THE ASTROPHYSICAL JOURNAL, 328:714–725, 1988 May 15

RADIAL PULSATION IN LUMINOUS HOT HELIUM STARS

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Received 1987 September 28; accepted 1987 November 9

ABSTRACT

Radial pulsations in hydrogen-deficient stars have been studied by calculating linear nonadiabatic pulsation frequencies and by reexamining the observed properties for systematic effects. The observed periods of the variable hydrogen-deficient stars decrease as a function of effective temperature. The observational surface gravity–effective temperature diagram seems to indicate that nonvariable extreme helium stars have smaller luminosity-to-mass ratios than variable helium stars.

The nonadiabatic linear pulsation analysis shows that because of the extremely nonadiabatic pulsations, there exist unstable (overstable) pulsation modes even in very high effective temperature models up to $T_{\rm eff} \sim$ 30,000 K. They are excited by the usual kappa and gamma mechanisms mainly in the second helium ionization zone. Applying the theoretical results to the pulsating luminous hot helium stars, we obtain pulsation masses. The masses of the visible components of hydrogen-deficient binaries ($\sim 1.2\ M_\odot$) are systematically higher than the masses of the single extreme helium stars (~ 0.8–$0.9\ M_\odot$).

Subject headings: stars: hydrogen-deficient — stars: pulsation

I. INTRODUCTION

Small-amplitude light and radial velocity variations of the R Coronae Borealis variable RY Sgr have long been identified with radial pulsation (Alexander *et al.* 1972). Similar small-amplitude light variations have been observed in other R Coronae Borealis (RCrB) stars in extreme helium (EHe) stars (Hill 1969), and in the hydrogen-deficient binary (HdB) stars (Eggen, Kron, and Greenstein 1950). All three groups of stars are characterized by extremely low surface hydrogen abundances ($n_{\rm H}/n_{\rm He} \sim 10^{-4}$) and low surface gravities. It has been known for some time that the behavior of the small-amplitude variations in some RCrB stars is roughly periodic (e.g., Fernie, Sherwood, and DuPuy 1972) and has hence been identified with pulsation. Evidence for periodicity in the variations of the HdB KS Per was found by Osawa, Nishimura, and Nariai (1963) and in the EHe HD 160641 by Landolt (1975). In recent years more substantial evidence for periodicity in the light curves of EHe and HdB stars has become available, beginning with the detection of a ~ 21 day period in the EHe BD $+1°4381$ (Jeffery and Malaney 1985).

Although similar in many aspects, the RCrB, EHe, and HdB stars represent quite distinct groups of stars. The EHe and RCrB stars are apparently single stars (Jeffery, Drilling, and Heber 1987) with high surface abundances of C and N (e.g., Schönberner and Wolf 1974; Schönberner 1978). The RCrB stars are distinguished by the presence of a large infrared excess (Feast and Glass 1973) and by occasional deep light minima (Pigott 1797). The HdB stars have low surface C abundances and are all single-lined spectroscopic binaries (Jeffery, Drilling, and Heber 1987).

The theoretical problem of pulsation in luminous helium stars has been addressed by several authors (Trimble 1972; Wood 1976; Cox *et al.* 1980; Saio, Wheeler, and Cox 1984; Saio and Wheeler 1985; Saio 1986; Proffitt and Cox 1987; Weiss 1987). In these stars, with $L/M \sim 10^4\ L_\odot/M_\odot$, the thermal time scale is close to the dynamical time scale, giving rise to strong nonadiabaticity in the dynamical properties of the star. The strong nonadiabaticity significantly modifies the instability boundary in the H-R diagram. Below a critical luminosity the blue edge of the instability strip resembles that for the Cepheids, but it becomes almost horizontal in the H-R diagram at the critical luminosity (Cox *et al.* 1980; Saio, Wheeler, and Cox 1984). However, although hydrogen-deficient (Hdef) stars are observed at effective temperatures ranging from ~ 5000 to $\sim 32{,}000$ K, only a limited number of linear nonadiabatic pulsation calculations have been carried out for $T_{\rm eff} > 10{,}000$ K.

In view of recent developments in the observation of Hdef stars, the overall properties of Hdef stars are examined for systematic effects. New linear nonadiabatic pulsation calculations are presented, and an interpretation of the observational material is attempted.

II. OBSERVED PROPERTIES OF HYDROGEN-DEFICIENT STARS

In Table 1 the best available values for effective temperatures ($T_{\rm eff}$), surface gravities (g) and photometric "periods" (Π) have been collated. It is necessary to qualify these data in particular because of the inhomogeneity of the source material. For many RCrB stars only a blackbody temperature has been derived from the stellar flux distribution observed in the visual. For the remainder, effective temperatures and surface gravities have been derived by the method of fine analysis from the correspondence of ionization equilibria and excitation temperature in (g, $T_{\rm eff}$)-space (e.g., Schönberner 1975). For the EHe and HdB stars, effective temperatures have been obtained from the UV + visual flux distributions and line-blanketed model

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TABLE 1

PROPERTIES OF VARIABLE LUMINOUS HELIUM STARS

Star (1)	T_{eff} (K) (2)	References for col. (2) (3)	log g (4)	References for col. (4) (5)	Π (days) (6)	References for col. (6) (7)
		R CrB Stars				
S Aps[1]	4000[a]	1	...		120 ± 20	16
S Aps[2]	4000[a]	1	...		40 ± 10	17
UW Cen	6800 ± 600	2	0.0 ± 0.5	2	43 ± 3	16
R CrB	7000 ± 500	3, 4	0.4 ± 0.5	3, 4	46 ± 5	18
RY Sgr	7100 ± 600	3	0.1 ± 0.7	3	38.6 ± 0.2	19
XX Cam	7100 ± 500	3, 4	0.1 ± 0.5	3, 4	[b]	
MV Sgr	15400 ± 500	5	2.5 ± 0.3	9	[c]	20
		EHe Stars				
BD +1°4381	9500 ± 400	5	...	...	22 ± 2	21, 22
BD −1°3438	10900 ± 600	5	1.3 ± 0.3	10	7 ± 2	23
LS IV −1°2	11900 ± 400	5	...	...	11 ± 1	24
PV Tel (HD 168476)	12400 ± 400	5	1.3 ± 0.3	11	(7)	25
LS II +33°5	14800 ± 700	5	...	...	3–4	20, 26
HD 124448	15500 ± 500	5	2.5 ± 0.2	9	[d]	20, 27
BD +10°2179	17700 ± 600	5	2.5 ± 0.3	12	[d]	28, 29
LSS 3184	22000 ± 800	6	3.2 ± 0.3	6	[b]	20
BD −9°4395	23000 ± 700	5	2.6 ± 0.2	13	3–11	30, 31
V2076 Oph (HD 160641)	31900 ± 1500	5	...		0.7–1.2	32, 33
		HdB Stars				
KS Per	9000 ± 500	5	1.0 ± 0.5	14	31 ± 2	34
υ Sgr	10500 ± 500	5	1.0 ± 0.5	15	20.3 ± 2.1	35
CPD −58°2721	14000 ± 500	7	(1.2 ± 0.3)[e]	7	9 ± 2	36
LSS 4300	14400 ± 500	8	1.4 ± 0.3	8	[c]	37

[a] Blackbody temperature.
[b] Variability unknown.
[c] Period unknown.
[d] Not variable.
[e] Parentheses indicate that value is uncertain or unpublished.

REFERENCES.—(1) Kilkenny and Whittet 1984; (2) Giridhar and Rao 1986; (3) Schönberner 1975; (4) Cottrell and Lambert; (5) Drilling *et al.* 1984; (6) Heber, Jonas, and Drilling 1986; (7) K. Morrison and U. Heber 1987 private communication; (8) Schönberner and Drilling 1984; (9) Jeffery *et al.* 1987; (10) Schönberner 1978; (11) Walker and Schönberner 1981; (12) Heber 1983; (13) Kaufmann and Schönberner 1977; (14) Nariai 1967; (15) Hack and Pasinetti 1963; (16) Kilkenny and Flanagan 1983; (17) Kilkenny 1983; (18) Fernie 1982; (19) Kilkenny 1982; (20) Landolt 1986*b*; (21) Jeffery and Malaney 1985; (22) Morrison *et al.* 1987*b*; (23) Jeffery, Hill, and Morrison 1986; (24) Morrison 1987; (25) P. W. Hill 1987, private communication; (26) Morrison and Willingale 1987; (27) Lynas-Gray and Jeffery 1988, in preparation; (28) Grauer, Drilling, and Schönberner 1984; (29) Hill, Lynas-Gray, and Kilkenny 1984; (30) Jeffery *et al.* 1985; (31) Landolt and Grauer 1986; (32) Lynas-Gray *et al.* 1987; (33) Walker and Kilkenny 1980; (34) Osawa, Nishimura, and Nariai 1963; (35) Malcolm and Bell 1986; (36) Morrison *et al.* 1987*a*; (37) Landolt 1986*a*.

atmospheres (e.g., Drilling *et al.* 1984). Surface gravities were mostly obtained by matching He line profiles and ionization equilibria in (g, T_{eff})-space (e.g., Schönberner and Wolf 1974). However, the surface gravity for υ Sgr was obtained using the method of coarse analysis. The surface gravities for BD − 1°3438 and CPD − 58°2721 are regarded as representative, while they currently remain unpublished.

The photometric properties of the Hdef stars have recently been reviewed by Landolt (1986*b*) and Hill (1987). Here we summarize the salient points and comment on some particular cases. Of the periods reported in Table 1, only that for RY Sgr is given with a precision of less than 1%. The amplitude of the light variation is ~0.5 mag and hence has been well observed over many decades (e.g., Kilkenny 1982; Marraco and Milesi 1982). In the cases of the RCrB stars, the periods are for variations at "maximum light." No account is taken of the irregular deep light minima characteristic of the class. It is a general property that the light curves of Hdef stars are only quasi-periodic (e.g., RY Sgr, Alexander *et al.* 1972; R CrB, Fernie 1982) in that the length and shape of successive cycles varies about some mean value.[3] In addition, the mean amplitude of the light variations is ~0.1 mag, periods range from a few to a hundred or so days, and the majority of EHe stars are fainter than 10th magnitude. Therefore, the "errors" quoted in Table 1 indicate ranges in the actual photometric periods as well as an observational uncertainty due to the difficulty of acquiring accurate light curves.

a) S Aps

After finding evidence for a secular period change (Kilkenny and Flanagan 1983), Kilkenny (1983) reported evidence for a sudden change in the period from ~120 to ~40 days between 1967 and 1971. These are referred to as S Aps[1] and S Aps[2] in Table 1. Kilkenny suggested that the period change could be linked to a change in the pulsation mode.

[3] This property is consistent with the nonlinear pulsation models of Trimble (1972) and Saio and Wheeler (1985).

b) BD +1°4381

Initially found to have a period of ~21 days (Jeffery and Malaney 1985), values of 21.5, 23, and 24 days based on the assumption of a strictly periodic variation have been reported (Jeffery, Hill, and Morrison 1986; Morrison 1987). Jeffery, Hill, and Morrison (1986) noted that the behavior of the light curve may not be strictly regular, and since there is no evidence for strict periodicity in any other Hdef star, the apparent variety of the foregoing results finds a natural explanation in a quasi-periodic variation. A mean period of between 21 and 22 days seems likely (Morrison *et al.* 1987*b*).

c) BD −9°4395 and V2076 Oph = HD 160641

Both stars were found to be variable by Landolt (Landolt 1975; Landolt and Grauer 1986). Recent studies have shown their light curves to be complex and best represented as a multiperiodic variation. Their periods are not typical of those expected for radial pulsations and, with the absence of color variations, have been interpreted as evidence of nonradial pulsations (Jeffery *et al.* 1985; Lynas-Gray *et al.* 1987). The range of detected periods is indicated in Table 1.

d) HD 124448 and BD +10°2179

Reports of microvariability exist for both stars (Walker and Kilkenny 1980; Bartolini *et al.* 1982). However, recent studies have failed to confirm this (Hill, Lynas-Gray, and Kilkenny 1984; Grauer, Drilling, and Schönberner 1984; Landolt 1986*b*), and these objects are therefore assumed not to vary.

e) PV Tel = HD 168476

Walker and Kilkenny (1980) and Walker and Hill (1985) report variability over a long time scale. Unpublished data (P. W. Hill 1987, private communication) indicate that a period of 6–7 days may be present.

f) υ Sgr

Photometric variations taken as evidence for eclipses by Eggen, Kron, and Greenstein (1950) are more probably due to pulsation (Malcolm and Bell 1986), since their time scale is much shorter than that of the spectroscopic binary period.

g) LSS 4300

A recent report of variability in the HdB LSS 4300 has been made by Landolt (1986*a*).

h) DY Cen, V Cra, WX Cra, RT Nor, RZ Nor, VZ Sgr, RS Tel

Reports of periods in a number of RCrB stars have been made mainly from visual observations by amateur observers (e.g., Bateson 1974, 1975). The documentary evidence is unfortunately not extensive, and since this study is concerned mainly with the hotter Hdef stars, these objects are not considered further.

III. SYSTEMATICS OF THE PROPERTIES OF HYDROGEN-DEFICIENT STARS

Figure 1 shows the positions of Hdef stars in a (g, $T_{\rm eff}$)-diagram. Surface gravities and effective temperatures provide luminosity-to-mass ratios for these objects. Typical values are illustrated in Figure 1, where the Eddington limit for pure electron scattering in a singly ionized helium atmosphere is also shown. The range of L/M ratios displayed by Hdef stars is roughly $3.6 < \log (L/L_\odot)(M/M_\odot) < 5.1$. At least two of the Hdef stars with low L/M ratios (HD 124448 and BD +10°2179) are not known to vary.

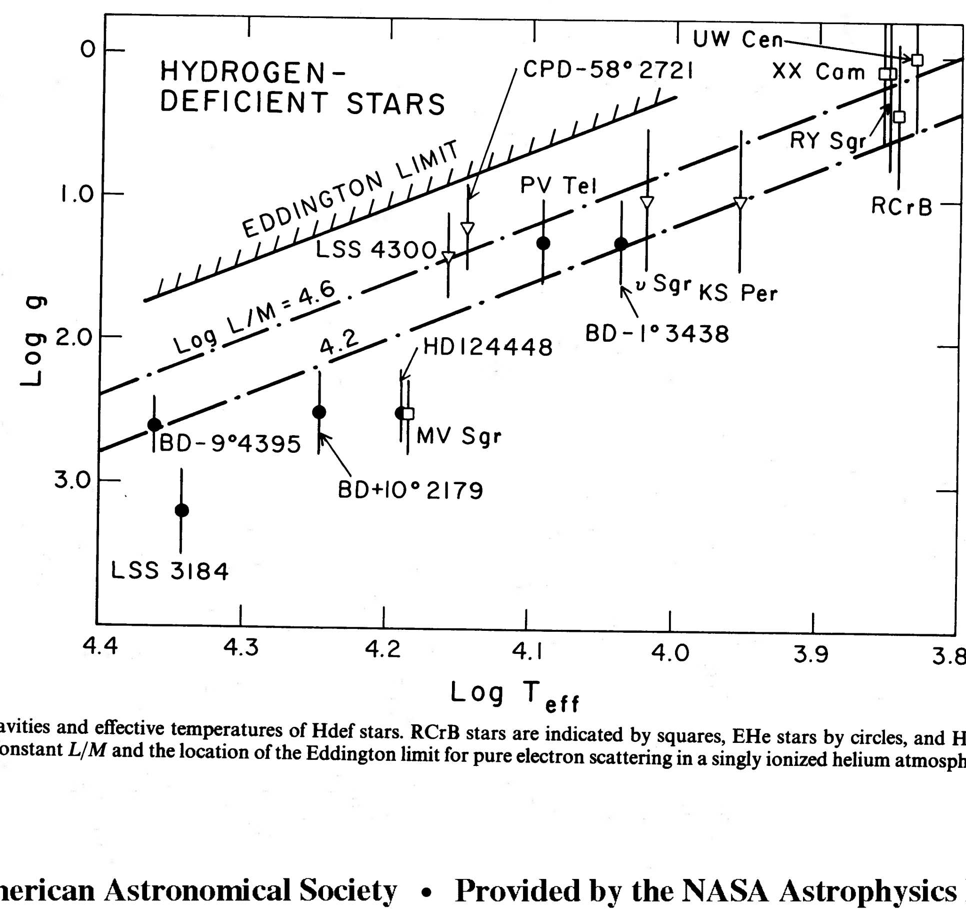

FIG. 1.—Surface gravities and effective temperatures of Hdef stars. RCrB stars are indicated by squares, EHe stars by circles, and HdB stars by triangles. Also shown are the lines of constant L/M and the location of the Eddington limit for pure electron scattering in a singly ionized helium atmosphere.

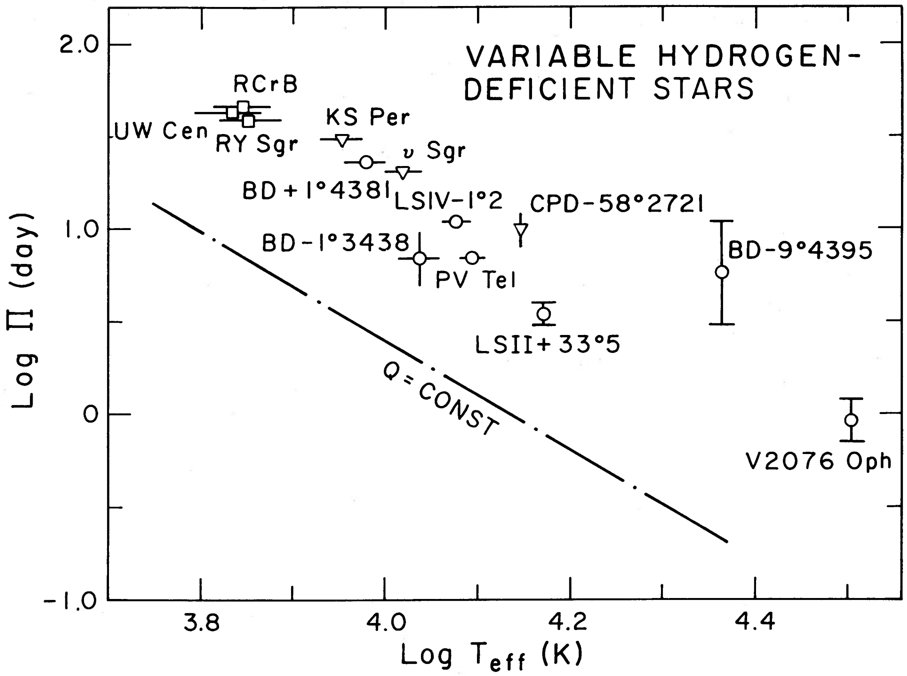

FIG. 2.—Photometric (pulsation) periods and effective temperature of Hdef stars. Symbols as for Fig. 1. The dashed line indicates the gradient of the period-temperature relation expected from a constant Q-value.

Figure 2 shows the observed photometric periods of Hdef stars as a function of $T_{\rm eff}$. It is instructive to find that, with one or two exceptions, there is an approximately linear relationship between the periods and $T_{\rm eff}$ of Hdef stars. This is consistent with the assumption that the photometric periods are due to radial pulsation, because it is predicted for homologous stars with similar L and M by the well-known period–mean density relation (Ritter 1879)

$$\Pi(\rho/\bar{\rho}_\odot)^{1/2} = Q \,, \qquad (1)$$

where $\bar{\rho}_\odot$ is the mean density of the Sun. The gradient of the (Π, $T_{\rm eff}$)-relation predicted by equation (1) with a constant Q-value is illustrated in Figure 2.

IV. LINEAR NONADIABATIC RADIAL PULSATION

a) Results of the Numerical Calculations

To investigate the theoretical pulsation properties of the Hdef stars, the calculations of Saio, Wheeler, and Cox (1984) have been extended to cover a wider range of model parameters. Two compositions were adopted, $(Y, X_c) = (0.9, 0.1)$ to represent the EHe stars, and $(Y, Z) = (0.97, 0.03)$ to represent the HdB stars where Y, X_c, Z stand for the mass fractions of helium, carbon, and heavy elements (including carbon), respectively. The EHe composition is the same as that used in Saio, Wheeler, and Cox (1984). The opacity values for the HdB composition were taken from Cox and Tabor (1976). Masses, luminosities, and effective temperatures were selected from the following grid:

$$M \in (0.6, 1.0, 1.4),$$

$$L \in (0.8, 1.126, 2, 3, 4) \times 10^4 \,,$$

$$T_{\rm eff} \in (7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 25, 30) \times 10^3 \text{ K} \,,$$

where (and hereafter) mass and luminosity are expressed in solar units.

The linear nonadiabatic Q-values (defined by eq. [1]) for the unstable (overstable) modes obtained are shown in Figure 3 (EHe mixture) and Figure 4 (HdB mixture), in which the most unstable mode for each model is shown by the filled or circled symbol. (In order to see the continuous behavior of the Q-values as a function of $T_{\rm eff}$, the Q-values for some models other than the grid are also plotted.) These figures show that there exist unstable modes even in the models with very high effective temperature compared with the blue edge of the Cepheid instability strip ($T_{\rm eff} \sim 6000$ K for $L \sim 10^4$). This gives theoretical support for the assumption that the light variations observed for some EHe and HdB stars are caused by stellar pulsations.

All the unstable modes are excited by the usual kappa and gamma mechanisms (e.g., Cox 1974) mainly in the second helium ionization zone, except for some short-period modes in low-$T_{\rm eff}$ models, for which the first ionization zone is important, especially for the HdB mixture. Usually, these excitation mechanisms are not strong enough to excite pulsations in hot stars. Cox *et al.* (1980) found that if the luminosity-to-mass ratio is high enough, the instability region extends blueward considerably. Figures 3 and 4 show that the extension of the instability reaches the region as blue as $T_{\rm eff} \sim 30{,}000$ K.

We did not calculate the frequencies for the HdB models with $T_{\rm eff} > 20{,}000$ K, because for these models very strong (maybe unphysical) damping appears in the optically thin surface region, where we employed the diffusion approximation. Although such strong dissipation appears in the high-$T_{\rm eff}$ models even for the EHe mixture, the effects on the pulsation frequencies and on the growth rates are not significant.

Figures 5*a* and 5*b* show the Q-values and the amplitude growth rate, $-\sigma_I/\sigma_R$, of some modes (including stable ones) as functions of $T_{\rm eff}$ for the 1 $M_\odot$ EHe models with $L = 2 \times 10^4$ (Fig. 5*a*) and 3×10^4 (Fig. 5*b*), where σ_R and σ_I represent the

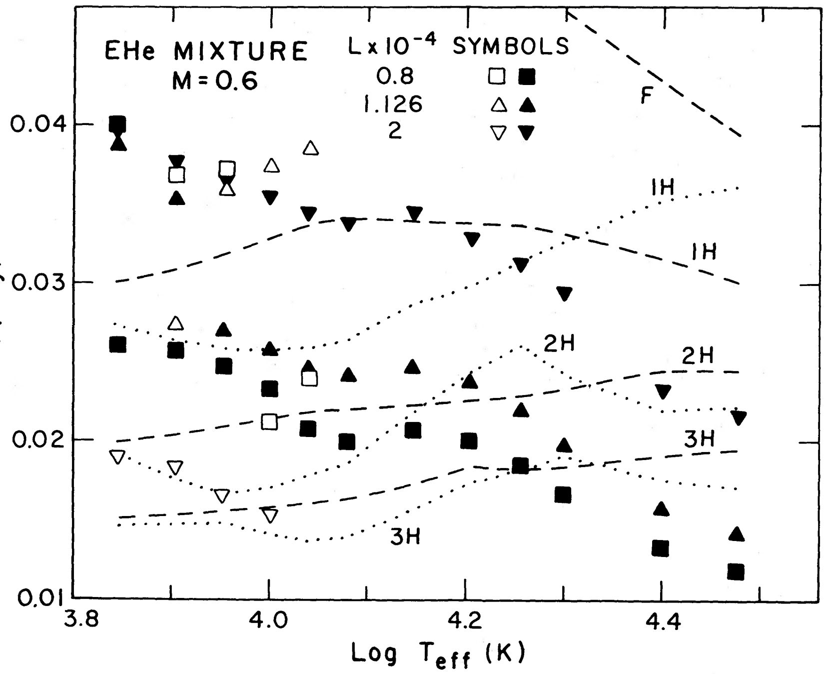

FIG. 3*a*

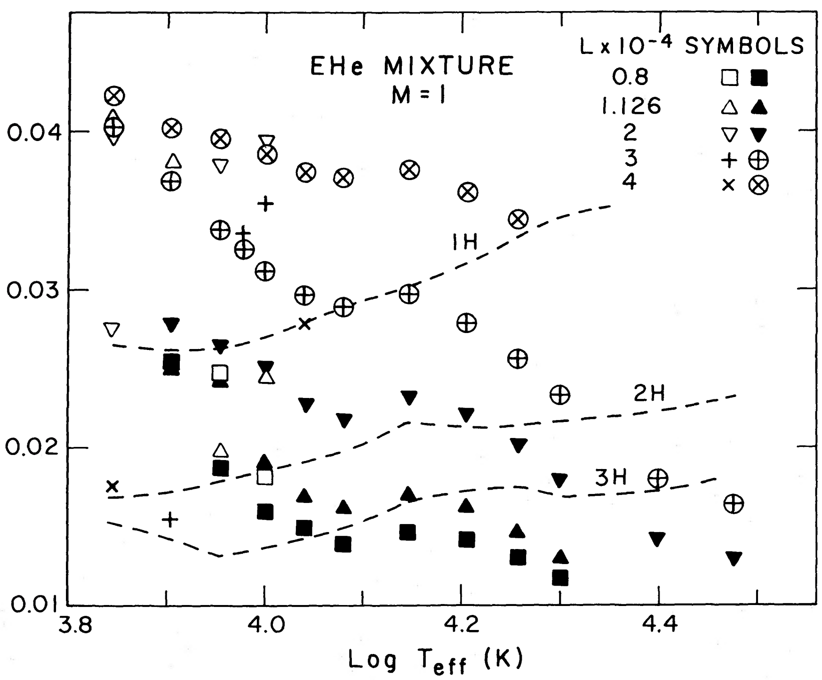

FIG. 3*b*

FIG. 3.—Q-values of unstable modes for EHe mixture $(Y, X_c) = (0.9, 0.1)$. The most unstable mode for each model is shown by a filled or circled symbol. (*a*) $M = 0.6$. The adiabatic Q-values for $L = 8 \times 10^3$ and $L = 2 \times 10^4$ are shown by the dashed and the dotted lines, respectively. (*b*) $M = 1.0$. The adiabatic Q-values for $L = 3 \times 10^4$ are shown by the dashed lines. (*c*) $M = 1.4$. The adiabatic Q-values for $L = 4 \times 10^4$ and $L = 8 \times 10^3$ are shown by the dashed and the dotted lines, respectively.

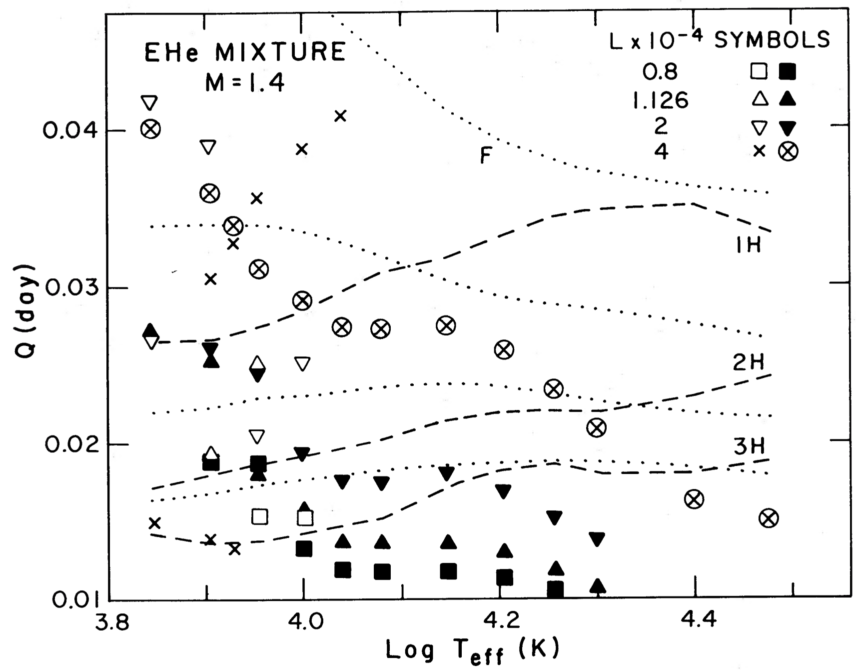

FIG. 3c

real and the imaginary parts of the pulsation frequencies [the temporal dependence of pulsation is assumed to be exp $(i\sigma t)$]. The modes are labeled A, B, and C (after Worrell 1986). Mode A's can be unstable for $T_{\rm eff} \lesssim 10^4$ K. We refer to the modes which are unstable even in the high-$T_{\rm eff}$ models as mode B's. The frequencies of mode C's are nearly complex conjugates of those of mode B's in the high-$T_{\rm eff}$ models. If the luminosity is high enough, the mode B's are extremely unstable (the amplitude growth time, $1/\sigma_I$, is comparable to or shorter than the period $2\pi/\sigma_R$) and are the most unstable modes for the entire $T_{\rm eff}$ range considered. The mode A's are the most unstable modes for $T_{\rm eff} < 9000$ K in relatively low-luminosity models (see Figs. 5*a*, 5*b*). These properties of the pulsation modes are similar to those found by Worrell (1986) for RV Tau and 89 Her stars, which have hydrogen-rich envelopes.

The Q-values of the mode B's increase as luminosity increases and/or the stellar mass decreases. The least-squares fit presented below shows that the Q-values of the mode B's are

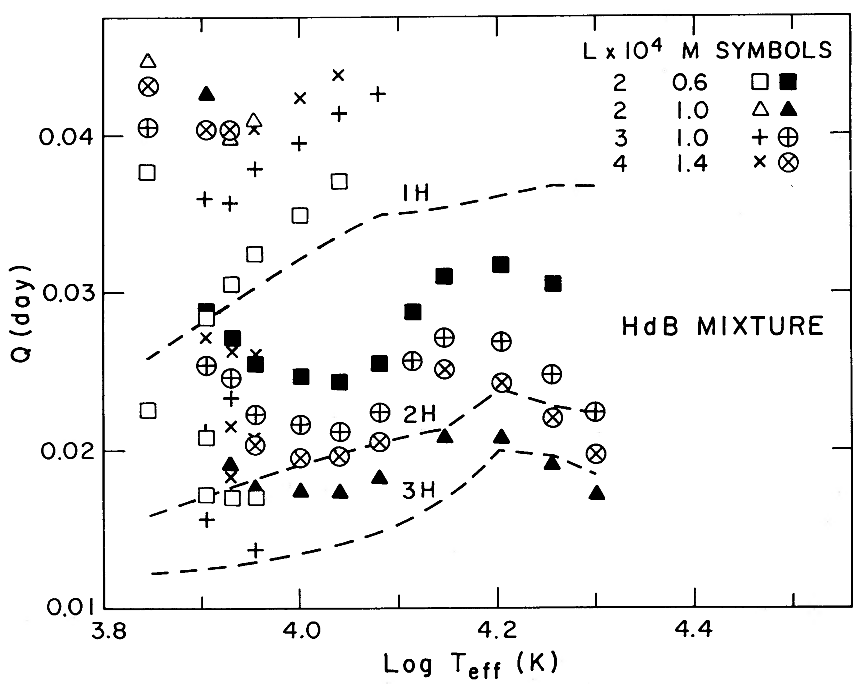

FIG. 4—Q-values of unstable modes for HdB mixture $(Y, Z) = (0.97, 0.03)$. The most unstable mode for each model is shown by a filled or circled symbol. The adiabatic Q-values for $(M, L) = (1.4, 4 \times 10^4)$ are shown by the dashed lines.

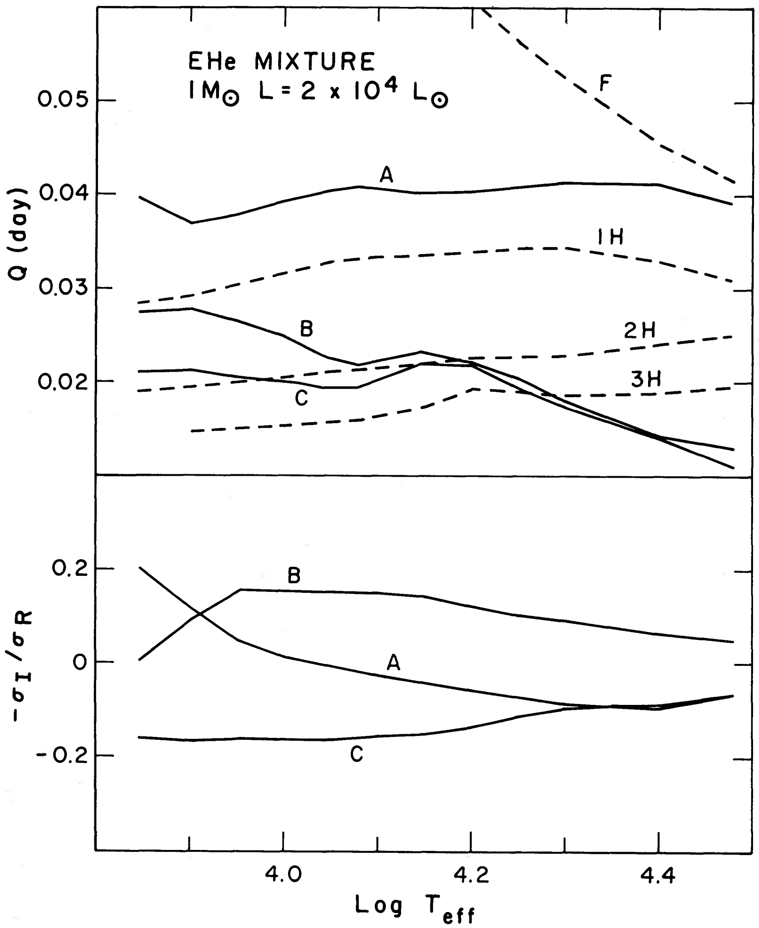

FIG. 5a

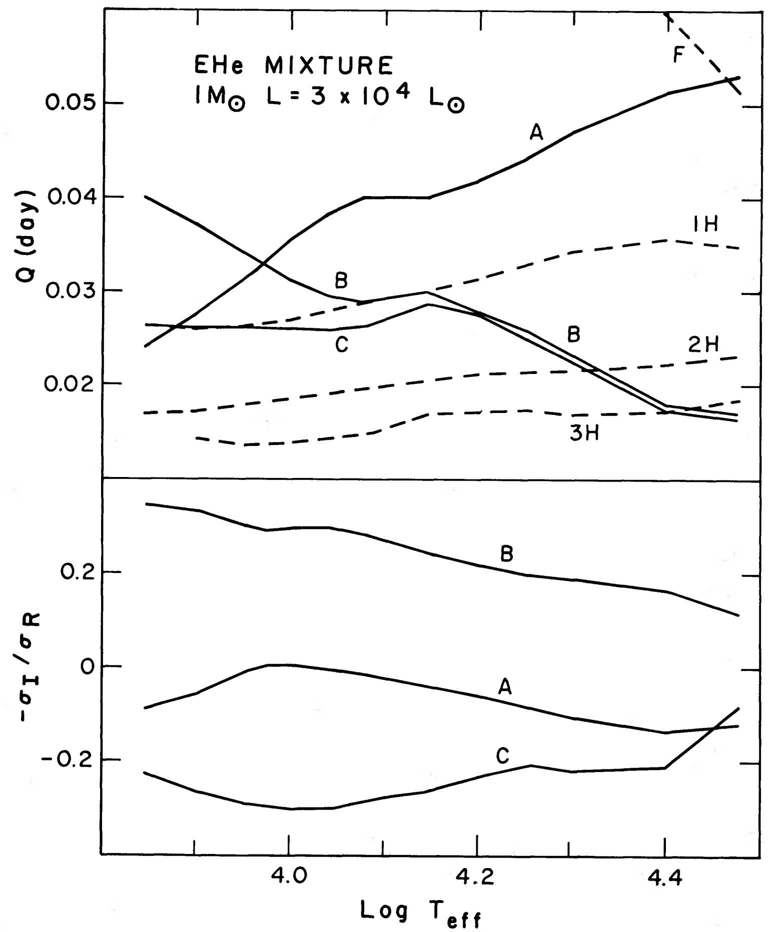

FIG. 5b

FIG. 5.—Q-values and growth rate $-\sigma_I/\sigma_R$ of selected modes of 1 $M_\odot$ EHe stars as functions of effective temperature. The adiabatic Q-values are shown by the dashed lines. (a) $L = 2 \times 10^4$; (b) $L = 3 \times 10^4$.

approximately proportional to $\alpha \equiv L^{0.6}/M^{0.8}$ for a given $T_{\rm eff}$. The Q-values of mode B's generally decrease as $T_{\rm eff}$ increases, which corresponds to the shifting of the helium ionization zones toward the surface. There is a peak in the Q-values around $T_{\rm eff} \sim 16{,}000$ K, for which the first helium ionization zone is close to the photosphere. For the models with $T_{\rm eff}$ lower than this peak, the Q-values of the mode B's are considerably different between EHe and HdB mixtures because the opacity in the region exterior to the first helium ionization zone is sensitive to the metal abundance. Above $T_{\rm eff} \sim 14{,}000$ K, this region vanishes at the stellar surface, so that the Q-values are similar for both compositions.

It is apparent in Figures 3 and 4 that the luminosity and mass dependence of the Q-values of the modes other than the mode B's are quite different from that for the mode B's. For the models with sufficiently high values of α, the Q-values of the most unstable modes enter into the region between the adiabatic fundamental and the first-overtone modes as the effective temperature decreases. On the other hand, for the models with small α, all the modes become stable around $T_{\rm eff} \sim 7000$ K. This corresponds to the blueward extension of the blue edge of the instability region for the "fundamental" mode which was first found by Cox *et al.* (1980). It is interesting to note that the mass dependence of the critical luminosity for the blueward extension of the blue edge (see Fig. 6 in Saio, Wheeler, and Cox 1984) is represented by $\alpha \sim 220$.

For models with small α, the Q-values of the most unstable modes are very small. The outer mechanical boundary conditions of $\partial(\delta P_{\rm gas}/P_{\rm gas})/\partial r \to 0$ may not be appropriate for these short-period modes, where δ means the Lagrangian perturbation due to pulsation. It is quite possible that these modes are not excited in a real star because of the effect of dissipation in the outer atmosphere. This may be consistent with the observational fact that the nonvariable EHe stars tend to have small L/M. Further investigation is necessary to clarify the property of pulsations in an optically thin outer atmosphere.

b) *Fitting Formulae*

In order to apply the theoretical results to the observed period of pulsations, it is convenient to use a fitting formula which approximately represents the theoretical periods of the unstable modes as a function of luminosity, mass, and $T_{\rm eff}$. The effective temperatures of the variable EHe and HdB stars are higher than $\sim$9000 K (Table 1). For the models with $\alpha > 300$ the Q-values of the most unstable modes are continuous functions of $T_{\rm eff}$ in the range 9000 K $\lesssim T_{\rm eff} \lesssim$ 30,000 K. The method of least squares is used to derive fitting formulae for these Q-values (days) as follows. For the EHe mixture, we obtain, for 9000 K $< T_{\rm eff} <$ 20,000 K,

$$\log Q = 0.5956 \log L - 0.7879 \log M - 4.173 - 0.5184\xi + 4.400\xi^2 - 13.325\xi^3 \; (\pm 0.016) , \qquad (2)$$

where

$$\xi \equiv \log T_{\rm eff} - 4 \qquad (3)$$

and the standard deviation of the fit is given in parentheses. Since the first two terms of equation (2) are very close to log α, we redefine α as

$$\log \alpha \equiv 0.5956 \log L - 0.7879 \log M . \qquad (4)$$

Assuming the same dependence on luminosity and mass as above, we obtain, for 20,000 K $\leq T_{\rm eff} \leq$ 30,000 K,

$$\log Q = \log \alpha - 3.571 - 3.408\xi + 3.361\xi^2 \; (\pm 0.016) . \qquad (5)$$

For the HdB mixture, we obtain, for 9000 $K \leq T_{\rm eff} \leq$ 20,000 K,

$$\log Q = \log \alpha - 4.339 - 0.014\xi + 7.616\xi^2 - 24.819\xi^3 \; (\pm 0.013) . \qquad (6)$$

Again, the same dependence on luminosity and mass as in equation (2) is assumed in obtaining equation (6).

The efficacy of approximations (2), (5), and (6) is illustrated in Figure 6, where the quantity log (Q/α) of the most unstable modes is plotted as a function of effective temperature.

Since the Q-value is defined by equation (1), we have

$$\log \Pi = \log Q + 0.75 \log L - 0.5 \log M - 3\xi - 0.710 . \qquad (7)$$

Thus equations (2), (5), and (6) become, respectively,

$$\log \Pi = 1.288\gamma_p - 4.883 - 3.518\xi + 4.400\xi^2 - 13.325\xi^3 \qquad (8)$$

(EHe; 9000 K $\leq T_{\rm eff} \leq$ 20,000 K),

$$\log \Pi = 1.288\gamma_p - 4.281 - 6.408\xi + 3.361\xi^2 \qquad (9)$$

(EHe; 20,000 K $\leq T_{\rm eff} \leq$ 30,000 K), and

$$\log \Pi = 1.288\gamma_p - 5.049 - 3.014\xi + 7.616\xi^2 - 24.819\xi^3 \qquad (10)$$

(HdB; 9000 K $\leq T_{\rm eff} \leq$ 20,000 K), where γ_p is defined as

$$\gamma_p \equiv 1.0448 \log L - \log M . \qquad (11)$$

It will be seen, therefore, that the periods of the most unstable modes of pulsating Hdef stars can be approximately represented as a function of L/M and $T_{\rm eff}$. Consequently, the detection of a pulsation period in a luminous Hdef star with known $T_{\rm eff}$ allows a direct estimate of its luminosity-to-mass ratio (assuming that its most unstable mode gives rise to the observed pulsation). The significance of this result for the observations described above will be explored in § VI.

V. THE MASS-LUMINOSITY RELATION FOR Hdef STARS

Without a direct method for determining the masses of Hdef stars available, recourse has to be made to indirect methods in order to interpret observations within the framework of stellar structure and evolution. Since luminosity-to-mass ratios are available (from two sources: surface gravity and pulsation period), a suitable mass-luminosity (M-L) relation will provide an estimate of both quantities. The current model for the structure of Hdef stars consists of a degenerate CO core surrounded by a helium-rich envelope constituting a small fraction of the stellar mass. Most of the luminosity is provided by a helium-burning shell at the base of the envelope (Plavec 1971; Schönberner 1977; Schönberner and Drilling 1983). Two methods for calculating the relation of the core mass (M_c) to luminosity for helium-shell burning stars have been investigated (Jeffery 1988; Saio 1988). These gave almost identical results. The M_c-L relation adopted in this paper is shown in Figure 7. In Saio's models the envelope mass is approximately less than 5% of the core mass for $M_c > 0.7$. Thus we will adopt the M_c-L relation in Figure 7 as the L-M relation of the pulsating Hdef stars for $M_c > 0.7$.

VI. APPLICATION

It has been demonstrated that luminosity-to-mass ratios for pulsating helium stars may be derived in two ways. The spectroscopically determined surface gravity and effective temperature give (of course)

$$\gamma_s \equiv \log L - \log M = 4 \log T_{\rm eff} - \log g - 10.608 , \qquad (12)$$

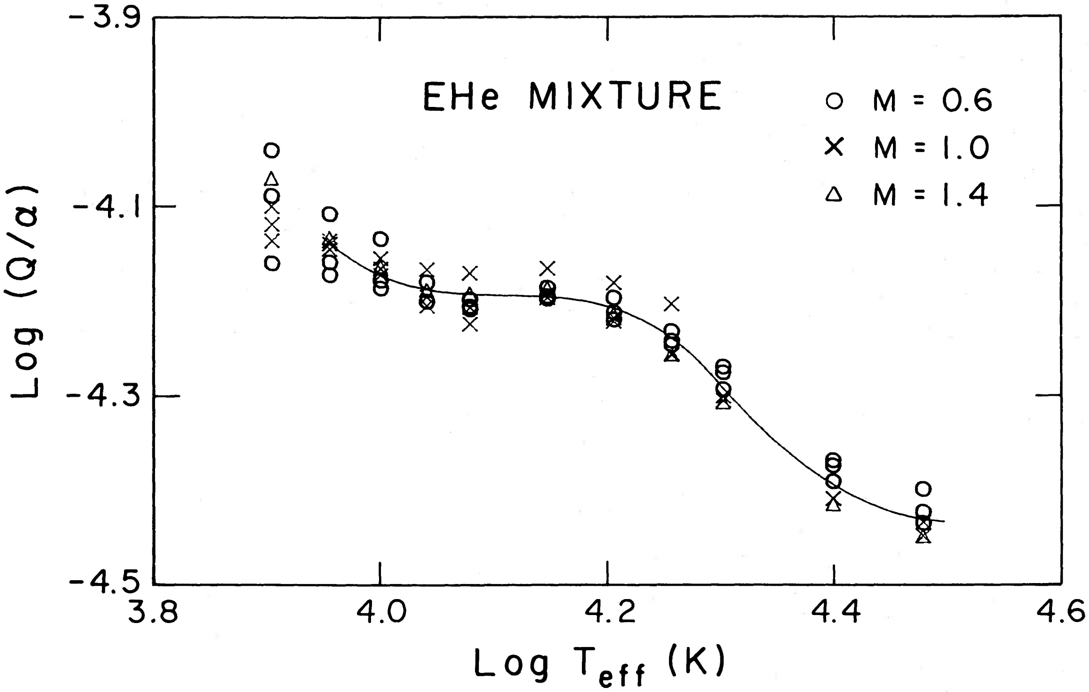

FIG. 6*a*

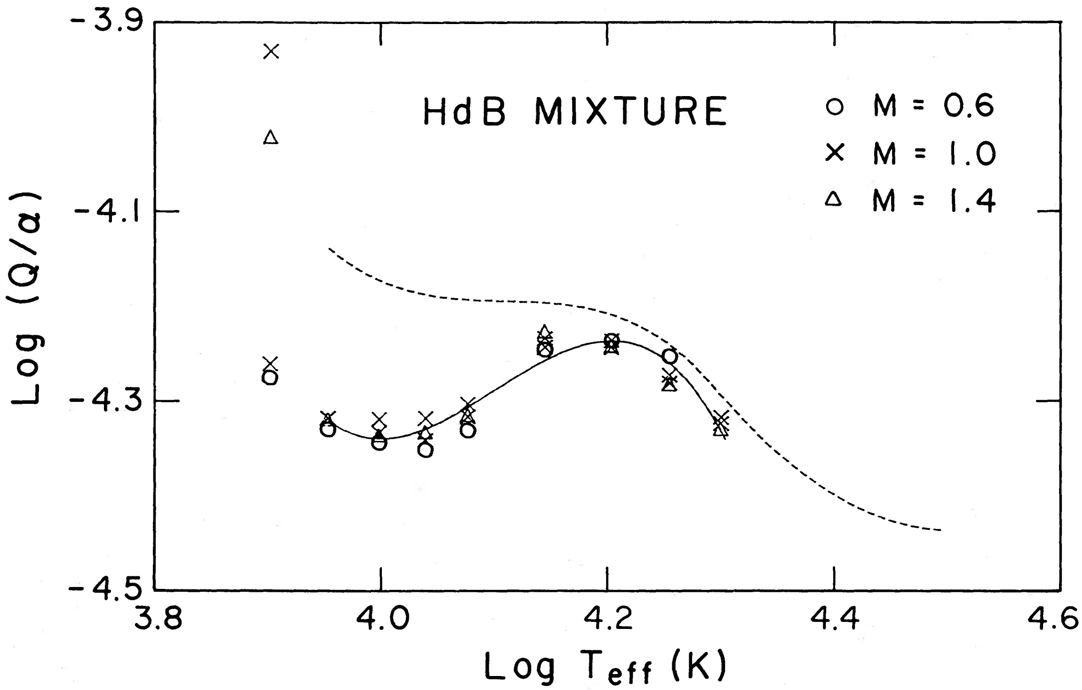

FIG. 6*b*

FIG. 6.—Transformed Q-values (Q/α) as a function of effective temperature for the most unstable pulsation modes with $\alpha > 300$ and $T_{\rm eff} \geq 8000$ K. The solid lines are obtained by least-squares fits. (*a*) EHe mixture $(Y, X_c) = (0.9, 0.1)$; (*b*) HdB mixture: $(Y, Z) = (0.9, 0.03)$. The dashed line is the relation for EHe mixture.

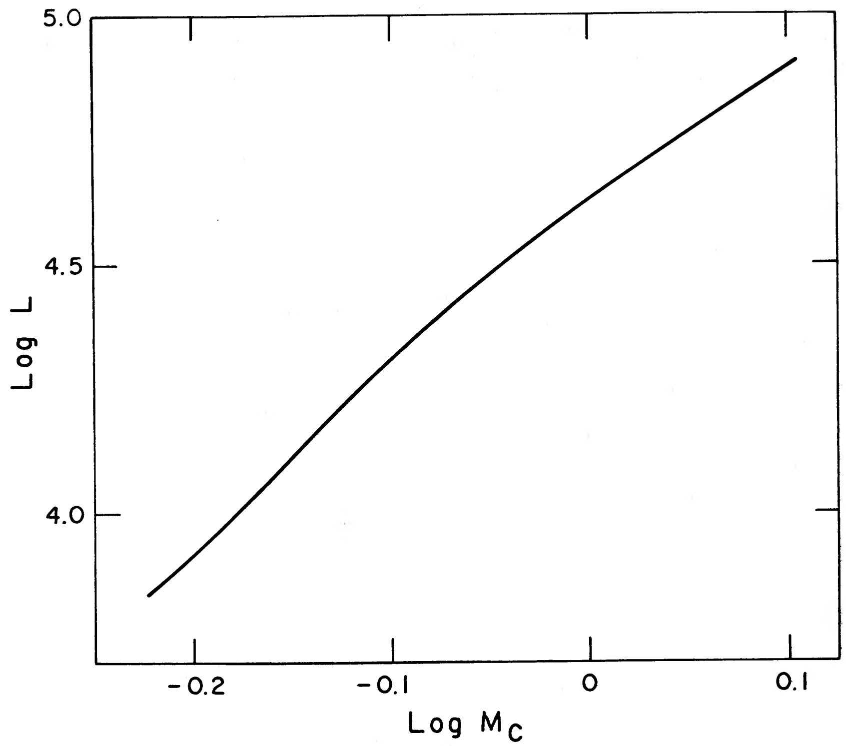

FIG. 7.—Theoretical core mass–luminosity relation for helium stars

while the pulsation period and $T_{\rm eff}$ give γ_p from equations (8)–(10). Observed values for γ_s and γ_p are given in Table 2. The theoretical values for γ_s and γ_p may be derived from the mass-luminosity relation shown in Figure 7, and these are shown in Figure 8. Comparing observed and theoretical values allows spectroscopic (M_s) and pulsation (M_p) masses to be obtained. These are also given in Table 2.

TABLE 2

OBSERVED VALUES

Star	γ_s	M_s	γ_p	M_p
UW Cen	4.72	1.13	[a]	[a]
R CrB	4.60	0.97	[a]	[a]
RY Sgr	4.70	1.10	[a]	[a]
XX Cam	4.62	0.99	[a]	[a]
MV Sgr	3.64	[a]	[b]	[b]
BD +1°4381	...	...	4.77	0.93
BD −1°3438	(4.24)[c]	(0.70)[c]	4.55	0.77
LS IV −1°2	...	...	4.79	0.94
PV Tel (HD 168476)	4.46	0.83	(4.68)[c]	(0.85)[c]
LS II +33°5	...	...	4.63	0.82
HD 124448	3.64	[a]	[d]	[d]
BD +10°2179	3.88	(0.54)[c]	[d]	[d]
LSS 3184	3.56	[a]	[e]	[e]
BD −9°4395	4.24	0.68	(5.15–5.58)[c]	(>1.3)[c]
V2076 Oph (HD 160641)	...	...	(5.04–5.22)[c]	(>1.3)[c]
KS Per	4.21	0.68	4.96	1.16
υ Sgr	4.48	0.85	4.98	1.20
CPD −58°2721	(4.78)[c]	(1.24)[c]	4.94	1.14
LSS 4300	4.62	0.99	[b]	[b]

[a] Not applicable.
[b] Period unknown.
[c] Parentheses indicate that value is uncertain.
[d] Not variable.
[e] Variability not known.

The agreement between M_s and M_p is remarkable in three cases (PV Tel, BD −1°3438, and CPD −58°2721). It is marred by the fact that the available pulsation period in one case (PV Tel) and the surface gravities in the other two cases are, as yet, provisional.

The shortcomings in the surface gravities for the remaining two stars with both M_s and M_p (KS Per and υ Sgr) have been referred to in § II. Disregarding M_s for these two stars, it is therefore instructive to find, on the basis of the pulsation periods alone, that the masses of the visible components of the HdB stars ($\sim 1.2\ M_\odot$) are systematically higher than those of the variable EHe stars ($\sim 0.9 \pm 0.1\ M_\odot$). This result also holds (but becomes less prominent) if equation (10) is replaced by equation (8) for deriving the masses of the HdB stars.

As noted previously, the nonvariable EHe stars (HD 124448 and BD +10°2179) have lower luminosity-to-mass ratios than their variable counterparts. The adopted *M-L* relation gives $M \lesssim 0.6$ for these two objects but is not well defined in this region (see § V). The only hot RCrB star with a spectroscopic analysis (MV Sgr) also comes into this category. At 12th magnitude its photometric variability is poorly known, except for that of its RCrB-type light curve.

Assuming that the reported periods for BD −9°4395 and V2076 Oph are due to radial rather than nonradial oscillations implies masses for these objects well in excess of 1.3 $M_\odot$, and luminosities well above the Eddington limit. This reinforces the implausibility of such an interpretation.

It should be noted that, at present, photometric periods may be determined with much greater precision than spectroscopic surface gravities. Therefore, the method presented for the measurement of pulsation masses represents a much more sensitive diagnostic for the interpretation of helium stars than was hitherto available.

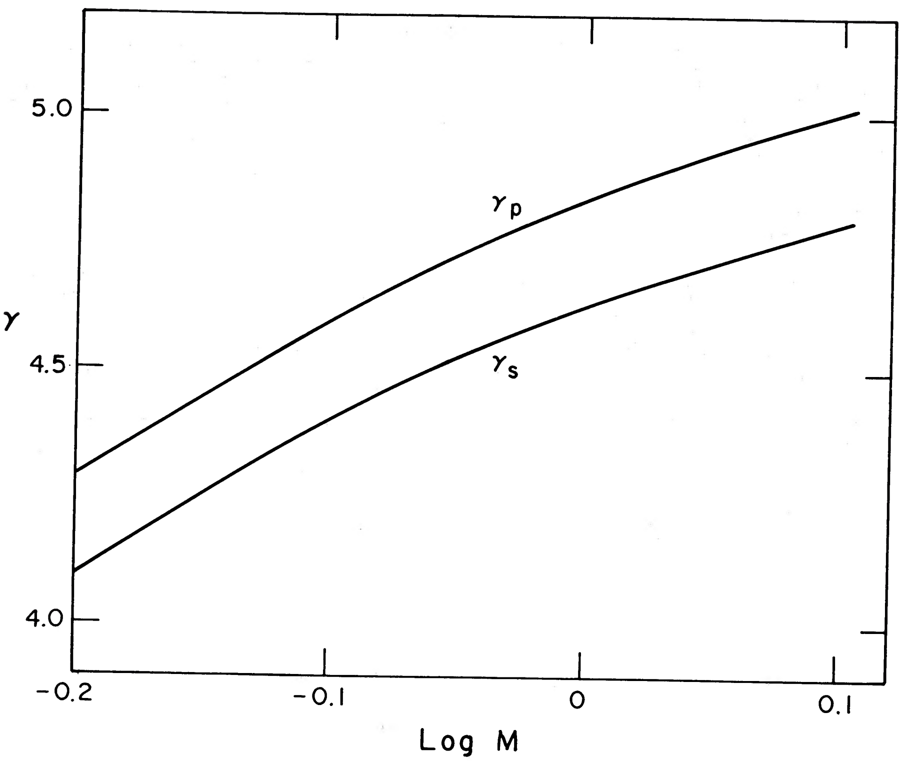

FIG. 8.—Luminosity-mass ratios, γ_p, γ_s (see text for definitions) as functions of mass

VII. CONCLUSION

Examining the observed properties of the hydrogen-deficient stars, we found that the periods of the variable hydrogen-deficient stars decrease as the effective temperature increases and that nonvariable hydrogen-deficient stars tend to have small luminosity-to-mass ratios.

Linear nonadiabatic radial pulsation analyses for various model parameters show the existence of the unstable modes even in the models with high effective temperature up to ~30,000 K. Although this theoretical result is consistent with the existence of the variable helium stars with high effective temperature, further investigations are necessary to clarify how to treat the outer part of the extended envelope of the hot helium stars.

Applying the fitting formulae for the periods of the most unstable modes and theoretical mass-luminosity relation, we have obtained pulsation masses. The results indicate that the masses of the visible components of hydrogen-deficient binaries are systematically larger than those of the extreme-helium stars.

This paper was inspired at the meeting on "Hydrogen-deficient Stars" (IAU Colloquium 87) held in Mysore, India. Both authors were able to attend through financial assistance from the scientific organizing committee. Travel assistance at a later stage from JILA, the University of St. Andrews, and Louisiana State University is also gratefully acknowledged. Most of the numerical calculations in this paper were carried out on the JILA VAX 8600.

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