	0.7	19.32	20.59	19.49	21.30	21.23
	0.8	19.86	21.14	19.99	21.72	21.72
$(m-M)_0$	0.7	13.54	14.18	12.97	14.57	14.49
	0.8	13.56	14.26	13.00	14.53	14.52
Adopted $(m-M)_0$	...	13.55	14.22	12.99	14.55	14.51
V_0(HB)	...	13.99	15.05	13.56	14.99	14.99
M_V(HB)	...	0.44	0.83	0.57	0.44	0.48

[a] Metallicities are from Zinn and West 1984; δ computed from [Fe/H] using calibration of Carney 1979.

[b] Results of main-sequence fitting from the sources below.

[c] Sources of V(MS), V(HB), $E(B-V)$: (1) Hesser *et al.* 1987; (2) Richer and Fahlman 1987; (3) Penny and Dickens 1986; (4) Bolte 1987; (5) Heasley and Christian 1986.

[d] $M_V(B-V) = M_V(\text{Hyades: } B-V) + \Delta M_V^{\mathrm{H}}(B-V)$; M_V(Hyades; 0.7) = 5.06 mag, M_V(Hyades; 0.8) = 5.62 mag, assuming $m-M = 3.30$ mag.

than we would have predicted from the statistical parallax and Baade-Wesselink results. What is the problem?

Let us first echo the comments made by others in their attempts to derive the distances to clusters using main-sequence fitting. The number of calibrating halo dwarfs is very small (eight), their individual distances are not very precise, and they show a large spread in metallicity. Distances derived to individual clusters are thus not, in general, more accurate than about ± 0.2 mag. Further, model isochrones are used to bring these calibrating stars to a common metallicity, and in some cases a distance is then derived by again relying on model isochrones to compare the field stars' and the clusters' main sequences. Any dependence on theory ought to be avoided in principle if at all possible, since one virtue of the distance determinations is that they may be used as tests of the theory. Other uncertainties in the derived M_V(RR) values from this method include those of the adopted reddening of the clusters, the determination of the location of the RR Lyrae gap in the clusters possessing few of these stars, and the accuracy of the main-sequence photometry.

These uncertainties, and those of the other methods, may account for some or all of the discrepancy in the derived $\langle M_V \rangle_{\mathrm{RR}}$ values. However, there is a second potential major yet subtle problem that must also be addressed. When, for example, field RR Lyraes are selected for study, chances are the sample will mostly involve stars that spend the greater part of their lives in the instability strip, and which will in general probably begin core helium burning there. However, a globular cluster chosen for study may have very few RR Lyraes because its zero-age horizontal branch is populated on either the blue or the red side of the instability strip. In this case, we would be comparing a population of stars with zero-age horizontal-branch (ZAHB) positions located predominantly in or near the instability strip (the field) with clusters whose variables enter the strip only after prolonged evolution away from their ZAHB position. Such stars will almost always be brighter on average than ZAHB RR Lyraes. Since four of the clusters in Table 17 have very few RR Lyraes in comparison with their total horizontal-branch populations (indicated in Table 17 by the number of known member RR Lyraes), we would expect their variables to be brighter than average, as the main-sequence fitting suggests. Put another way, if the field population ZAHB is populated like those of the five clusters in Table 17, any random selection of field RR Lyraes will select M5-like variables about 80% of the time.

To derive the most accurate globular cluster distance and hence $\langle M_V \rangle_{\mathrm{RR}}$, we should select only those clusters which have a horizontal branch populated in such a way that most of the core helium-burning stars are in or near the instability strip. Among the clusters of Table 17, only M5 qualifies. To derive as accurate a distance to the cluster as possible, we use *only one halo dwarf*, HD 103095. This star has the dual advantages of having essentially the same metallicity as M5 (−1.36: Carney 1979; −1.44: Peterson 1981) *and* the most accurate trigonometric parallax available, $\pi = 0\rlap{.}''117 \pm 0\rlap{.}''003$ (Jenkins 1963; Beardsley, Gatewood, and Kamper 1974; Heintz 1984), which translates into $M_V = 6.79 \pm 0.06$ mag at $B-V = 0.75$ mag. In this case, the distance uncertainties for M5 due to the calibrating star are very small, and the statistical corrections discussed, for example, by Lutz and Kelker (1973) are negligible. If we fit this star to the M5 main sequence of Richer and Fahlman (1987), who give $V_0 = 20.98$ mag at $(B-V)_0 = 0.75$ mag, we find $(m-M)_0 = 14.19 \pm 0.12$ mag, where we have convolved the distance uncertainty to HD 103095 with an estimated error of ± 0.10 mag in the V magnitude of the M5 sequence at the same color index. This distance modulus, in turn, yields $\langle M_V \rangle_{\mathrm{RR}} = 0.86 \pm 0.12$ for the variables in M5, in excellent agreement with the Baade-Wesselink and statistical parallax estimates for the field RR Lyraes.

To resolve the discordant results of the other four clusters with the field star analyses, we recommend two research efforts. First, more field halo dwarfs of varying metallicities must have accurate trigonometric parallax measurements. Such efforts can be undertaken with ground-based facilities and, of course, HIPPARCOS and the Hubble Space Telescope. Second, the Baade-Wesselink method should be applied to variables in globular clusters with different metallicities and horizontal-branch morphologies. Cohen and Gordon (1987) have already published the results of a first attempt using B, i photometry for the cluster M5. We have begun such work as well. Beck (1988) has used our technique as well as our VK photometry and echelle-Reticon spectra, to obtain $\langle M_V \rangle_i = 0.86 \pm 0.16$ mag for the variable V8 in M5. We plan to complete optical and infrared photometry and radial velocity observations of another variable in M5, as well as some in the very metal-poor cluster M92, during 1988.

v) *Comparison with Theory and Conclusions*

Our Baade-Wesselink results compare well with the recent theoretical calculations of Sweigart (1987). If we focus on his results at a temperature of 6600 K, which is the average of the seven variables we are considering here, we find a predicted spread of 0.35 mag in $M_{\rm bol}$ as we progress from [Fe/H] $= -0.5$ to [Fe/H] $= -2.2$, assuming a main-sequence helium mass fraction of 0.20 and restricting our attention to only zero-age horizontal-branch stars. The actual value of $\langle M_{\rm bol} \rangle$ depends on the helium abundance, but our observed spread (see Fig. 16) is certainly consistent with this simple prediction. Our results imply slightly fainter luminosities than those predicted by the $Y = 0.20$ models, but these values can be reconciled with slightly higher heavy-element mass fractions in the models—since it is known for the halo dwarfs that oxygen is enhanced relative to iron, our use of [Fe/H] may underestimate Z—or by assuming $Y \lesssim 0.20$. Resolution of this situation must await comparable studies of globular cluster variables, since there we have the perspective necessary to identify stars' evolutionary states with respect to the zero-age horizontal branch.

In conclusion, there seems to be a mounting body of evidence from the Baade-Wesselink, statistical parallax, and cluster main-sequence fitting techniques that the value of $\langle M_V \rangle_{\rm RR}$ may be closer to 0.85 mag than to the "canonical" value of 0.60 mag. There may be a slight trend with metallicity such that the metal-poor stars are more luminous.

vi) *Implications for Globular Cluster Ages*

In Paper V, we have briefly illustrated some of the implications of $\langle M_V \rangle_{\rm RR} = 0.9$ mag. For now, we concentrate only on M5, for the reasons discussed in § IV*a*(iv) above. The main-sequence fit implies $\langle M_V \rangle = 0.86 \pm 0.12$ mag, which agrees very well with the Baade-Wesselink results of Table 16, $\langle M_V \rangle = 0.85 \pm 0.06$ mag (standard error of the mean) and with the statistical parallax results of SRM (0.75 ± 0.2 mag) and Barnes and Hawley (1986) (0.80 ± 0.14 mag when placed on the reddening scale of Burstein and Heiles 1982). If we allow for the external, systematic errors inherent in our Baade-Wesselink results, the errors properly should be taken as ± 0.12 mag. If M5 is as metal-rich as [Fe/H] $= -1.12$, as suggested by Richer and Fahlman (1987) from their observation of the main-sequence $\delta(U-B)_{0.6}$ value and by Burstein, Faber, and Gonzalez (1986), the revised main-sequence fit for M5 would result in the RR Lyraes being brighter by 0.12 mag. Neglecting this latter correction, the weighted result for the RR Lyraes in M5 is $\langle M_V \rangle = 0.84 \pm 0.07$ mag, leading to a cluster distance modulus of 14.17 ± 0.07 mag. Richer and Fahlman's (1987) data show the dereddened turnoff at $V_0 = 18.75 \pm 0.1$ mag, from which it follows that the turnoff absolute visual magnitude is M_V(T.O.) $= 4.58 \pm 0.12$ mag. The empirical bolometric corrections for such hot metal-poor stars is -0.20 mag (Carney 1983), so $M_{\rm bol}$(T.O.) $= 4.38 \pm 0.12$ mag. We are now in a position to estimate the cluster's age, independent of any color-temperature calibrations or reliance on convection theory. All we require is the chemical composition of the M5 stars and a set of model isochrones. Hesser *et al.* (1987) and Caputo, Martinez Roger, and Paez (1987) have argued that the relevant main-sequence helium mass fraction for halo stars is $Y = 0.24$. For the iron-peak metallicities, let us adopt a compromise between the Burstein, Faber, and Gonzalez (1986) and Zinn (1985) metallicities: [Fe/H] $= -1.25 \pm 0.15$. This revised metallicity alters our derived turnoff luminosity slightly because of the slightly revised match between M5 and HD 103095, so $M_{\rm bol}$(T.O.) $= 4.36 \pm 0.12$ mag. The heavy element metallicity, [m/H], is probably higher than [Fe/H] $= -1.25$, however, for oxygen is generally enhanced in halo stars, with [O/Fe] ~ 0.5 at [Fe/H] $= -1.2$ (see Sneden 1985 for a review). The mean heavy-element metallicity of M5 thus becomes approximately [m/H] $= -1.0$, or $Z = 0.0017$. VandenBerg's (1985) isochrones for $Y = 0.25$, $Z = 0.0017$ extend to only $M_{\rm bol}$(T.O.) $= 4.18$ mag at an age of 15×10^9 yr, from which we infer that M5 must be at least a couple of billion years older still. Interpolation in the slightly more extensive $Y = 0.20$ and $Y = 0.30$ isochrones of VandenBerg and Bell (1985) also reveals that they do not extend to quite old enough ages. We turn, then, to the older Yale isochrones of Ciardullo and Demarque (1977). Although these isochrones are based upon a different ratio of convective mixing length to pressure scale height than the VandenBerg results, and even though they lack model atmospheres to compute observational color indices and bolometric corrections, they are more than adequate, since we do not require radius and temperature estimates. The stellar luminosities are all we need, and, as VandenBerg (1983) has pointed out, the Yale and Victoria calculations predict very similar mass-luminosity relations for metal-poor stars. The age we obtain from [m/H] $= -1.0 \pm 0.15$, $Y = 0.24$, and $M_{\rm bol}$(T.O.) $= 4.36 \pm 0.12$ mag is $(18 \pm 3) \times 10^9$ yr. This, we feel, is the most realistic age for a globular cluster obtained to date, since it avoids some of the "free parameters" of the usual isochrone-fitting process, as well as reliance upon convection theory.

We defer to future papers the question of an age spread among the globular clusters. As we pointed out in Paper V, if all the RR Lyraes have the same absolute visual magnitude, there must be a significant age spread as a function of metallicity. However, as we have noted here, it is quite possible that variable-poor clusters' RR Lyraes may be brighter than our field star calibrators because the former are in an evolutionary stage well advanced from the ZAHB, whereas the latter are most likely on or near their ZAHB locations. We feel the safest way to approach the question of differing ages is to determine distances to clusters individually. This will require extensive efforts to determine trigonometric parallaxes of field halo stars and Baade-Wesselink analyses of cluster variables. We are currently obtaining optical and infrared photometry and radial velocities for RR Lyraes in M5 and M92, and one of the Cepheids in M5 as well. If the Baade-Wesselink method can also be applied to the Cepheids, we should be able to determine easily distances to those clusters that contain such stars.

b) *Period-Luminosity Relations*

i) *Optical P-L-A Relation and the Sandage Effect*

According to van Albada and Baker (1971) and Sandage, Katem, and Sandage (1981),

$$\log P = 11.497 - 0.68 \log M + 0.84 \log L - 3.48 \log T_{\rm eff} , \tag{10}$$

where the mass M and the luminosity L are in solar units. For two variables possessing identical masses and effective temperatures, the difference between them can be derived from equation (10), so that

$$\Delta \log P = 0.84\,\Delta \log L = -0.34\,\Delta M_{\rm bol}\,, \qquad (11)$$

or

$$\Delta M_{\rm bol} = -3\,\Delta \log P\,. \qquad (12)$$

It is apparent that the star with the longer period will also be more luminous. According to Sandage (1982), the difference between the mean periods of RR Lyraes of two globular clusters is related to the difference between their metallicities by

$$\Delta \log P = -0.116\,\Delta[\mathrm{Fe/H}]\,. \qquad (13)$$

If equation (12) is valid for the difference between the mean periods as well, then equations (12) and (13) can be combined to yield

$$\Delta M_{\rm bol} = 0.348\,\Delta[\mathrm{Fe/H}]\,. \qquad (14)$$

If there is a relationship between the amplitude of variation of an RR Lyrae star in the B filter, A_B, and its effective temperature $T_{\rm eff}$ that is independent of metallicity, then equation (10) can be written as a period-luminosity-amplitude (P-L-A) relation. Sandage (1982) derived such a relation, using the variables in M3 to obtain the A_B-$T_{\rm eff}$ relation, and obtained

$$\Delta M_{\rm bol} = M_{\rm bol}(\mathrm{var}) - M_{\rm bol}(\mathrm{M3}) = -3[\log P + 0.129A_B + 0.088]\,, \qquad (15)$$

for any RR Lyrae star with period P and amplitude A_B, assuming that it possesses the same mass as the variables in M3. This can be rewritten to yield the expected magnitude difference between any two variables with the same mass,

$$\Delta M_{\rm bol} = -3[\Delta \log P + 0.129\,\Delta A_B]\,. \qquad (16)$$

For variables within a given globular cluster, a quantity $\Delta \log P_A$ can be defined (Sandage 1988) as

$$\Delta \log P_A = \log P - (-0.129A_B - \mathrm{const}) = \log P + 0.129A_B + \mathrm{const}\,, \qquad (17)$$

where the constant is the appropriate one for the P-L-A relation of that cluster. It follows from this that

$$\Delta M_{\rm bol} = -3\,\Delta \log P_A \qquad (18)$$

for stars within that cluster.

Table 18 presents the $\Delta M_{\rm bol}$ values computed from equations (14) and (16), along with the observed values, of the program stars with respect to the most metal-poor star, X Ari. It can be seen that the magnitude differences predicted for the most metal-rich stars, RS Boo and TW Her, are much larger than the ones we derived using the Baade-Wesslink analyses. One possible explanation of this is that these field stars are not ZAHB stars, so the Sandage relations may not apply to them. Sandage (1988) has shown that the vertical (i.e., magnitude) width of the horizontal branch of a cluster depends upon its metallicity, the widths of metal-rich clusters being larger than those of metal-poor clusters. As an example, he lists the widths of 47 Tuc and M15 as about 0.55 and 0.25 mag, respectively. If RS Boo is indeed 0.55 mag brighter than a ZAHB star of the same metallicity, and X Ari is a typical metal-poor ZAHB star, then the value of $\Delta M_{\rm bol}$ between the two ZAHB stars would be 0.84 mag; if, on the other hand, X Ari is 0.25 mag brighter than a metal-poor ZAHB star, then the magnitude difference would be 0.59 mag, which is essentially that predicted by equation (14). We note, however, that the horizontal-branch width of 47 Tuc from Hesser *et al.* (1987), $\sim$0.4 mag, is smaller than that of Sandage (1988). Further, the assumption that RS Boo is the full width above the ZAHB may not be valid, since the other metal-rich program stars, particularly SW Dra and UU Vir, appear to be more luminous than it. Both of these effects tend to make the magnitude difference smaller.

An alternative explanation of this discrepancy between observed and predicted $\Delta M_{\rm bol}$ values is that the absolute values of the coefficients in equations (14)–(18) may be too large. Sandage (1988) has determined from plots of magnitude differences versus $\Delta \log P_A$ for various clusters that the observed coefficient of equation (18) varies from -1.7 for M15 to -2.6 for M4. All of his values are greater than -3. He plausibly attributes the smaller coefficients to a variation of mass for cluster variables such that the brighter, longer period variables are less massive than the less luminous variables. It will be shown in § IV*c* that both X Ari and VY Ser may be less massive than the others, which would explain why the observed magnitude differences are smaller than those predicted from equations that were derived assuming that the variables possessed the same mass. The possibility also exists that equation (10) is not valid for the coolest stars, which may explain both Sandage's smaller masses and those derived in § IV*c*. Since there is no reason to expect that variables in different clusters possessing the same temperature should show this mass variation, equations (14)–(16) may still be valid for them. On the other hand, it is also possible that the amplitude-temperature relation may depend upon the metallicity, in which case equations (14)–(18) may not be valid. Caputo (1988) has suggested this to explain the lack of variation in brightness between variables of widely different metallicities in ω Cen. If the behavior noted by Sandage (1988) is due to this effect, then the coefficients of equations (14)–(18) may be smaller, particu-

TABLE 18

MAGNITUDE DIFFERENCES, $\Delta M_{\rm bol}$, FOR PROGRAM STARS

Difference[a]	RS Boo	SW Dra	TW Her	DH Peg	VY Ser	UU Vir
[Fe/H]	1.7	1.4	1.7	1.4	0.4	1.5
ΔA_B	0.39	−0.04	0.43	...	−0.50	0.24
$\Delta \log P$	−0.23696	−0.05806	−0.21206	...	0.04007	−0.13644
$\Delta M_{\rm bol}$:						
Observed	0.29 ± 0.12	0.10 ± 0.11	0.24 ± 0.11	0.25 ± 0.15	0.15 ± 0.11	0.15 ± 0.11
Eq. (14)	0.59	0.49	0.59	0.49	0.14	0.52
Eq. (16)	0.56	0.19	0.47	...	0.07	0.32

[a] Difference of quantity: Δ Quan(var) = Quan(var) − Quan(X Ari).

larly for metal-poor stars, and the predicted magnitude differences would also be smaller. Clearly, the possible metallicity dependence of the amplitude-temperature relation and the temperature range in which equation (10) is valid must be investigated before any conclusions can be drawn concerning the Sandage effect.

ii) *Infrared Period-Luminosity Relation*

On the basis of observations of cluster RR Lyrae stars, it has been proposed (Fernley, Longmore, and Jameson 1986; Longmore, Fernley, and Jameson 1986; Fernley *et al.* 1987) that there is a simple relationship between the period and the luminosity of RR Lyrae variables in the infrared region that is independent of temperature and metallicity effects. According to Fernley, Longmore, and Jameson (1986).

$$\langle M_K \rangle = -2.2 \log P - 0.97 . \quad (19)$$

Figure 17 depicts the variation of $\langle M_K \rangle_i$ versus log P for our seven program stars (the value of log P of the c-type star DH Peg has been adjusted by 0.127 following Iben 1974). The relationship

$$\langle M_K \rangle_i = -1.92(\pm 0.23) \log P - 0.74(\pm 0.15) \quad (20)$$

was obtained by fitting these values by least squares, weighting the values according to the inverse square of their random errors. The quoted errors in equation (20) were determined from the residuals; the zero-point error also includes the systematic uncertainties of these stars. Why do the two expressions differ? The difference in slope may arise from their use of single-phase observations of globular cluster variables in the derivation of their relation (Longmore, Fernley, and Jameson 1986). Since the shorter period stars may have larger amplitudes in the K as well as in the optical region (§ II*d*), the use of random phase observations may introduce a bias into the derived relation. On the other hand, our results are vulnerable to the evolutionary effects discussed previously, and could also be affected by the errors in our synthetic color index versus temperature relations being a function of metallicity. The difference in the zero point is determined primarily by the differences in the calibrating stars. Longmore, Fernley, and Jameson (1986) used Baade-Wesselink results (using an unknown number of color indices and magnitudes) for V445 Oph, X Ari, and VY Ser. Their results for the latter two stars are brighter than our results by 0.24 and 0.27 mag, respectively. All in all, the agreement must be considered good, and an improved version should be obtainable.

This possibly universal infrared period-luminosity relation for RR Lyrae stars has important applications in the determination of distances to star systems, such as distant globular clusters and the Magellanic Clouds, whose RR Lyrae stars may be bright enough to determine their $\langle K \rangle$ values but not bright enough to derive accurate K light curves necessary for the Baade-Wesselink method. More important, since the uncertainty in $\langle K \rangle$ due to an uncertainty in $E(B-V)$ is much smaller than that in $\langle V \rangle$, such a relation should be especially useful in determining the distances to heavily reddened globular clusters and to the Galactic center, as already discussed by Fernley *et al.* (1987). Clearly, the relation warrants further investigation.

c) *Masses and Kinematics*

Since the $T_{\rm eff}$ values derived from the static model atmospheres around phase 0.0 are probably not accurate, the values of $\langle T_{\rm eff} \rangle$ are probably not very accurate either. However, the mean radius, $\langle R \rangle$, derived from the integration of the radial velocity curve,

$$\langle R \rangle = \frac{1}{n} \sum_{i=1}^{n} R(\phi_1) = \frac{1}{n} \sum_{i=1}^{n} [R(\phi_0) + \Delta R(\phi, \phi_0)]$$
$$= R(\phi_0) + \langle \Delta R(\phi, \phi_0) \rangle , \quad (21)$$

should not be affected by this [note: $R(\phi_0) = R(\phi_n)$]. There-

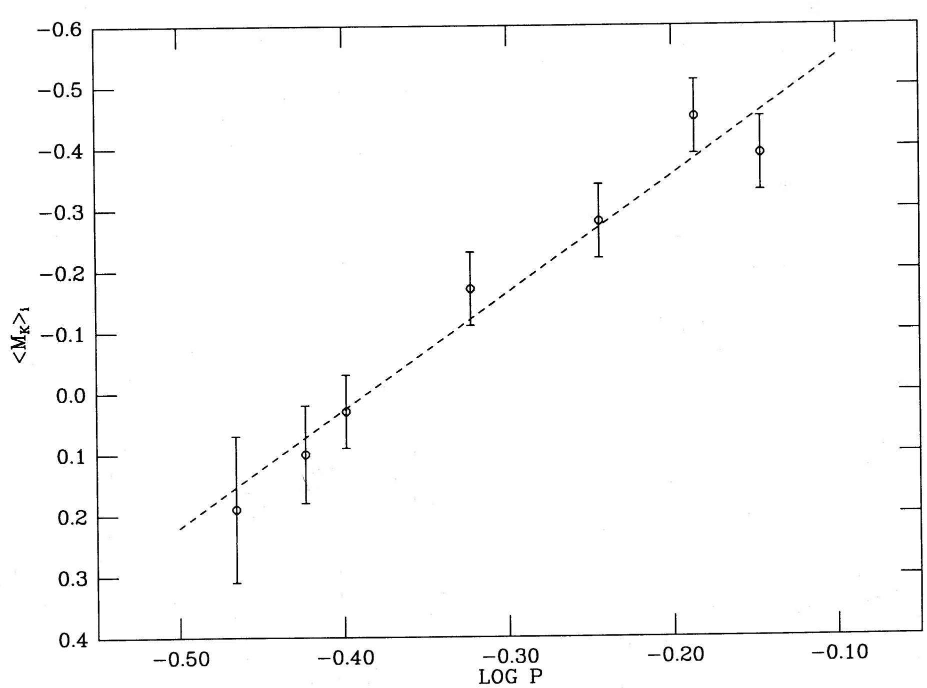

FIG. 17.—Intensity-averaged $\langle M_K \rangle$ versus log P. Error bars depict random uncertainties only; line represents eq. (20).

TABLE 19

GALACTIC VELOCITIES OF PROGRAM STARS

Star	U	V	W	V_{sp}	V_{rf}
X Ari	-25 ± 4	-235 ± 9	-9 ± 5	237 ± 9	31
RS Boo	-28 ± 16	50 ± 16	-13 ± 7	59 ± 16	272
SW Dra	46 ± 16	-51 ± 10	-15 ± 5	70 ± 13	176
TW Her	-45 ± 9	-1 ± 7	-17 ± 9	48 ± 9	224
DH Peg	42 ± 6	-44 ± 2	30 ± 4	68 ± 4	184
VY Ser	234 ± 62	-52 ± 87	55 ± 67	246 ± 64	293
UU Vir	86 ± 11	-51 ± 14	-27 ± 9	104 ± 12	191

fore, the mass M of an RR Lyrae star can be determined by rewriting equation (10) to obtain

$$\log M = \frac{-1.592 - \log P - 0.03 \log L + 1.74 \log R}{0.68}, \quad (22)$$

where the quantities M, L, and R are all in solar units. Table 16 presents the masses of the program stars generated from this equation. The values of the metal-rich stars are in good agreement with the values of Taam, Kraft, and Suntzeff (1976), and also with the value of 0.55 $M_{\odot}$ derived for double-mode RR Lyrae stars in the Oosterhoff I globular cluster M3 by Cox, Hodson, and Clancy (1983). However, the values of X Ari and VY Ser do not agree with that derived by Cox, Hodson, and Clancy (1983) for the double-mode pulsators in the metal-poor cluster M15, 0.65 $M_{\odot}$. We do not understand why this discrepancy exists. One possibility is that, as we have argued, we are comparing rather different stars: field variables that are near the ZAHB and cluster variables that are significantly evolved. Another possibility was pointed out by van Albada and Baker (1971): their pulsation equation may not be valid for $T_{\mathrm{eff}} <$ 6000 K because of convection effects; since both X Ari and especially VY Ser are relatively cool, then equation (22) might not yield accurate results for them. As was noted earlier, this could also explain why the coefficients of Sandage (1987) were not -3, an effect that was most pronounced for the metal-poor clusters. It is clear that the reliability of equation (22) at cooler temperatures needs to be further investigated. Also, hotter metal-poor variables should be analyzed to see whether their derived masses are in better agreement with that from the double-mode pulsators.

Table 19 lists the derived space velocities of the program stars, which were computed from the equations of Johnson and Soderblom (1987) using the proper motions from Hawley *et al.* (1986) and the distances and systemic velocities derived in our analyses. We note that our U values are positive in the direction of the Galactic anticenter, rather than toward the center as in Johnson and Soderblom (1987). No attempt was made to compute the Galactic orbits for these stars. The results of Saio and Yoshii (1979) can be used, however, as a guide to the different behavior of the orbits of these stars. It can be seen, for example, that X Ari has a highly eccentric orbit ($\epsilon = 0.88$, where ϵ is the eccentricity of Eggen, Lynden-Bell, and Sandage 1962) typical of a halo star, as expected from its metallicity, while the more metal-rich stars possess more circular orbits, with $\epsilon < 0.38$, with one (TW Her) having a very circular orbit ($\epsilon = 0.07$). Saio and Yoshii (1979) did not compute a Galactic orbit for VY Ser because its space velocity was so extreme; however, it can be seen from Table 19 that our estimate of its velocity is much smaller, probably because of our determination that it is less luminous and hence closer to Earth, so that its velocity may not be so extreme after all.

In conclusion, we again emphasize the importance of reducing the systematic biases present in all of the different techniques of deriving $\langle M_V \rangle_{\mathrm{RR}}$, so that this quantity can be more precisely determined. The possibility that this quantity is closer to 0.85 mag than to 0.60 mag should be seriously considered, since there seems to be considerable evidence to support this value. It also appears that there is a dependence of $\langle M_{\mathrm{bol}} \rangle$ upon [Fe/H] such that the metal-poor stars are more luminous, although this trend may not be as strong as that predicted by Sandage (1982). The validity of the application of the pulsation equation of van Albada and Baker (1971) to the cooler RR Lyraes, particularly the metal-poor ones, should be investigated to see whether this explains the discrepancies in masses noted here and in the work of Sandage (1988); the metallicity dependence, if any, of the amplitude-temperature relation should also be ascertained. Finally, we again stress the importance of the universal infrared period-luminosity relation noted originally by Longmore, Fernley, and Jameson (1986) and urge that the possibility and ramifications of such a relation should be fully explored.

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REFERENCES

Balona, L. A. 1977, *M.N.R.A.S.*, **178**, 231.
Barnes, T. G., III, and Hawley, S. L. 1986, *Ap. J. (Letters)*, **307**, L9.
Beardsley, W. R., Gatewood, G., and Kamper, K. W. 1974, *Ap. J.*, **194**, 637.
Beck, J. 1988, M.S. thesis, University of North Carolina.
Bolte, M. 1987, *Ap. J.*, **319**, 760.
Burki, C., and Meylan, G. 1986, *Astr. Ap.*, **156**, 131.
Burstein, D. 1987, private communication.
Burstein, D., Faber, S. M., and Gonzalez, J. J. 1986, *A.J.*, **91**, 1130.
Burstein, D., and Heiles, C. 1982, *A.J.*, **87**, 1165.
Butler, D. 1975, *Ap. J.*, **200**, 68.
Caputo, F. 1988, *Astr. Ap.*, **189**, 70.
Caputo, F., Martinez Roger, C., and Paez, E. 1987, *Astr. Ap.*, **183**, 228.
Carney, B. W. 1979, *Ap. J.*, **233**, 211.
———. 1983, *A.J.*, **88**, 623.
Carney, B. W., and Jones, R. 1983, *Pub. A.S.P.*, **95**, 246.
Carney, B. W., and Latham, D. W. 1984, *Ap. J.*, **278**, 241 (Paper I).
———. 1987, *A.J.*, **93**, 116.
Ciardullo, R. S., and Demarque, P. 1977, *Trans. Yale Univ. Obs.*, Vol. **33**.
Clube, S. V. M., and Dawe, J. A. 1978, in *IAU Colloquium 80, The HR Diagram*, ed. A. G. Davis Philip and D. S. Hayes (Dordrecht: Reidel), p. 53.
Cohen, J. G., and Gordon, G. A. 1987, *Ap. J.*, **318**, 215.
Cox, A. N., Hodson, S. W., and Clancy, S. P. 1983, *Ap. J.*, **266**, 94.
Eggen, O. J. 1977, *Ap. J.*, **215**, 812.
Eggen, O. J., Lynden-Bell, D., and Sandage, A. R. 1962, *Ap. J.*, **136**, 748.

Eggen, O. J., and Sandage, A. R. 1959, *M.N.R.A.S.*, **119**, 255.
Elias, J. H., Frogel, J. A., Matthews, K., and Neugebauer, G. 1982, *A.J.*, **87**, 1029.
Fernley, J. A., Longmore, A. J., and Jameson, R. F. 1986, in *Lecture Notes in Physics*, Vol. **274**, *Stellar Pulsation*, ed. A. N. Cox, W. M. Sparks, and S. G. Starrfield (Berlin: Springer), p. 239.
Fernley, J. A., Longmore, A. J., Jameson, R. F., Watson, F. G., and Wesselink, T. 1987, *M.N.R.A.S.*, **226**, 927.
Hanson, R. B. 1979, *M.N.R.A.S.*, **186**, 875.
———. 1980, in *IAU Symposium 85, Star Clusters*, ed. J. E. Hesser (Dordrecht: Reidel), p. 71.
Hawley, S. L., Jeffreys, W. H., Barnes, T. G., III, and Wan Lai. 1986, *Ap. J.*, **302**, 626.
Heasley, J. N., and Christian, C. A. 1986, *Ap. J.*, **307**, 738.
Heck, A., and Lakaye, J. M. 1978, *M.N.R.A.S.*, **184**, 17.
Heintz, W. D. 1984, *Pub. A.S.P.*, **96**, 557.
Hesser, J. E., Harris, W. E., VandenBerg, D. A., Allwright, J. W. B., Shott, R., and Stetson, P. B. 1987, *Pub. A.S.P.*, **99**, 739.
Iben, I., Jr. 1974, *Ann. Rev. Astr. Ap.*, **12**, 215.
Jameson, R. F. 1986, *Vistas Astr.*, **29**, 17.
Jenkins, L. F. 1963, *General Catalogue of Trigonometic Stellar Parallaxes* (New Haven: Yale University Observatory).
Johnson, D. R. H., and Soderblom, D. R. 1987, *A.J.*, **93**, 864.
Jones, R. V. 1987, Ph.D. thesis, University of North Carolina, Chapel Hill.
———. 1988, *Ap. J.*, **326**, 305 (Paper IV).
Jones, R. V., Carney, B. W., and Latham, D. W. 1988, *Ap. J.*, **326**, 312 (Paper V).
Jones, R. V., Carney, B. W., Latham, D. W., and Kurucz, R. L. 1987*a*, *Ap. J.*, **312**, 254 (Paper II).
———. 1987*b*, *Ap. J.*, **314**, 604 (Paper III).
Kanyo, S. 1980, *Inf. Bull. Variable Stars*, No. 1832.
Kukarkin, B. V., *et al.* 1970, *General Catalog of Variable Stars* (3d ed.; Moscow: Sternberg State Astronomical Institute of Moscow State University).
Laird, J. B., Carney, B. W., and Latham, D. W. 1988, *A.J.*, in press (LCL).
Landolt, A. U. 1983, *A.J.*, **88**, 439.
Latham, D. W. 1985, in *IAU Colloquium, Stellar Radial Velocities*, ed. A. G. Davis Philip and D. W. Latham (Schenectady: Davis), p. 21.
Latham. D. W., Mazeh, T., Carney, B. W., McCroskey, R. E., Stefanik, R. P., and Davis, R. J. 1988, *A.J.*, submitted.
Longmore, A. J., Fernley, J. A., and Jameson, R. F. 1986, *M.N.A.S.*, **220**, 279.
Longmore, A. J., Fernley, J. A., Jameson, R. F., Sherrington, M. R., and Frank, J. 1985, *M.N.R.A.S.*, **216**, 873.
Lu, P. K., Demarque, P., van Altena, W., McAlister, H., and Hartkopf, W. 1987, preprint.
Lub, J. 1977, *Astr. Ap. Suppl.*, **29**, 345.
———. 1979, *A.J.*, **84**, 383 (Lub).
———. 1987, in *Stellar Pulsation*, ed. A. N. Cox, W. M. Sparks, and S. G. Starrfield (Berlin: Springer), p. 218.
Lutz, T. E., and Kelker, D. H. 1973, *Pub. A.S.P.*, **85**, 573.
Manduca, A., Bell, R. A., Barnes, T. G., III, Moffett, T. J., and Evans, D. S. 1981, *Ap. J.*, **250**, 312.
McDonald, L. H. 1977, Ph.D. thesis, University of California, Santa Cruz.
McNamara, D. H., Helm, T. M., and Wilcken, S. K. 1970, *Pub. A.S.P.*, **82**, 293.
Mould, J. R. 1976, *Ap. J.*, **210**, 402.
Murray, C. A. 1983, *Vectoral Astrometry* (Bristol: Hilger), p. 297.
Oke, J. B. 1966, *Ap. J.*, **145**, 468.
Oke, J. B., and Bonsack, S. J. 1960, *Ap. J.*, **132**, 417.
Oke, J. B., Giver, L. P., and Searle, L. 1962, *Ap. J.*, **136**, 393.
Oosterhoff, P. Th. 1946, *Bull. Astr. Inst. Netherlands*, **10**, 103 (No. 369).
Penny, A. J., and Dickens, R. J. 1986, *M.N.R.A.S.*, **220**, 845.
Peterson, R. C. 1981, *Ap. J.*, **244**, 989.
Preston, G. W. 1959, *Ap. J.*, **130**, 507.
Preston, G. W., and Paczyński, B. 1964, *Ap. J.*, **140**, 181.
Richer, H. B., and Fahlman, G. G. 1987, *Ap. J*, **316**, 189.
Saio, H., and Yoshii, Y. 1979, *Pub. A.S.P.*, **91**, 553.
Sandage, A. 1969, *Ap. J.*, **158**, 1115.
———. 1970, *Ap. J.*, **162**, 841.
———. 1981, *Ap. J.*, **248**, 161.
———. 1982, *Ap. J.*, **252**, 553.
———. 1988, preprint.
Sandage, A., Katem, B., and Sandage, M. 1981, *Ap. J. Suppl.*, **46**, 41.
Sanford, R. 1949, *Ap. J.*, **109**, 208.
Savage, B. D., and Mathis, J. S. 1979, *Ann. Rev. Astr. Ap.*, **17**, 73.
Sneden, C. 1985, in *Production and Distribution of* C, N, *and* O *Elements*, ed. I. J. Danziger, F. Matteucci, and K. Kjär (Garching: ESO), p. 1.
Stothers, R. 1983, *Ap. J.*, **274**, 20.
Strugnell, P., Reid, N., and Murray, C. A. 1986, *M.N.R.A.S.*, **220**, 413 (SRM).
Sturch, C. 1966, *Ap. J.*, **143**, 774.
Sweigart, A. V. 1987, *Ap. J. Suppl.*, **65**, 95.
Szeidl, B. 1976, in *IAU Colloquium 29, Multiple Periodic Variable Stars*, ed. W. S. Fitch (Dordrecht: Reidel), p. 133.
Taam, R. E., Kraft, R. P., and Suntzeff, N. 1976, *Ap. J.*, **207**, 201.
Tifft, W. G. 1964, *Ap. J.*, **139**, 451.
van Albada, T. S., and Baker, N. 1971, *Ap. J.*, **169**, 311.
VandenBerg, D. A. 1983, *Ap. J. Suppl.*, **51**, 29.
———. 1985, *Ap. J. Suppl.*, **58**, 711.
VandenBerg, D. A., and Bell, R. A. 1985, *Ap. J. Suppl.*, **58**, 561.
Varsavsky, C. M. 1960, *Ap. J.*, **131**, 623.
Wan, L., Mao, Y-Q., and Ji, D-S. 1981, *Ann. Shanghai Obs.*, No. 2.
Woolley, R., and Aly, K. 1966, *Roy. Obs. Bull.*, No. 114.
Wyatt, W. F. 1985, in *IAU Colloquium 88*; *Stellar Radial Velocities*, ed. A. G. Davis Philip and D. W. Latham (Schenectady: Davis), p. 123.
Zinn, R. 1975, *Ap. J.*, **293**, 424.
Zinn, R., and West, M. J. 1984, *Ap. J. Suppl.*, **55**, 45.

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