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ENERGIZED WINDS

DAVID EICHLER
Department of Physics and Astronomy, University of Maryland; and Department of Physics, Ben Gurion University
AND
YUAN-KUEN KO
Department of Physics and Astronomy, University of Maryland
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ABSTRACT

The general theory is developed of winds whose energy flux increases along the flow trajectories, but without momentum deposition. It is found, under rather general conditions, that the sonic point occurs where $E/(A^{3/4})$ is a maximum, where A is the cross-sectional area of a streamline bundle, and E is the specific energy. The results are used to derive general expressions for the critical external flux needed to excite a wind from a gravitationally bound object. The mass flux is derived taking into account the self-shielding by the wind against the external flux. The analysis is applied in a somewhat modified form to the Cyg X-3 system where the external energy flux is assumed to be cosmic rays from the compact object and where bremsstrahlung losses in wind are included. The observed rapid period change is shown to be expected if the compact object is putting out 10^{38} erg s^{-1} or more. The wind is automatically on the order of a radiation length thick and is therefore an efficient converter of cosmic rays to gamma rays, as required by reported ultra-high–energy gamma-ray observations. The possibility is raised that close companions can be completely vaporized by compact objects.

Subject headings: shock waves — stars: binaries — stars: winds

I. INTRODUCTION

Transonic winds from astrophysical objects are typically driven by energy derived from the central object. In the case of the solar wind, energy from the convection zone is deposited into the corona and conducted further downstream (Parker 1958). Energy addition to the solar wind above the coronal base may lead to more than one critical point (Holzer 1977) and is needed to produce high-speed streams with reasonable mass flux and coronal base pressure (Leer and Holzer 1980).

However, in some instances winds can be stimulated from without; e.g., evaporative flows from clouds embedded in a hot gas (Cowie and McKee 1977, Balbus and McKee 1982), comet comae, fusion pellets, and winds from companion stars to accreting neutron stars (e.g., Arons 1973, Basko and Sunyaev 1973, McCray and Hatchett 1975, Basko *et al.* 1977). The orbital period change $\dot{P}/P \sim 10^{-6}$ yr (Mason and Sanford 1979; Elsner *et al.* 1980) of the Cyg X-3 system, whose total mass is $\sim 3\ M_\odot$, suggests that mass is being lost from the companion more rapidly than would be typical of a $\sim 1\ M_\odot$ star. A natural conclusion is that the mass loss is stimulated by the compact object. The hypothesis that the millisecond pulsar was spun up by accretion and the absence of orbital modulation suggests the possibility that the companion star was almost entirely dissolved by the luminosity of the neutron star in its accretion phase.

If the energy driving the wind comes from without, and is absorbed by the wind material during the outflow and converted to heat, then, in the absence of cooling, the specific energy of the flow increases along the flow. This same result can be achieved in other ways, e.g., combustion in a jet or rocket engine, or possibly magnetic field dissipation in a coronal hole. In the analysis, we will be as general as possible; we let the specific energy of the flow $E(r)$ be a free function throughout much of the analysis with the understanding that it increases with distance r along the flow trajectory.

We show in § II that the sonic point is generally within the region of significant energy deposition or at its boundary. In § III, we assume the energy is deposited into a thin layer and show that the jump conditions across the layer imply that the fluid jumps to a Mach number of 1 just downstream of the deposition layer, as implied by the results of § II, if the deposited energy per unit area exceeds some critical amount. We use this result to a general estimate of the incident flux needed to excite a wind from a stellar surface in terms of the star's mass and the penetration grammage of the incident flux.

In § IV, we calculate the expected mass flux from the star bombarded by a "supercritical" energy flux, and compare the results to the measured $\dot{P}/P$ of Cyg X-3. The rate of mass loss is established by the ability of the outflow to shield the surface from the bombarding radiation, and shielding is included self-consistently in the analysis. Because we include shielding, we obtain a much lower value for $\dot{m}$ than previous estimates (Stecker, Harding, and Barnard 1985).

II. THE WIND EQUATION

We now investigate the outflows in which the specific energy increases with radius. Suppose for simplicity that the flow is approximately radial. In a steady state, the conservation of mass, energy flow, and the equation of motion give the following complete set of equations for $u(r)$, $P(r)$, and $\rho(r)$ (fluid velocity, pressure, and mass density):

$$\rho u A(r) = C_1\ , \tag{1}$$

$$\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{P}{\rho} - \frac{Gm}{r} = E(r)\ , \tag{2}$$

$$\rho u \frac{du}{dr} + \frac{dP}{dr} + \frac{Gm\rho}{r^2} = 0 . \tag{3}$$

Here $A(r)$ is the cross-sectional area of a given streamline at radius r and $E(r)$ is the total energy per unit mass of the outflowing fluid which varies with r because of energy deposition of cosmic rays, i.e., $dE/dr > 0$.

By taking $d/dr = (d/dA)(dA/dr)$ to euqation (2), using equation (1) to eliminate ρ, and combining equation (3) to eliminate dP/dr, we have the following equation.

$$\frac{1}{uA}\left(\frac{du^2}{dA} + \frac{1}{2}\frac{u^2}{A}\right)\frac{dA}{dr} + \frac{d}{dA}\left(\frac{E}{uA}\right)\frac{dA}{dr} = -Gm\left[\frac{d}{dr}\left(\frac{1}{uAr}\right) + \frac{5}{2}\frac{1}{uAr^2}\right] . \tag{4}$$

Here we assume that $\gamma = 5/3$ because we want to investigate whether the wind could be stimulated without the assistance of heat conductivity.

The first and second terms on the left-hand side of equation (4) can be written, respectively, as

$$\frac{1}{uA}\left(\frac{1}{\sqrt{A}}\frac{d(u^2\sqrt{A})}{dA}\right)\frac{dA}{dr} = \frac{1}{uA^{3/2}}\frac{d(u^2\sqrt{A})}{dr}$$

and

$$-\frac{1}{2}\frac{E}{u^3A^{3/2}}\frac{d(u^2\sqrt{A})}{dr} + \frac{1}{uA^{1/4}}\frac{d}{dr}\left(\frac{E}{A^{3/4}}\right) .$$

The right-hand side of equation (4) can be written as

$$-\frac{1}{uA^{3/2}}\frac{Gm}{r}\left[\frac{3}{2}\left(\frac{\sqrt{A}}{r} - \frac{d\sqrt{A}}{dr}\right) - \frac{1}{2u^2}\frac{d(u^2\sqrt{A})}{dr}\right] .$$

So equation (4) becomes

$$\left[1 - \frac{1}{2u^2}\left(E + \frac{Gm}{r}\right)\right]\frac{d(u^2\sqrt{A})}{dr} + \frac{3Gm}{2r}\left(\frac{\sqrt{A}}{r} - \frac{d\sqrt{A}}{dr}\right) = -A^{5/4}\frac{d}{dr}\left(\frac{E}{A^{3/4}}\right) . \tag{5}$$

Given the assumption $\gamma = 5/3$, the energy equation gives

$$1 - \frac{1}{2u^2}\left(E + \frac{Gm}{r}\right) = \frac{3}{4}\left(1 - \frac{1}{M^2}\right) ,$$

where M is the Mach number defined by $M^2 = u^2\rho/\gamma P$. We then have

$$\frac{3}{4}\left(1 - \frac{1}{M^2}\right)\frac{d(u^2\sqrt{A})}{dr} + f(r) = -A^{5/4}\frac{d}{dr}\left(\frac{E}{A^{3/4}}\right) , \tag{6}$$

$$f(r) = \frac{3Gm}{2r}\left(\frac{\sqrt{A}}{r} - \frac{d\sqrt{A}}{dr}\right) . \tag{7}$$

Further simplification is possible if gravity is negligible *or* in the case of spherical flow $A = r^2$. In these cases, $f(r) = 0$. Equation (6) becomes

$$\frac{d(u^2\sqrt{A})}{d(E/A^{3/4})} = -\frac{4}{3}\frac{A^{5/4}}{1 - 1/M^2} . \tag{8}$$

Assuming $d[u^2(A)^{1/2}]$ increases with radius, which is the case here, equation (8) implies that $E/A^{3/4}$ reaches its maximum when $M^2 = 1$. No increase in $E/A^{3/4}$ can happen beyond that point. We conclude that the stellar flow will reach the sonic point at the point where $E/A^{3/4}$ is a maximum, then it will become supersonic. In general, the initial conditions are adjusted so that $M^2 = 1$ is attained at the maximum value of $E/A^{3/4}$. In a one-dimensional flow with $A =$ constant, $M^2 = 1$ where E reaches its final (maximum) value.

In § IV, we will use this result in a one-dimensional model as a boundary condition at the point beyond which E is constant.

III. CRITICAL POWER FOR WIND EXCITATION

We now calculate the change in the flow quantities under the effect of energy deposition of cosmic rays when they hit the atmosphere of the star. For simplicity, we make the approximation that the energy flux F of the cosmic rays is deposited in the form of a δ-function at $r = R$. The steady state, spherically symmetric radial fluid equations, for general γ, are then

$$\frac{1}{r^2}\frac{d}{dr}(\rho u r^2) = \dot{m}_f\,\delta(r - R) , \tag{9}$$

$$\frac{1}{r^2}\frac{d}{dr}\left[\rho u r^2\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{P}{\rho} - \frac{Gm}{r}\right)\right] = F\,\delta(r - R) , \tag{10}$$

$$\rho u \frac{du}{dr} + \frac{dP}{dr} + \frac{Gm\rho}{r^2} = \epsilon\, \delta(r - R) . \tag{11}$$

If the mass flux and momentum flux of the cosmic rays can be neglected, i.e., $\dot{m}_f \approx 0$, $\epsilon \approx 0$, then the above equations can be further written in the following form by taking the integral of r from $R - \Delta$ to $R + \Delta$ with $\Delta \to 0$:

$$\rho u R^2 = C_1 , \tag{12}$$

$$\frac{1}{2}(u_2^2 - u_1^2) + \frac{\gamma R^2}{(\gamma - 1)C_1}(P_2 u_2 - P_1 u_1) = \frac{FR^2}{C_1} , \tag{13}$$

$$C_1(u_2 - u_1) + R^2(P_2 - P_1) = 0 , \tag{14}$$

where u_1, P_1 and u_2, P_2 are the flow velocity and pressure before and after energy deposition. C_1 is the rate of mass flow, which is constant in this case. Note that gravity does not affect the jump conditions.

We can solve equations (12)–(14) for the flow quantities after energy deposition in terms of the flow quantities before the energy deposition:

$$\Delta u = u_2 - u_1 = \left[\frac{\gamma R^2 P_1}{(\gamma + 1)C_1} - \frac{u_1}{\gamma + 1}\right] \pm \sqrt{\left[\frac{\gamma R^2 P_1}{(\gamma + 1)C_1} - \frac{u_1}{\gamma + 1}\right]^2 - \frac{2(\gamma - 1)FR^2}{(\gamma + 1)C_1}} , \tag{15}$$

$$\Delta P = P_2 - P_1 = -\frac{C_1}{R^2}\Delta u . \tag{16}$$

Equations (15) and (16) can be simplified in terms of M_1^2:

$$\Delta u = \frac{u_1}{\gamma + 1}\left[\left(\frac{1}{M_1^2} - 1\right) \pm \sqrt{\left(\frac{1}{M_1^2} - 1\right)^2 - \frac{2(\gamma^2 - 1)FR^2}{C_1 u_1^2}}\right] , \tag{17}$$

$$\Delta P = -\frac{C_1 u_1}{(\gamma + 1)R^2}\left[\left(\frac{1}{M_1^2} - 1\right) \pm \sqrt{\left(\frac{1}{M_1^2} - 1\right)^2 - \frac{2(\gamma^2 - 1)FR^2}{C_1 u_1^2}}\right] . \tag{18}$$

Here $M_1^2 = u_1^2 \rho_1/\gamma P_1 = u_1 C_1/\gamma P_1 R^2$ is the Mach number for the flow before energy deposition. The upper sign is the solution for $M_1^2 > 1$ and the lower sign for $M_1^2 < 1$, subject to the limiting condition that if $F = 0$, $\Delta u = \Delta P = 0$. We now choose the lower sign since we are interested in the case $M_1^2 < 1$.

There exists a critical power $\mathscr{L}_{\rm crit}$ of the cosmic rays, for which the square-root term in equation (18) vanishes, given by

$$\mathscr{L}_{\rm crit} = F_{\rm crit} R^2 = \frac{C_1 u_1^2}{2(\gamma^2 - 1)}\left(\frac{1}{M_1^2} - 1\right)^2 , \tag{19}$$

such that

$$u_2 = u_1 + \Delta u = \frac{u_1}{\gamma + 1}\left(\frac{1}{M_1^2} + \gamma\right) , \tag{20}$$

$$P_2 = P_1 + \Delta P = \frac{C_1 u_1}{\gamma(\gamma + 1)R^2}\left(\frac{1}{M_1^2} + \gamma\right) . \tag{21}$$

When $\mathscr{L} \geq \mathscr{L}_{\rm crit}$, the Mach number after energy deposition, M_2, is always exactly unity, regardless of by how large a margin $\mathscr{L}$ actually exceeds $\mathscr{L}_{\rm crit}$:

$$M_2^2 = \frac{u_2^2 \rho_2}{\gamma P_2} = \frac{u_2 C_1}{\gamma P_2 R^2} = 1 . \tag{22}$$

According to the arguments in § II, $\mathscr{L}_{\rm crit}$ is the necessary flux needed to excite a sonic transition, and equation (19) gives the critical power of cosmic rays needed for wind excitation. We can estimate it in terms of the star's parameters by the following arguments. Assume that the wind starts from a nearly static atmosphere so the $M_1^2 \ll 1$. Then

$$\mathscr{L}_{\rm crit} \sim \frac{C_1 u_1^2}{2(\gamma^2 - 1)} \frac{1}{M_1^4} = \frac{\gamma^2}{2(\gamma^2 - 1)} \frac{P_1^2 R^4}{C_1} . \tag{23}$$

The quantity P_1 must be at least the hydrostatic pressure ηg, where η is the penetration depth $\int \rho dr \sim \rho_2 R$ of cosmic rays. Then

$$C_1 = (\rho_2 R)(u_2 R) \sim \eta R\left(\frac{u_1}{\gamma + 1}\frac{1}{M_1^2}\right) ,$$

whence

$$\eta R \frac{\gamma}{\gamma + 1}\frac{P_1 R^2}{C_1} \sim \frac{\gamma}{\gamma + 1}\frac{\eta^2 g R^3}{C_1}$$

and

$$C_1 \sim \left(\frac{\gamma}{\gamma+1}\eta^2 g R^3\right)^{1/2} .$$

Therefore

$$\begin{aligned} \mathscr{L}_{\rm crit} &\sim \frac{\gamma^2}{2(\gamma^2-1)}\sqrt{\frac{\gamma+1}{\gamma}}\,\eta g^{3/2} R^{5/2} ,\\ &\sim \eta R^{-1/2}(Gm)^{3/2} \ (\text{for } \gamma = \tfrac{5}{3}) ,\\ &\sim 5\times 10^{35}\eta_2 R_{11}^{-1/2}(m/M_\odot)^{3/2}\ \text{ergs s}^{-1} . \end{aligned} \tag{24}$$

Here η_2 and R_{11} mean that we use $\eta \sim 10^2$ g cm^{-2}, $R \sim 10^{11}$ cm in the numerical estimate of $\mathscr{L}_{\rm crit}$.

The following alternative way of estimating $\mathscr{L}_{\rm crit}$ yields essentially the same result as equation (24). For $\gamma = 5/3$ the total energy has to be positive in order to have the wind solution (e.g., Holzer and Axford 1970). The energy equation $u^2/2 + \gamma P/(\gamma-1)\rho - Gm/r = E(r)$, with $E > 0$ at $R + \Delta$ demands that

$$\frac{\mathscr{L}_{\rm crit}}{C_1} \gtrsim \frac{Gm}{R} . \tag{25}$$

The critical condition that when $\mathscr{L} = \mathscr{L}_{\rm crit}$, $M_2^2 = 1$ implies

$$\frac{\mathscr{L}_{\rm crit}}{C_1} \sim 2u_2^2 \left(\text{for } \gamma = \frac{5}{3}\right) . \tag{26}$$

Using equation (20) to estimate u_2 in the manner done above, and still assuming $M_1^2 \ll 1$ and $P_1 \sim \eta g$, we have

$$u_2 \sim \frac{5}{8}\frac{\eta g R^2}{C_1}\left(\text{for } \gamma = \frac{5}{3}\right) . \tag{27}$$

Equations (25)–(27) give

$$\mathscr{L}_{\rm crit} \sim \eta R^{-1/2}(Gm)^{3/2} ,$$

which yields the same estimate for $\mathscr{L}_{\rm crit}$ as the first argument.

IV. CALCULATION OF $\dot{m}$ FOR BINARY X-RAY SYSTEMS

Here we seek to include the shielding of the cosmic rays from the surface by the evaporated material above. It is this self-regulating effect that establishes the rate of mass loss. We study this with a one-dimensional model. We will determine, within this simple geometric model, the integration constants of the flow given the parameters of the cosmic-ray flux and its interaction with the outflowing matter. For simplicity, we neglect gravity, which is valid when $\mathscr{L} \gg \mathscr{L}_{\rm crit}$. We will find that $\mathscr{L}$ for Cyg X-3 may be somewhat, but not necessarily much, greater than $\mathscr{L}_{\rm crit}$, in fact, but our estimates should be valid to within a factor of unity. In a one-dimensional flow,

$$\rho u = C_1 , \tag{28}$$

$$P + \rho u^2 = P + C_1 u = P_0 , \tag{29}$$

$$\frac{1}{2}u^2 + \frac{\gamma}{\gamma-1}\frac{P}{\rho} = E(r) . \tag{30}$$

Assume the energy deposition is of the form

$$\frac{dE}{dr} = f(r)\frac{\rho}{\sigma C_1} . \tag{31}$$

Here $f(r)$ is the cosmic-ray energy flux at position r and is assumed, for simplicity, to undergo exponential attenuation when penetrating the stellar atmosphere, i.e.,

$$f(r) = Fe^{-\tau(r)} , \tag{32}$$

where

$$\tau = \frac{1}{\sigma}\int_r^{R_0} \rho\, dr' , \tag{33}$$

and σ is the grammage which gives $\tau = 1$. Since we consider the shielding of the cosmic rays from the surface of the star, the simplified picture is that the cosmic rays start to interact with the outflowing matter from the companion star at $r = R_0$ and get to the surface of the star $r = r_0$. Therefore $\tau(R_0) = 0$ and F is the energy flux of cosmic rays at R_0 where the penetration starts. From equation (33) $d\tau/dr = -\rho/\sigma$. This leads equation (31) to $dE/dr = -(F/C_1)\exp[-\tau(r)](d\tau/dr)$. Therefore, $E(r)$ can be written as

$$E(r) = \frac{F}{C_1} e^{-\tau(r)} + \text{constant} . \tag{34}$$

We assume that at large depth, $\tau \to \infty$, the flow begins with zero energy, hence the constant in equation (34) is taken to vanish. Eliminating u and P from equations (28)–(30), one obtains

$$e^{-1/\sigma \int_r^{R0} \rho\, dr'} = \frac{C}{\rho} - \frac{D}{\rho^2} , \tag{35}$$

where

$$C = \frac{\gamma}{\gamma - 1} \frac{P_0 C_1}{F} , \tag{36}$$

$$D = \frac{\gamma + 1}{2(\gamma - 1)} \frac{C_1^3}{F} . \tag{37}$$

The differential form of equation (35) is

$$\frac{1}{\sigma} \rho\, dr = d \log \left(\frac{C}{\rho} - \frac{D}{\rho^2}\right) . \tag{38}$$

Equation (38) can be integrated to get $\rho(r; C_1, P_0)$ as a function of r and the parameters C_1 and P_0:

$$\frac{1}{\sigma}(r - r_0) = \frac{2}{\rho} + \left(\frac{2\gamma}{\gamma + 1} \frac{P_0}{C_1^2}\right) \log \left(1 - \frac{\gamma + 1}{2\gamma} \frac{C_1^2}{P_0} \frac{1}{\rho}\right) , \tag{39}$$

where r_0 is the position where $u = 0$, $\rho = \infty$, which we associate with the surface of the star.

In § II, we have shown that $E/A^{3/4}$ reaches its maximum when $M^2 = 1$. In our one-dimensional model here, where we take A = constant; this implies a boundary condition that $M^2 = 1$ at R_0, because E attains its maximum at this point and remains constant beyond. We can thus determine C_1, P_0 as well as the mass-loss rate of the stellar flow from the above equations by imposing the following two boundary conditions at R_0:

Condition (*a*).—$M^2 = 1$, then

$$u_m^2 = \frac{\gamma P_m}{\rho_m} = \frac{\gamma(P_0 - C_1 u_m)u_m}{C_1} ,$$

whence

$$u_m = \frac{\gamma P_0}{(\gamma + 1)C_1} , \qquad \rho_m = \frac{(\gamma + 1)C_1^2}{\gamma P_0} , \tag{40a}$$

where u_m, P_m, ρ_m are the flow quantities at $r = R_0$.

Condition (*b*).—$\tau = 0$, $C_1 E = F$, then from equation (35)

$$1 = \frac{C}{\rho_m} - \frac{D}{\rho_m^2} , \tag{40b}$$

which yields

$$\frac{\gamma + 1}{2(\gamma - 1)} C_1 u_m^2 - \frac{\gamma}{\gamma - 1} P_0 u_m + F = 0 , \qquad u_m = \frac{\gamma P_0}{(\gamma + 1)C_1} \pm \frac{(\gamma - 1)}{(\gamma + 1)C_1} \sqrt{\left(\frac{\gamma}{\gamma - 1} P_0\right)^2 - 4 \frac{\gamma + 1}{2(\gamma - 1)} C_1 F} . \tag{40c}$$

From equations (40), we have

$$\left(\frac{\gamma}{\gamma - 1} P_0\right)^2 = \frac{2(\gamma + 1)}{(\gamma - 1)} C_1 F ; \tag{41}$$

also, equation (39) can be written as

$$\frac{1}{\sigma}(r - r_0) = \frac{2}{\rho} + \frac{2}{\rho_m} \log \left(1 - \frac{\rho_m}{2\rho}\right) , \tag{42}$$

$$\text{at } r = R_0 , \quad \frac{1}{\sigma}(R_0 - r_0) = \frac{2}{\rho_m}(1 - \log 2) = \frac{0.6}{\rho_m} . \tag{43}$$

One can thus determine from these equations how the parameters C_1 and P_0 depend on the rate of energy deposition of the cosmic rays:

$$C_1^3 = \frac{0.72(\gamma - 1)\sigma^2 F}{(\gamma + 1)(R_0 - r_0)^2} , \tag{44}$$

$$P_0^3 = \frac{2.4(\gamma + 1)(\gamma - 1)^2 \sigma F^2}{\gamma^3 (R_0 - r_0)} . \tag{45}$$

One can also estimate the mass-loss rate of the stellar flow: from equation (43),

$$\rho_m = \frac{0.6\sigma}{R_0 - r_0} = 0.6 \times 10^{-9} \frac{\sigma_2}{R_{11}} \text{ g cm}^{-3} , \tag{46}$$

and from equations (28), (30) and (34), subject to the boundary condition (*b*),

$$u_m \sim \left[\frac{2(\gamma - 1)}{\gamma + 1} \frac{F}{\rho_m}\right]^{1/3} = 1.3 \times 10^8 \frac{\mathscr{L}_{38}^{1/3}}{R_{11}^{1/3} \sigma_2^{1/3}} \text{ cm s}^{-1} , \tag{47}$$

whence

$$\dot{m} \sim \rho_m u_m (R_0 - r_0)^2 = 7.8 \times 10^{20} \mathscr{L}_{38}^{1/3} R_{11}^{2/3} \sigma_2^{2/3} \text{ g s}^{-1} ,$$

i.e.,

$$\dot{m}/m = 1.2 \times 10^{-5} \mathscr{L}_{38}^{1/3} R_{11}^{2/3} \sigma_2^{2/3} (M_\odot/m) \text{ yr}^{-1} . \tag{48}$$

In estimating u_m, we use $F \sim \mathscr{L}/\pi(R_0 - r_0)^2$. (The factor of π here is a crude approximation.) All the above estimates are in units of $\mathscr{L}_{38}$, R_{11}, σ_2, which are defined as $\mathscr{L}/(10^{38}$ ergs s^{-1}), $R/(10^{11}$ cm), $\sigma/(10^2$ g cm^{-2}), respectively. The above calculation assumes that bremsstrahlung cooling is negligible. However, we find that if $\sigma_2 \gtrsim 1$, both the bremsstrahlung cooling time scale and the photon diffusion time are shorter than the hydrodynamic time scale. In this case, the energy of the cosmic rays is essentially transferred to radiation which then leaks out of the star. The wind, if it exists, is driven by radiation pressure. The equation of motion in one dimension is now

$$m_p u \frac{du}{dr} = \frac{f(r)\sigma_T}{c} , \tag{49}$$

where as before (eqs. [32] and [33]),

$$f(r) = F e^{-\tau(r)}$$

and

$$\tau(r) = \frac{1}{\sigma} \int_r^{R_0} \rho \, dr' .$$

In the right-hand side of equation (49), σ_T is the Thomson cross section, $f(r)$ denotes the outward radiant energy flux through radius r, which is equated with the inward cosmic-ray flux. Since the case is radiation dominated, the gas pressure can be neglected. Also note that gravity is now neglected as in the previous case.

Together with equation of continuity (28), the mass-loss rate $\dot{m}$ can be calculated as follows. Taking d/dr on both sides of equation (49) and using $d\tau/dr = -\rho/\sigma = -C_1/\sigma u$, we have

$$\frac{d}{dr}\left(u \frac{du}{dr}\right) = \frac{C_1}{\sigma} \frac{du}{dr} .$$

This can be integrated twice using the boundary conditions that, at $r = r_0$, (1) the force of radiation exerted on the gas $m_p u \, du/dr = 0$ and (2) $u = 0$. This yields

$$u = \frac{C_1}{\sigma} (r - r_0) \tag{50}$$

and

$$\rho = \frac{\sigma}{r - r_0} . \tag{51}$$

At $r = R_0$, $\tau = 0$, equation (49) becomes

$$u \frac{du}{dr}\bigg|_{R_0} = \frac{F\sigma_T}{m_p c} \tag{52}$$

From equation (50), equation (52) is then

$$\left(\frac{C_1}{\sigma}\right)^2 (R_0 - r_0) = \frac{F\sigma_T}{m_P c} .$$

Therefore,

$$C_1 = \left[\frac{\sigma^2 F \sigma_T}{m_P c(R_0 - r_0)}\right]^{1/2} . \tag{53}$$

hence,

$$\dot{m} \sim \rho u(R_0 - r_0)^2 = C_1(R_0 - r_0)^2 = \sigma \left(\frac{F\sigma_T}{m_P c}\right)^{1/2} (R_0 - r_0)^{3/2} . \tag{54}$$

Expressed in terms of Eddington luminosity,

$$\mathscr{L}_{Edd} = \frac{Gmm_P 4\pi c}{\sigma_T} = 1.26 \times 10^{38}(m/M_\odot) \text{ ergs s}^{-1} , \tag{55}$$

and with

$$F \sim \frac{\mathscr{L}}{\pi(R_0 - r_0)^2} ,$$

$$\dot{m} = \sigma\left(4Gm \frac{\mathscr{L}}{\mathscr{L}_{Edd}}\right)^{1/2} (R_0 - r_0)^{1/2}$$

$$= 7.3 \times 10^{20} \sigma_2 R_{11}^{1/2} \left(\frac{\mathscr{L}}{\mathscr{L}_{Edd}}\right)^{1/2} \left(\frac{m}{M_\odot}\right)^{1/2} \text{ g s}^{-1} , \tag{56}$$

i.e.,

$$\dot{m}/m = 1.15 \times 10^{-5} \sigma_2 R_{11}^{1/2} (\mathscr{L}/\mathscr{L}_{Edd})^{1/2} (m/M_\odot)^{-1/2} \text{ yr}^{-1} . \tag{57}$$

Thus, for super-Eddington dragging, we obtain a value for $\dot{m}$ similar to its value when radiative cooling is negligible.

The physical reason that $\dot{m}$ is insensitive to the physics of the outflow is that, given the negative feedback due to shielding, the thickness of the wind tends to be of the order of the cosmic-ray penetration depth.

To summarize, we have shown that under geometric simplifications, $\dot{m}$ is of $\gtrsim 10^{-5}\ M_\odot$ yr^{-1}, if $\mathscr{L}_{38} \gtrsim 1$. This value should be compared to the observations of $\dot{P}/P$ for the Cyg X-3 system (Mason and Sanford 1979, Elsner *et al.* 1980) which typically indicate $\dot{m}/m \lesssim 3 \times 10^{-6}$ yr^{-1}. The predicted $\dot{m}$, though much smaller than previous estimates, is still somewhat larger than the observations imply. This could be attributed to the gross geometric simplification we have made. Actually, because the star is irradiated only from one side at a time, horizontal pressure gradients must develop, and most of the deposited energy could be dissipated in winds that blow across the surface of the star and do not cause efficient mass loss.

For Cyg X-3 type parameters, with $\sigma_2 \gtrsim 1$, we find that any wind is driven by radiation pressure, hence $\mathscr{L}$ must be greater than $\mathscr{L}_{Edd}$ (since the luminosity is thermalized to $\sim 10^5$ K, resonant atomic absorption may decrease the effective $\mathscr{L}_{Edd}$). The thermal radiation may account for the modulated IR flux observed from Cyg X-3, but we do not attempt a detailed fit here.

However, hard X-rays and possibly soft γ-rays are also emitted by the compact object. Their penetration depth is only ~ 1 g cm^{-2}. This would excite a much hotter, more tenuous wind, according to equations (46) and (47), and bremsstrahlung losses may not entirely kill such a wind. Further discussion of this possibility is deferred to a later paper (Ruderman *et al.* 1988).

V. CONCLUSIONS AND DISCUSSION

We have shown that spherical energized wind becomes transonic where $E/r^{3/2}$ is a maximum. In simple models, in which E increases over very little change in r, we used this result to establish a boundary condition of $M^2 = 1$, where E is a maximum. Within these geometrically simplified models, we calculated the critical energy power $\mathscr{L}_{crit}$ needed to excite a hydrodynamic wind from the surface of a gravitationally bound object, and the mass flux that would be excited as a function of $\mathscr{L}$ when $\mathscr{L} \gg \mathscr{L}_{crit}$. We find that, because of shielding effects, the estimate of $\dot{m}$ is insensitive to physical conditions of the wind and most sensitive to penetrating power of the energy carrier.

Cyg X-3 (and perhaps other similar sources) can be understood to be distinct among binary X-ray sources in that its (reportedly) high cosmic-ray luminosity combined with its unusually short orbital radius yields an energy flux at the companion's surface that is sufficient to excite a dense wind. This implies winds which, because excited from outside the stellar surface, are thick enough to marginally permit the carrier to penetrate to the surface. Thus, a wind excited by cosmic rays will be thicker than one excited by X-rays, i.e., it will be several radiation lengths thick and will automatically serve as an efficient converter.

The star, given the thickness of the wind, could be much smaller than the X-ray light curve suggests. Indeed, the inferred rapid mass loss from the system is predicted by the theoretical considerations presented here, and the companion star is expected to be dissolved over time. An interesting question is whether the companion will eventually dissolve completely via the bootstrap process of wind-fed accretion powering cosmic rays (or X-rays) that drive the wind. This question is discussed at greater length elsewhere (Ruderman *et al.* 1988).

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DAVID EICHLER: Astronomy Program, University of Maryland, College Park, MD 20742

YUAN-KUEN KO: Department of Physics, University of Maryland, College Park, MD 20742