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LINEAR POLARIZATION OF MILLIMETER-WAVE EMISSION LINES IN CLOUDS WITHOUT LARGE VELOCITY GRADIENTS

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ABSTRACT

We present a model for the linear polarization of optically thin molecular line emission from dark molecular clouds and observational upper limits for HCO^+ and $H^{13}CO^+$. The model abandons the large velocity gradient (LVG) assumption used in the work of Goldreich and Kylafis and Deguchi and Watson. It predicts a polarization of $\sim 2\%$ in the $J = 1 \rightarrow 0$ line of $H^{13}CO^+$ in the most favorable case, taking into account the effect of the hyperfine structure which reduces the observed polarization by a factor of 3. In the case of HCO^+, our model predicts polarizations up to $\sim 7\%$, but because the optical depth of HCO^+ lines is expected to be large, multiple scattering which has not been included in this work will significantly reduce any observed polarization. We have searched for linear polarization of HCO^+ and $H^{13}CO^+$ lines in four dark clouds with only negative results. Typical 3 σ upper limits achieved are 2%–10%. The instrumentation employed in the observations sets a lower limit of $\sim 1\%$ for the smallest detectable polarization. However, because the lines observed are weak, the integration time required to achieve this limit is very large. A significant improvement in sensitivity is therefore required for linear polarization measurements of millimeter-wave emission lines to be an efficient way for studying magnetic fields inside dense molecular clouds.

Subject headings: interstellar: magnetic fields — interstellar: molecules — polarization

I. INTRODUCTION

Magnetic fields can play a very important role in the evolution of molecular clouds. Methods of measuring interstellar magnetic fields have been discussed recently by Heiles (1986), who points out three basic techniques to measure the direction of the magnetic field component perpendicular to the line of sight: polarization of starlight, linear polarization of synchrotron radiation, and linear polarization of radio-frequency spectral lines. Of these methods only the last one is likely to permit the study of the magnetic field deep within dense, dark molecular clouds. Magnetic field components parallel to the line of sight can be studied using either Faraday rotation or the Zeeman effect. Zeeman splitting of the 21 cm H I line was detected both in absorption against strong radio continuum sources (Verschuur 1968, 1969, 1970), and in emission (Troland and Heiles 1982). Zeeman splitting of the 18 cm OH absorption lines has also been detected recently against Ori B, W22, and Cas A (Heiles and Stevens 1986), and against NGC 2024 and W22 (Kazès and Crutcher 1986). For millimeter-wavelength lines, however, Doppler line widths are much larger than expected Zeeman splittings, making this type of observation of little use.

The fact that interstellar millimeter lines may possess a significant degree of linear polarization was first pointed out by Goldreich and Kylafis (1981). They consider a simple two-level system in the strong field limit, and using the Sobolev (or Large Velocity Gradient) approximation (Sobolev 1960; Castor 1970; Lucy 1971), predict that the linear polarization of emitted radiation may attain a maximum of $\sim 7\%$ for an optical depth $\tau \approx 1$. The direction of polarization may be either perpendicular or parallel to the projection of the magnetic field on the plane of the sky and may thus be used to trace the magnetic field direction. In a second paper (Goldreich and Kylafis 1982), the authors consider both the strong and weak field limits and predict even larger polarizations up to $\sim 14\%$. These predictions stand in contradiction with the observations. The extensive search performed by Wannier, Scoville, and Barvainis (1983) in a number of sources (including hot cores, as well as the centers and edges of dark clouds) did not result in any positive detection of polarized emission, although the observational limits achieved were often a factor of 3 or more below the theoretical predictions. A recent search for linear polarization in ammonia line emission from OMC-1 (Barvainis and Wootten 1987) was also unsuccessful. Subsequent calculations by Deguchi and Watson (1984) for multilevel systems do not qualitatively change the results of Goldreich and Kylafis. While polarization is decreased by a factor of 2, a few percent linear polarization is still expected to be common for millimeter-wave emission lines. The suggestion that the emergent polarization is reduced by a lack of uniformity in magnetic field structure over length scales comparable to the resolution of existing millimeter telescopes has not been confirmed by optical polarization measurements, which often show an impressive degree of large-scale order (Moneti *et al.* 1984; Jones, Hyland, and Bailey 1984; McCutcheon *et al.* 1986). This suggests that either the cloud parameters chosen in previous computations are unrealistic or that the LVG approximation is not valid for most dark clouds.

Morris, Lucas, and Omont (1985) present a model for molecular emission from expanding circumstellar envelopes in which molecular excitation is dominated by infrared transitions to vibrationally excited states, followed by decay to higher rotational levels of the ground vibrational state. The detailed computations suggest that the polarization of emergent radiation may be as large as 5% and can thus be detected by existing large millimeter telescopes.

The goal of the present paper is to present an equivalent model for linearly polarized emission of optically thin lines in dark clouds without embedded infrared sources, which abandons the LVG assumption. Molecular clouds generally consist of a high-density core surrounded by an envelope with density decreasing with the distance from the core. If the density contrast between the core and the outer portion of the envelope is large, the excitation temperature can drop rapidly with radius. Gradients in the excitation temperature through the cloud can be expected to produce an anisotropic radiation field away from the cloud center. If the excitation temperature gradient is large enough, this anisotropy may produce significant differences in the population of different Zeeman sublevels of a given rotational level J, resulting in the polarization of emergent radiation. In § II, we develop this theoretical model. In § III, we present observations of $H^{13}CO^+$ and HCO^+ obtained at FCRAO. The significance of the upper limits obtained is discussed in § IV.

II. THEORETICAL MODEL

Consider the emission at an offset, p, from the center of a spherical cloud (Fig. 1). In the following discussion the magnetic field is assumed to be either zero or radial; the implications of a nonradial magnetic field are discussed at the end of this section. The Zeeman splitting is assumed to be smaller than the Doppler line width (preventing production of circular polarization) and much larger than the natural line width. The last condition refers to the "strong field limit" (Goldreich and Kylafis 1981) and is generally satisfied for radio-frequency lines and some infrared lines. The model cloud has a 1.5 pc radius with 0.05 pc radius constant density core, a r^{-2} density distribution outside the core, a uniform turbulent velocity dispersion equal to 1 km s^{-1} FWHM, and a uniform kinetic temperature $T_k = 15$ K. The above parameters were chosen to match those of L1450 (Guélin, Langer, and Wilson 1981).

Typical values for molecular hydrogen densities in the core used in the computations are 5×10^4–5×10^5 cm^{-3}. In general, the molecule selected for the computations should be widely distributed in dark clouds so that its fractional abundance will be reasonably constant through the core and envelope of the cloud. Further, because all radiative rates are proportional to μ^2, where μ is the permanent dipole moment, the largest gradients in the excitation temperature are expected for molecules having a large dipole moment, which are relatively difficult to thermalize by collisions. Molecules with a small dipole moment, such as CO, are close to thermalization even in the outer parts of the envelope and have a relatively uniform excitation temperature. Finally, because the multiple scattering associated with opacity effects can be expected to reduce the net polarization of an emitted spectral line, the molecular transition selected for modeling should be optically thin; multiple scattering is not treated quantitatively in the present work.

The ion $H^{13}CO^+$ with a dipole moment $\mu = 3.3D$ (Woods *et al.* 1975) seems to be a very good candidate for polarized emission. The fractional abundance of HCO^+ varies from 0.3 to 1.0×10^{-8} (Irvine, Goldsmith, and Hjalmarson 1987, and references therein). Therefore, the fractional abundance of $H^{13}CO^+$ should be on the order of $X = 10^{-10}$. This is sufficiently low that one can expect the optically thin emission required for the single scattering model to be applicable. There is evidence that HCO^+ emission in dark clouds is extended and includes low-density regions as well as higher density cloud cores (e.g., Langer *et al.* 1975; Baudry *et al.* 1981). This situation is optimum for detecting polarized emission by the scattering process which we discuss in the present paper. Other molecules which might be considered include HCN and $H^{13}CN$, which have dipole moments and rotational constants similar to those of HCO^+ and $H^{13}CO^+$. The polarization of the HCN and $H^{13}CN$ lines may be strongly affected, however, by the complex

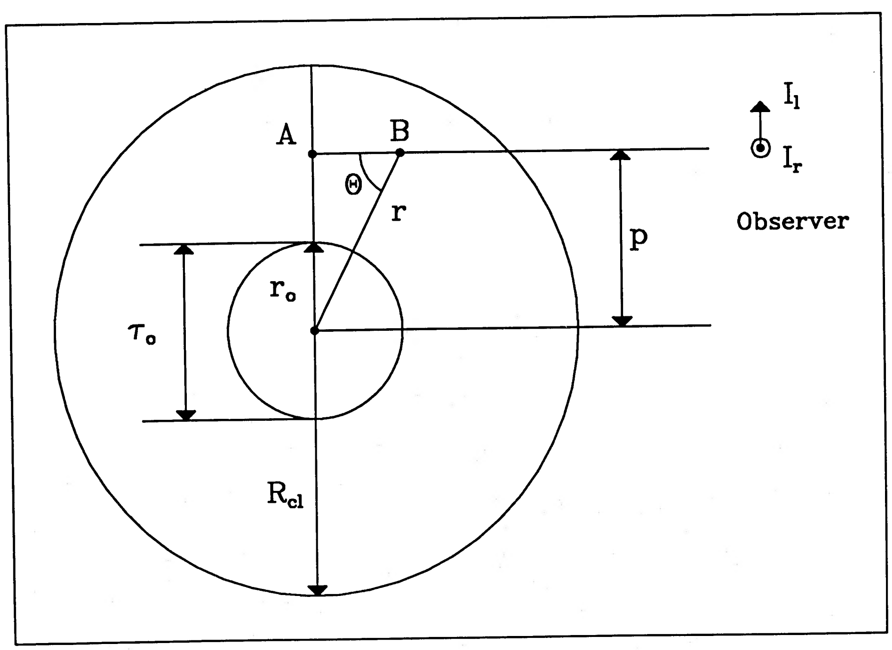

FIG. 1.—Geometry of the cloud with a radial magnetic field. The cloud and the core radius are denoted by R_{cl} and r_0, respectively, p is the offset from the cloud center, and τ_0 is the optical depth through the core. For any position along the line of sight the quantization axis makes an angle, Θ, with the direction of observation. At the point of the closest approach, A, $\Theta = 90°$ and the upper limit for the polarization is given by eq. (12).

hyperfine structure of these molecules. In addition to magnetic dipole hyperfine structure due to the proton and ^{13}C (similar to that found in HCO^+ and $H^{13}CO^+$ [see § II]), they have electric quadrupole hyperfine structure due to the presence of the nitrogen atom. This splitting is detectable in dark clouds which have narrow line widths (e.g., Irvine and Schloerb 1984). Although we did not make calculations for these particular molecules, we expect the emergent polarization to be significantly reduced by their complex hyperfine structure, and we thus consider HCO^+ and $H^{13}CO^+$ to be better candidates for detection of polarized emission.

In order to calculate the emergent polarization the following procedure was applied:

1. Using the Monte Carlo method described by Bernes (1979), the excitation temperature of HCO^+ and $H^{13}CO^+$ within the cloud was first found without accounting for different Zeeman sublevels of the molecules.

2. Differences in the sublevel populations were then calculated assuming that the molecules are excited by an isotropic component of the radiation field, equivalent to a blackbody with the temperature $T_{\rm iso}$, together with excess emission from the central cloud core with excitation temperature T_0 and optical depth at line center τ_0. The values of T_0 and τ_0, obtained for the $J = 1 \rightarrow 0$ transition from the Monte Carlo calculations, were assumed to be constant for all transitions. The value of $T_{\rm iso}$ was then determined such that for fixed T_0 and τ_0 the average excitation temperature between $J = 0$ and the sublevels of $J = 1$ was equal to the value obtained from the Monte Carlo calculations, $T_{\rm ex}$.

Under the above assumptions the rate equation for the population of a sublevel M of the rotational level J of a linear molecule with $^1\Sigma$ ground electronic state without hyperfine structure, denoted $n_{J,M}$, may be written as

$$\frac{dn_{J,M}}{dt} = \sum_{M'=-(J-1)}^{J-1} [n_{J-1,M'} A_{J,M\rightarrow J-1,M'}(J_{\rm iso} + J_0 \alpha) - n_{J,M} A_{J,M\rightarrow J-1,M'}(1 + J_{\rm iso} + J_0 \alpha)]$$
$$+ \sum_{M'=-(J+1)}^{J+1} [n_{J+1,M'} A_{J+1,M'\rightarrow J,M}(1 + J_{\rm iso} + J_0 \alpha) - n_{J,M} A_{J+1,M'\rightarrow J,M}(J_{\rm iso} + J_0 \alpha)] . \quad (1)$$

In the above expression we have taken

$$J_{\rm iso} = \left[\exp\left(\frac{h\nu_{JJ'}}{kT_{\rm iso}}\right) - 1\right]^{-1} , \quad (2)$$

and

$$J_0 = \left\{\left[\exp\left(\frac{h\nu_{JJ'}}{kT_0}\right) - 1\right]^{-1} - J_{\rm iso}\right\} Y W(r) ; \quad (3)$$

$W(r)$ is the dilution factor, given by

$$W(r) = \frac{\Omega}{4\pi} = \frac{1}{2}\left\{1 - \left[1 - \left(\frac{r_0}{r}\right)^2\right]^{1/2}\right\} , \quad (4)$$

where r_0 is the radius of the core, r the distance from its center to the observation point, Ω is the solid angle of the core as observed from point A (Fig. 1). The optical depth correction is denoted by Y:

$$Y = \int \Phi_\nu[1 - \exp(-\tau_\nu)]\, d\nu , \quad (5)$$

where $\tau_\nu = \tau_0 \exp[-(\nu - \nu_0)^2/2\sigma^2]$, ν_0 is the line center frequency, and the normalized line profile, Φ_ν, is assumed to be Gaussian. The integral is carried out over the line.

The spontaneous emission coefficient for a $J', M' \rightarrow J, M$ transition is given by

$$A_{J',M'\rightarrow J,M} = A_{J'\rightarrow J} | \langle J, 1, M, M' - M | J', M' \rangle |^2 , \quad (6)$$

where $\langle J, 1, M, M' - M | J', M' \rangle$ is the appropriate Clebsch-Gordan coefficient and

$$A_{J'\rightarrow J} = \frac{512\pi^4 \mu^2 B^3 J'^4}{3hc^3(2J' + 1)} \delta_{J'-1,J} , \quad (7)$$

μ being the permanent dipole moment, and B the rotational constant of the molecule in units of frequency. The factor

$$\alpha = \tfrac{3}{2}(1 - \delta_{M,M'}) \quad (8)$$

in equation (1) excludes transitions with $\Delta M = 0$ induced by radiation from the core. The choice of quantization axis in the radial direction results in different selection rules for transitions induced by the isotropic radiation and those induced by radiation from the core. In the limit of a very small core, photons emitted from the core travel along the quantization axis, so that the photon spin projection is $+1$ or -1 and never 0. In fact, because of the finite size of the core, there is a finite probability that the photon spin projection is zero. Therefore, our results are upper limits to the polarization of the emitted radiation.

The polarization of the emitted radiation seen by the observer is defined as

$$P = \frac{I_r - I_l}{I_r + I_l} , \quad (9)$$

where I_r, I_l are the intensities in the perpendicular and parallel directions as defined in Figure 1. For optically thin emission

$$I_{r,l} = \int [S_{r,l} - \tfrac{1}{2}B(T_{bb})]\, d\tau_{r,l} , \tag{10}$$

where $B(T_{bb})$ is the Planck function at the temperature of the cosmic background, $T_{bb} = 2.7$ K. The source functions, $S_{r,l}$, and the absorption coefficients for the polarized radiation are given by Deguchi and Watson (1984). (Notice that eq. [5] of Deguchi and Watson should be divided by 2 in order that for $\Theta = 0$, k_r and k_l be equal). The integral is carried out from 0 to the maximum optical depth for a given impact parameter. It is clear that integration along the line of sight reduces the emergent polarization, and an upper limit for P is given by taking the results for $\Theta = \pi/2$ only, which yields

$$P = \frac{S_r k_r - S_l k_l - (1/2)B(T_{bb})(k_r - k_l)}{S_r k_r + S_l k_l - (1/2)B(T_{bb})(k_r + k_l)} . \tag{11}$$

For the $J = 1 \rightarrow 0$ transition, which is expected to have the highest degree of polarization, equation (11) takes the simple form

$$P = \frac{n_{11} - n_{10}}{n_{11} + n_{10} - 2n_{00} \exp\left(-\dfrac{h\nu}{kT_{bb}}\right)} . \tag{12}$$

The computations were performed for five values of the central hydrogen density, $n_0 = 5 \times 10^4$, 10^5, 2×10^5, 4×10^5, and 8×10^5 cm^{-3}. Values of T_0, τ_0, Y, and the line center optical depth from the center to the edge of the cloud, τ_1, for the different models are given in Table 1. Levels with $J \leq 4$ were included in the Monte Carlo computations and the collisional cross sections were taken from Monteiro (1985). These are generally of the order of few times 10^{-10} cm^3 s^{-1}, about a factor of 2 larger than those of Green (1975), after correcting for the difference in reduced mass between hydrogen and helium. The collisional rates between different sublevels of a given rotational level, J, are generally considered to be much smaller than those between different J levels (Oka 1973; Bomsdorf and Mader 1983; Brechignac *et al.* 1980).

Because the spontaneous emission rate for the $J = 1 \rightarrow 0$ transition is $A_{1\rightarrow 0} = 3 \times 10^{-5}$ s^{-1}, the critical density, for which the spontaneous emission rate is equal to the collisional deexcitation rate, is of the order of 10^5 cm^{-3}. Figure 2 shows the excitation temperature of the $J = 1 \rightarrow 0$ transition of $H^{13}CO^+$ in our model cloud as a function of radius for different central densities. Because the excitation temperature drops rather rapidly with radius, the core of the cloud is well defined, and the two component model adopted for the radiation field seems to be a good approximation.

The results of the computations for a kinetic temperature $T_k = 15$ K and a fractional abundance $X = 10^{-10}$, appropriate for $H^{13}CO^+$, are presented in Figure 3*a* and Table 1 (models 1A–1E). For a given central density, polarization of the emitted radiation is a slowly increasing function of the offset from the cloud center, p. It decreases for small p because T_{ex} approaches T_0. For low central densities ($n_0 < 10^5$ cm^{-3}), P is basically independent of the value of n_0. As n_0 increases above 10^5 cm^{-3}, the polarization decreases due to the near-thermalization of the relevant level populations. It should be pointed out, however, that for small central densities, the predicted line intensity is low, which makes detection more difficult. In addition, for small central densities the brightness temperature, presented in Figure 4, drops rather rapidly with the offset from the center of the cloud. Although as shown in Figures 3*a–d*, the *polarization* is predicted to be greater for larger offsets from the cloud center, the drop in *intensity* and lower signal to noise attainable will limit the distance to which one can profitably search for polarized emission.

In order to check how our results are affected by different model parameters, we varied T_0, τ_0, and T_{iso} successively. Changes of

TABLE 1

PARAMETERS OF MONTE CARLO MODELS[a]

Model	n_0	X	T_0[b]	τ_0[c]	τ_1[d]	Y[e]
1A	5×10^4	1×10^{-10}	6.04	0.34	0.45	0.210
1B	1×10^5	1×10^{-10}	9.53	0.37	0.64	0.226
1C	2×10^5	1×10^{-10}	14.32	0.41	0.89	0.247
1D	4×10^5	1×10^{-10}	17.25	0.58	1.26	0.327
1E	8×10^5	1×10^{-10}	16.48	1.06	1.93	0.504
2A	1×10^5	2×10^{-10}	10.23	0.66	1.18	0.362
2B	2×10^5	2×10^{-10}	15.03	0.74	1.61	0.394
3A	5×10^4	5×10^{-10}	7.46	1.26	1.88	0.561
3B	1×10^5	5×10^{-10}	11.56	1.34	2.53	0.603
4A	2.5×10^4	1×10^{-9}	5.49	1.90	2.40	0.695
4B	5×10^4	1×10^{-9}	8.65	2.03	3.27	0.715

NOTE.—The velocity distribution of molecules is assumed to be a Gaussian with FWHM equal to 1 km s^{-1} in all models, and the kinetic temperature of the gas is 15 K.

[a] The cloud parameters which are varied in the models are n_0, central density of the model cloud (cm^{-3}); and X, molecular fractional abundance.

[b] Central excitation temperature (K).

[c] Optical depth through the core.

[d] Optical depth from the center to the edge of the cloud.

[e] Optical depth correction (eq. [5]).

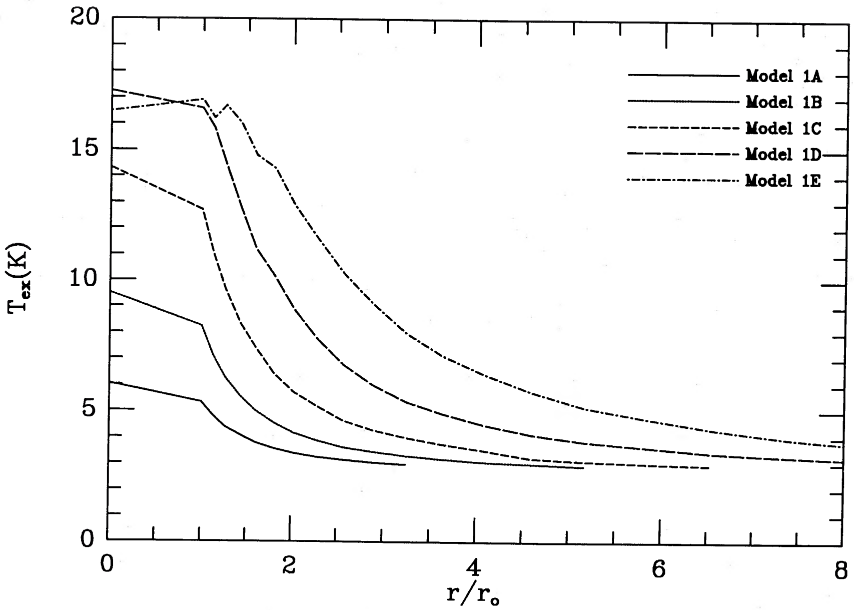

FIG. 2.—The excitation temperature of $J = 1 \rightarrow 0$ transition of $H^{13}CO^+$ as a function of distance from the center of the cloud, for models with different central densities: $n_0 = 5 \times 10^4, 10^5, 2 \times 10^5, 4 \times 10^5$ and 8×10^5 cm s^{-1} for models 1A to 1E, respectively.

0.1 K in T_0 and 0.1 in τ_0 produce a change of $\sim 0.1\%$ in P for all offsets. Variations in the value of T_{iso} had the most important impact, especially for low excitation temperatures. A change of 0.01 K in T_{iso} can produce a change in P of $\sim 0.05\%$ for $T_{ex} \approx 3$ K. We also checked the effect of optical depth on the observed polarization, treating the fractional abundance, X, as a free parameter, and running additional models for $X = 2 \times 10^{-10}$, 5×10^{-10}, and 10^{-9} (models 2, 3, and 4, respectively). For any particular value of X we observe the same trends as for $X = 10^{-10}$. Polarization in the single scattering model increases with X. The highest degree of polarization is predicted for $X = 10^{-9}$, approximately the fractional abundance of HCO^+. In this case, however, even for values of n_0 as small as 2.5×10^4, the optical depth from the center to the edge of the cloud, τ_1, is larger than 1 and multiple scattering will reduce the observed polarization significantly. For smaller values of n_0, the excitation temperature through the cloud is too small for observation of polarized emission. Because P is a slowly varying function of the offset, p, beam averaging should not significantly affect the predicted polarization.

It was assumed in the computations, that T_0 and τ_0 are constant for all transitions. In order to check the validity of this assumption we calculated another model with density, kinetic temperature, and fractional abundance equivalent to model 1A, assuming one value of T_0 and τ_0 for the $J = 1 \rightarrow 0$ transition and another for all higher transitions. Two values of T_{iso} were fitted to obtain correct excitation temperatures for the $J = 1 \rightarrow 0$ and $J = 2 \rightarrow 1$ transitions. The emergent polarization was found to be smaller by $\sim 10\%$–15% than that in model 1A. As the effect of transitions higher than $J = 2 \rightarrow 1$ for the polarization of $J = 1 \rightarrow 0$ line is expected to be insignificant, we conclude that the results presented in Figures 3*a*–3*d* are really upper limits for the polarization of emergent radiation.

The calculations presented above were made under the assumption that any magnetic field in the cloud is either zero or radial. In the presence of a nonradial magnetic field, the radial direction is no longer the appropriate choice for the quantization axis. The magnetic field term in the Hamiltonian causes mixing of different Zeeman sublevels and the only natural choice for the quantization axis is the magnetic field direction, which in general makes an angle, ψ, with the propagation direction. In this case the polarization of $J = 1 \rightarrow 0$ transition is given by

$$P = \frac{\sin^2 \psi (n_{11} - n_{10})}{n_{11}(1 + \cos^2 \psi) + n_{10} \sin^2 \psi - 2n_{00} \exp[-(h\nu/kT_{bb})]}, \tag{13}$$

where the sublevel populations are calculated with the quantization axis in the direction of the magnetic field. For $\psi = 90°$ it is equivalent to equation (12). The difference in sublevel populations itself depends on the angle ψ. It varies as $0.5(3 \cos^2 \psi - 1)$ (Morris, Lucas, and Omont 1985). In the most extreme case, $\psi = 0$ (magnetic field parallel to the line of sight), the photon spin projection is always zero, so that one can expect significant differences in sublevel populations, but unpolarized emitted radiation.

The observed polarization will be further reduced by the hyperfine structure of HCO^+ and $H^{13}CO^+$. An attempt was made to estimate the effect of the hyperfine structure on the polarization of scattered radiation for a simple two-level system with $J = 0$ and $J = 1$ levels only and no background radiation field. In the case of a molecule with a nuclear spin $I = \frac{1}{2}$ (HCO^+), the polarization of radiation scattered at the angle $\pi/2$ is reduced from 100% to 43%, while for a molecule with two nuclear spins, $I_1 = I_2 = \frac{1}{2}$ ($H^{13}CO^+$), it is reduced to $\sim 28\%$. Our predicted results for HCO^+ should thus be scaled down by about a factor of 2, and results for $H^{13}CO^+$ by about a factor of 3. The maximum polarization of $H^{13}CO^+$ is then smaller than 2% in the most favorable case with $n_0 < 10^5$ cm^{-3} and may just be barely detectable. The effect of the hyperfine structure is quite significant. It may be, therefore, reasonable to look for a molecule without hyperfine structure. The dipole moment of simple molecules is, however, smaller than that

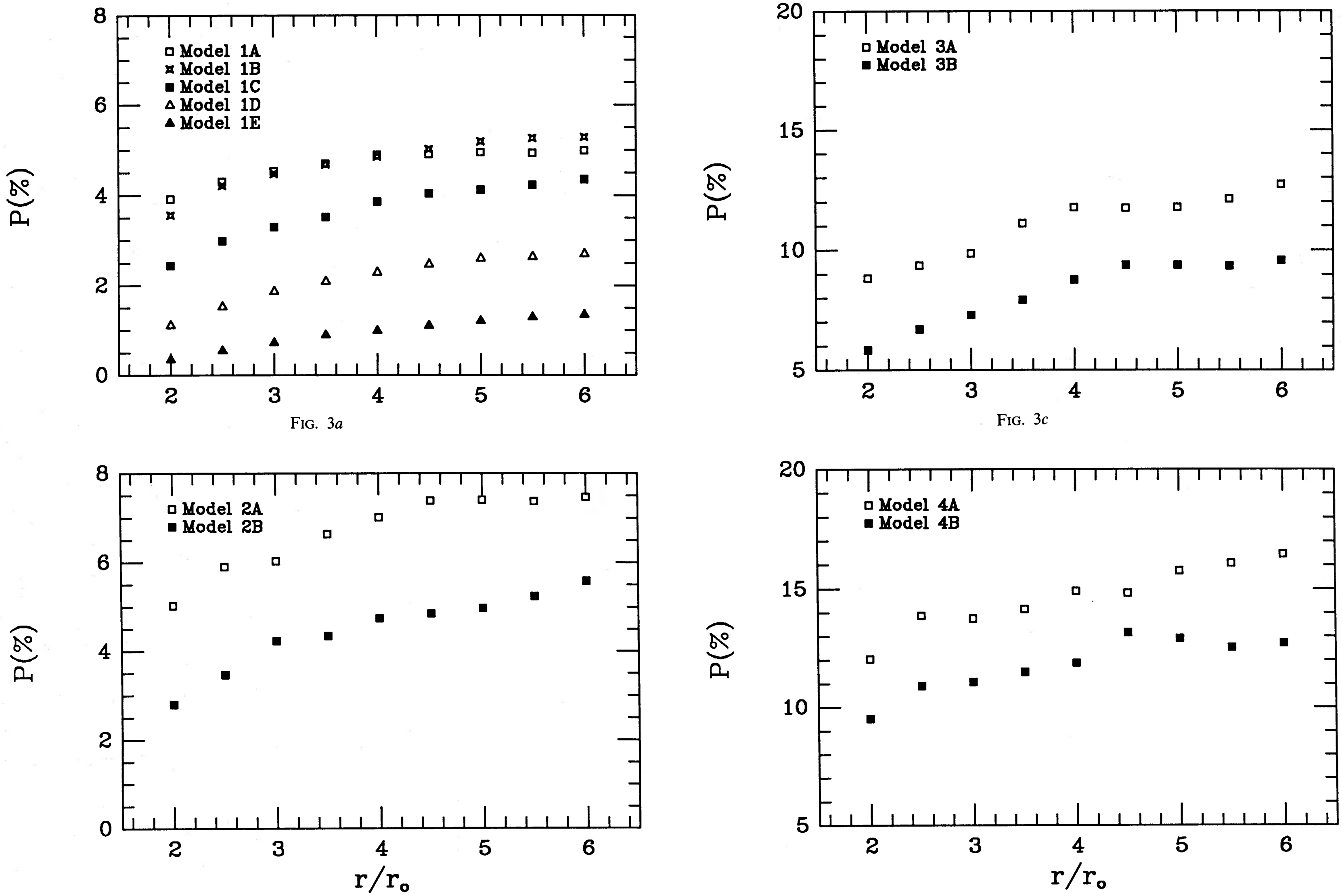

FIG. 3a
FIG. 3b
FIG. 3c
FIG. 3d

FIG. 3.—(a–d) Predicted linear polarization for different models from Table 1; $P > 0$ means $I_r > I_l$. The results do not include any effects of the hyperfine structure which will reduce observed polarization, as discussed in text.

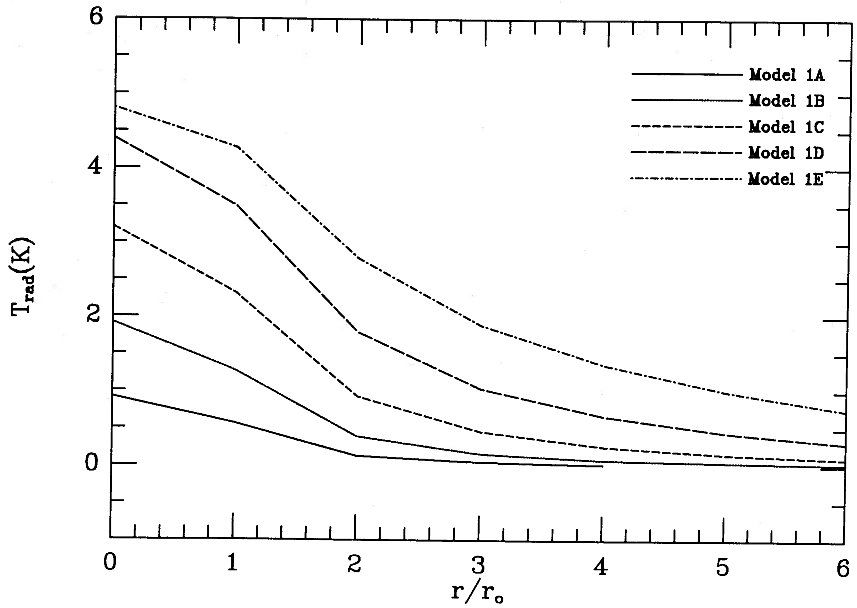

FIG. 4.—The predicted radiation temperature (equivalent to T_A for perfectly coupled antenna having infinitesimal beamwidth) of $J = 1 \rightarrow 0$ transition of $H^{13}CO^+$ as a function of the offset from the center of the cloud, for models with different central densities.

of HCO^+, and thermalization of relevant rotational levels occurs at lower densities. For example, the critical density of the $J = 1 \rightarrow 0$ transition of CS is ~ 17 times smaller than that of HCO^+.

III. OBSERVATIONS

Polarization measurements of the $J = 1 \rightarrow 0$ transitions of HCO^+ (89.188545 GHz) and $H^{13}CO^+$ (86.754340 GHz) in dark clouds were made during one four-day observing session in 1986 March using the FCRAO 14 m telescope at New Salem, Massachusetts. Additional observations of the Kleinmann-Low nebula in Orion were made in 1987 May. The receiver used during the observations employed a cooled Schottky diode mixer and a quasi-optical single sideband filter with cooled image termination. The typical single sideband system temperature (referred to above the Earth's atmosphere) was ~ 500 K. We used a 256 channel filterbank backend, each channel having a resolution of 100 kHz, equivalent to 0.33 km s^{-1}. A detailed description of the FCRAO polarimeter and data taking procedures may be found in Wannier *et al.* (1983) and Barvainis and Predmore (1985). Jupiter and Saturn were used as calibration sources. The beam size was determined to be 58″ FWHM at 86.8 GHz. Assuming source temperatures of 179 K for Jupiter and 153 K for Saturn (Ulich *et al.* 1980), the aperture and beam efficiencies were determined to be 0.49 and 0.57, respectively. The resulting conversion factor from antenna temperature T_A^* to unpolarized flux density for a source filling the antenna main beam is then 33.0 Jy K^{-1}.

The HCO^+ line was observed at 10 positions in four sources and the $H^{13}CO^+$ line was observed at four positions in three sources (Table 2). The average integration time was ~ 2 hr for each position. Assuming 3 σ to be an acceptable detection level, we conclude that no evidence of polarization was detected. The upper limits for HCO^+ and $H^{13}CO^+$ observations are presented in Table 2. Figures 5*a*–5*d* show the positions in L1450 and N2264 where our best limits were achieved. (Detailed maps of the sources we observed can be found in Guélin, Langer and Wilson 1983.) The coordinates of the HCO^+ peak emission are $(-1, 0)$ for L134N and $(-3, 0)$ for TMC-2. $H^{13}CO^+$ emission peaks at the following positions: (0, 1.5) for L134N, (1.5, -6) for L1450, and (0, 0) for N2264. Thus, all of our positions correspond to projected offsets of 1′.5–7′ from the peak of the emission. The value of p/r_0 (Fig. 1) ranges from 1 to 10, assuming the distances given by Guélin, Langer, and Wilson (1982): 100 pc for L134N and TMC-2, 300 pc for L1450, and 800 pc for N2264, together with 0.05 pc for a typical core radius.

In order to check whether the achieved limits are really statistical and not corrupted by instrumental errors, additional observations were made of the Kleinmann-Low nebula in Orion, where the emission is not expected to be polarized because of the high density in the cloud. Figure 6*a* presents the averaged result of over 10 hr of integration on different days and at different parallactic angles of the source. The formal 1 σ limits achieved for the polarization are on the order of 0.15% at the line center. A residual polarization of ~ 0.5% is present over the whole line. If it is assumed that the emission is unpolarized, 0.5% then represents the instrumental limit of the polarimeter. In order to check the validity of this assumption, we made a series of independent measurements on different days, at different parallactic angles. Because the HCO^+ line in Orion KL is very strong ($T_A^* \approx 10$ K), limits as low as 0.2% can be achieved in ~ 2 hr of integration. If the observed polarization were real, we would have obtained similar polarization patterns for all parallactic angles. Figures 6*b* and 6*c* present typical results of observations made on the same day, before transit (average parallactic angle $\approx -20°$) and after transit (average parallactic angle $\cong 25°$). The fact that the polarization pattern is completely different convinces us that we are dealing with an instrumental effect. The residual polarization may be produced, for example, by changes in telescope pointing produced by rotation of the polarimeter. For off-center observations of a centrally peaked cloud, a small change in pointing may produce a relatively large difference in the antenna temperature between the parallel and

TABLE 2

OBSERVATIONAL LIMITS FOR FRACTIONAL LINEAR POLARIZATION

Source	Position[a]	Limit[b]	t_{int}[c]	T_A[d]
	HCO^+ Emission			
L134N	(+0, +5)	10.1	3.1	0.29
	(+2, +0)	12.0	3.9	0.26
	(−4, +2)	16.6	2.1	0.31
L1450	(+0, +2)	9.0	2.1	0.44
	(−2, −6)	2.1	2.4	1.74
	(+0, +0)	2.2	2.3	2.80
TMC-2	(+0, +0)	6.0	2.2	1.69
N2264	(−4, +0)	20.1	2.7	0.55
	(+2, +0)	6.2	2.7	2.08
	(−2, +0)	3.1	2.5	1.35
	$H^{13}CO^+$ Emission			
L134N	(+0, +0)	29.1	4.1	0.38
L1450	(+0, +0)	4.0	4.2	0.80
	(+0, −6)	8.1	3.5	0.51
N2264	(+2, +0)	14.7	0.9	0.53

[a] Offset from central position in minutes of arc in right ascension and declination. The coordinates of the (0, 0) positions are L134N: $\alpha = 15^h51^m30^s$, $\delta = -2°43'31''$; L1450: $\alpha = 3^h25^m56^s$, $\delta = 31°10'38''$; N2264: $\alpha = 6^h38^m25^s$, $\delta = 9°32'29''$; TMC-2: $\alpha = 4^h29^m32^s$, $\delta = 24°16'55''$.

[b] 3 σ in a single channel at the line center having 0.33 km s^{-1} velocity resolution (%).

[c] Integration time (hours) including reference position for total intensity measurements.

[d] Peak antenna temperature (K) corrected for atmospheric absorption but not corrected for coupling efficiency, which is determined to be 0.57 for a source filling the main beam.

perpendicular polarizations. The smallest detectable polarization, which taking into account the instrumental errors discussed above is on the order of 1%, is still below typical predictions of our theoretical model.

IV. CONCLUSION

We have presented a model for the linear polarization of optically thin radio-frequency lines from dark molecular clouds which abandons the large velocity gradient assumption. It is based on the asymmetry of the radiation field produced by a moderately optically thick, relatively highly excited dense core of the cloud occupying a restricted solid angle as seen from positions along an offset line of sight. The maximum fractional polarization predicted by LVG models has varied from 14% (Goldreich and Kylafis 1982) down to 7% (Deguchi and Watson 1984). For $H^{13}CO^+$ the present model predicts smaller values of fractional polarization, with a maximum of 5% ignoring the hyperfine structure, which has the effect of reducing the linear polarization by a factor of 3. The final fractional polarization in our model is ~2% in the most favorable case with magnetic field in the radial direction. Our calculations predict a maximum fractional polarization of 15% for HCO^+, but the effects of multiple scattering, which have not been included in the calculations, will significantly reduce the observed polarization of this optically thick line. The predicted HCO^+ polarization will also be reduced due by a factor of 2 to the hyperfine structure.

The previous Sobolev calculations predict a 7% peak polarization for *any* molecular transition, provided that the optical depth is moderate and anisotropic. The 5% maximum polarization in our model corresponds to the most favorable situation, with the density at the center of the cloud comparable to the critical density of the $J = 1 \rightarrow 0$ transition of the molecule studied, and it drops rapidly as the central density of the model increases (assuming the same density distribution). Although the difference between our results and the results of the previous Sobolev calculations is rather small for the maximum possible polarization (5% vs. 7%), it becomes important for transitions of molecules with smaller dipole moments, which in our model are unpolarized. Because our model is based on different assumptions about cloud structure than are the previous calculations, the discrepancy between predicted polarizations for molecules with small dipole moments is not an indication of inaccuracy in either treatment.

We have carried out observations of HCO^+ and $H^{13}CO^+$ lines in a number of sources, which reveal no evidence of linear polarization. The limits achieved are comparable to or somewhat higher than our theoretical predictions. However, our observations together with those of Wannier *et al.* (1983) exclude the possibility of the large polarization predicted by previous LVG models.

With the instrumental errors of ~0.5%, the smallest polarization that could be detected is on the order of 1%, which would require 1 σ statistical uncertainy of 0.3% or less. Even for the strongest HCO^+ lines from dark clouds, the statistical errors achieved in the present work are a factor of 3 higher than this. In order to reduce the statistical uncertainty to the point where the instrumental errors become the limiting factor, one would have to increase the integration time by an order of magnitude. Therefore, ~20 hr of integration for each position would be required in order to definitely answer the question of whether HCO^+ emission from dark molecular clouds is linearly polarized at the 1% level. In the case of $H^{13}CO^+$ the situation is even worse, because the lines are substantially weaker. Although theoretically the quality of the instrumentation available is high enough to obtain limits at the

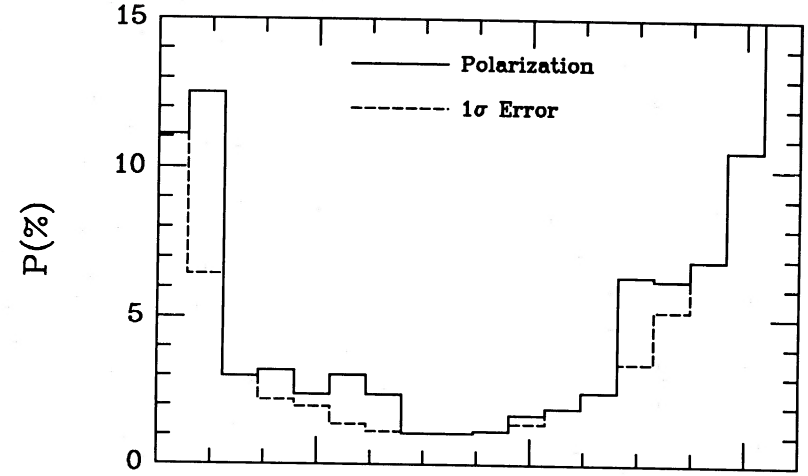

FIG. 5a

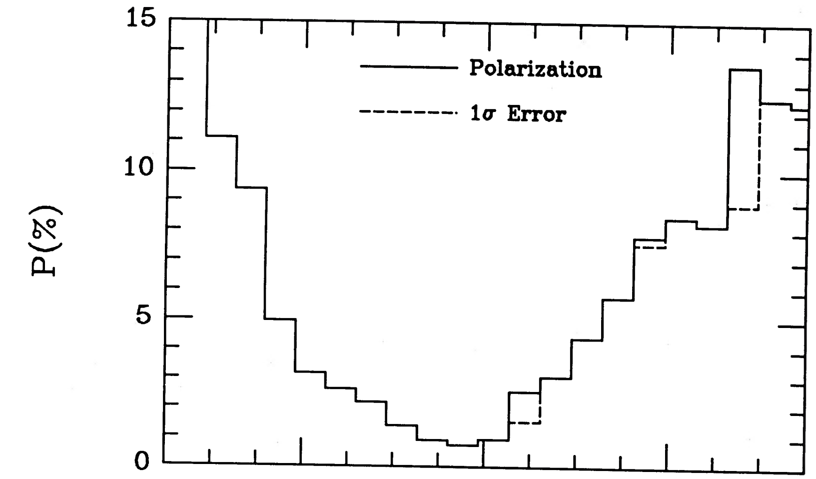

FIG. 5c

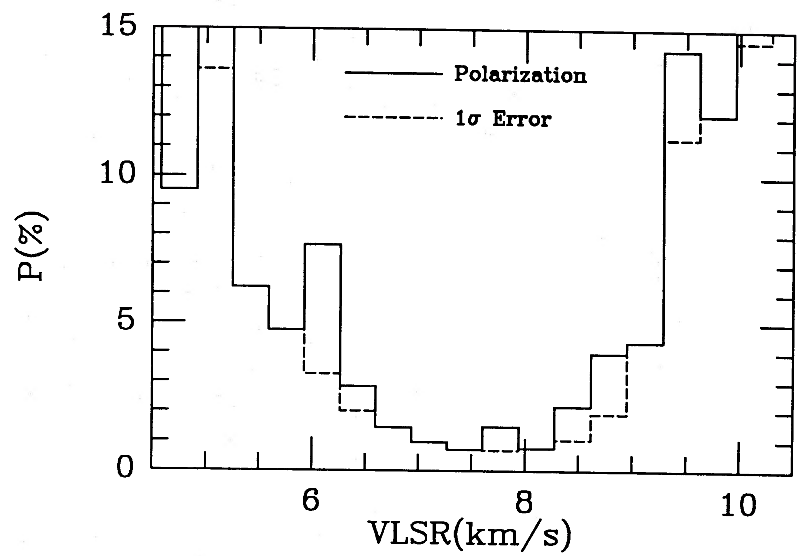

FIG. 5b

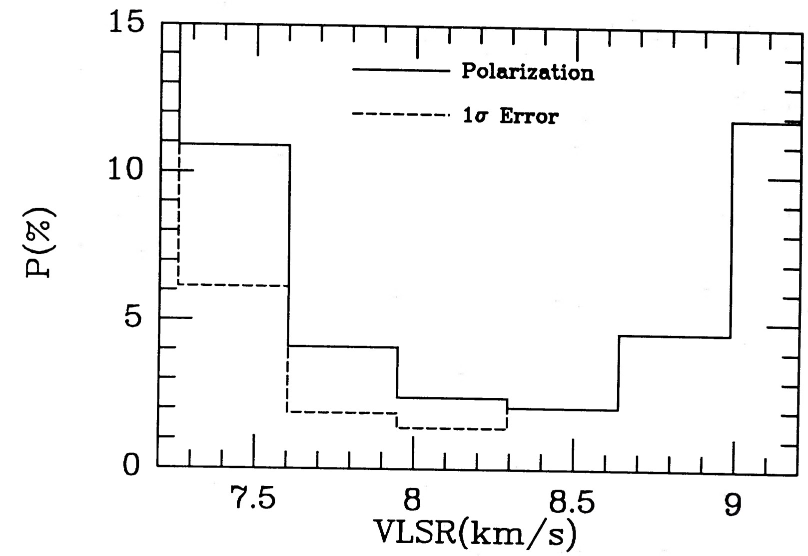

FIG. 5d

FIG. 5.—(*a–d*) The observed limits for linear polarization of HCO^+ and $H^{13}CO^+$ emission: (*a*) N2264 (−2, 0), HCO^+; (*b*) L1450 (−2, −6), HCO^+; (c) L1450 (0, 0), HCO^+; (*d*) L1450 (0, 0), $H13CO^+$. For these positions there is a possible indication of a detection at the 2 σ level, but we do not consider this to be significant. The total integration time of ∼20 hr for each HCO^+ position would be required to reduce statistical errors to the level below the instrumental limits (see discussion in § III). Note that in L1450 the peak of the emission is at the (1.5, 0) position so that Figs. 6*c* and 6*d* are off-peak observations.

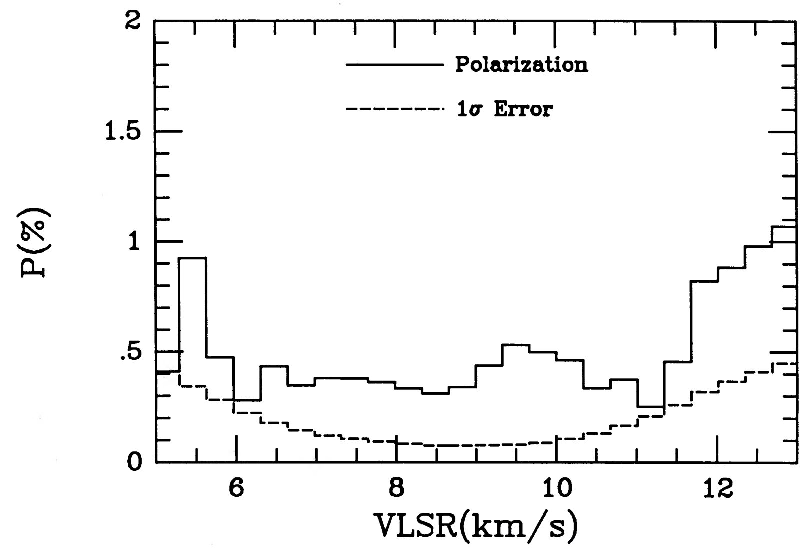

FIG. 6a

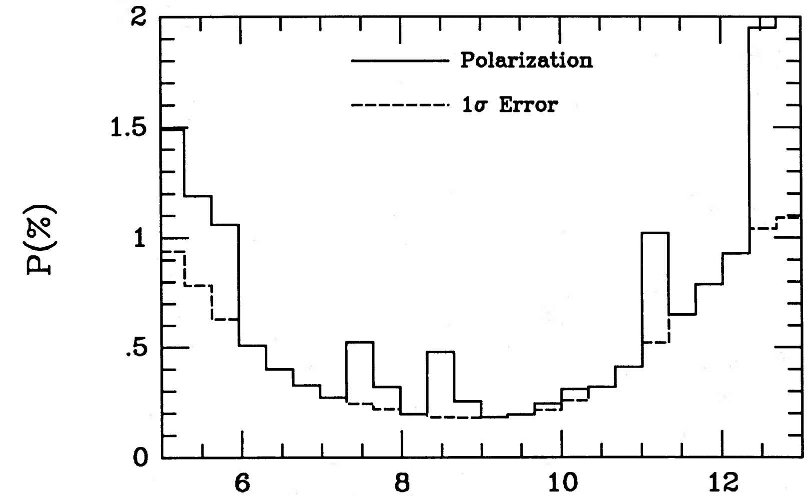

FIG. 6b

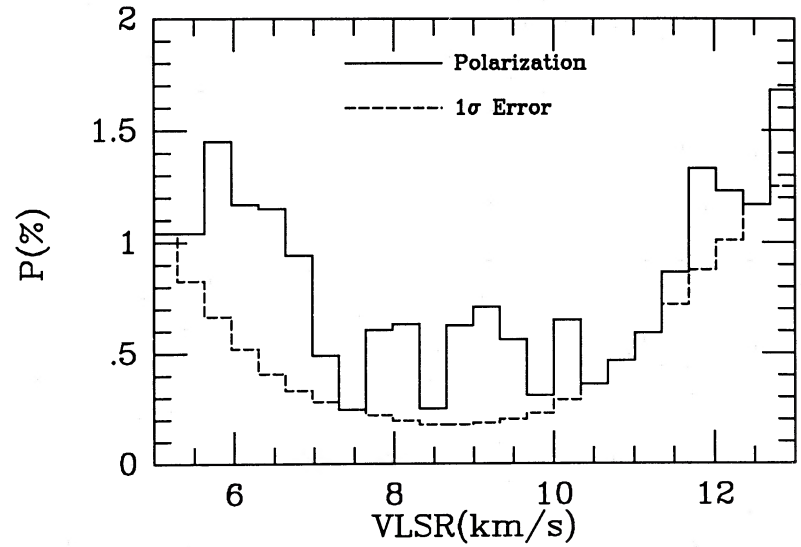

FIG. 6c

FIG. 6.—(*a–c*) The observed polarization of HCO^+ emission from Orion KL ($\alpha = 5^h32^m47^s$, $\delta = -5°24'20''$). Because of high density and optical depth, the emission is expected to be unpolarized. The observed polarization suggests that 0.5% is the instrumental limit for FCRAO polarimeter. (*a*) Representation of the averaged result of 10 hr of integration on different days and for different parallactic angle. (*b–c*) Independent observations with 2 hr integration time, made on the same day, at different parallactic angles, before and after transit, respectively. The difference in observed polarization pattern between (*b*) and (*c*) confirms our assumption that the HCO^+ emission from Orion KL is unpolarized and that the residual polarization of ∼0.5% observed is caused by instrumental errors.

level of 1%, the amount of time required to obtain these limits is very large. We therefore conclude that even if the polarization of radio frequency lines from dark molecular clouds is discovered in the future, considerably improved sensitivity will be required to make it a practical method for studying magnetic field structures within these regions.

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