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21 CENTIMETER LINE WIDTH DISTANCES OF CLUSTER GALAXIES AND THE VALUE OF H_0

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ABSTRACT

Locally calibrated blue and infrared Tully-Fisher (TF) relations are applied to an essentially (i.e., 82%) complete sample of 81 Sab-Sm galaxies which are bona fide members of the Virgo Cluster. Because of the proximate completeness of the sample a nearly *unbiased* Virgo modulus of $(m - M)^0 = 31.60 \pm 0.15$ can be derived. The result is in perfect agreement with independent recent determinations which gives additional justification to the adopted local calibrators. The distance modulus from the blue TF relation carries higher weight because infrared H magnitudes are available for only 25 Virgo members. The remaining H magnitudes are derived from a well defined $(B-H)/\log \Delta v_{21}$ relation; nevertheless, the 25 members suffice to show that the bTF and iTF relations give almost identical distance moduli from any selected Virgo subsample. The intrinsic scatter about the two TF relations is $\sigma_M = 0.7$ mag. This is considerably larger than the observed scatter in the UMa Cluster and in 10 more distant clusters for which B and H magnitudes and 21 cm line widths are available; their reduced scatter discloses that the hitherto investigated samples merely contain the brightest spirals at any given line width. Distance determinations of these clusters can therefore only be achieved by fitting the upper envelopes of their TF relations onto the blue and infrared upper envelopes of the Virgo Cluster. The resulting distances define a linear expansion law with a small scatter. The linearity of the expansion can be extended for the range from $v \approx 200$ km s^{-1} out to $v \approx 20{,}000$ km s^{-1} using external data. The present cluster data require a Hubble constant of $H_0 = 56.6 \pm 0.9$ km s^{-1} Mpc^{-1} which implies through first-ranked cluster galaxies $51 < H_0(\text{global}) < 56$ at very large distances; the external error of these values is probably less than 20%. A search for Local Group motion toward the microwave background dipole remains inconclusive.

Subject headings: galaxies: clustering — galaxies: distances — radio sources: 21 cm radiation — cosmology

I. INTRODUCTION

Ever since the log (21 cm line width)/absolute magnitude relation was introduced to determine distances of spiral galaxies (Tully and Fisher 1977; hereafter TF), it has yielded discrepant distances and hence contradictory results on the Hubble constant H_0 (e.g., Sandage and Tammann 1976; TF; de Vaucouleurs 1983). The situation has not been improved by introducing infrared H magnitudes (Aaronson, Huchra and Mould 1979; Sandage and Tammann 1976) instead of the originally used blue magnitudes, even though the former minimize the effects of uncertainties in the internal absorption corrections. Differences in the adopted distances of calibrating galaxies explain a divergence in H_0 between various authors of the order of 20% but do not account for factors of up to 2 which occur in the literature. The idea that the dilemma is caused by selection effects in the mostly flux-limited samples is becoming more and more accepted (Bottinelli *et al.* 1986, 1987; Feast 1987; Fröhlich 1987; Giraud 1985, 1987*a*, *b*; Kraan-Korteweg, Cameron, and Tammann 1986; Teerikorpi 1975, 1984).

The patchy distribution of galaxies, however, denies an analytical correction to the selection effects. In addition, the value of the intrinsic magnitude scatter σ_M of the relation—which determines the size of the selection bias—is disputed. While some authors claim that the intrinsic scatter is as small as 0.3 mag, Rubin *et al.* (1985), using first-class rotation curves, show that it is rather 0.85 mag for a fixed maximum rotation velocity. A lower limit of $\sigma_M > 0.4$ mag follows from an observed $\sigma(\log \text{mass}) = 0.15$ at $v_{\max} = \text{const.}$, even if the mass-to-light ratio were the same for all spirals (van Albada 1987). The latter two values of σ_M refer to blue magnitudes, but also for the infrared Rubin *et al.* (1985) find a minimum scatter of $\sigma_M \approx 0.7$ mag. A scatter of this size causes most severe selection effects in either color. It implies that a distant galaxy which is 3 σ brighter than the mean and which therefore enters preferentially any flux-limited catalog will be 2.1 mag brighter than the average, nearby calibrators of equal line width.

The fact that the TF relation is not independent of morphological type poses an additional difficulty (Roberts 1978; Rubin *et al.* 1980, 1985; Meisels 1983; Giraud 1986, 1987*a*; Kraan-Korteweg, Cameron, and Tammann 1986; but see de Vaucouleurs *et al.* 1982). It is difficult to distinguish type segregation from a nonlinearity of the TF relation which has been proposed instead in the case of the infrared TF relation by Aaronson and Mould (1983) and by Aaronson *et al.* (1986). That type segregation is present will be shown below (§ IV) from a nearly complete sample of Virgo Cluster members and also in a forthcoming paper on field galaxies (Kraan-Korteweg, Cameron, and Tammann 1988). The type segregation requires, of course, great caution when nearby calibrators are compared to galaxy samples of different type mixture.

Due to the intrinsic scatter—and, to a lesser extent to the type dependence—it is not surprising that quite different slopes and zero points of the TF relation are derived from different samples. More recent determinations are: field galaxies in the blue (Giraud 1985; Bottinelli *et al.* 1986) and infrared (Aaronson *et al.* 1982), galaxies in groups in the blue (Richter

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and Huchtmeier 1984; Huchtmeier and Richter 1987) and in the infrared (Aaronson and Mould 1983), as well as Virgo galaxies in the blue (Richter and Huchtmeier 1984; Bottinelli *et al.* 1987; Hoffman, Helou, and Salpeter 1987, Hoffman *et al.* 1987) and in the infrared (Mould, Aaronson and Huchra 1980). The different results are clearly an effect of different sample definitions. Samples of field galaxies are generally cut by apparent blue magnitude, but are frequently also influenced by the availability of 21 cm line widths and, in the case of the infrared TF relation, of H magnitudes. For Virgo Cluster samples the cut in apparent magnitude corresponds to an absolute magnitude limit which still leaves room for a bias in line width (Teerikorpi 1987; Bottinelli *et al.* 1987). Samples of galaxies in various groups combine the selection effects of field and cluster galaxies, unless the different groups are sampled to a fixed limit in M_B. The field sample of Huchtmeier and Richter (1987) has particularly favorable statistical properties because it is *complete* out to a reasonably well-defined *distance* limit, i.e., $v_0 < 500$ km s^{-1}. Although the sample is unique as to the range in line widths, luminosities, and morphological types, its small volume and the low space density of luminous spirals lend relatively little statistical weight to the brightest galaxies which prevail in most field and cluster samples.

The demonstration of selection effects in flux-limited samples is easy. For instance, the field galaxies with infrared H photometry available from Aaronson *et al.* (1986) yield TF relations which shift to brighter magnitudes with increasing distance (Kraan-Korteweg, Cameron and Tammann 1986). This is clearly an effect of the Malmquist bias which denies the sample the less luminous galaxies at larger distances. Neglecting the effect leads to a value of H_0 which increases continuously with distance (Tammann 1987) which, of course, contradicts the linear Hubble expansion law required by brightest galaxies in groups and clusters and by supernovae Ia.

To overcome at least the Malmquist effect of flux-limited samples the use of the inverse TF relation was suggested with the absolute magnitude as the independent variable and log Δv_{21} as the dependent one (Kraan-Korteweg, Cameron, and Tammann 1986). The condition for a successful application of this method is that the sample is chosen independent of line width. This condition is not fulfilled usually as will be discussed in § V and for field galaxies in a subsequent paper (Kraan-Korteweg, Cameron, and Tammann 1988). A severe drawback of this method is that any particular galaxy sample which is flux limited will have its individual slope and zero point. This makes the calibration of the inverse TF relation of such a sample by means of (differently selected) nearby calibrators extremely difficult.

A breakthrough in the analysis of the TF relation for field galaxies has been reached by Sandage (1988*b*). From the combination of galaxy samples which are cut by different apparent magnitude limits he could show (1) that the samples are compatible only if the expansion field *is* linear in good approximation, (2) that the intrinsic magnitude dispersion at a given line width is as large as 1.0 mag in the blue and 0.9 mag in the infrared TF relation, and (3) that the data require a Hubble constant in the range of $50 < H_0 < 70$ km s^{-1} Mpc^{-1}.

Aaronson (1987) has emphasized that the result from the infrared TF relation of $H_0 \approx 90$ km s^{-1} Mpc^{-1} is not based on field galaxies, but relies entirely on cluster galaxies in Virgo and 10 more distant clusters. In this paper we reinvestigate these data in order to decide whether there is a real difference of H_0 for field and cluster galaxies, or whether the cluster data rather support Sandage's (1988*b*) results from field galaxies. The present analysis has become possible by a new, *complete* sample of Virgo Cluster members.

We proceed as follows. In § II the 13 local calibrators of type Sb to Sm are introduced; they are used to determine the zero point *and* the slope of the mean blue TF relation (bTF) and of the infrared (H) TF relation (iTF). A new, strong correlation between 21 cm line width and color ($B-H$) is discussed in § III. The corresponding relation from the local calibrators, which is confirmed by three additional samples, is used to calculate H magnitudes for a number of Virgo Cluster galaxies for which only B magnitudes and 21 cm line widths existed so far. In § IV an almost *complete* sample of Virgo members, which matches the local calibrators in type and line width distribution, yields a bTF and iTF distance determination of the cluster; the sample gives an independent determination of the slope and exhibits the type segregation of the TF relation as well as the full scatter which amounts to $\sigma_M = 0.7$ mag for the bTF *and* for the iTF relation. The foreground-background problem of the cluster is discussed and found not to be responsible for the scatter. In § V the available data on the Ursa Major Cluster and 10 more distant clusters are investigated. These clusters show a magnitude scatter of typically only $\sigma_M \approx 0.4$ mag, which proves that they lack the fainter members at any given line width. This precludes a comparison with the *mean* TF relation from the Virgo cluster. An *upper-envelope* fit of the bTF and iTF relation is used instead; the resulting distances are tabulated. A Hubble constant of $H_0 = 57 \pm 9$ km s^{-1} Mpc^{-1} is derived in § VI from the Virgo Cluster as well as from the more distant clusters. This is in excellent agreement with Sandage's (1988*a*, *b*) results for *field* galaxies. Hubble ratios H_i for individual clusters are tested against the microwave background (MWB) dipole. These results are discussed in § VII as well as the global value of H_0. The conclusions are summarized in the last section.

II. THE TF RELATION FROM LOCAL CALIBRATORS

a) The Data

For the calibration of the TF relation all local Sb–Sm galaxies are used with independently known distances, m_B and m_H magnitudes, and with inclinations $i \geq 45°$. The inclination restriction is conventionally imposed in order to minimize the line width correction to edge-on orientation. The price of the inclination restriction is large internal-absorption corrections in the case of the bTF. But as these corrections are applied consistently to all calibrating galaxies and to the cluster galaxies, any existing errors will be of second order only.

Good (inclination-corrected) 21 cm line widths at the 20% intensity level are available for 13 galaxies which meet the above conditions. The line width data are adopted from Aaronson *et al.* (1982). A comparison with the values listed in Huchtmeier *et al.* (1983) shows an insignificant mean deviation of $\Delta \log \Delta v_{21} = 0.002$ with a scatter of $\sigma_{\log \Delta v} = 0.02$. The scatter is partly due to observational errors of the 21 cm profile and partly to uncertainties of the inclination. It translates into a magnitude dispersion of only $\sigma_{M_B} = 0.13$ mag and $\sigma_{M_H} = 0.20$ mag for an individual galaxy.

NGC 224 (M31) is retained as calibrator although some doubts have been cast on its H photometry (Manousoyanniki and Chincarini 1986). Its exclusion would have a negligible effect on the following conclusions.

The relevant data are set out in Table 1 for the 13 calibrating

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TABLE 1

THE CALIBRATORS OF THE TULLY FISHER RELATION

Name (1)	Type (2)	$(m - M)^0$ (3)	$M_{B_T}{}^{o,i}$ (4)	M_H (5)	$\log \Delta v_{21}$ (6)
NGC 224	Sb	24.2	−21.08	−23.29	2.75
NGC 598	Sc	24.4	−18.71	−20.02	2.40
NGC 247	Sc	26.8	−17.92	−19.11	2.36
NGC 253	Sc	27.5	−20.12	−22.76	2.64
NGC 7793	Sd	27.5	−18.25	−19.61	2.38
NGC 2366	SBm	27.8	−16.77	−16.98	2.10
NGC 2403	Sc	27.8	−19.51	−21.35	2.48
IC 2574	Sm	27.8	−17.11	−17.73	2.14
NGC 4236	Sd	27.8	−18.45	−18.72	2.30
NGC 3031	Sb	28.7	−21.69	−24.32	2.73
Ho IV	Sm	29.2	−16.44	−16.71	2.03
NGC 5204	Sd	29.2	−17.91	−18.90	2.19
NGC 5585	Sd	29.2	−18.26	−19.05	2.32

Col. (2). Abbreviated galaxian type from the RSA (Sandage and Tammann 1981) or Kraan-Korteweg 1986.

Col. (3). True distance modulus (Tammann 1987).

Col. (4). Absolute *B* magnitude calculated from the distance modulus (col. [3]) and the corrected apparent B_T magnitudes (as listed in Kraan-Korteweg 1986). The galactic intrinsic absorption corrections follow the precepts of the RSA.

Col. (5). Same as col. (4) for *H* magnitudes. The corrected apparent *H* magnitudes are taken from Aaronson *et al.* (1982).

Col. (6). Logarithm of the 21 cm line width at the 20% intensity level and corrected for inclination as given by Aaronson *et al.* (1982).

galaxies. The adopted distances represent values of minimum controversy being averages of various distance determinations published in the literature over the last decade (cf. Tammann 1987). The members of the NGC 2403 group (NGC 2366, IC 2574, and NGC 4236) and of the M101 group (NGC 5204 and NGC 5585) have relatively few direct distance determinations; they are assumed to lie at the distance of NGC 2403 and M101, respectively. M101 itself cannot be used here because of its low inclination.

The 13 Sb–Sm calibrators are well distributed over a wide range of line widths ($100 < \Delta v_{21} < 600$ km s^{-1}) and absolute magnitude (-16.5 mag $< M_{B_T}{}^{oi} < -22.0$ mag and -16.5 mag $< M_H < -24.5$ mag).

The mean distance zero point determined by the 13 calibrators agrees reasonably with previous authors. The six calibrators in common with those used by Sandage and Tammann (1974*a*, *b*) are here brighter by only $\Delta(m - M)^0 = 0.22 \pm 0.19$ mag on average. The nine calibrators used by de Vaucouleurs (1979) for his distance scale are less distant on average by $\Delta(m - M)^0 = 0.39 \pm 0.13$ mag, and the three calibrators used by Aaronson *et al.* (1986) are brighter in this paper by $\Delta M = 0.18 \pm 0.05$ mag. These modest offsets demonstrate that resulting distance scales should agree to within 10%–20%, whatever calibration is used. It is not possible to specify the external error of the zero point adopted here, but the fact that the distances in Table 1 incorporate the progress of many authors during the last decade makes it improbable that the present zero point is off by more than 0.2 mag (amounting to 10% in distance).

b) *The Blue Tully-Fisher Relation*

A TF relation of the form

$$M_{B_T}{}^{oi} = a \cdot \log \Delta v_{21} + b \tag{1}$$

yields with the corresponding data in Table 1 the following least squares solution (cf. Fig. 1):

$$M_{B_T}{}^{oi} = -6.69 \cdot \log \Delta v_{21} - (2.77 \pm 0.10) . \tag{2}$$

The high correlation coefficient of $r = 0.97$ implies that the inverse regression—with $M_{B_T}{}^{oi}$ as the independent variable—leads to a very similar result, i.e., $a = -7.06$ and $b = -(1.90 \pm 0.11)$. The two solutions differ by only 0.15 mag at the extreme line widths of $\log \Delta v_{21} = 2.0$ and 2.8.

In the following we will adopt the slope of equation (2) for the bTF. In § IV we find it in agreement with the slope deter-

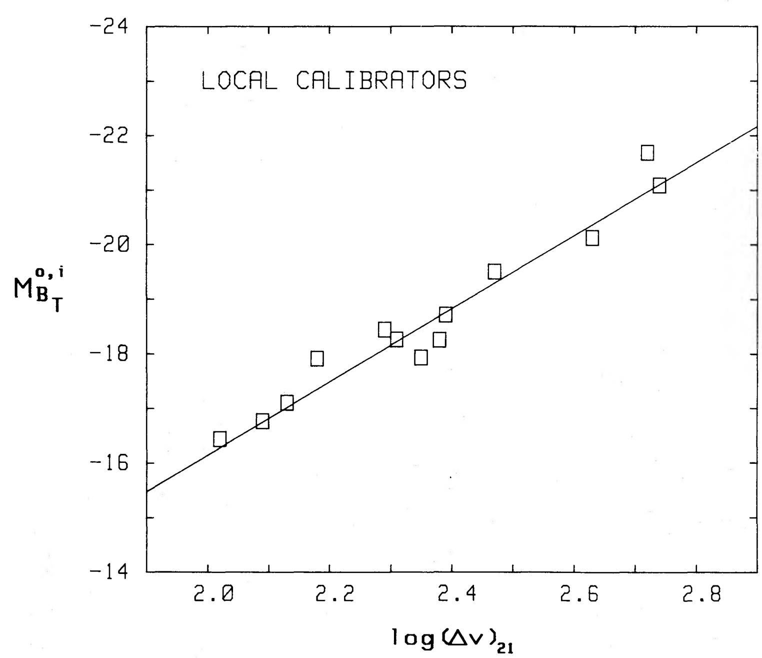

FIG. 1.—The blue Tully-Fisher relation for 13 calibrating galaxies from Table 1. The linear regression from eq. (2) is shown.

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mined from Virgo members. Within the statistical errors the value also compares favorably with earlier determinations, e.g., $a = -6.25$ (Tully and Fisher 1977) and $a = -6.88$ (Sandage and Tammann 1976). Based on a sample of spiral and Im galaxies with less restrictive inclinations Richter and Huchtmeier (1984) found $a = -7.17$. The steeper slope of $a = -7.60$, recently determined by Huchtmeier and Richter (1987) for a distance-limited sample, is so heavily weighted toward Im galaxies that it is not directly comparable. If this sample is restricted, however, to Sab–Sm galaxies with $i \geq 45°$ the slope becomes $a = -6.68$! The considerably flatter slope of $a = -5.0$ adopted by de Vaucouleurs and his collaborators (de Vaucouleurs *et al.* 1982; Bottinelli *et al.* 1980, 1983, 1986, 1987) is almost entirely explained by the different internal absorption correction and to a much lesser extent by their large correction on Δv_{21} for turbulent velocities; their internal absorption correction is nearly independent of galaxian type, contrary to the precepts of the Revised Shapley-Ames Catalog (RSA) and more recent *IRAS* data (de Jong and Brink 1987). The very steep slope of $a \approx -10$ by Rubin *et al.* (1980, 1982, 1985) finds its explanation in the specific sample selection which was optimized to determine first-class rotation curves and favors very luminous galaxies at large distances. Hence this sample should not be compared with the present calibrators or any other differently chosen sample.

The magnitude scatter of the 13 calibrators about equation (2) amounts to only $\sigma_{M_B} = 0.38$ mag. It will be shown in § IV that this value is fortuitously small and that the true scatter of the bTF relation, as revealed by the complete Virgo Cluster sample, is rather $\sigma_{M_B} \approx 0.7$ mag. We consider the small scatter of the local calibrators as a statistical fluctuation. In fact, the probability amounts to 15% that the value of 0.38 mag from only 13 data points lies within the distribution of $\sigma_{M_B} \approx 0.7$ mag.

c) *The Infrared Tully-Fisher Relation*

Using the calibrators of Table 1 and their H magnitudes gives a least- squares solution for the iTF relation of

$$M_H = -10.12 \cdot \log \Delta v_{21} + (4.12 \pm 0.14) . \quad (3)$$

As for the bTF relation, the inverse regression differs only slightly from the above solution: $a = -10.56$ and $b = +(5.15 \pm 0.14)$. The difference of the two solutions at log $\Delta v_{21} = 2.0$ and 2.8, respectively, is the same as in the blue ($\Delta M = 0.15$ mag). The scatter about the mean relation is somewhat larger here, i.e., $\sigma_M = 0.50$ mag. This scatter agrees prima facie with the value derived by Aaronson and Mould (1983) for differently defined samples, viz. $\sigma_M = 0.52$ mag for a specific sample of field galaxies, $\sigma_M = 0.42$ mag for galaxies in groups, and $\sigma_M = 0.45$ mag for bright Virgo galaxies. However, as will be shown in § IV for a complete sample of Virgo spirals, the true scatter is likely to be as high as 0.7 mag. The difference between $\sigma_M = 0.50$ mag, which rests on only 13 galaxies, and the true value of $\sigma_M = 0.7$ mag is statistically insignificant. A χ^2 test shows that a parent distribution with $\sigma_M = 0.7$ mag yields in 25% of the cases the smaller value of $\sigma_M = 0.50$ mag if $n = 13$.

The slope of equation (3) is in good agreement with other authors who found values of -9.4 to -10.2 from nearby galaxies and cluster galaxies (Aaronson, Huchra, and Mould 1979; Mould, Aaronson, and Huchra 1980; Aaronson, Mould and Huchra 1980; Sandage and Tammann 1984). The value of $a = -10$ is generally adopted for the iTF relation, and Aaronson, Huchra, and Mould (1979) even suggested a theoretical justification for this value. If steeper slopes have been found for field galaxies, i.e., $a = -11.5$ (Aaronson and Mould 1983) and $a = -11.1$ (Rubin *et al.* 1985), it is again an indication for selection effects of flux-limited samples.

First doubts as to the linearity of the iTF relation have been raised by Aaronson *et al.* (1982) and Aaronson and Mould (1983). Aaronson *et al.* (1986) have proposed a quadratic form of the relation which they assign to the increasing influence of noncircular motions in late spirals. An attempt to fit a quadratic law to the data in Table 1 gives only an insignificantly small quadratic term which, in addition, has the opposite sign to the one found by Aaronson *et al.* (1986). The linear relation of equation (3) is therefore adopted (cf. Fig. 2).

III. THE $(B-H)$/LINE WIDTH RELATION

At the expense of interrupting the flow of arguments we intromit here a discussion of the color–line-width relation. The existence of a correlation between Δv_{21} and $(B-H)$ is implied by the different slopes of the bTF and iTF relations. The relation of two distance-independent variables contains, of course, no distance information, but it is useful to test the consistency of the adopted bTF and iTF relations, provided the $(B-H)$/line width relation can be derived from independent samples. Moreover the relation allows to predict H magnitudes for galaxies whose H photometry is not yet available. The calculated H magnitudes will prove useful in § IV to make the iTF relation of Virgo galaxies more transparent.

A correlation between the color $(B-H)$ and the 21 cm line width has previously been found by Tully, Mould and Aaronson (1982), Bothun (1984), and Bothun *et al.* (1985*b*). Using a hybrid color $(B-H)$ the correlation is reinvestigated here. To be specific, the total B-magnitudes are corrected for galactic and internal absorption (B_T^{oi}); the H magnitudes are identical to the corrected $H^c_{-0.5}$ values given by Aaronson *et al.* (1982, 1986) and Mould (1986); the line widths are measured at the 20% intensity level throughout; only disk galaxies with $i \geq 45°$ are considered.

In the following the $(B-H)/\log \Delta v_{21}$ relation is derived from the local calibrators in Table 1. The universality of the relation is, of course, not proved from 13 calibrators alone; the relation will be tested therefore against three additional samples, i.e., for members of the Virgo and UMa Clusters, for a large sample of field galaxies, and for galaxies in 10 distant clusters.

1. The 13 calibrators define a linear regression of (cf. Fig. 3*a*)

$$(B-H) = 3.43 \log \Delta v_{21} - (6.89 \pm 0.09) . \quad (4)$$

The solution is fortuitously close to the algebraic difference of the bTF (eq. [2]) and iTF (eq. [3]) relations. The scatter about the mean relation amounts to only $\sigma_{B-H} = 0.32$ mag.

2. For 25 Virgo Sa–Im members with known B magnitudes (Binggeli, Sandage, and Tammann 1985; Kraan-Korteweg 1986) line widths and H magnitudes are available (Mould, Aaronson, and Huchra 1980; Aaronson *et al.* 1982). This sample of cluster galaxies is increased by the 24 Sb–Im galaxies which Aaronson *et al.* (1982) assign to the UMa Cluster; their B magnitudes come from the RSA ($n = 19$) and from Fisher and Tully (1981; $n = 5$). The $(B-H)$ versus log Δv_{21} relation is plotted in Figure 3*b* for the combined sample. Both clusters individually follow very closely the mean relation of

$$(B-H) = 3.23 \log \Delta v_{21} - (6.42 \pm 0.05) . \quad (5)$$

The scatter of $\sigma_{B-H} = 0.32$ mag is the same as for the 13 calibrators.

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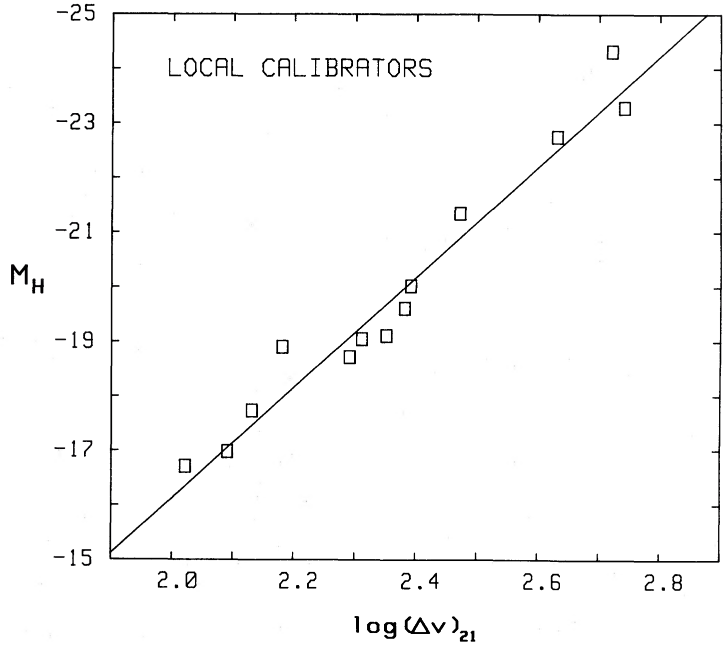

FIG. 2.—The infrared Tully-Fisher relation for 13 calibrating galaxies from Table 1. The linear regression from eq. (3) is shown.

3. For 232 field and group galaxies of type Sa–Im (excluding the clusters Virgo, UMa, and Fornax) line widths and H-magnitudes (Aaronson *et al.* 1982) as well as B magnitudes are available. Roughly half of the latter come from the RSA or RC2; the remaining B magnitudes have lower weight, because they are transformed from the Harvard of Zwicky systems (cf. RSA Appendix; Kraan-Korteweg 1986). The field galaxies define the following relation (cf. Fig. 3*c*)

$$(B-H) = 3.64 \log \Delta v_{21} - (7.43 \pm 0.03) . \quad (6)$$

Because of the inclusion of B magnitudes of lower weight, the scatter is here somewhat larger, i.e., $\sigma_{B-H} = 0.48$ mag, but the agreement of equation (6) with equation (4) is very good over the relevant range.

4. For 145 galaxies in 10 clusters with redshifts $4000 < \langle v_0 \rangle < 11{,}000$ km s^{-1} line widths and H magnitudes are known from Aaronson *et al.* (1986). The available B magnitudes are of second quality only; they are compiled from various sources and had to be transformed into the B_T-system. In addition, the galaxian types are mostly unknown. This adds another uncertainty to the absorption corrected B_T^{oi}-values, because the correction is type-dependent. These somewhat unsatisfactory data are plotted in Figure 3*d*; they define a mean relation of

$$(B-H) = 2.63 \log \Delta v_{21} - (4.90 \pm 0.04) . \quad (7)$$

The scatter is still only $\sigma_{B-H} = 0.54$ mag, but the relatively small range in Δv_{21} necessitates a low correlation coefficient of $r = 0.53$. We take the difference between equation (7) and equation (4) not as an indication of real physical differences—which would cast considerable doubt on the general applicability of the TF method—but rather as a confirmation of the lower quality of the data going into equation (7).

The analysis of Bothun (1984) and Bothun *et al.* (1985*b*) cannot be directly compared with the present results because these authors use a color $(B_{-0.5} - H_{-0.5})$ which differs systematically from our hybrid color $(B-H)$ (cf. Tully, Mould, and Aaronson 1982); yet Bothun *et al.* (1985*b*) reach the same basic conclusion: within the errors the field and cluster relations are the same. They suggest, however, that the individual clusters have significantly different zero points. This would preclude the existence of an universal bTF and/or iTF relation, questioning thus the usefulness of the TF relation for the determination of distances. Their zero-point offset is partly due to the very high galactic absorption adopted for Abell 539, and partly due to the type-independent corrections for internal absorption, which causes spurious zero points because of the different type mixture in different clusters. If the B magnitudes are corrected for galactic and internal absorption following the precepts of the RSA, no significant zero-point differences are found.

The conclusion, then, is that the color–line-width relation, stemming from four differently selected samples, holds universally over a wide range of line widths ($2.0 < \log \Delta v_{21} < 2.8$). The range of validity for different types is not well determined because samples (1) to (4) contain few Sa and Sm–Im galaxies, but the relation is well tested for Sab to Sd galaxies. We will adopt in the following equation (4) from the local calibrators. This is justified by samples (2) and (3) which, if taken together, give an almost identical solution. The relation is useful in several respects.

a) The slope of the $(B-H)/\log \Delta v_{21}$ relation, which rests on the 13 calibrators *and* on external data (samples 2 and 3), provides a consistency check on the adopted bTF (eq. [2]) and iTF (eq. [3]) relation. It is evident, for instance, that the latter two relations cannot have the same slope in view of the nonzero slope of equation (4). In fact, equation (4) requires that any change in slope of equation (2) is accompanied by a corresponding change in slope of equation (3), and vice versa.

b) The best quality data indicate a dispersion of the $(B-H)/\log \Delta v_{21}$ relation of $\sigma_{B-H} = 0.32$ mag. If the observational error of the H magnitudes is 0.15–0.20 mag (Aaronson *et al.* 1982) and of the line widths 0.03–0.04 dex, then the remaining mean observational error of a B_T^{oi} magnitude is only 0.20–

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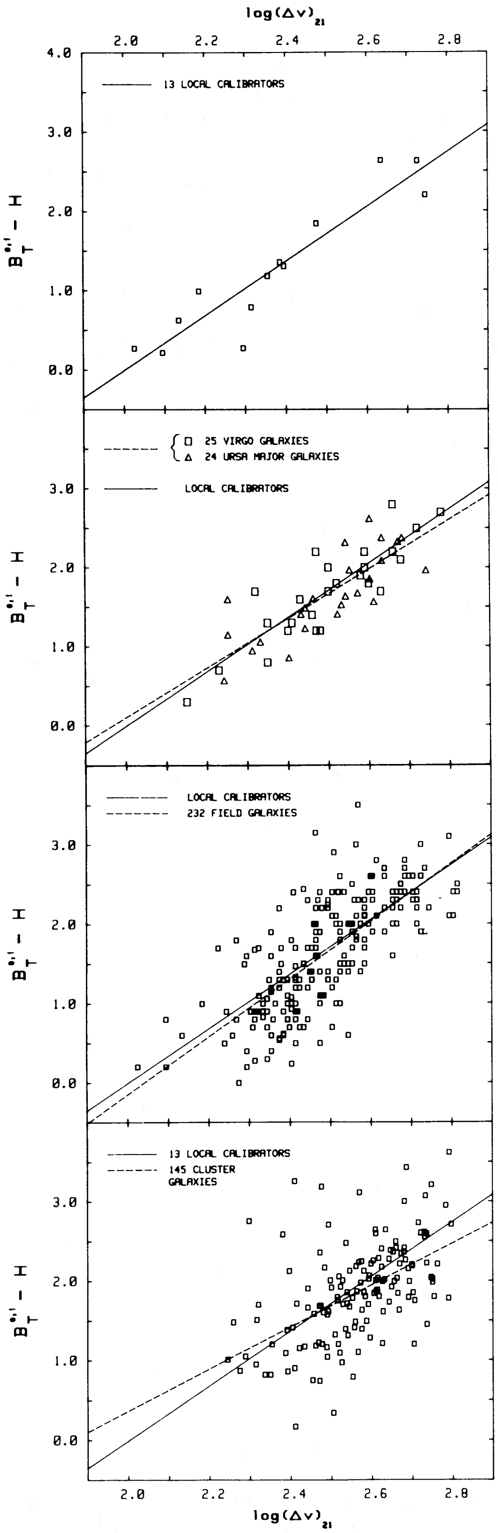

FIG. 3.—The $(B-H)/\log \Delta v_{21}$ relation. (*a*) For 13 calibrating galaxies; (*b*) For the Virgo and UMa cluster galaxies; the dotted line is the best fit to the data, the full line is repeated from Fig. 3*a*; (*c*) For 232 field galaxies; their least-squares fit is shown as a dotted line, the full drawn line is repeated from Fig. 3*a*; (*d*) For the 145 spirals in 10 distant clusters; they define the dotted line, while the full drawn line is repeated from Fig. 3*a*.

0.26 mag. This is an upper limit, because some of the observed scatter must be intrinsic. It is surprising that the admissible error of the B magnitudes *after* correction for galactic *and* internal absorption should be so small. It indicates, in any case, that the here adopted absorption corrections are satisfactory approximations. It has been claimed that the iTF relation is *a priori* superior to the bTF relation, because the former is very little affected by absorption. From the present result this conclusion is not obvious.

c) The scatter of $\sigma_{B-H} = 0.32$ mag from samples (1) and (2) can be used to test the quality of other observational data. If the scatter about the appropriate mean $(B-H)/\log \Delta v_{21}$ relation is significantly larger, it may be taken as an indication for nonperfect observational data.

d) The $(B-H)/\log \Delta_{v21}$ relation in combination with the TF relation implies a correlation between $(B-H)$ and absolute magnitude M (cf. Tully, Mould and Aaronson 1982; Bothun 1984; Bothun *et al.* 1985*b*). Concurrently the 13 calibrators in Table 1 give the regression

$$M_B = -1.68(B-H) - 16.52 \, , \quad (\sigma_B = 0.71 \text{ mag}) \, . \tag{8}$$

The scatter is considerably larger than for the bTF and the iTF relation for the same calibrating galaxies (eqs. [2] and [3]). The color $(B-H)$ seems therefore a less effective luminosity indicator than the 21 cm line width. However, it will be shown in § IV that for cluster galaxies the full scatter of the TF relation is $\sigma_M = 0.7$ mag and it is even larger for field galaxies (Sandage 1988*b*). Hence the M_B or $M_H/(B-H)$ relation could still be competitive. Further tests are required, yet reliable unbiased samples with H magnitudes are not available. The field sample (3) is flux-limited and suffers other selection effects, while the cluster samples (2) and (4) are highly incomplete (cf. next section for Virgo galaxies); they can therefore provide neither a reliable slope of the $M_B/(B-H)$ relation nor its true scatter.

e) The rather tight correlation between color $(B-H)$ and line width $\log \Delta v_{21}$ of individual galaxies can be combined with Meisel's (1983) dependence of *average* line width Δv_{21} on the Hubble type to predict that the sequence of Sb-Sm galaxies have on average progressively bluer $(B-H)$ colors. Exactly this mean color-type dependence has been pointed out for Sa, Sb, and Sc galaxies by Rubin *et al.* (1985).

f) It is possible to predict H magnitudes from equation (4) for all galaxies with known line widths and B magnitudes. This possibility is exploited in the next section.

IV. THE VIRGO CLUSTER AND THE TULLY-FISHER RELATIONS

a) *The Sample*

In the past few years the available data for Virgo Cluster galaxies have increased immensely, making an investigation of the TF relation possible with an unprecedented completeness and range of variables.

The Virgo Cluster catalog (Binggeli, Sandage, and Tammann 1985; VCC) contains 169 certain members of type Sa–Im whose inclination is $i \geq 45°$ according to the best available evidence and which have no tidal distortion and a minimum of hydrogen confusion. The number of 169 includes 10 galaxies whose cluster membership has only recently been confirmed on the basis of new redshift determinations. Plain blue compact dwarf (BCD) galaxies and questionable types were not considered. For 108 of these galaxies 21 cm line widths are given in the literature, mainly by Huchtmeier and

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TABLE 2

THE TYPE DISTRIBUTION OF THE VIRGO SAMPLE

	Sa	Sab,b	Sbc-d	Sdm,m	Im ≤ 16.5	Im > 16.5	Total
VCC Galaxies	13	10	55	16	40	25	159
New	1	...	1	5	3	...	10
With Δv_{21}	7	9	43	19	33	8	119
Percentage %	50	90	77	90	77	32	(70)
Outside VCC	...	...	9	...	...	...	9

Richter (1986), Haynes and Giovanelli (1986), and Hoffman, Helou, and Salpeter (1988). The sample is increased by nine unquestionable members within 10° of the cluster center but outside the area of the VCC. They are taken from Huchtmeier and Richter (1986) and constitute by themselves a subsample complete to $B_T{}^{\circ} = 14.9$ mag; their types and inclinations are given in the same source. The type distribution of the whole sample is shown in Table 2.

It can be seen in the Table that only 50% of the Sa galaxies of the complete sample have line widths Δv_{21}; for reasons given below, Sa galaxies will be excluded anyhow. For Im galaxies the detection rate decreases with increasing magnitude from 77% to less than 32%. Because the irregulars are not important for the distance determination of the Virgo Cluster, this incompleteness will have no consequences. Of the remaining 87 VCC galaxies of type Sab–Sm 71 have Δv_{21} data, i.e., this subsample, on which hinges the distance determination below, is complete to 82%. Most of the 18% without line width information have been observed but were not detected so far. The present sample may be considered unbiased and complete in good approximation. The 16 missing Sbc–Sm galaxies would shift the present mean magnitude of $B_T{}^{\circ} = 13.68$ mag to 13.80 mag. This shift would correspond to an expected distance underestimate of 0.12 mag *if* the missing galaxies had the same line width on average. But since their mean line width is supposedly somewhat narrower, the effect on the distance should be even smaller.

$B_T{}^{oi}$ magnitudes are taken from the RSA when available; for the fainter galaxies the $B_T{}^{\circ}$ magnitudes of the VCC are corrected for internal absorption following the convention of the RSA. (The galactic absorption of the Virgo Cluster is assumed to be $A_B = 0.00$ mag. H magnitudes (here short for corrected $H^c{}_{-0.5}$ magnitudes) are taken from Mould, Aaronson, and Huchra (1980) and Aaronson *et al.* (1982). Inclinations come mainly from de Vaucouleurs and Pence (1979) and for the fainter galaxies from Binggeli, Sandage, and Tammann (1985). The observational data of the Virgo sample are compiled in Table 3. It should be noted that the values of Δv_{21} are measured at the 20% level; they are corrected for inclination. The H magnitudes denoted by an asterisk (*) are actually observed; the remaining ones are *calculated* from equation (4).

At this point the reader may wonder about the usefulness of calculated H magnitudes; their information content is borrowed from the B magnitudes and they cannot provide independent distance information. However, they are valuable to reveal the *true mean* iTF relation and its *true* scatter and therefore to judge on the relative merits of the bTF and iTF relations, as discussed in the next paragraph.

b) The Blue and Infrared TF Relation for Sa–Im Galaxies

The TF relations for the Virgo Cluster depend strongly on the sample selection. We illustrate this point for two different samples, the one consisting of all the 25 Virgo members from the VCC whose H magnitudes have been *observed* [this data set happens to contain all subsets of Virgo galaxies on which previous claims are based that the Virgo modulus is small, i.e., $(m - M)^0 < 31.2$], the other comprising the 128 galaxies in Table 3.

The bTF relations for the small ($n = 25$) and the large ($n = 128$) Virgo samples are shown in Figures 4*a* and 4*b*. The *systematic* difference between the two samples is striking. Not only is the scatter much larger in the complete sample ($\sigma_B = 0.82$ mag or 0.77 mag if one strongly deviating galaxy is excluded, versus $\sigma_B = 0.61$ mag), but also its zero point is fainter by 0.52 mag! Clearly the small sample is not suited to define the mean bTF relation nor its scatter. A blind least-squares fit to the small sample would yield a severely biased bTF relation and—if directly compared to the calibration in equation (2)—to a much too small Virgo modulus. We postpone a discussion of the *reasons* for this bias to § V, but note in passing that the least-squares fit to the *unbiased* sample ($n = 127$ with the strongly deviating galaxy omitted) yields

$$B_T{}^{oi} = -6.37 \log \Delta v_{21} + (28.01 \pm 0.07) \tag{9}$$

with a high correlation coefficient of $r = 0.95$ and no significant deviation from linearity. The slope of equation (9) is close to that of the calibrators in equation (2) although the types span here a wider range. Hoffman, Helou, and Salpeter *et al.* (1988) found for a somewhat different, large Virgo sample of Sb–Im galaxies a slope of only -5.06 which is explained by their nearly type-independent absorption correction (cf. § II) and their larger number of irregulars; in addition their weaker inclination restrictions $i \geq 24°$ causes an even larger scatter ($\sigma_B = 0.9$ mag).

One could argue at this point that the bTF relation with its large intrinsic scatter is indeed vulnerable to selection effects, but that the iTF relation had a much smaller intrinsic scatter and hence were a far superior instrument. Such an argument, however, is against all available evidence. The iTF relations of the two samples from above are plotted in Figures 4*c* and 4*d*. Again the small sample ($n = 25$) shows much smaller scatter and defines a brighter zero point ($\Delta M = 0.51$ mag) than the large sample. True, the majority of the H magnitudes in the large sample are not observed, but only calculated through equation (4). However, the $(B-H)/\log \Delta v_{21}$ relation with its small intrinsic scatter and its insensitivity against sample selection, *cannot* artificially blow up the scatter of the iTF relation and can cause even less a *systematic* zero point shift. One is therefore forced to the conclusion that the fit to the data in Figure 4*d*

$$H = -9.81 \log \Delta v_{21} + (34.94 \pm 0.07) \tag{10}$$

(with $r = 0.98$, and $\sigma_H = 0.77$ mag, excluding one galaxy) is a good approximation to the true iTF relation. This conclusion is further supported by the following consideration. The small, completely observed B and H samples ($n = 25$) yield very similar distance moduli for the Virgo Cluster, but the complete set of 128 observed B magnitudes proves this modulus to be biased. Therefore the modulus from the 25 H magnitudes must also be incorrect. Indeed, if one includes in addition the calculated H magnitudes one obtains a H modulus which is in perfect agreement with the large-sample B-modulus (cf. § IV).

It is noteworthy that the scatter about the iTF relation (eq. [10]) is as large as in the bTF relation. While we have argued that the calculated H magnitudes carry little systematic error,

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TABLE 3

VIRGO GALAXIES FOR THE DETERMINATION OF THE BLUE AND INFRARED TULLY FISHER RELATION

VCC (1)	Name (2)	Type (3)	log Δv_{21} (4)	$B_T^{o,i}$ (5)	$H_{-0.5}^c$ (6)
...	NGC 4064	Sc	2.36	11.62	10.4
17	UGC 7150	Im	2.15	14.93	14.4
34	IC 3033	Sc	2.24	14.23	13.4
66	NGC 4178	Sc	2.47	11.26	10.14*
67	IC 3044	Sc	2.17	13.40	12.8
83	IC 3049	Im	2.15	14.73	14.2
87		Sm	2.15	14.87	14.4
92	NGC 4192	Sb	2.68	9.85	7.77*
...	UGC 7249	Sd	2.24	14.13	13.3
120	NGC 4197	Scd	2.47	12.60	11.0
145	NGC 4206	Sc	2.50	11.98	10.34*
152	NGC 4207	Sc	2.43	12.97	11.5
157	NGC 4212	Sc	2.59	11.44	9.4
159		Im	1.97	14.81	14.9
162	IC 3074	Sd	2.41	13.60	12.2
167	NGC 4216	Sb	2.72	9.76	7.33*
169		Im	1.70	16.23	17.3
187	NGC 4222	Scd	2.40	13.04	11.7
217	UGC 7307	Im	1.89	15.12	15.5
226	NGC 4237	Sc	2.58	12.12	10.2
241	IC 3105	Sd	2.08	13.88	13.6
318	IC 776	Scd	2.39	13.51	12.2
322	IC 3142	Im	2.04	14.8	14.7
328		Im	2.07	16.6	16.4
350		Im	1.82	17.0	17.6
381		Im	1.75	16.3	17.2
446		Im	1.90	15.2	15.6
453		Sm	2.10	15.6	15.3
460	NGC 4293	Sa	2.64	10.38	8.2
465	NGC 4294	Sc	2.40	12.02	10.77*
477		Im	1.69	16.4	17.5
483	NGC 4298	Sc	2.66	11.65	9.4
497	NGC 4302	Sc	2.59	11.72	9.7
509	UGC 7423	Sm	2.27	14.71	13.8
512	UGC 7421	Sm	2.13	15.0	14.6
522	NGC 4305	Sa	2.66	12.58	10.3
524	NGC 4307	Sbc	2.69	11.98	9.6
530		Im	1.87	15.6	16.1
565		Im	1.91	15.5	15.8
566		Sm	2.09	15.5	15.2
570	NGC 4313	Sab	2.45	11.56	10.0
576	NGC 4316	Sbc	2.53	12.85	11.1
620	IC 3239	Sm	2.08	14.8	14.6
630	NGC 4330	Sd	2.50	12.27	10.6
713	NGC 4356	Sc	2.48	13.20	11.6
740		Sm	2.18	15.4	14.8
...	NGC 4376	Scd	2.29	13.22	12.3
792	NGC 4380	Sab	2.52	11.64	9.78*
793		Im	1.85	16.3	16.8
809	IC 3311	Sc	2.25	13.68	12.8
826	UGC 7512	Im	1.93	14.7	15.0
827	IC 3322A	Sc	2.47	12.89	10.75*
836	NGC 4388	Sab	2.60	10.68	8.86*
851	IC 3322	Sc	2.33	13.31	12.2
865	NGC 4396	Sc	2.32	12.33	11.3
873	NGC 4402	Sc	2.49	11.88	10.2
888		Im	2.09	14.7	14.4
912	NGC 4413	Sab	2.41	11.56	10.2
958	NGC 4419	Sab	2.53	11.12	9.3
963		Im	1.88	17.0	17.4
979	NGC 4424	Sa	2.09	11.50	11.2
980	IC 3365	Scd	2.21	13.70	13.0
995	IC 3371	Sc	2.21	14.44	13.7
1001		Im	1.77	16.4	17.2
1011	UGC 7567	Sdm	2.24	14.55	13.8
1013		Im	1.97	16.2	16.3
1021	IC 3374	Im	2.04	14.77	14.7
1060		Sm	2.13	14.76	14.3
1091	RMB 169	Sbc	2.28	14.01	13.1
1110	NGC 4450	Sab	2.59	10.33	8.06*
...	NGC 4455	Sc	2.23	12.27	11.62*
1179	IC 3412	Im	2.17	14.51	14.0
1189	IC 3414	Sc	2.24	13.25	12.5
1193	NGC 4466	Sc	2.27	13.94	13.0
1208		Sm	2.06	14.9	14.7
1249	UGC 7636	Im	1.87	14.51	15.0
1356	IC 3446	Sm	2.28	14.6	13.7
1374	IC 3453	Im	2.11	14.74	14.4
1379	NGC 4498	Sc	2.35	12.14	10.77*
1401	NGC 4501	Sbc	2.78	9.76	7.15*
1410	NGC 4502	Sm	2.35	14.31	13.1
1427	IC 3471	Im	2.12	15.08	14.7
1455		Im	1.78	16.5	17.3
1468		Im	2.12	14.7	14.3
1508	NGC 4519	Sc	2.48	11.92	10.75*
1516	NGC 4522	Sbc	2.46	12.00	10.60*
1552	NGC 4531	Sa	2.66	11.54	9.3
1554	NGC 4532	Sm	2.43	11.98	10.45*
1555	NGC 4535	Sc	2.63	10.14	8.46*
1566	IC 3517	Sd	2.19	14.28	13.7
1569	IC 3520	Scd	2.16	14.43	13.9
1575	IC 3521	Sm	2.28	13.77	12.8
1585	IC 3522	Im	2.15	14.9	14.46*
1596		Im	1.89	16.9	17.3
1624	NGC 4544	Sc	2.35	13.24	12.1
1644		Sm	2.02	17.0	17.0
1654	IC 3562	Im	2.02	14.69	14.7
1686	IC 3583	Sm	2.20	13.68	13.0
1690	NGC 4569	Sab	2.59	9.39	7.45*
1699	IC 3589	Sm	2.18	13.85	13.3
1725		Sm	2.17	14.28	13.7
1753		Im	1.93	17.2	17.5
1760	NGC 4586	Sa	2.53	11.54	9.8
1789		Im	2.12	14.82	14.4
1791	IC 3617	Sm	2.20	14.40	13.7
1804		Im	2.01	15.2	15.2
1811	NGC 4595	Sc	2.34	12.49	11.4
1816		Im	1.75	15.9	16.8
1859	NGC 4606	Sa	2.38	11.89	10.6
1868	NGC 4607	Scd	2.40	13.03	11.7
1918		Im	1.96	15.15	15.3
1929	NGC 4633	Scd	2.48	13.21	11.6
1931		Im	2.31	15.0	14.0
1943	NGC 4639	Sb	2.65	11.57	9.4
1952		Im	1.93	15.7	16.0
1965		Im	1.82	16.2	16.8
...	NGC 4651	Sc	2.66	10.89	8.66*
1987	NGC 4654	Sc	2.58	10.71	8.80*
1992	UGC 7906	Im	2.19	15.3	14.7
2007	IC 3716	Im	1.99	14.9	15.0
2023	IC 3742	Sc	2.36	13.34	12.1
2034		Im	2.01	15.6	15.6
2037		Im	1.77	15.5	16.3
2070	NGC 4698	Sa	2.66	11.16	8.40*
...	NGC 4713	Sc	2.41	11.74	10.44*
...	NGC 4758	Sc	2.32	12.90	11.23*
...	IC 3881	Sc	2.35	12.59	11.79*
...	NGC 4808	Sc	2.50	11.99	9.96*

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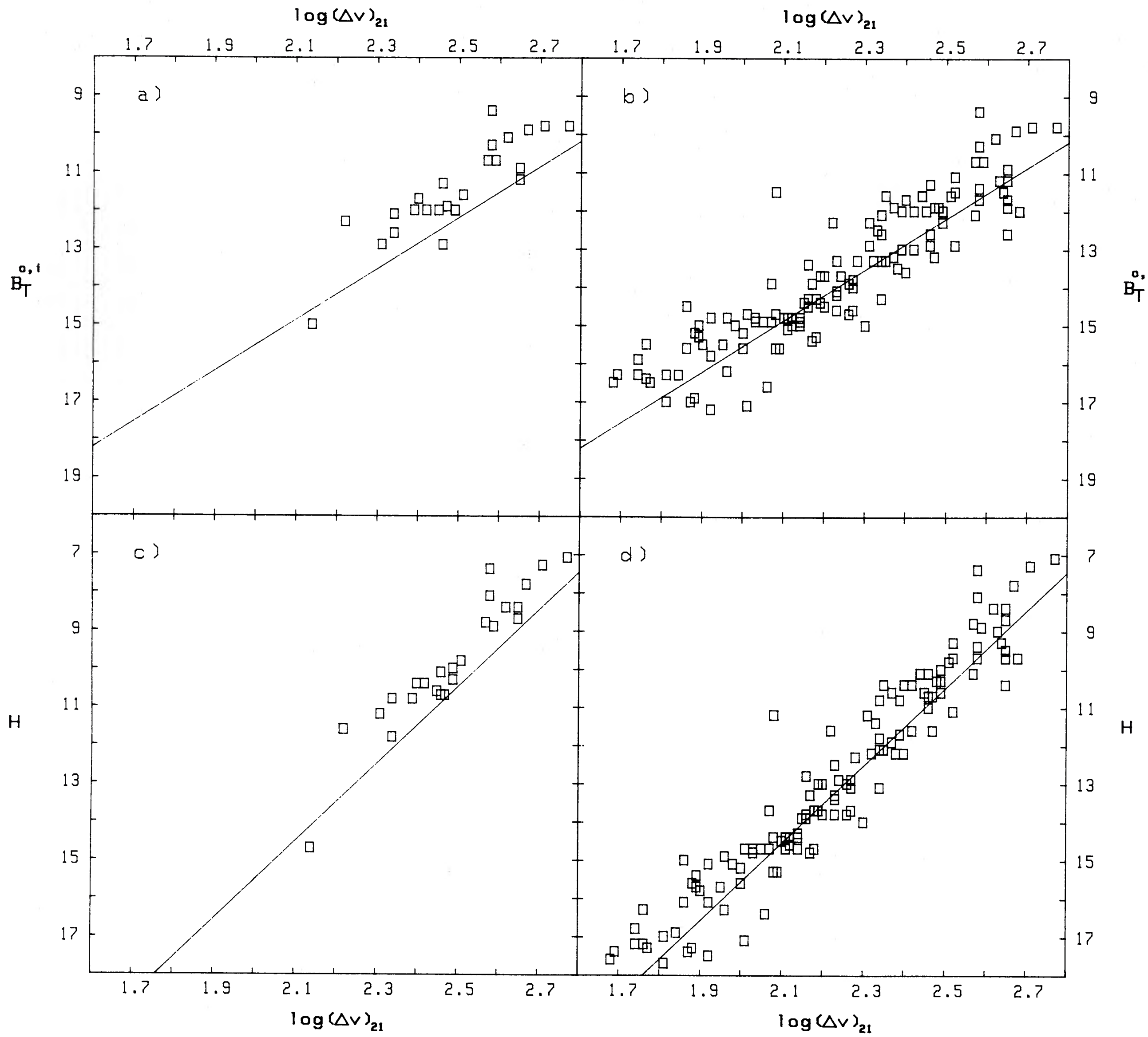

FIG. 4.—The TF relation for Virgo cluster galaxies of Type Sa–Im. (*a*) and (*b*) show the bTF relation for the small ($n = 25$) and large ($n = 128$) sample, respectively. Eq. (2) has been drawn as a fiducial line. The small sample comprises all galaxies on which most Virgo bTF relations have so far been based. Figs. (*c*) and (*d*) show the corresponding data for the iTF. The fiducial line is from eq. (3). The 25 *H* magnitudes in (*c*) are actually observed, the additional ones in (*d*) are calculated through eq. (4). Samples (*a*) and (*c*) are clearly unsuitable to define the *mean* TF relation.

they are possibly affected by somewhat larger random errors than the observed magnitudes. The *true* scatter of the iTF relation may therefore be smaller than $\sigma_H = 0.77$ mag, but the evidence here is fully compatible with Sandage's (1988*b*) conclusion that the iTF scatter is only less than 12% smaller than that of the bTF relation. Therefore there is no justification for the claim that the iTF relation was a much superior tool over the bTF relation for the determination of distances. In contrary, the steeper slope of the iTF relation (cf. eqs. [2] and [3]) makes it more vulnerable to any observational errors of the 21 cm line width.

Before equations (9) and (10) are used for a distance determination of the Virgo Cluster, the TF relations should be tested against any dependence on the Hubble type. Does Figure 4 conceal any type segregation?

c) Type Dependence and Scatter of the TF Relations

A good understanding of the dependence of the bTF and iTF relations on morphological type is important for any successful application of the TF method. In order to test for the type dependence we turn first to the bTF relation because of the large sample of available *B* magnitudes.

The data of the upper panel of Figure 4 are replotted in five panels in Figure 5, but now binned according to type. Because each type bin covers only a relatively narrow interval of log Δv_{21}, we have not deemed it justified to calculate separate bTF relations for individual bins. Instead, the same reference line is drawn in each panel of Figure 4 so as to guide the eye. The presence of type differences is indeed obvious. There is, for instance no evidence for Sa galaxies (first panel) to follow any

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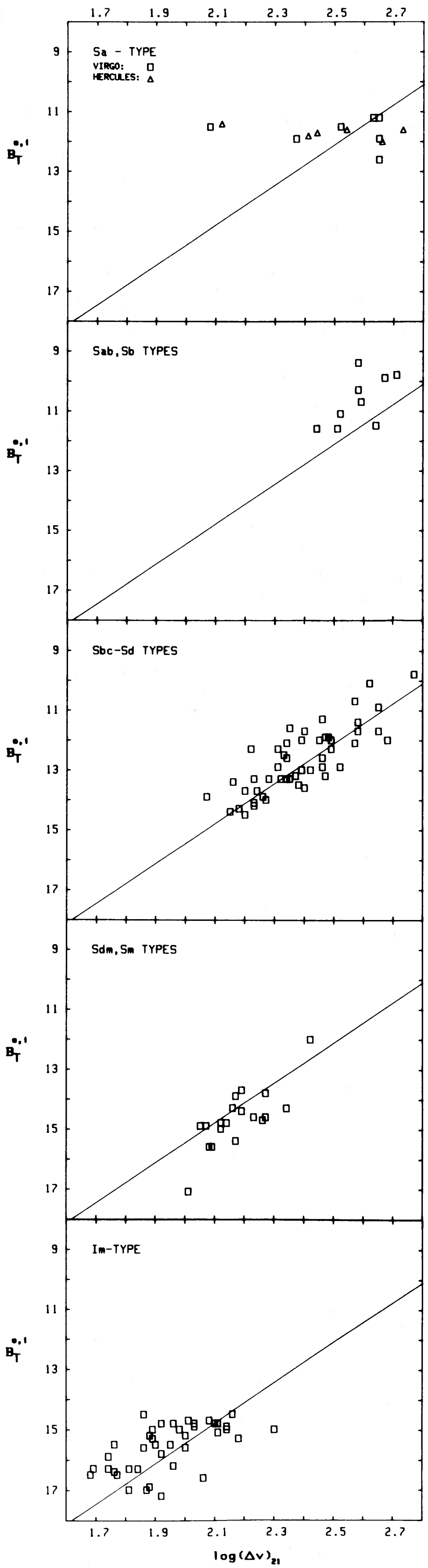

FIG. 5.—The bTF relation of Virgo Cluster members binned according to galaxy types. The reference line is from eq. (2) and an adopted distance modulus of 31.60. The panel of the Sa galaxies shows also those in the Hercules Cluster. The latter are arbitrarily shifted by 2.91 mag (which cannot correspond to the true difference of the cluster moduli).

TABLE 4

DEVIATIONS OF DIFFERENT HUBBLE TYPES FROM A MEAN bTF RESPECTIVELY iTF RELATION

	COMPLETE VIRGO SAMPLE			ONLY 25 SPIRALS		
TYPE (1)	n (2)	Δ_B (3)	σ_B (4)	n (5)	Δ_B (6)	Δ_H (7)
Sab	7	−1.00	0.60	4	−1.31	−1.12
Sb	3	−0.50	0.64	2	−0.87	−0.94
Sbc,c	38	−0.17	0.70	16	−0.33	−0.63
Scd,d	13	+0.14	0.43	1	...	...
Sdm,m	19	+0.42	0.65	1	...	...
Im	41	(−0.20)	(0.82)	1	...	...

relation between line width and magnitude. The Virgo Sa's, which are complimented by those in the Hercules cluster (cf. § V), are spread over a wide range in line width without a trace of luminosity correlation. This conclusion is supported by a large sample of field Sa's from the RSA which shows at best a very weak TF relation (Huchtmeier, Sandage, and Tammann 1988). The absence of correlation requires the exclusion of Sa's for any application of the TF relation. The TF relation of Sb galaxies alone cannot be established satisfactorily because they exist only over a range in absolute magnitude of 4 mag (Kraan-Korteweg, Sandage, and Tammann 1984; Sandage, Binggeli, and Tammann 1985). If fitted to the TF relation of equation (2) respectively (3) Sb galaxies exhibit a scatter of $\sigma_M = 0.7$ mag. If Sb galaxies were assigned simply a mean absolute magnitude the scatter would be $\sigma_M \approx 1$ mag, which can be considerably reduced by dividing them into luminosity classes (Sandage 1988*a*). The TF relation provides therefore for Sb galaxies no important gain. Sbc–Sm galaxies cover a wide absolute magnitude interval of ∼6 mag. For this type bin the TF relation is most useful. But the scatter remains as high as $\sigma_M = 0.7$ mag. An additional difficulty is—as inspection of panels 2–4 of Figure 5 reveals—the systematic shift of the TF zero point from Sab/b to Sdm/m galaxies. This monotonous shift holds even for narrower type bins, but within the rather large errors they indicate no significant change of slope of the bTF relation. The zero-point deviations of the different type bins relative to an arbitrary zero point are significant; they are listed in Table 4 (left side). Also listed is the magnitude scatter σ_M about a bTF relation of constant slope and fitted to the zero point of the respective type bin. The scatter of the individual type bins is marginally smaller than the one about the mean relation ($\sigma_M = 0.74$ mag) for the combined sample of Sab–Sm galaxies. When interpreting the data in Table 4 one should remember that the Virgo sample is not fully complete yet, and that the missing galaxies may still alter the figures somewhat. But the agreement with external data is quite good: while Giraud (1986) found for the zero point difference between field galaxies of type Sab/b and Scd/d $\Delta_B \approx 1.0 \pm 0.4$ mag, the corresponding value in Table 4 is $\Delta_B = 0.99$ mag.

The calculated H magnitudes are useless in establishing any zero-point deviations of the iTF relation because they will merely reflect the type segregation of the bTF relation. Nevertheless, the 25 galaxies with observed H magnitudes suffice to disclose a similar tendency. The zero-point shifts Δ_H and Δ_B of

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this bright subsample are given on the right side of Table 4. The conclusion from these numbers is that a significant type dependence exists also for the iTF relation, although it may be somewhat weaker than for the bTF relation. Yet the type independence of the $(B-H)$/line width relation implies automatically that a type segregation in the blue must be accompanied by a similar segregation in the infrared.

It is possible to minimize the zero-point differences of the bTF relation for Sab/Sm spirals by adopting a mean relation of the form

$$B_T^{oi} = -7.67 \log \Delta v_{21} + 31.07 \; . \tag{11}$$

But this relation reduces the zero-point deviations by only one-third and, more importantly, *it holds for only one specific mixture of types.* A better approach is ascertained by using the calibrated TF relations of equations (2) and (3) throughout and to *ensure that the type mixture of the calibrators matches the one of the sample under investigation.*

The Im sample, illustrated in the fifth panel of Figure 5, is quite incomplete. Taken at face value the Im's reverse the trend of zero-point shifts (cf. Table 4) and they show considerable scatter, i.e., $\sigma_M = 0.82$ mag about their mean. While it is believed that most of the scatter of the earlier types is intrinsic, a considerable part of the scatter of Im's may have to be attributed to observational errors. For the fainter Im's from the VCC the quoted B magnitude error is $\sigma_M = 0.5$ mag. In addition the inclination of irregulars are notoriously uncertain. Finally, corrections of the line widths due to turbulence are neglected here; they would shift the Im galaxies systematically to the left in Figure 4, enhancing their overluminosity relative to a mean TF relation. For these reasons little weight should be given to the Im's for a distance determination of the Virgo Cluster.

The best distance determination, therefore, must come from a restricted sample of Sab–Sm cluster members. This type restriction is particularly attractive because the types of the calibrators in Table 1 lie within this type interval.

d) The Distance of the Virgo Cluster

The necessary data for a successful determination of the Virgo Cluster distance are now at hand. The bTF and iTF relations as defined by the local calibrators in equations (2) and (3) can be compared with a corresponding and nearly complete sample of Sab–Sm Virgo members.

But before entering into the results some precautions should be discussed because it is mandatory for any distance determination that only equivalent data be compared. The possibility that field and cluster galaxies do not follow the same TF relation will be discussed in § VII. Moreover, it follows from the type segregation discussion above that the type mixture of the calibrators and of the complete Virgo sample will influence the distance determination of the cluster. To control this problem the average type $\langle T \rangle$ of a galaxy sample is introduced, where T is the coded galaxian type as defined in RC2. Also the mean line width $\langle \log v_{21} \rangle$ will be used to ensure the compatibility of the calibrators and the Virgo sample.

In Table 5 three different solutions for the distance modulus of the Virgo Cluster are shown. We consider the blue data first: Solution 1 has somewhat earlier type $\langle T \rangle$ than the calibrators defining equation (2); in fact no Sab galaxy is among the calibrators. This solution is therefore given low weight. Solution 2—without Sab galaxies—matches the calibrators well in $\langle T \rangle$ and mean line width $\langle \log \Delta v_{21} \rangle$. Also the exclusion of the Sab galaxies with their large zero-point deviation reduces the steepness of the slope of the Virgo spirals (eq. [11]) to $a = -7.25$ which is closer to the adopted slope. The second solution is therefore our preferred one. Solution 3 considers all Virgo galaxies with $2.0 < \log \Delta v_{21} < 2.8$, which includes also Sab and some of the brighter Im galaxies. The difficulties with the latter type have already been discussed. But it is reassuring to see that solutions 2 and 3 agree so closely, independent of whether the large Virgo sample is cut in galaxy type or in line width.

The chosen method to derive distances, i.e., forcing the slope of the calibrators onto the Virgo data, bears no influence on the solution. A reevaluation of the distance modulus by adopting either the least-squares solution of the different Virgo samples (with their individual slopes and zero points), or the inverse relation or even the orthogonal relation all confirm—with minute variations—the same blue Virgo distance modulus of $(m-M)^0 = 31.60 \pm 0.15$.

It is not possible to determine an *independent* Virgo modulus from the iTF relation because most of the H magnitudes in Table 3 are derived from the corresponding B magnitudes. But still the iTF modulus provides a most valuable check on the internal consistency of the available data. A fit of the H magnitudes to the iTF relation defined by the calibrators (eq. [3]) yields Virgo moduli for the three cases from above as shown in Table 5. The agreement with the blue moduli is so perfect that the preferred solution of $(m-M)^0_{\rm Virgo} = 31.60 \pm 0.15$ gains additional weight.

One might object that the infrared modulus depended heavily on the calculated H magnitudes and that it were hence an artifact. But it can be shown that samples which contain only galaxies with truly measured magnitudes in both wavebands always yield nearly identical results from the bTF and iTF relations. For instance, the best fit of the 25 Virgo

TABLE 5

DISTANCE ESTIMATES TO THE VIRGO CLUSTER

Sample	n	$\langle T \rangle$	$\langle \log \Delta v \rangle$	$(m-M)_B \pm \epsilon$	σ_B	$(m-M)_H \pm \epsilon$	σ_H	obs. H mag
Calibrators	13	6.2	2.37					
Sab–Sm	80	5.7	2.38	31.51 ± 0.13	0.74	31.54 ± 0.13	0.73	30%
Sb–Sm	73	6.0	2.37	31.60 ± 0.13	0.70	31.62 ± 0.12	0.69	27%
Sab–Im ($2 < \log \Delta v < 2.8$) ...	96	6.5	2.33	31.59 ± 0.15	0.74	31.61 ± 0.12	0.73	25%
Sb–Sm:								
B	15	5.7	2.37	31.90 ± 0.21	0.71	31.93 ± 0.16	0.62	27%
10°-B	59	6.1	2.36	31.51 ± 0.13	0.69	31.51 ± 0.09	0.69	25%
6°-B	45	6.0	2.37	31.59 ± 0.14	0.67	31.61 ± 0.10	0.66	20%

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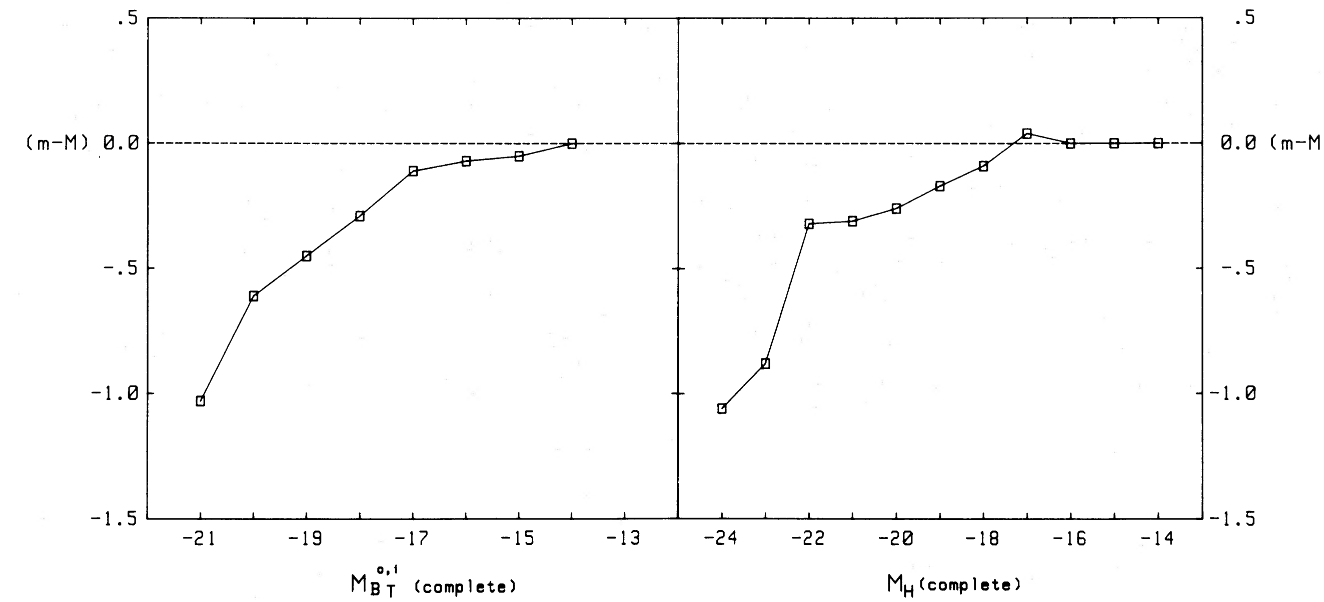

FIG. 6.—Illustration of the Teerikorpi effect. The distance moduli of the Virgo Cluster are plotted against the cutoff magnitude of the sample. The zero point of the abscissa is set arbitrarily to agree with the asymptotic value which is nearly identical for the bTF (*right panel*) and iTF (*left panel*) relations. Observed and calculated H magnitudes are used in the right panel; the two lowest points rest on observed H magnitudes only, the fraction of calculated magnitudes increases for the higher points. The diagram shows that a reliable approximation to the unbiased $(m - M)^0$ requires samples that are complete to 6 mag below the top.

members for which actually observed H magnitudes exist give blue and infrared moduli of 30.98 and 30.97, respectively. The relative agreement is again perfect. The latter moduli are, of course, totally meaningless because only galaxies were considered which are exceptionally bright for their line width.

The foregoing example illustrates again the enormous importance of the sample selection. It has sometimes been held that selection bias can be prevented by considering cluster samples which are complete to a given magnitude limit. That this is not necessarily correct is shown in Figure 6. Here the blue and infrared Virgo moduli are plotted which are obtained by cutting our Virgo sample at various magnitude levels.

The horrifying fact is that the distance moduli increase asymptotically to the true value as one goes to fainter and fainter magnitude limits. For instance, a cut 3.5 mag below the brightest members gives blue and infrared distance moduli which fall short of the true value by ~ 0.3 mag. The asymptotic approach of the true modulus is the result of (1) the intrinsic scatter of the TF relation and (2) the negative slope of this relation. A horizontal cut in any line width/magnitude diagram denies the sample an important fraction of galaxies which are too faint for their line width. This selection problem can be solved analytically for complete samples if the following three parameters are well known: (*a*) the slope of the TF relation, (*b*) the *intrinsic* scatter of the relation, and (*c*) the true completeness limit. Such an analytical analysis has been presented by Teerikorpi (1987) who called the effect the "cluster population incompleteness bias."

That the Teerikorpi effect is not important for the adopted Virgo modulus can be shown in three ways. First, Figure 6 indicates that the asymptotic distance is reached with the present sample. It should be noted, however, that a reliable distance determination requires complete samples over 6 mag, i.e., over the total range over which spiral galaxies exist! Second, our nearly complete Sab-Sm Virgo sample is actually *not* a magnitude-limited sample. because the faintest spiral is significantly brighter than the VCC completeness limit of $B_T \approx 18.0$ mag. Third, the close agreement of solutions 2 and 3 where the latter is cut in line width and therefore not subjected to this bias leaves little room for a remaining bias.

The Virgo Cluster is composed of two subclusters, A and B (Binggeli, Tammann, and Sandage 1987). Cluster B may be more distant than A (see e.g., de Vaucouleurs and Corwin 1986; for a discussion see Binggeli, Tammann, and Sandage 1987). This possibility is tested here. All cluster galaxies within 2° of NGC 4472 are assigned to cluster B, the others to cluster A. Two cases are considered for A: (1) all galaxies within 10° of the Virgo Cluster center but not in B, (2) all those within 6° of the center but not in B. The distances of these aggregates are determined by applying the calibrating equations (2) and (3) to the respective Sb–Sm spirals. The results from the bTF and iTF relations are listed in Table 5. Our preferred solution of A comes from the 6° sample, i.e., $(m - M)_A{}^0 = 31.60 \pm 0.14$, because the outer spirals of the 10° solution may not be sampled to an equally faint magnitude limit as in the area of the VCC. The solution for B is $(m - M)_B{}^0 = 31.91 \pm 0.21$. The difference is $\Delta(m - M)^0 = 0.31 \pm 0.21$ (the calculated error allows for the fact that the error of 0.10 of the calibrators does not enter into the relative distance between A and B). This difference gives a marginal indication for B being about 15% more distant than A. We return to this possibility in § VII. In the present context, the most important result is that the scatter about the blue and infrared TF relations is not significantly reduced by considering A and B separately. We take this as proof that the large scatter of the TF relations is *not* caused by a front-to-background effect, but that it is mainly intrinsic.

The main conclusion from this section is: $(m - M)^0_{\rm Virgo} = 31.60 \pm 0.12$. This result rests on an almost, but not yet fully complete sample of Virgo spirals. As already discussed, the remaining $\sim 18\%$ of Sb–Sm galaxies can increase the distance modulus by 0.12 mag at most. The present result should therefore be taken as a close lower limit to the true Virgo distance.

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TABLE 6

ADOPTED CLUSTER DISTANCES AND HUBBLE RATIOS

Cluster (1)	n (2)	$\langle v_{220} \rangle$ (3)	iTF $(m-M)^0$ (4)	bTF $(m-M)^0$ (5)	Mean $(m-M)^0$ (6)	(Mpc) (7)	H_i (8)
Virgo	87	1196	31.6	31.6	31.6	20.9	57.2
UMa	24	1270	31.8	31.9	31.8_5	23.4	54.3
Pisces	20	5114	34.8_5	34.5	34.6_8	86	59.5
A400	7	6988	35.7	35.6	35.6_5	135	51.8
A539	9	8500	36.0_5	35.8	35.9_3	153	55.6
Cancer	22	4903	34.8	34.6	34.7	87	56.4
A1367	20	6644	35.1	35.1	35.1	105	63.3
Coma	13	7143	35.7	35.5	35.6	132	54.1
Zw 74-23	13	6229	35.2	35.2	35.2	110	56.6
Hercules	11	11212	(36.2)	(36.0)	(36.1)	(166)	(67.5)
Pegasus	22	3880	34.0_5	34.3	34.1_8	68	57.0
A2634/66	11	8610	35.8	36.0	35.9	151	57.0

V. DISTANCES OF 11 CLUSTERS BEYOND VIRGO

In 11 clusters Aaronson *et al.* (1982, 1986) have determined H magnitudes for some of the brighter galaxies, which appear to be cluster members. We do not rediscuss the membership assignment (cf. Sandage 1988*b*), but accept the galaxy data to be representative on average for the individual clusters. But we want to discuss the reasons why these data led Aaronson *et al.* (1986) to very small cluster distances and thereby to an amazingly high Hubble constant ($H_0 = 90$ km s^{-1} Mpc^{-1}). That indeed their distance scale is too short has been implied in the previous section, where the complete Virgo data demanded an increase of the modulus by 0.8 mag as compared to their earlier solution. The question whether they picked up an additional rubber band factor between Virgo and the 11 more distant clusters remains to be answered.

The 11 clusters, together with the Virgo Cluster, are listed in Table 6. The small number of galaxies for which H magnitudes were measured is given in column (2). Fitting these cluster galaxies to their adopted iTF relation, Aaronson *et al.* (1986, Table 6) found a scatter between $\sigma_M = 0.34$ mag and $\sigma_M = 0.71$ mag for individual clusters, with a mean value of $\sigma_M = 0.46$ mag. It is alarming that this scatter is significantly smaller than that found for the Virgo cluster in the previous section. Even if some of the Virgo scatter were due to observational errors, these could only be larger for the more distant clusters. There are only two solutions to the puzzle. Either different clusters have different scatter about the mean iTF relation—in which case the nonuniversality of the TF relation makes it a useless method for distance determinations—or, more plausible, the clusters beyond Virgo suffer a strong selection effect which reveals only a spuriously small scatter, similar to the *observed* H magnitudes of the Virgo Cluster. As optimists we adopt the latter solution. But how is it possible that the true scatter is not revealed even though some of the clusters are sampled as deep as 5 mag? The explanation is given by Bothun *et al.* (1985*a*): the cluster galaxies were not selected optically but according to line width. Line widths of relatively bright galaxies were available *before* the galaxies were selected for the distance determination.[2] The desire to cover as wide a range in Δv_{21} as possible led then quite naturally to the brightest galaxies at any given line width. Hence the galaxies were sampled along the *upper envelope* of the TF relation.

A straightforward distance determination from these biased data is not possible. It is therefore proposed to fit the *upper envelopes* to the available cluster galaxies. We take as the fiducial upper envelope the 2 σ line of the iTF relation as defined by all Sa–Sm Virgo members (cf. the Virgo panel in Fig. 7). The 1 σ deviation is adopted as $\sigma_H = 0.7$ mag. It is to be expected that 95% of any appropriately defined, but complete cluster population lies below the upper 2 σ envelope.

This procedure is uncontroversial because regardless whether or not the data are biased against the fainter galaxies at any Δv_{21}, it must yield on average the correct distance. The price of an upper-envelope fit is that a strict solution cannot be derived from a mathematical routine as long as the total parent population is unknown; it is therefore not as objective a procedure as the fitting of mean relations. We have independently fitted by eye the 2 σ upper envelope from the Virgo Cluster to the more distant clusters, and we are convinced that the personal error of the fitting is less than 0.2 mag.

Our adopted upper-envelope fits for the iTF relation are shown in Figure 7 using the 2 σ envelope of Virgo as the fiducial line. The Virgo Cluster data from Figure 4*c* are repeated in the upper left corner of Figure 7 showing the 25 galaxies with observed H magnitudes. They cling indeed nicely to the 2 σ upper envelope. The moduli differences $\Delta(m - M)$ between the Virgo Cluster and the additional 11 clusters in Figure 7 can now be obtained by simply shifting the Virgo upper envelope to the respective cluster data. The absolute cluster moduli follow then directly from $(m - M)^0_{\rm Virgo} = 31.60$, as determined in § IV. The resulting moduli are listed in Table 6, column (4).

The analogous procedure was applied to the upper envelopes of the bTF relation (Fig. 8). The B_T^{oi} magnitudes of the cluster galaxies were discussed already in § III. With the probable exception of UMa they are of somewhat lower quality. The resulting distance moduli in Table 6, column (5), carry therefore not the same weight as those from the iTF fit. However, the results from the iTF and bTF relations agree to within ~0.1 mag.

The resulting distance moduli from the two upper-envelope fits and the corresponding linear distances are given in Table 6, columns (6) and (7). Adopting an internal error of the Virgo modulus of 0.15 mag and a random error of 0.2 mag for the imperfect definition of the upper envelope, one finds a mean error of $\sigma_{m-M} \approx 0.25$ mag for a single cluster. The mean zero

[2] Note that the opposite selection procedure was used here for the Virgo cluster, where first an optically defined, *complete* galaxy sample was selected for which *then* as many line widths as possible were compiled.

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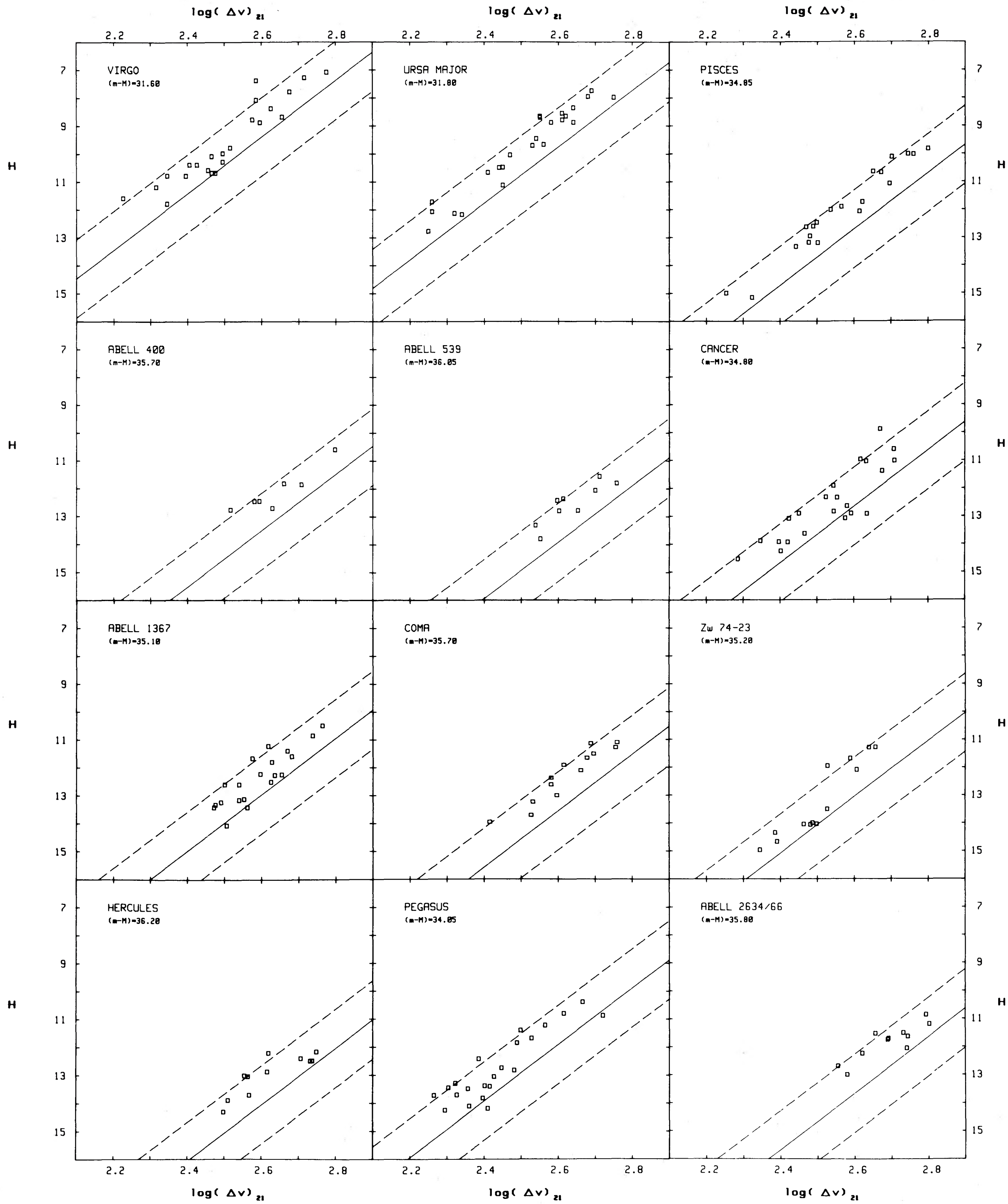

FIG. 7.—The upper-envelope fits of the iTF relation for Virgo, UMa, and 10 more distant clusters. The 2 σ confidence band, as determined by the complete Sb–Sm data in Virgo (*upper left panel*), is shown. The upper 2 σ envelope of Virgo is fitted to the individual clusters. The resulting distance moduli are in Table 6. (In the Virgo panel only the observed H magnitudes including the Sa, Sab galaxies are shown).

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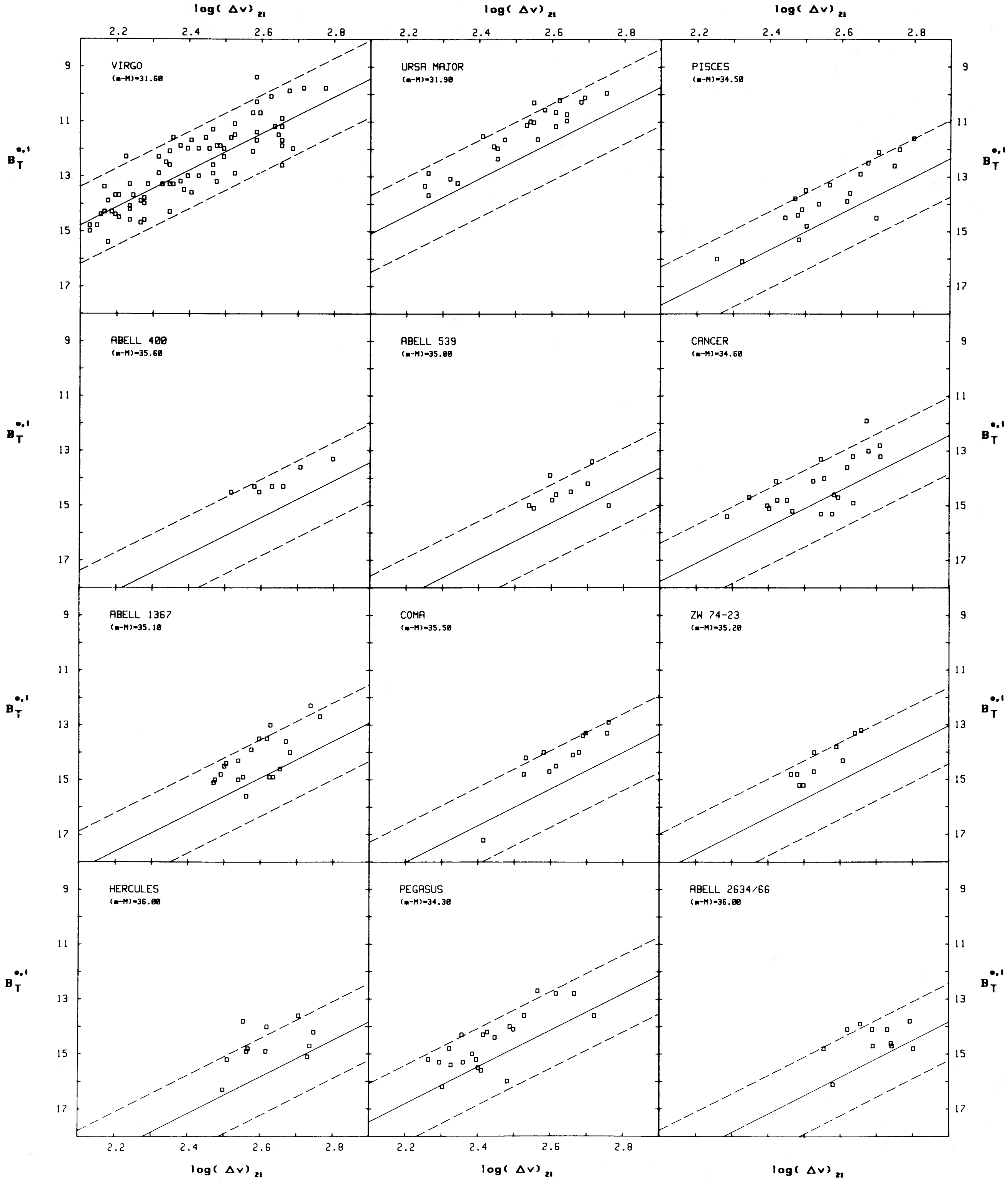

FIG. 8.—Same as Fig. 7, but for blue magnitudes. Only the (nearly) complete Virgo sample of Sa–Sm spirals (*upper left panel*) reveals the intrinsic scatter of the bTF relation. (Six Virgo members lie off-scale to the left).

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point of the distance scale, as defined by the nine clusters with $v > 3500$ km s^{-1} (excluding Hercules), has therefore an internal error of probably less than 0.2 mag.

The modulus differences $\Delta(m - M)$ between the individual clusters and the Virgo Cluster have been listed by Aaronson *et al.* (1986). Their mean value for nine clusters beyond $v = 3500$ km s^{-1} (without Hercules) is $\Delta(m - M) = 3.36$ mag. From Table 4, column (6) we find 3.61 mag instead. The difference of the differences is 0.25 mag which is only ~ 1.2 times our estimated error. We believe that our upper-envelope fitting to the available data is a much safer and preferable method. The increase in distance is however only about 12%. The reason is that the galaxies with H photometry in the more distant clusters *and* in the Virgo cluster are about equally biased, and that a mean TF line through the biased points gives reasonably good *relative* distances.

An analytical correction for the selection bias in clusters has led Bottinelli *et al.* (1987, Table 3) and Bottinelli, Gouguenheim, and Teerikorpi (1988, Table 3) to a mean distance difference between the same nine clusters and Virgo of $\Delta(m - M) = 4.00$ mag in the blue and $\Delta(m - M) = 3.6$ mag in the infrared. These values are corrected for the Teerikorpi effect under rather idealized assumptions (cf. § IV). But because the correction enters only differentially into the moduli differences, the near agreement between their mean value of $\Delta(m - M) = 3.80$ mag and our value of $\Delta(m - M) = 3.61$ mag is significant. This lends additional support to the upper-envelope fitting procedure.

It should be added that the meaning of a distance determination to the Cancer Cluster is not quite clear, because Bothun *et al.* (1983) have identified the clustering as an unbound collection of groups.

More severe are our problems with the most distant cluster in Hercules. Our formal distance of $(m - M)^0 \approx 36.1$, only 4.4 mag more than Virgo and 0.4 mag more than Coma, seems untrustworthy. While Buta and Corwin (1986) imply a distance difference between Hercules and Virgo of 4.2 mag, Aaronson *et al.* (1986) and Bottinelli *et al.* (1987) obtain from the TF method a difference of 4.45 mag and 5.2 mag. The cluster contains a relatively large fraction of Sa galaxies (cf. Fig. 5; Buta and Corwin 1986), which are unsuitable for the TF relation. In addition the observational parameters are clearly of lower quality at the distance of Hercules. For instance the 10 values of $\log \Delta v_{21}$ in common in Bothun (1981) and Giovanelli, Chincarini, and Haynes (1981) have a mean scatter of 0.14, which alone introduces a scatter in magnitudes of $\sigma_M = 0.9$ mag respectively 1.4 mag into the bTF and iTF relations. Of course, the fitting of upper envelopes, which always depends on a few points, is particularly sensitive to observational errors. For these reasons we refrain from quoting a distance value for the Hercules Cluster.

After the exclusion of the Hercules Cluster 11 clusters remain in Table 6. They will be used in the next section for a determination of H_0.

VI. HUBBLE RATIOS OF 11 CLUSTERS

In the previous section the TF distances to 11 clusters are given (Table 6). They span the velocity range $1200 < v_{220} < 8600$ km s^{-1}. The mean cluster velocities are taken mainly from Aaronson *et al.* (1986); their quoted mean error is typically 90 km s^{-1} which is negligible for the following conclusions. The velocity of the Virgo Cluster comes from Binggeli, Tammann, and Sandage (1987); because this paper excludes any contamination by nonmembers the resulting cluster velocity carries high weight, its statistical error being 67 km s^{-1}. The velocity of the UMa Cluster rests on the 24 galaxies which were used for the distance determination; due to the small velocity dispersion of this cluster the velocity mean is determined to within 36 km s^{-1}. All cluster velocities were corrected for our local infall velocity of 220 km s^{-1} toward the Virgo Cluster (Kraan-Korteweg 1985; Tammann and Sandage 1985). They are listed in Table 6, column (3).

The Hubble ratios H_i of the individual clusters follow directly from columns (3) and (7) in Table 6. They are given in Column (8). The mean of the 11 determinations of H_i is $H_0 = 56.6 \pm 0.9$ km s^{-1} Mpc^{-1}, which is interpreted as the best estimate from the cluster data. The very small formal error of H_0—resting on only 11 clusters—should be taken as a chance result of small number statistics. A more realistic error estimate is obtained by remembering from § V that the mean zero-point error of the distance scale is ~ 0.2 mag. This corresponds to a 1 σ confidence range of $52 < H_0 < 62$ km s^{-1} Mpc^{-1} at the effective distance of the clusters under investigation.

The result gains additional support from the fact that the values of H_i show no dependence on distance. The ensuing *linearity* of the expansion field in the relevant distance range ($1200 < v_{220} < 8600$ km s^{-1}) is in perfect agreement with the independent evidence from first-ranked galaxies in groups and clusters (Sandage 1975) and from the maximum luminosity of Type Ia supernovae (Tammann 1987). A conspicuous and sensitive presentation of the linearity of the present distance scale is rendered by the Hubble diagram in Figure 9, where the cluster velocities are plotted versus their distances on a *linear* scale. The consequences of the small scatter about the mean relation—even if fortuitous to some degree—on the peculiar velocities of the 11 clusters is investigated in § VII.

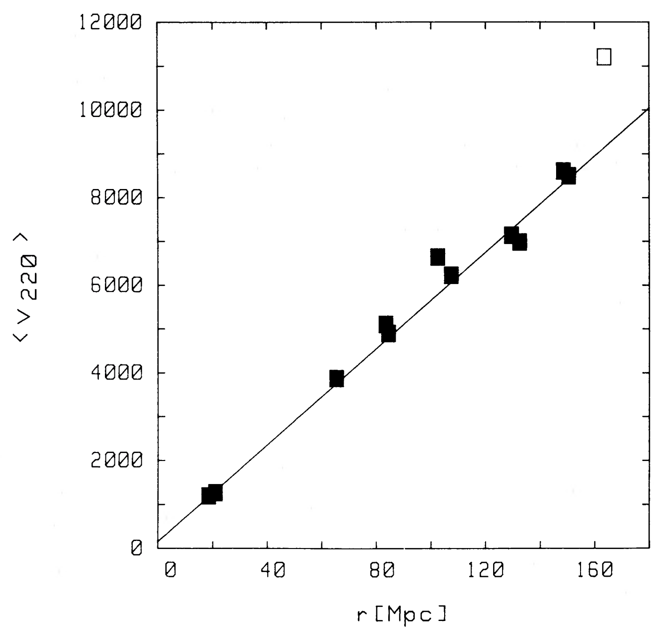

FIG. 9.—The Hubble diagram on a linear scale for 12 clusters with TF distances. The uncertain Hercules cluster is shown as an open symbol. The best fit through the data, $v_{220} = 54.9r + 153$ (km s^{-1}), is shown. Note the small scatter ($\sigma_v = 342$ km s^{-1}) and the high degree of linearity.

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VII. RESULTS

For the first time the TF relation could be applied to a (nearly) complete sample of Virgo Cluster members. The resulting blue *and* infrared TF relations have an intrinsic scatter of $\sigma_m = 0.7$ mag, which is only slightly less than the scatter derived for field galaxies (Sandage 1988*b*). The large scatter of the TF relation makes it a rather delicate tool for distance determinations, because it is enormously vulnerable to selection bias. The scatter is larger than claimed in earlier papers and explains the widely diverging distance moduli published for the Virgo Cluster.

In addition, different type mixtures define different TF relations (§ IV). The restriction to a narrow type interval brings no improvement, because the slope of the relation would then be poorly determined. A necessary requirement for a successful application of the TF relation is therefore an approximate agreement on the type and/or line width distribution between the sample at unknown distance and the calibrators, which are used for the zero point of the TF relation. Luckily the 13 calibrators in Table 1 and the Virgo sample in Table 3 fulfill this requirement (after exclusion of the Sa, Sab, and Im galaxies).

The bTF relation of the 13 calibrators yields for the complete Virgo sample a best modulus of $(m - M)^0_{\rm Virgo} = 31.60 \pm 0.15$. The solution is quite stable against details of the sample definition as long as the calibrators and the Virgo sample agree approximately in mean type and mean line width.

For only 25 relatively bright members of the Virgo sample H magnitudes have been observed. If the mean iTF relation, as defined by any local calibrators, were applied to this biased subsample, an arbitrarily small Virgo modulus would be derived (e.g., Aaronson and Mould 1983). We have used the well-defined dependence of $(B-H)$ on line width (eq. [4]) to calculate H magnitudes for all galaxies of the Virgo sample for which no direct H photometry exists. The resulting scatter of the iTF relation is 0.7 mag, i.e., as large as for the bTF relation. This result is essentially confirmed by field galaxies for which Sandage (1988*b*) has shown that the scatter of the iTF relation is only slightly smaller than that of the bTF relation. There is therefore no reason to prefer the iTF over the bTF relation. Occasional claims to the contrary are based on biased samples. If the observed and calculated H magnitudes are combined for all Virgo galaxies of type Sb–Sm the iTF relation alone yields a distance modulus of 31.62 ± 0.12. The perfect agreement with the result of the bTF relation not only adds confidence in the calculated H magnitudes, but also lends support to the adopted Virgo modulus of $(m-M)^0 = 31.60 \pm 0.15$.

The Virgo Cluster contains two major subclusterings A and B; the possibility of B being more distant than A has been discussed by Binggeli, Tammann, and Sandage (1987). If the bTF and iTF relations are separately applied to A and B, one finds that B is more distant by $\Delta(m - M)^0 = 0.31 \pm 0.21$. This is hardly significant and the two subclusters could well be at the same distance. However, a self-consistent Virgocentric infall model with a local infall velocity of 220 km s^{-1} puts B—with a mean velocity of $\langle v_0 \rangle = 824$ km s^{-1}—at 1.2 times the distance of A (Kraan-Korteweg 1986). This corresponds to $\Delta(m - M)^0 = 0.40$ and agrees well within the error with our derived distance difference; the latter may indeed be slightly too low because of the remaining difference in average type $\langle T \rangle$. A higher local infall velocity would move B even farther into the background and would worsen the agreement with the observed distance difference. Hence, if B is truly more distant than A, relatively small local infall velocities are more probable. Taken separately, clusters A and B exhibit roughly the same scatter of the bTF and iTF relations as the combined sample (cf. Table 5); this proves that the large scatter of the complete sample is not caused by a front-to-background effect.

The indicated error of the adopted Virgo modulus reflects only internal errors. Probably the two most important external error sources are (1) distance errors of the group galaxies (i.e., the calibrators) and the generally hydrogen-poor cluster galaxies (i.e., the Virgo Cluster). Although individual distances in Table 1 may still be subject to revisions, the mean zero point from the calibrators seems to be secure at the 0.2 mag level (cf. § II). The question whether the same TF relation holds for field and cluster galaxies can be tested only by comparing independent determinations of the Virgo modulus. Table 7 compiles four recent, independent methods. The Virgo moduli agree to within the indicated internal errors and they give a weighted mean modulus of $(m - M)^0_{\rm Virgo} = 31.62 \pm 0.15$. This value is identical to the present value of 31.60, and there remains hardly any room for external errors beyond ~ 0.3 mag.

Between 7 and 24 H magnitudes per cluster are available in 11 clusters beyond Virgo. They each define an iTF relation with a scatter ranging from $\sigma_M = 0.34$ mag to 0.71 mag which—on average—is significantly less than the full scatter of the complete Virgo sample. This is contrary to the *a priori* expectation, because the Virgo galaxies are carefully selected bona fide cluster members, while the membership assignment of the more distant clusters is quite rough (Sandage 1988*b*). The only plausible explanation for the reduced scatter is selection bias. Clearly the cluster galaxies were not sampled down to an apparent blue magnitude limit (which would still leave some selection bias; cf. Teerikorpi 1987; Bottinelli *et al.* 1987) but they were selected with preexisting knowledge of their 21 cm line widths and the desire to cover a wide range in Δv_{21}. As

TABLE 7

DIFFERENT DETERMINATIONS OF THE VIRGO MODULUS

Method	$(m - M)^0_{\rm Virgo}$	Calibrators used	Source
TF relation	31.60 ± 0.15	13 nearby galaxies	Here
SNe Ia	31.94 ± 0.44	Various methods	Tammann 1987
Gl. Clusters	31.43 ± 0.30	Galaxy (+ Local Group)	van den Bergh *et al.* 1985
Novae	31.57 ± 0.43[a]	M31	Pritchet and van den Bergh 1987
D_n/σ	31.64 ± 0.20[b]	M31	Dressler 1987*a*

[a] Adjusted by 0.12 mag for absorption of M31 (Sandage and Tammann 1987)
[b] May have to be increased by ~ 0.12 mag (Sandage and Tammann 1987)

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TABLE 8

THE ABSOLUTE MAGNITUDE OF FIRST-RANKED GALAXIES WHOSE PARENT CLUSTERS HAVE TF DISTANCES

Cluster	V_c^{T} [a]	$(m-M)^0$ [b]	M_V
Virgo	8.21	31.60	−23.39
A 539	12.89	35.93	−23.04
A 1367	12.10	35.10	−23.00
Coma	11.58	35.60	−24.07:
Pegasus	11.26	34.18	−22.92
Mean (without Coma):			-23.09 ± 0.10
Mean (with Coma):			-23.27 ± 0.20

[a] From Sandage and Hardy 1973.
[b] From Table 6.

magnitude $\langle M_V \rangle$. From the mean Hubble line $m_V = 5 \log cz - 6.83$ of Sandage and Hardy (1973) follows for, e.g., $cz = 20{,}000$ km s^{-1} an apparent magnitude of $m_v = 14.68$ mag. Combining this value with $\langle M_V \rangle = -23.09 \pm 0.10$ from Table 8 yields a distance modulus of $(m-M)^0 = 37.77$ or a linear distance of $r = 358$ Mpc and hence $H_0 = 20{,}000/358 = 56$ km s^{-1} Mpc^{-1}. Including the Coma Cluster, whose brightest member may be overly luminous, one obtains $\langle M_V \rangle = -23.27 \pm 0.20$ mag and $H_0 = 51$ km s^{-1} Mpc^{-1}. These two values agree so well with the above result ($H_0 = 56.6$ km s^{-1} Mpc^{-1}) that any systematic change of H_0 must be quite small beyond $cz \approx 10{,}000$ km s^{-1}.

Do the 10 distant clusters reflect the Local Group's space motion which is required to explain the dipole moment of the MWB? Our peculiar MWB velocity of ~630 km s^{-1} can be decomposed into a (local) Virgocentric velocity vector of 220 km s^{-1} and a second velocity vector of 495 ± 60 km s^{-1} in the direction of ($\alpha = 153.^\circ5 \pm 5.^\circ4$, $\delta = -41.^\circ5 \pm 3.^\circ7$) (Tammann and Sandage 1985). This is in the direction of Hydra, but the question whether the Hydra (Centaurus) supercluster *is* the accelerator remains, of course, open. If the accelerator would lie within the bounds of the effective distance ($v_{220} \approx 6000$ km s^{-1}) of the nine distant clusters (excluding here again the uncertain Hercules cluster), they formed a rest frame which should reflect our Hydracentric motion (correcting first for our Virgocentric infall). In that case the velocity residuals $\Delta v = v_{220}(\text{observed}) - \langle H_0 \rangle r$ had to show the appropriate correlation with cos a; here $\langle H_0 \rangle$ is the mean Hubble constant of the nine clusters, r is the distance of a given cluster and a is the angle between this cluster and the Hydra apex. The relevant data are shown in Figure 10. No correlation is visible between the velocity residuals and the position in the sky. If the data are taken *at face* value the nine clusters do *not* form a rest frame, but they partake of our motion toward the Hydra apex. This conclusion is opposite to the result from the old TF cluster distances (Aaronson *et al.* 1986; Mould 1986), but it agrees qualitatively with the solution of Dressler and collaborators (Dressler 1987*b*) and Lynden-Bell (1986) from independent data. The latter authors suggest that a volume of radius ~10,000 km s^{-1} partakes of a bulk motion of several hundred km s^{-1}. A later interpretation of the data led Lynden-Bell *et al.* (1988) to the conclusion of large-scale streaming motions toward a common "great attractor" at $v_0 \approx 4500$ km s^{-1} in Centaurus. This model, which leaves still unexplained the observed MWB dipole, implies a distance- and direction-dependent peculiar motion of the Sun with respect to the present nine clusters. But a realistic appraisal of the present data permits no firm conclusion. The distances of the nine clusters must be further improved and their number probably increased to decide to what extent they provide a rest frame.

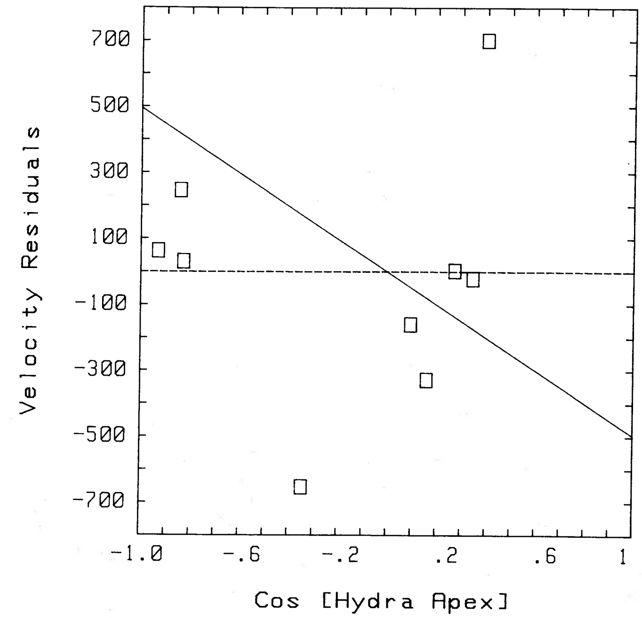

FIG. 10.—The velocity residuals of nine clusters with $4000 < v_{220} < 8500$ km s^{-1} against the angular distance from the expected apex in Hydra. The sloped line gives the mean relation *if* the Local Group were moving with a MWB velocity of 495 km s^{-1} (after correction for the Virgocentric infall) with respect to the nine clusters. No such motion is revealed by the data.

VIII. CONCLUSIONS

The blue and infrared TF relations, due to their large intrinsic scatter ($\sigma_M \approx 0.7$ mag), can successfully be applied only to very well defined samples. A particularly suitable, almost complete sample is provided by *all* spiral galaxies which are certain members of the Virgo Cluster. Based on bTF and iTF relations, which are calibrated using 13 nearby galaxies with known distances, a Virgo modulus of $(m-M)^0_{\text{Virgo}} = 31.60 \pm 0.15$ is derived. Judging from independent evidence this result carries an external error of less than 0.3 mag.

The quoted value of the Virgo modulus is not vulnerable to the TF type segregation—which is certainly present—because it is based on the restricted subsample of Sb–Sm galaxies which coincides with the type interval of the calibrators. In addition, Virgo members and calibrators agree well in mean line width. But the present distance determination leaves still room for improvement. The data for the undisturbed Virgo members with $i \geq 45°$ are not fully complete yet. The cluster contains in the relevant type range 90 Sb–Im galaxies, but for only 75 of those line widths are available. The missing galaxies will probably increase the distance modulus, but by an insignificant amount. In addition, only a fraction of the Virgo spirals have actually observed infrared magnitudes; for the majority of the cluster members H magnitudes had to be calculated from a well-defined $(B-H)/\log \Delta v_{21}$ relation (cf. § III). Future observations of these magnitudes will add to the weight of the present result. But the available H magnitudes suffice to show that the bTF and iTF relations yield closely matching Virgo distances. Besides these desiderata the improvement of the accuracy of the observational parameters is probably of lesser importance.

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TABLE 8

THE ABSOLUTE MAGNITUDE OF FIRST-RANKED GALAXIES WHOSE PARENT CLUSTERS HAVE TF DISTANCES

Cluster	$V_c^{T\ a}$	$(m - M)^{0\ b}$	M_V
Virgo	8.21	31.60	−23.39
A 539	12.89	35.93	−23.04
A 1367	12.10	35.10	−23.00
Coma	11.58	35.60	−24.07:
Pegasus	11.26	34.18	−22.92
Mean (without Coma):			−23.09 ± 0.10
Mean (with Coma):			−23.27 ± 0.20

[a] From Sandage and Hardy 1973.
[b] From Table 6.

magnitude $\langle M_V \rangle$. From the mean Hubble line $m_V = 5 \log cz - 6.83$ of Sandage and Hardy (1973) follows for, e.g., $cz = 20{,}000$ km s^{-1} an apparent magnitude of $m_v = 14.68$ mag. Combining this value with $\langle M_V \rangle = -23.09 \pm 0.10$ from Table 8 yields a distance modulus of $(m - M)^0 = 37.77$ or a linear distance of $r = 358$ Mpc and hence $H_0 = 20{,}000/358 = 56$ km s^{-1} Mpc^{-1}. Including the Coma Cluster, whose brightest member may be overly luminous, one obtains $\langle M_V \rangle = -23.27 \pm 0.20$ mag and $H_0 = 51$ km s^{-1} Mpc^{-1}. These two values agree so well with the above result ($H_0 = 56.6$ km s^{-1} Mpc^{-1}) that any systematic change of H_0 must be quite small beyond $cz \approx 10{,}000$ km s^{-1}.

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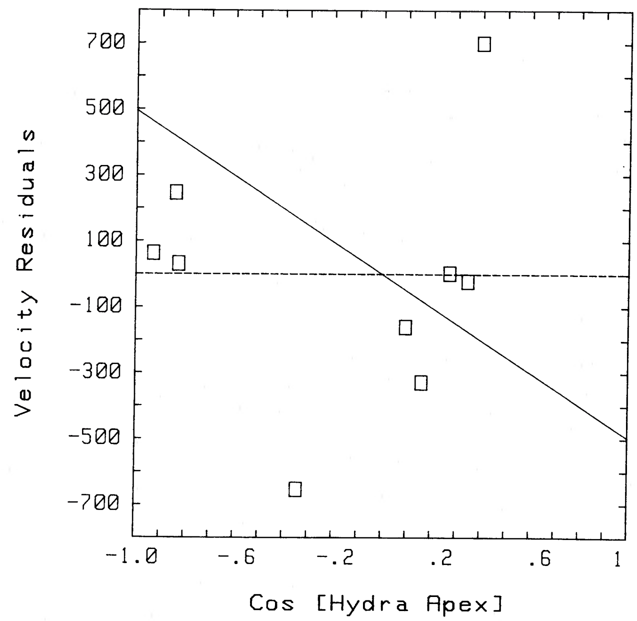

FIG. 10.—The velocity residuals of nine clusters with $4000 < v_{220} < 8500$ km s^{-1} against the angular distance from the expected apex in Hydra. The sloped line gives the mean relation *if* the Local Group were moving with a MWB velocity of 495 km s^{-1} (after correction for the Virgocentric infall) with respect to the nine clusters. No such motion is revealed by the data.

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The unbiased relation for spirals between the color $(B-H)$ and the *absolute magnitude* is still poorly defined; if its formal intrinsic scatter of $\sigma_M \approx 0.7$ mag should be confirmed it would become a competitive distance indicator.

Some H photometry is available in the UMa Cluster and 10 more distant clusters, but it covers only the brightest galaxies at given line width in each cluster. This is clearly revealed by their small scatter in the TF relation. A fitting of the upper 2 σ envelope, as determined by the Virgo Cluster data, to the individual clusters—for the iTF as well as the bTF—still yields consistent distances. They are somewhat preliminary, but the observation of the necessary complete spiral sample down to ~ 6 mag below the brightest member in each cluster remains an enormous task for the future.

The Virgo Cluster, the UMa Cluster, and nine more distant clusters (disregarding the uncertain Hercules Cluster) define a value of the Hubble constant of $H_0 = 57 \pm 1$ km s^{-1} Mpc^{-1}. For the brightest members of five of these clusters good V_T^c photometry is available from Sandage and Hardy (1973). Their TF distances define a mean absolute magnitude of -23.09 mag $< M_V < -23.27$ mag. If this value is inserted into Sandage and Hardy's mean Hubble relation, which is linear out to very large distances, the global value of H_0 becomes $51 < H_0 < 56$ km s^{-1} Mpc^{-1} with an estimated external error of less than 20%. The statistical weight of this result could be increased by V_T^c photometry of the brightest members of the remaining four clusters with TF relations.

The linearity of the very local expansion field, as found by Sandage (1987), for the distance range $200 < v_{220} < 1200$ km s^{-1} is perfectly continued by the present data for the range $1000 < v_0 < 8500$ km s^{-1} and well beyond this limit by the Hubble diagram of first-ranked cluster galaxies. The random velocity deviations of the clusters with TF distances from a perfectly linear Hubble expansion are less than 400 km s^{-1} in one dimension. No conclusion can be drawn as to our peculiar motion with respect to the rest frame, the latter being still poorly defined by the present clusters.

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