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BIASED GALAXY FORMATION IN THE NEUTRINO SCENARIO

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ABSTRACT

We consider the timing problem of the neutrino-dominated cosmological scenario, i.e., the inferred young age for the large-scale structure versus the observationally indicated older age of galaxies and quasars. It can be resolved by "antibiasing" of galaxy formation. By applying a variety of alternative prescriptions for galaxy formation to N-body simulations of the neutrino scenario, we find that the clustering of galaxies could be suppressed relative to the neutrinos to an extent that eliminates the timing problem. It requires that galaxies form preferentially in the flat "sheets" relative to the denser "filaments" and compact clusters. This could emerge either from differences in the efficiency of galaxy formation itself or from feedback influence from a first generation of objects (e.g., quasars) which suppresses the formation of incipient galaxies locally. We present various illustrative examples in order to quantify the requirements from such antibiasing mechanisms and discuss their physical plausibility. Other aspects of the neutrino cosmology are discussed in view of the proposed antibiasing, concluding that we see no fatal flaw in this picture.

Subject headings: cosmology — galaxies: clustering — galaxies: formation — neutrinos

I. INTRODUCTION

The claimed "detection" of mass $m_\nu \sim 30$ eV for the electron neutrino (Lyubimov *et al.* 1980) triggered theoretical work on its cosmological implications (e.g., Bond, Efstathiou, and Silk 1980) which led to the neutrino-pancake scenario. Its advantages were noticed immediately: (i) it can accommodate a critical density, $\Omega = 1$, while a baryonic universe is limited to $\Omega_b \leq 0.06h^{-2}$ by primordial nucleosynthesis constraints (Yang *et al.* 1984) and thus requires fine tuning of the initial conditions; (ii) it is consistent with the observed upper limits on the anisotropies of the microwave background, which pose a difficulty for a baryonic universe with "adiabatic" fluctuations (see Kaiser and Silk 1987); (iii) it has a natural scale of $\lambda_\nu \sim 40$ Mpc (depending on the neutrino mass) which can explain the gross features of the large-scale structure. As a modification of the old Zel'dovich "pancake" scenario (Zel'dovich 1970; Sunyaev and Zel'dovich 1972), it predicts a large-scale "cellular" structure of flat "sheets," elongated "filaments" and compact "knots" surrounding big regions of low densities, in qualitative agreement with the observational indications for a similar pattern of superclusters, rich clusters, and "voids" in the distribution of galaxies (e.g., Oort 1983; Einasto *et al.* 1984).

Since these initial successes, the neutrino model has lost popularity; first, because it has been realized that the claimed detection of neutrino mass was premature, but mostly because numerical simulations have shown that the scenario suffers from a timing/scaling difficulty (Klypin and Shandarin 1983; White, Frenk, and Davis 1983; Centrella and Melott 1983; Dekel and Aarseth 1984). The simulated neutrino two-point correlation function, $\xi_\nu(r)$, steepens in time as the pancakes develop and reaches the logarithmic slope of $\gamma = 1.8$, which is the slope observed for the galaxy-galaxy correlation function, $\xi_g(r)$ (e.g., Davis and Peebles 1983), soon after the formation of the first pancakes. If this time in the simulations is identified with the present epoch, then the pancakes, and therefore the first galaxies, must have formed at $z \leq 2$ (see Fig. 1 for the evolution of ξ_ν; the first pancakes form at $a \sim 3$). This is incompatible with the indications that galaxies started forming before $z = 3$, based on the existence of quasars at $z > 3$, the actual detection of a galaxy at $z = 3.2$ (Djorgovski *et al.* 1985), the old ages deduced for globular clusters (Renzini 1986, and references therein), and the evolutionary models of galaxies (Wyse 1985, and references therein).

If the galaxies indeed form only in the collapsed superstructures, with a similar efficiency in each of them, they would be even more clustered than the neutrinos—at any given time $\xi_g(r)$ is at least as steep and with a higher amplitude than $\xi_\nu(r)$ (see Fig. 2 below; also White, Frenk, and Davis 1983). This makes the discrepancy even more severe by introducing a scaling problem: the galaxy clustering length, r_0, defined by $\xi_g(r = r_0) = 1$, is at that time in the simulations already greater than $5(\Omega h^2)^{-1}$, whereas the one observed for galaxies is only $r_0 \approx 5h^{-1}$ Mpc; these values could agree only if $\Omega h > 1$ (h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}), which is unlikely.

Two recent developments, observational and theoretical, have revived interest in the neutrino scenario and made it worthwhile to search for ways to overcome the above difficulty. First, it is debatable whether certain observed features of the large-scale structure are easily reproducible by the cold dark matter (CDM) cosmology which is so successful in explaining galaxies (e.g., Blumenthal *et al.* 1984; Dekel and Silk 1986; Davis *et al.* 1985). Among these, one can list the possible indications for large streaming velocities on very large

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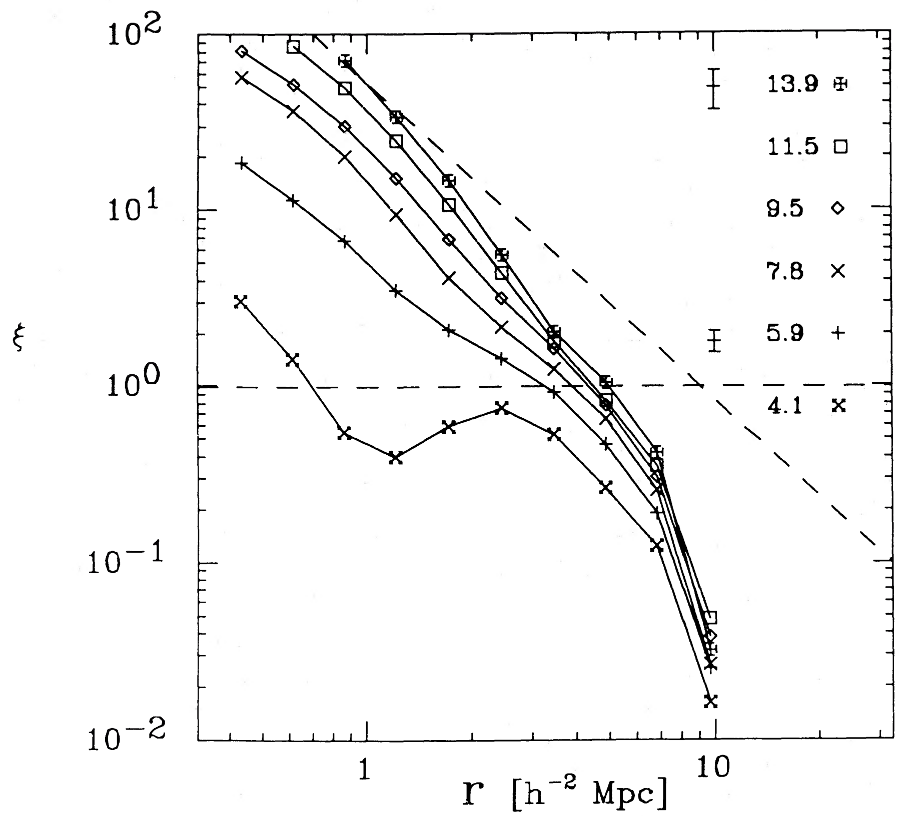

FIG. 1.—Time evolution of the two-point correlation function for the neutrinos, averaged over the three simulations. Each curve corresponds to a different time denoted by the corresponding expansion factor. One standard deviation of our ensemble of simulations is represented by the error bars on the right, for two different times.

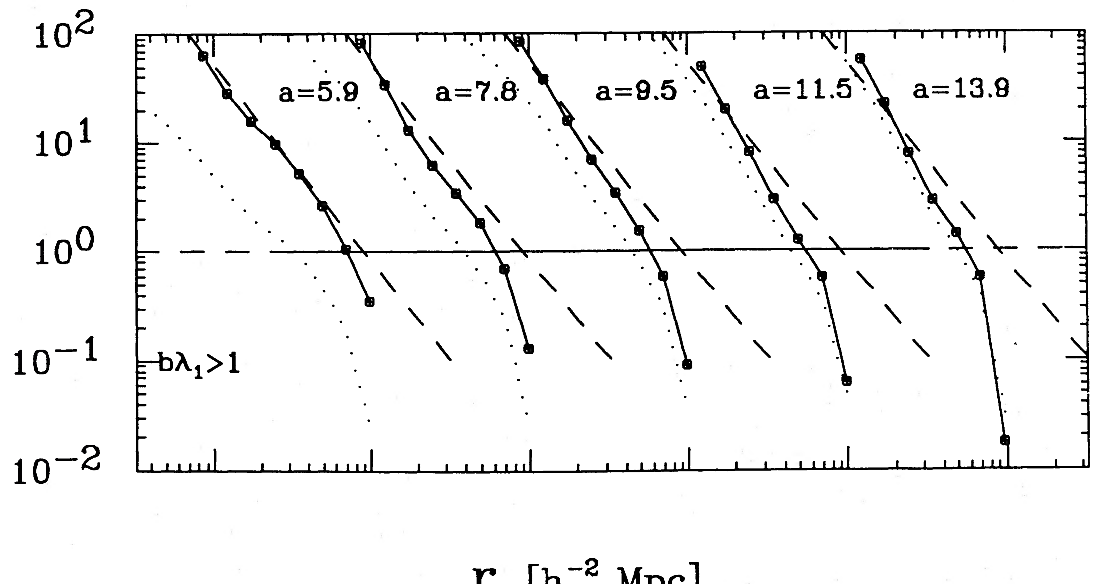

FIG. 2.—The two-point correlation function for the galaxies selected by the criterion $b\lambda_1 \geq 1$ (the trivial bias), averaged over the three simulations, in comparison with the neutrino correlation function (*dotted lines*). The curves corresponding to the different times are shifted horizontally by one decade relative to each other.

scales (Collins, Joseph, and Robertson 1986; Burstein *et al.* 1986; for theoretical discussion, see Blumenthal, Dekel, and Primack 1988; Bond 1986), the existence of $\sim 100h^{-1}$ Mpc elongated superclusters of galaxies (e.g., Giovanelli, Haynes, and Chincarini 1986), never seen in CDM realizations, and huge regions void of bright galaxies (e.g., Kirshner *et al.* 1981; de Lapparent, Geller, and Huchra 1986; but see White *et al.* 1987), and in particular the excess clustering of rich clusters (e.g., Bahcall and Soneira 1983; Batuski and Burns 1985; Tully 1986; see, e.g., Barnes *et al.* 1985; Blumenthal, Dekel, and Primack 1988; White *et al.* 1987).

Second, motivated in large by the desire to reconcile the observations with $\Omega = 1$, the idea that the spatial distribution of galaxies does not necessarily follow the underlying mass distribution has become an acceptable option (Dekel and Rees 1987, and references therein). While studying possible physical mechanisms that could affect the efficiency of galaxy formation as a function of position, time, or both, it became evident that such biased galaxy formation could occur in many ways. In particular, it would be naive to assume that the biasing in the neutrino scenario is only the "trivial" one, in which the galaxies form with a uniform probability in all the collapsed regions. It is possible that the present population of galaxies is *less* clustered than what is predicted by the trivial bias, and perhaps even less clustered than the neutrinos themselves. If, for example, galaxy formation is somehow more efficient in the flat sheets than in the denser filaments and knots, the result is an antibias which would ease the constraint on the neutrino scenario (see Shapiro 1984; Shandarin 1987). In particular, the slope of the galaxy correlation function is a sensitive function of the global dimensionality of the superclusters (e.g., Dekel and Aarseth 1984), so a biasing away from the rich clusters and filaments might be helpful.

It is possible that the flat geometry of the sheets provide preferential conditions for galaxy formation. Alternatively, the bias might be due to feedback from a first generation of objects (e.g., quasars) which affect the formation of further galaxies. Such a feedback could, in principle, either help or suppress galaxy formation, and it could be effective either locally or at a distance. In order to ease the constraint on the neutrino scenario it must *suppress* galaxy formation *locally* in the regions of highest density.

The aim of this paper is to test possible antibiasing mechanisms which may help the neutrino scenario. Motivated by physical considerations, we assume various simple criteria for galaxy formation or for its suppression, or for both, and simulate the resultant evolution of the galaxy distribution using N-body simulations of the neutrino scenario. The goal is to estimate the properties required from successful antibiasing mechanisms, and to learn how sensitive they might be to the choice of parameters, in order to be able to evaluate the plausibility of such mechanisms.

In § II (and in the Appendix) we describe our simulations of the neutrino scenario and analyze the clustering of galaxies which form according to simple local geometrical and time criteria. In § III we discuss and simulate biasing scenarios where galaxy formation is suppressed by feedback influence from earlier objects. In § IV we consider other aspects of the neutrino cosmology, and in § V we conclude and discuss our results.

II. NEUTRINOS AND AUTONOMOUS BIASING

a) The Neutrino Simulations

We first ran an ensemble of three conventional neutrino simulations in a flat Einstein–de Sitter universe. For the initial neutrino spectrum of fluctuations we adopted (Bond and Szalay 1983)

$$|\delta_k|^2 = Ak^n 10^{-2(k/k_\nu)^{1.5}} , \qquad (1)$$

where A is the time-dependent amplitude and $n = 1$, assuming a primordial scale-invariant spectrum (Zel'dovich 1972). The characteristic wavenumber, $k_\nu \equiv 2\pi/\lambda_\nu$, corresponds to the free-streaming damping length

$$\lambda_\nu \approx 41(m_\nu/30 \text{ eV})^{-1} \text{ Mpc} \approx 13(\Omega h^2)^{-1} \text{ Mpc} . \qquad (2)$$

To construct the initial conditions, $N \approx 8000$ equal-mass particles were initially distributed uniformly inside a sphere of radius $2.5\lambda_\nu$, at the points of a cubic grid, so as to suppress initially any undesired small-scale noise. Then the position of each particle and its velocity were perturbed by a superposition of small-amplitude plane waves, assuming random phases and wavevectors and normally distributed amplitudes, which provide a random Gaussian realization of the spectrum of equation (1). The detailed procedure is described in the Appendix. The evolution of the system was followed in the linear regime by the approximation of Zel'dovich (1970; see eq. [A2]), until a stage where the rms density contrast on an arbitrary scale, $\lambda_u = 8h^{-1}$ Mpc, reached a value of $b \approx 0.15$. Then the cosmological expansion factor a was set equal to 1, and the N-body simulation started. The softened equations of motion were integrated directly using the comoving code developed by Aarseth (1984), with a softening parameter $\epsilon = 0.075\lambda_\nu a^{-1}$ (as measured in comoving units).

The timing problem of the neutrino scenario is apparent in Figure 1 which displays the familiar time evolution of the neutrinos pair correlation function, $\xi_\nu(r)$, calculated as in Dekel and Aarseth (1984) and averaged over our ensemble of simulations. The logarithmic slope of $\xi_\nu(r)$, which steepens in time, matches the observed slope of the galaxy pair correlation, $\gamma = 1.8$, at an expansion factor a between 5.9 and 7.8, and it becomes steeper at later times. If this epoch is considered to be the present epoch, the collapse of the first galaxies, which occurs at $a \approx 2.5$ (see below), would correspond to a redshift $z \leq 2$, which is too recent to explain the early galaxies and quasars.

b) Biased Galaxy Formation

It would be wrong to assume that the spatial distribution of galaxies traces the underlying neutrino mass distribution; the gas dissipates into the centers of the potential wells created by the gravitating neutrinos, and the efficiency of galaxy formation is probably different from place to place. It would be reasonable to assume though that this efficiency is, even if indirectly, a function of the local density of the neutrinos. Then we can apply rough but plausible criteria for galaxy formation based on the simulated neutrino distribution, without actually simulating the complex gasdynamic processes associated with the gas collapse and fragmentation.

We use the initial conditions and the Zel'dovich approximation to obtain criteria for identifying a simulated particle with a luminous galaxy. The half comoving spacing between the points in the simulations is $\sim 1.3(\Omega h^2)^{-1}$ Mpc, chosen to be comparable to the comoving radius of a normal galactic halo. This choice guarantees that by searching for galaxies at the positions of the simulated particles we do not miss any big galaxy, and, on the other hand, each galaxy is typically represented by only one particle. The local number density of bright galaxies is on the order of $\sim 0.01h^3$ Mpc^{-3}, so we expect

$\sim 1400h^{-3}$ bright galaxies in the volume of each of our simulations.

For each given biasing scenario we reproduce the evolution of the spatial distribution of galaxies by applying the assumed criteria for galaxy formation to the N-body simulations and then check consistency with the observational constraints. We identify independently two stages in the simulation with two corresponding cosmological times: the present epoch, a_0, and the epoch a_* characteristic of the onset of quasar formation. The present epoch is determined by the requirement that the logarithmic slope of the galaxy two-point correlation function, at least near $\xi_g \sim 1$, be -1.8. The quasars are assumed to be associated with the young galaxies, so a_* is chosen as the epoch when the first galaxies form (see the detailed procedure below). The crucial constraint comes from comparing $1 + z_* = a_0/a_*$ with $z > 3$. (A certain quasar density evolution is obtained from such a procedure for given criteria for galaxy formation and an assumed lifetime for a quasar; one can alternatively choose a_* to be the time when the number of quasar reaches a maximum, which is to be identified observationally with $z \sim 2$.)

c) *The Trivial Bias*

According to the Zel'dovich (1970) approximation, the local density at an Eulerian position,

$$\boldsymbol{r}(\boldsymbol{q}, t) = a(t)[\boldsymbol{q} - b(t)\boldsymbol{\psi}(\boldsymbol{q})] , \tag{3}$$

which is a function of the Lagrangian position $\boldsymbol{q}$ and the time t, is given by

$$\rho = \bar{\rho}\left[\det\left(\frac{\partial r_i}{\partial q_j}\right)\right]^{-1} = \bar{\rho}\left[\det\left(\delta_{ij} - b\,\frac{\partial \psi_i}{\partial q_j}\right)\right]^{-1} , \tag{4}$$

where $\bar{\rho}$ is the mean density and $b(t) \propto t^{2/3}$ is the linear fluctuation growth rate if $\Omega = 1$. After the deformation tensor $\partial\psi_i/\partial q_i$ is diagonalized locally (the tensor is symmetric under the assumption of no rotation), with eigenvalues $\lambda_i(\boldsymbol{q})$ defined such that $\lambda_1 \geq \lambda_2 \geq \lambda_3$, the density can be written as

$$\rho = \bar{\rho}(1 - b\lambda_1)^{-1}(1 - b\lambda_2)^{-1}(1 - b\lambda_3)^{-1} . \tag{5}$$

Consider a point which is a local positive maximum of λ_j: the density there increases as $b(t)$ grows, approaching infinity at some critical time in which $b\lambda_j = 1$. This corresponds to a collapse along the local principal axis j. In the case of neutrinos one expects this collapse to be coherent over the characteristic scale λ_ν and therefore to produce superstructures on that scale. The formation epoch is determined by $b\lambda_1 = 1$.

The classical pancake picture (Sunyaev and Zel'dovich 1972) suggests that galaxies form only in the collapsed ($\sim \lambda_\nu$) regions; the collapse into a singular plane produces shock waves that overtake an ever larger fraction of the collapsing gas and thus enable the radiative cooling of the infalling gas that accumulates behind them. This introduces a "trivial" bias in the formation of galaxies; only that matter which is shock heated and condensed will be able to fragment gravitationally to form galaxies. To approximate this biasing we use the criterion $b\lambda_1 \geq 1$ as a necessary and sufficient condition, assuming that galaxies form whenever and wherever a collapse to a singularity occurs at least along one direction.

Alternatively, in some cases we appeal to a three-dimensional density criterion and require that $\delta \equiv \delta\rho/\rho$ in equation (5) exceeds the critical value $\delta_c = 61$. The latter is the value one obtains from a spherical "top hat" model,

$$\delta = \frac{9}{2}\frac{(\theta - \sin\theta)^2}{(1 - \cos\theta)^3} - 1 , \tag{6}$$

at the conformal time $\theta = 3\pi/2$ which corresponds to collapse to one-half the radius at turnaround. This provides only a rough estimate because the actual collapse is not spherically symmetric. We use this procedure only to simulate a time lag in feedback biasing (see below).

The two-point correlation functions of the galaxies selected by the criterion $b\lambda_1 \geq 1$, $\xi_g(r)$, are shown in Figure 2 in comparison with $\xi_\nu(r)$. At $a = 5.9$ the logarithmic slope of ξ_g is roughly -1.8, like the observed one, and at later times ξ_g becomes steeper (and even steeper than ξ_ν), so $a \approx 5.9$ must be regarded as the present epoch a_0. The dotted lines in Figure 3 describe the growth of the number of galaxies in these trivial bias scenarios. The first quasars form at $a_* \approx 2.5$, i.e., at $z_* \approx 1.4$. This low value is in a clear conflict with the presence of galaxies and quasars at $z > 3$. Thus, the trivial bias makes the problem worse. A bias of the *opposite* sign is required. The spatial distribution of galaxies in a slice cut from this model is presented in Figure 4*a*. Large voids are seen between the well-defined elongated pancakes.

d) *Geometrical Biasing*

The geometry of the structures, at least near the critical formation epoch, is determined by the ratios of the eigenvalues. Flat sheets will be formed where $\lambda_1 \gg \lambda_2$, elongated filaments where $\lambda_1 \sim \lambda_2 \gg \lambda_3$ (at the intersections of sheets), and compact clusters where $\lambda_1 \sim \lambda_2 \sim \lambda_3$ (at the knots where filaments intersect). Associated streaming velocities will then carry material along the sheets toward the filaments and along the filaments toward the knots on time scales which are determined by the same ratios of the eigenvalues. The density in the flat sheets is lower than the density in the filaments and the knots, as can be seen in the projections of Figure 4.

The efficiency of galaxy formation may be strongly affected by the geometry of the large-scale structure in which the galaxies form. In the standard dissipative pancake picture the gas is heated by falling into the pancakes, and it later cools and fragments after crossing the planar shock wave (Sunyaev and Zel'dovich 1972; Shapiro, Struck-Marcell, and Melott 1983; Shapiro and Struck-Marcell 1985; Bond *et al.* 1984). However, the shock's pattern in the filament and knot is more complicated; the gas may be heated several times by different shocks coming from different directions, and the result might be different. There are also feedback processes (§ III below) which may work differently in the sheets and in the filaments and knots.

To mimic a general difference between the fragmentation efficiency in the different geometrics, we use the criterion $b\lambda_1 \geq 1$ as a necessary condition, but allow galaxies to form only where

$$\lambda_2/\lambda_1 \leq \eta , \tag{7}$$

where η is an arbitrary tunable parameter smaller than unity. This is equivalent to limiting galaxy formation to the sheetlike pancakes, rather than the filaments or knots.

In Figure 5 we show ξ_g for galaxies selected with $\eta = 0.2$. Based on the slope of ξ_g, the latest time which can be identified with the present epoch is $a_0 \approx 11.5$. From Figure 3, $a_* \approx 2.5$, so $z_* > 3$ as required. The number density of Abell clusters is also appropriate (see Table 1). Hence this geometrical pancake bias is successful at relieving the timing difficulties of the unbiased neutrino model.

e) *Temporal Biasing*

A similar effect could, in principle, be achieved if the efficiency of galaxy formation changes as a simple function of time

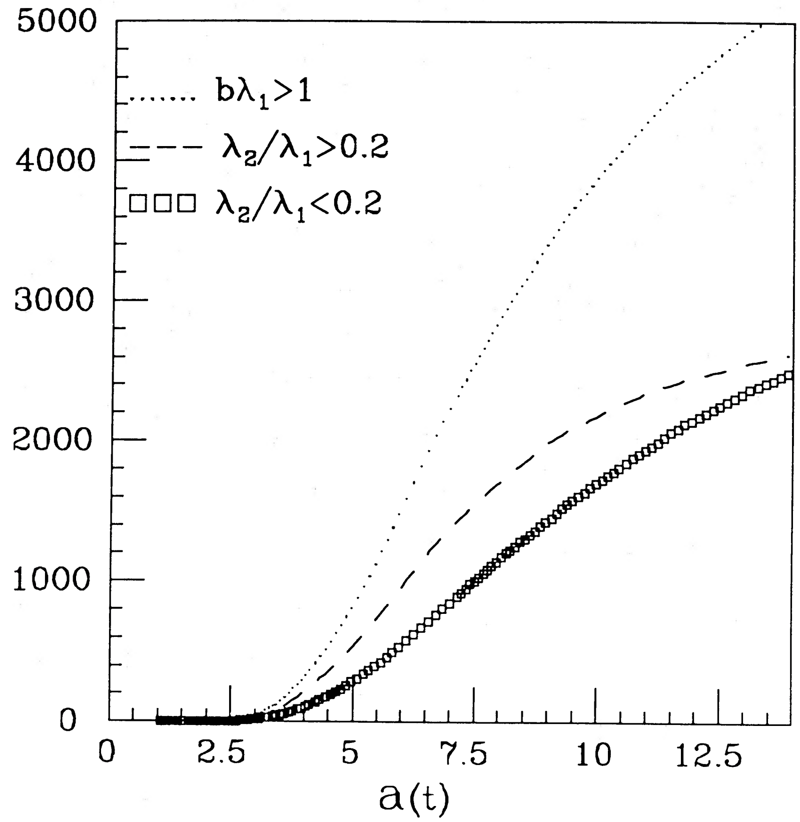

FIG. 3.—The evolution of the number of galaxies (*dotted line*) selected by $b\lambda_1 \geq 1$. The galaxies are divided into galaxies in sheets (*patterned line*) with $\lambda_2/\lambda_1 \leq 0.2$, and galaxies in filaments and clusters (*dashed line*) with $\lambda_2/\lambda_1 > 0.2$.

rather than as a function of position. As can be seen in Figure 3, cluster points ($\lambda_2/\lambda_1 > 0.2$) tend to collapse before filament points, which tend to collapse before the points in sheets ($\lambda_2/\lambda_1 < 0.2$). Hence, if galaxies are somehow prevented from being formed before a certain epoch (when $b = b_{\min}$), they preferentially occur in the lower density sheets and therefore display reduced clustering. The corresponding criteria for galaxy formation would be

$$b\lambda_1 \geq 1 \quad \text{and} \quad b \geq b_{\min} . \tag{8}$$

However, we find that even with a delay in the epoch of galaxy formation until after $a_{\min} = 6$, we can only push the present epoch to $a \sim 11.5$. If $a_* = a_{\min}$, one has $z_* \sim 1$, which does not help solving the timing problem. The temporal biasing may

TABLE 1

NUMBER OF RICH CLUSTERS ($a = 11.5$), $\bar{n}_{\rm obs}(h = 0.75) = 2.5E - 6$

Scenario	Number of Particles	$N_{\min}$	$N_{\rm cluster}$	$\bar{n}(h = 0.75)$	$v_{\rm tot}(h^{-1}$ km s$^{-1})$
Neutrinos	7984	73	7	8.6 E-6	2152
	8051		7	8.6 E-6	1892
	8012		6	7.3 E-6	1352
Trivial bias	4242	40	13	16 E-6	2098
	4756		14	17 E-6	1767
	4420		9	11 E-6	1572
Geometrical bias	1512	14	2	2.4 E-6	1989
	1653		3	3.6 E-6	1466
	1653		2	2·4 E-6	1504
Feedback bias	1879	18	2	2.4 E-6	1829
$R_{\text{non-comov}} = 0.4$	2089		1	1.2 E-6	1681
$b\lambda_1 > 1$	2004		...	...	...
Feedback bias	1198	11	3	3.6 E-6	2080
$R_{\max} = 0.1, t_{\rm cool} \sim \frac{1}{2}t_0$	1330		5	6.1 E-6	1291
$b\lambda_1 \geq 1$	1297		2	2.4 E-6	822
Feedback bias	1146	11	1	1.2 E-6	2124
$R_{\max} = 0.1, R \propto 1/a$	1183		3	3.6 E-6	2044
$b\lambda_1 \geq 1$	1144		...	...	...

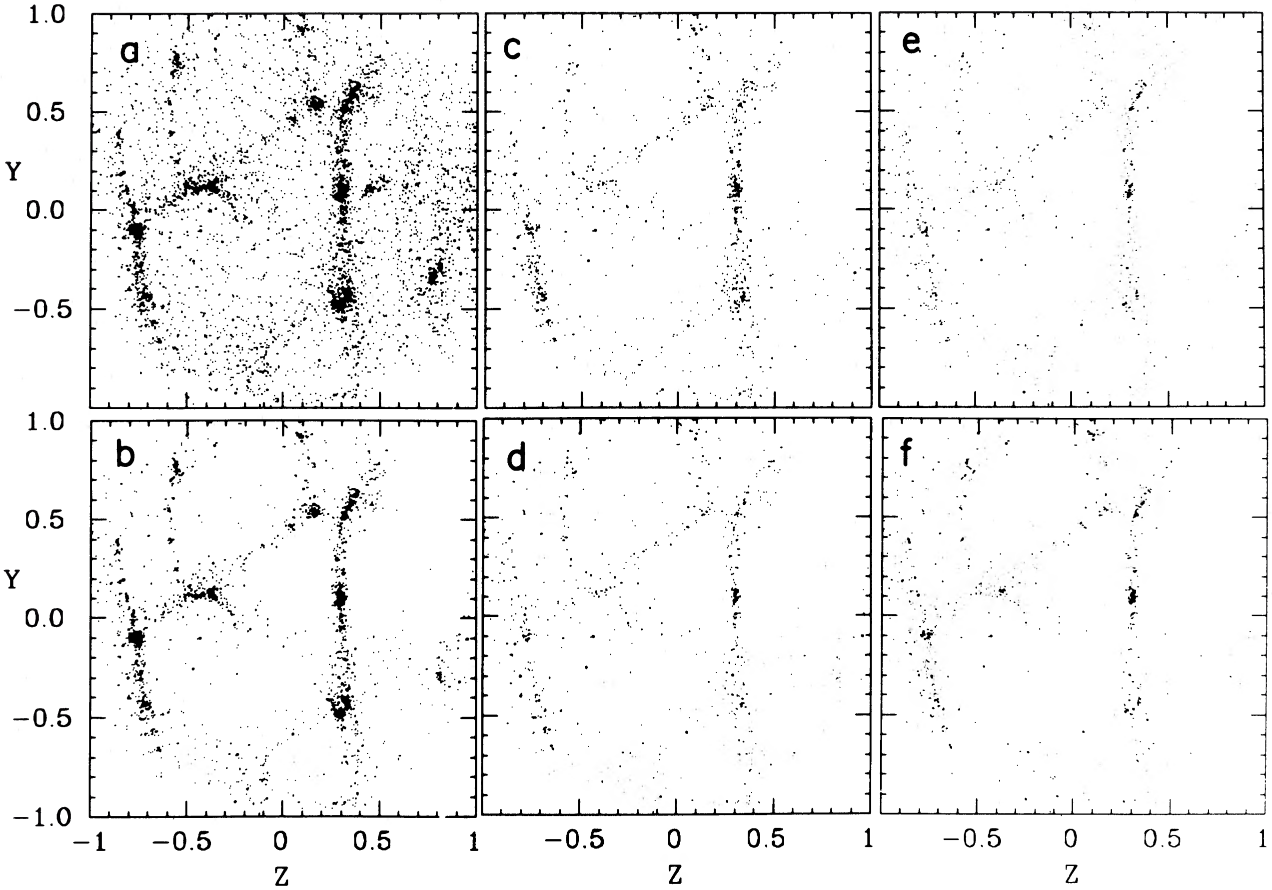

FIG. 4.—Projections of the distribution of galaxies in a slice of thickness $2.5\lambda_\nu = 32.5h^{-2}$ Mpc, cut from one simulation at $a = 11.5$. The galaxies result from the following schemes: (*a*) neutrinos, (*b*) trivial bias ($b\lambda_1 > 1$), (*c*) geometrical bias, (*d*) feedback case a, (*e*) feedback case b, (*f*) feedback case c.

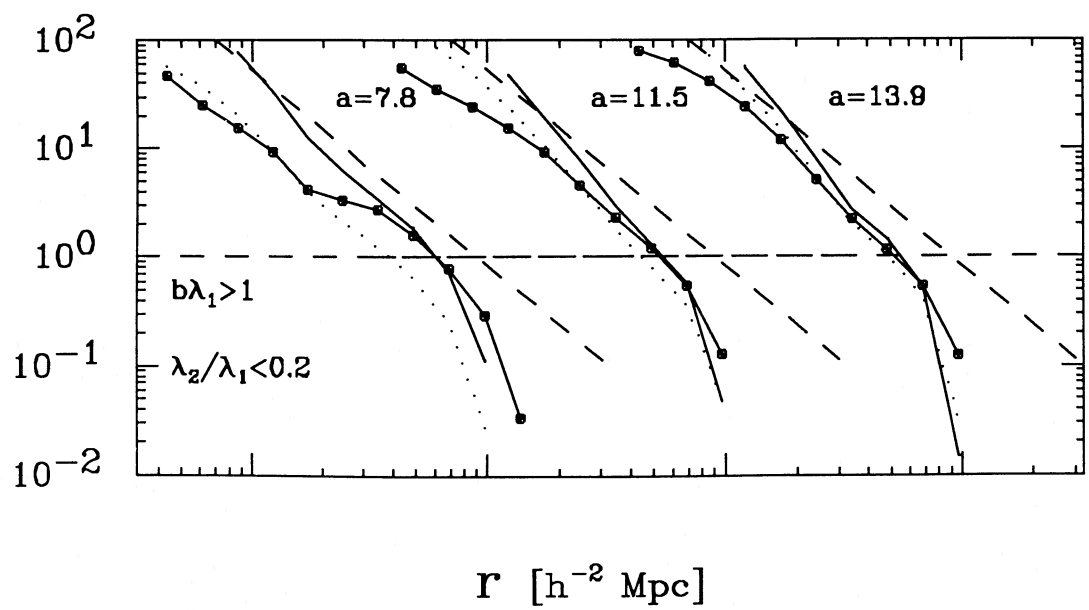

FIG. 5.—The two-point correlation function for galaxies according to the geometrical bias, $\lambda_2/\lambda_1 \leq 0.2$

work only if a first generation of quasars, at $a_* \approx 2.5$ ($z = 4$), inhibited the formation of all other galaxies until $a_{\min} \approx 6$ ($z = 1$).

III. FEEDBACK EFFECTS

a) *Physical Processes*

The formation of bright galaxies could be suppressed if enough energy or momentum from earlier sources (e.g., quasars) is fed in an appropriate form into the protogalactic gas. It would be worth summarizing here the physical feedback mechanisms while pointing to the ones that might be especially relevant to the neutrino scenario (for more general and detailed discussion see Rees 1985; Dekel and Rees 1987; Braun and Dekel 1987). The relevant features are the *range* of influence and its *time* dependence. Unlike the requirements from biasing in the cold dark matter picture, there is no need in the case of neutrinos to appeal to very energetic mechanisms that would affect the formation of galaxies at large distances, because the large-scale structure is already built-in, reflecting the neutrino coherence length λ_ν. The required negative bias should be rather *local*, within the clustering comoving length of $\sim 5h^{-1}$ Mpc. A negative influence at larger distances might neutralize the effect and should therefore be avoided. Most important for a local effect is a *time lag* between the time when the sources are turned on and the latest epoch when incipient galaxies could still be quenched. On the other hand, the local effect should not be too effective, so that some rich clusters still form.

The major relevant issues are the form of the energy *emitted* from the sources, the *coupling* to the gas, and the actual *suppression* mechanism.

The suppression could eventually arise from *inhibiting the condensation* of the gas in the dark matter potential wells at an early stage or by *affecting the IMF* such that the resultant galaxy is fainter. The condensation could be inhibited by increasing the internal energy density of the gas above the virial energy of the potential well—either by heating the gas or by exerting pressure through trapped momentum carriers (photons or nonthermal particles). An alternative way of inhibiting the condensation of the gas is by generating a drift velocity larger than the escape velocity relative to the dark matter due to a pressure gradient, which could result from slow diffusion to large distances. Most extravagant in energy could be blast waves that push the gas rapidly over large distances. After the source dies, the suppression might continue for a while; thermal pressure decays on a cooling time scale and bulk velocities decay as a^{-1} due to the expansion of the universe.

The required energy per baryon in order to suppress the condensation of a protogalaxy of baryonic mass M_b in an $\Omega = 1$ universe is

$$\epsilon \approx 160(1+z)h^{2/3}\left(\frac{M_b}{10^{11}\ M_\odot}\right)^{2/3}\left(\frac{\Omega_b}{0.1}\right)^{-2/3} \text{eV} . \quad (9)$$

This kind of energy, even taking into account an efficiency factor of $\sim 10^{-3}$ in the coupling of the energy to the gas, is consistent with the observed upper limits on the backgrounds of radiation or cosmic rays (Rees 1985).

If the sources are quasars or active galaxies, the relevant energy could be carried away by UV radiation, by fast particles, or by blast waves.

The coupling of UV photons to the gas could neither be achieved by Thomson scattering off electrons because the universe is optically thin for such scattering at late epochs, nor by photoionization which could not heat the gas much above $\sim 10^4$ K because any energy excess would be shared among several atoms (Rees 1985). (Compton-heated hard X-rays are excluded because of upper limits on the X-ray background.) The most promising process is thus scattering off Lyman lines, which could trap the pressure exerting photons in regions rich with neutral hydrogen. Neutral hydrogen might be available either before the universe got ionized ($z > 3$?), or in regions that could maintain a temperature $\leq 10^4$ K. The former would provide negative temporal biasing which we found to be not very satisfactory (§ II*e*), but regions of the latter kind arise naturally inside Zel'dovich pancakes because of cooling behind shocks.

Trapped Lyα radiation can provide significant radiation pressure in highly photoionized regions if they maintain a low temperature of $\leq 10^4$ K such that collisional ionization is not important. The high rate of recombination to the excited energy levels of hydrogen would provide a high density of line photons which could be trapped inside a protogalaxy for a long enough time and thus exert a significant pressure. The ionization front produced by the ionizing radiation from the quasar propagates toward its maximum extent—the Strömgren radius. The radiation pressure is maximal at this radius, and it is enough to prevent gas condensation in potential wells in the local neighborhood of the quasar, provided that the density is high enough. The maximum radius of suppression is

$$R_{\text{crit}} = 0.8 \text{ Mpc } Q_{56} T_4^{0.13} T_{v4}^{-2} , \quad (10)$$

and the minimal number density is

$$n_{\text{crit}} = 25 \times 10^{-4} \text{ cm}^{-3}\ Q_{56}^{-1} T_4^{0.6} T_{v4}^3 , \quad (11)$$

where Q_{56} is the number of ionizing photons emitted per second in units of 10^{56} (which corresponds to $L \sim 10^{46}$ ergs s^{-1}), T_4 is the gas temperature in units of 10^4 K, and T_{v4} is the virial temperature characterizing the potential well in which the gas condensation is to be suppressed. The mean cosmological density at $z \sim 3$ is only $\sim 10^{-4}$ cm^{-3}, but in pancakes the density is typically a few tens times the mean, and the temperature remains cool at $\sim 10^4$ K. Thus, a suppression of $\sim 10^8\ M_\odot$ subgalactic objects, which are the typical fragments in a pancake (Shandarin, Doroshkevich, and Zel'dovich 1983, and references therein), is possible within a sphere of radius ~ 1 Mpc. This corresponds to a radius of ~ 4 Mpc today—comparable to the galaxy clustering length (see a detailed calculation by Braun and Dekel 1987).

The diffusion of Lyα photons could provide pressure gradients and drift velocities, but this is not necessarily a local effect.

The UV radiation could affect the IMF by photodissociating the H_2 molecules, thus drastically reducing the cooling efficiency and bias the IMF toward bigger stars. This could lead to the disruption of the protogalaxies via supernovae (Silk 1985). Since the UV flux drops with the distance squared, this would also tend to cause a local antibias.

The coupling of fast particles to the gas is via collisions with the electrons and is controlled by the unknown magnetic field structure. Relatively slow particles ($v \leq 0.1c$) would slow down effectively in a Hubble time and heat the gas. Relativistic ions ($v > 0.1c$), if coupled to the gas with a mean free path much smaller than a protogalaxy, would exert pressure effectively, but they may propagate to distances larger than the clustering length thus neutralizing the antibiasing effect. The IMF might be biased toward faint stars because of metal enrichment due to fast particles (Couchman and Rees 1986).

b) The Suppression Radius

A rough way to estimate the radius of the sphere around the quasar inside which suppression is possible is by considering the available energy. We assume that each quasar emits its energy in a constant rate, $L \equiv 10^{46} L_{46}$ ergs s^{-1}, during its lifetime τ, starting at t_*. The total energy output from the quasar until time t is

$$E(t) = 6.3 \times 10^{62} L_{46} \left(\frac{t - t_*}{2 \times 10^9 \text{ yr}} \right) \text{ ergs} . \quad (12)$$

Assuming that the output energy is equally shared among the surrounding baryons with an efficiency factor f, feeding ϵ eV per baryon, the affected sphere has a radius

$$R(t) = 6.4 L_{46}^{1/3} \left(\frac{f}{0.01} \right)^{1/3} \left(\frac{\epsilon}{100 \text{ eV}} \right)^{-1/3} \times h^{-2/3} \left(\frac{\Omega_b}{0.1} \right)^{-1/3} \left(\frac{t - t_*}{2 \times 10^9 \text{ yr}} \right)^{1/3} (1 + z)^{-1} \text{ Mpc}. \quad (13)$$

As long as the quasar is active, the feedback radius is increasing in time in comoving coordinates, reaching a maximum value when $t - t_* = \tau$.

If the gas inside this sphere is heated, the suppression would continue until the gas could cool. At the relevant temperature of $\sim 10^4$ K the gas is partially ionized and the cooling is radiative, mainly via recombination of H and He and via Lyα line cooling. The cooling time is given by

$$t_{\text{cool}} = \frac{3kT}{\Lambda(T) n_b} \approx 1.5 \times 10^{11} \left(\frac{\epsilon}{100 \text{ eV}} \right) \left(\frac{\Omega_b}{0.1} \right)^{-1} h^{-2} (1 + z)^{-3} \text{ yr} , \quad (14)$$

where we have approximated the cooling rate by

$$\Lambda(T) \approx 10^{-24} T^{1/2} \text{ ergs s}^{-1} \text{ cm}^3 , \quad (15)$$

which is a reasonable approximation when the contribution from heavy elements is negligible. We have also assumed in equation (14) that the density in a protogalaxy is ~ 5.5 times the background density (as at a maximum expansion in a "top hat" model). With $\epsilon \approx 100$ eV per baryon, and $\Omega_b \approx 0.1$, we get $t_{\text{cool}} \approx 3h^{-2}(2 \times 10^9$ yr) at $z = 3$ and a value 4^3 times larger today.

If the suppression of galaxy formation is due to radiative pressure from a quasar of constant luminosity, where the pressure is roughly inversely proportional to the square of the distance from the quasar, it determines a critical radius, fixed in time, inside which the suppression is effective. This radius is decreasing in comoving coordinates as a^{-1}.

c) Feedback Simulations and Results

We assume the quasars to be identified with young galaxies, and we find them in the simulations by applying the same selection criteria that were used to find galaxies. The feedback influence which suppresses the formation of galaxies also suppresses the formation of more quasars, which eventually weaken the feedback, so there is a competition between opposite effects; the optimal parameters are to be determined by the simulations.

We first try a simple model in which the radius of influence around each quasar is fixed in time. The optimal radius is a compromise between the need for strong suppression within clusters and the worry from affecting distant regions thus neutralizing the antibiasing. A fixed radius of influence decays as a^{-1} in comoving coordinates, so the feedback effect weakens in time. In practice we decide at the end of each small time interval whether a particle should turn into a galaxy (or a quasar) by checking whether $b\lambda_1 \geq 1$ (or $\delta \geq 61$) and making sure that it is not located inside an active range of influence of another quasar. The fixed radius is chosen to be $\sim 5h^{-1}$ Mpc (in comoving units normalized to the present) at $a_* = 2.5$, the onset

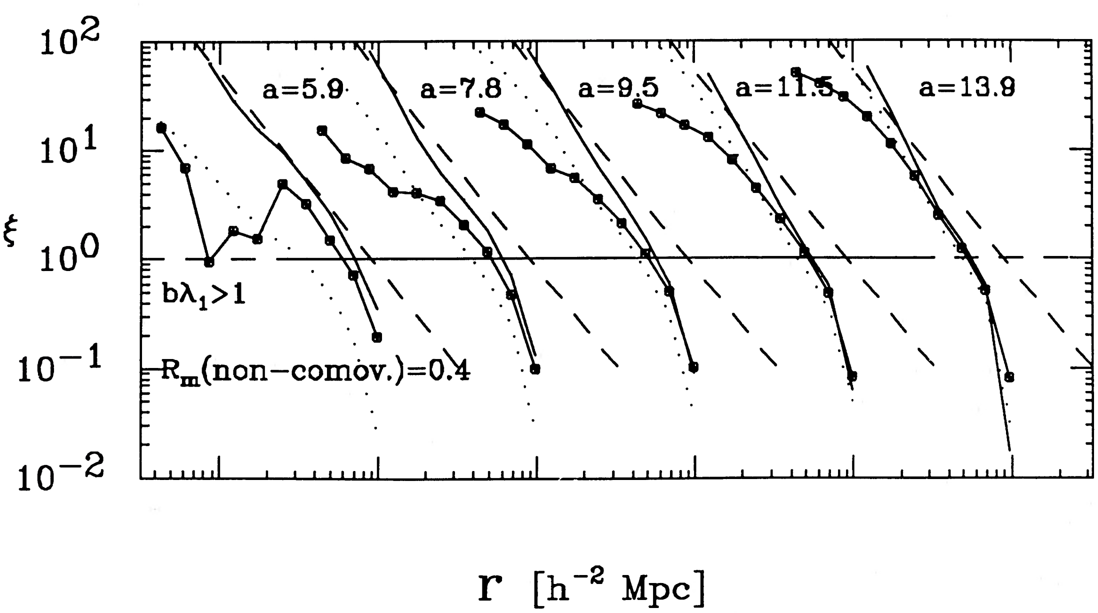

FIG. 6.—The two-point correlation function for galaxies according to a feedback bias with a fixed radius of influence

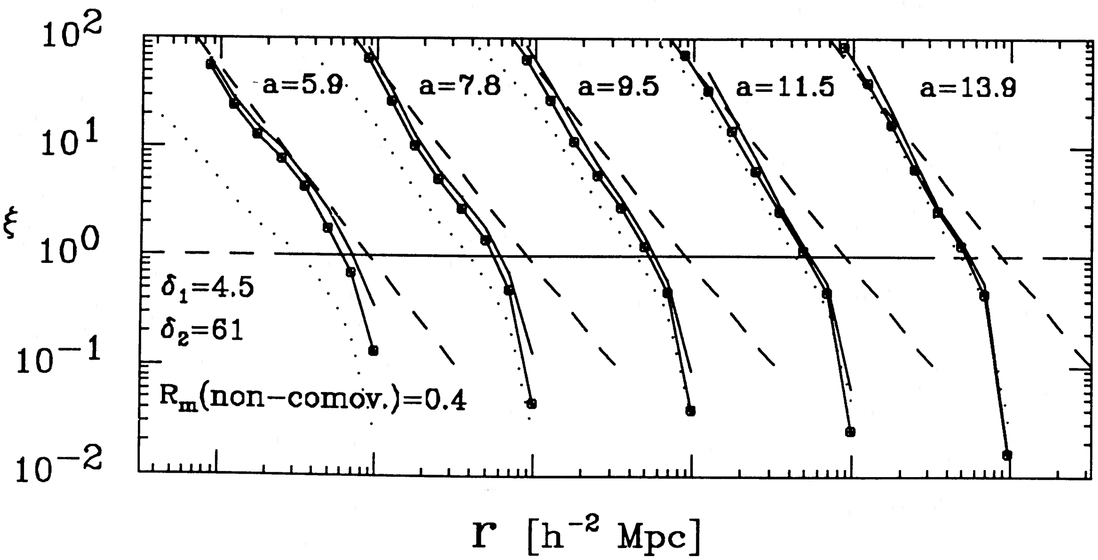

FIG. 7.—The two-point correlation function for galaxies ($\delta > 61$) according to a feedback bias with a time lag: protogalaxies are not affected if they have turned around ($\delta \geq 4.5$) by the time the feedback influence turned on at their position.

of galaxy (quasar) formation. The cooling time is assumed to be negligible. The slope of the resultant correlation function, as shown in Figure 6, admits $a_0 \approx 11.5$–13.9.

Next we introduce a significant time lag by assuming that a protogalaxy which starts feeling a feedback influence only after it had turned around ($\delta > 4.5$) is immune from the suppression effect. In this case, as shown in Figure 7, the suppression becomes less effective, as expected.

Finally we tried growing suppression radii, taking cooling into account. Figure 8 displays three cases with $\tau = 6 \times 10^8$ yr. In case a (Fig. 8*a*) the suppression radius was (in comoving coordinates)

$$R(t) = \begin{cases} R_{\rm max} & t_* < t < t_* + \tau + t_{\rm cool} \,, \\ 0 & \text{later} \,, \end{cases} \tag{16a}$$

and $t_{\rm cool} \sim t_0/2$. In case b (Fig. 8*b*) we assumed

$$R(t) = \begin{cases} R_{\rm max}\left(\dfrac{t - t_*}{\tau}\right)^{1/3} & t_* < t < t_* + \tau \,, \\ R_{\rm max}\left(\dfrac{t}{t_* + \tau}\right)^{-2/3} & \text{later} \,, \end{cases} \tag{16b}$$

and in case c (Fig. 8*c*)

$$R(t) = \begin{cases} R_{\rm max}\left(\dfrac{t - t_*}{\tau}\right)^{1/3} & t_* < t < t_* + \tau \,, \\ R_{\rm max} & t_* + \tau < t < t_* + \tau + t_{\rm ccol} \,, \\ 0 & \text{later} \,, \end{cases} \tag{16c}$$

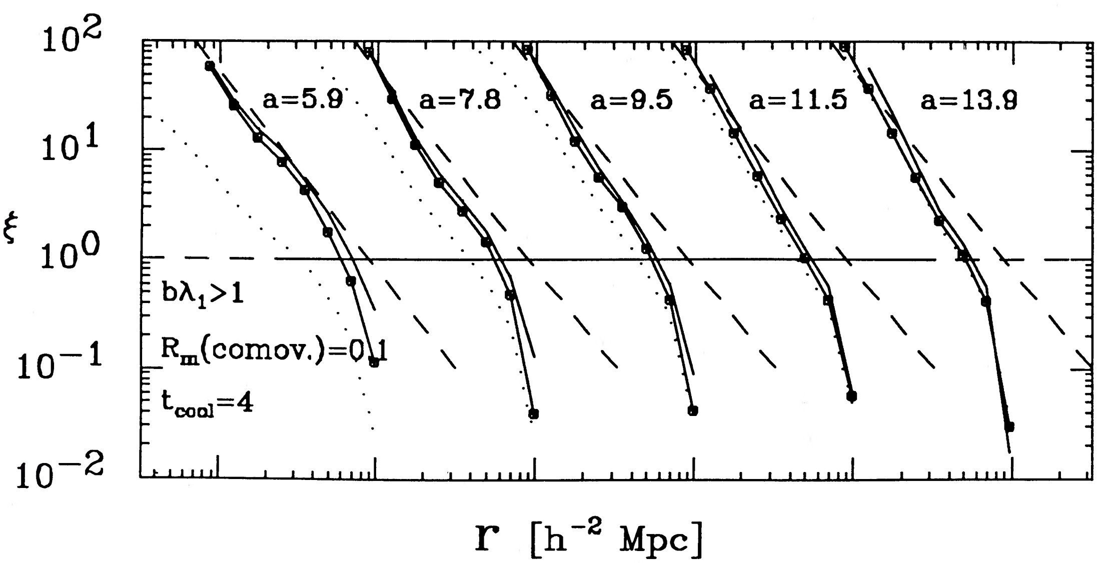

FIG. 8*a*

FIG. 8.—The two-point correlation function of galaxies according to a feedback bias in the cases a, b, and c (see text)

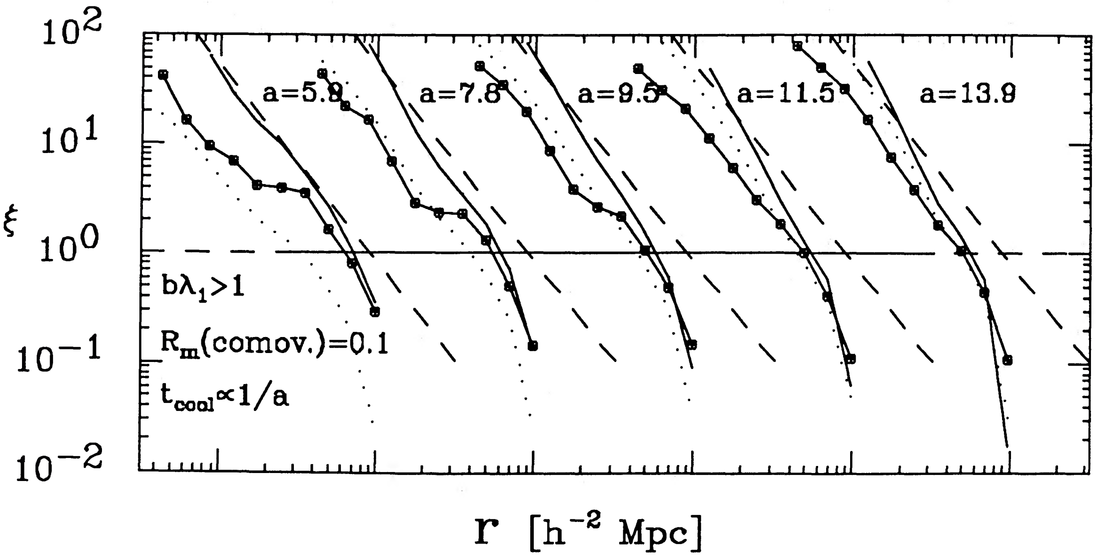

FIG. 8*b*

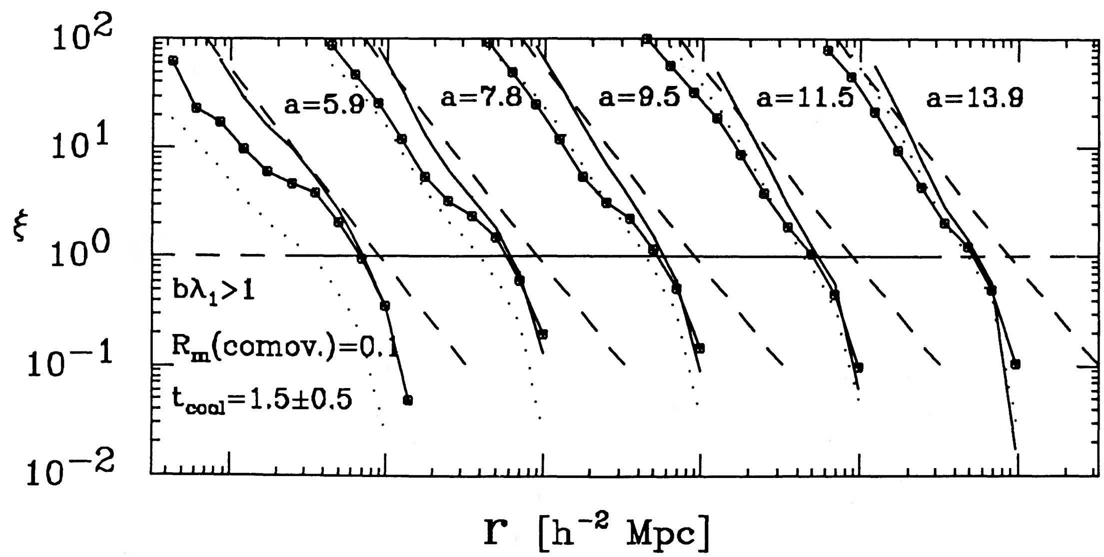

FIG. 8*c*

with t_{cool} chosen for each protogalaxy at random in the range (2–4) × 10^9 yr. For the feedback to be effective, the maximum comoving radius R_{max} must be on the order of the galaxy correlation length. We choose $R_{max} \approx 3h^{-1}$ Mpc, in agreement with the estimates of the previous sections, but we find the results not to be very sensitive to the exact value of R_{max}.

Figure 4 displays the projected distribution of galaxies in these scenarios, in comparison with that of the neutrinos and the galaxies of the trivial bias. The antibiased nature of these models is apparent where the compact clusters and dense filaments, which are seen in the neutrinos and trivial-bias models, are smeared out significantly.

All the models of Figure 8 give $a_0 \approx 11.5$, and in Figures 8*a* and 8*b* even $a_0 \approx 13.9$ is acceptable. The evolution of the number of objects in case b is shown in Figure 9. All the above scenarios display a growing quasar number density between $a \approx 2.5$ and $a \approx 4$, a flat maximum, and a slowly decreasing density later on. With $a_0 \approx 12$ this is qualitatively consistent with the indicated flat maximum in the quasar density near $z \leq 3$ (e.g., Schmidt, Schneider, and Gunn 1986). The lack of a well-defined maximum, together with the large observational uncertainties, prevents a quantitative comparison between the predicted and observed quasar density evolution, apart from requiring that the onset of quasar formation occurred at $z \sim 3$–4.

IV. OTHER ASPECTS OF THE NEUTRINO SCENARIO

a) Rich Clusters

The observed rich clusters, both as associations of galaxies and as X-ray sources, could provide further constraints on the cosmological scenario and on the bias mechanism. To find clusters in the simulations, either in the neutrino distribution or in the distribution of galaxies, we use an algorithm which assigns all the neighboring particles which are separated by less than a critical separation, *d*, to a given cluster (e.g., Dekel,

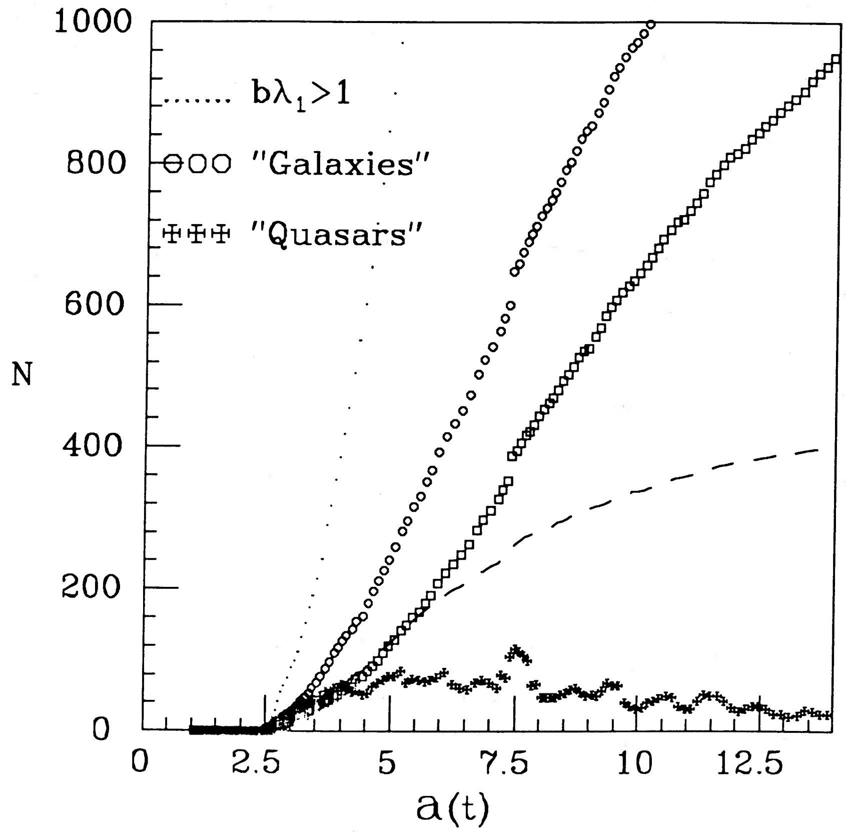

FIG. 9.—Evolution of the number of galaxies and quasars for feedback bias, case b. Notations is as in Fig. 3

West, and Aarseth 1984). This procedure selects clusters with a characteristic overdensity $n/\bar{n} \sim (d/\bar{d})^{-3}$, where $\bar{d}$ is the mean separation between neighbors. To match roughly the Abell (1958) criteria for clusters of richness $R \geq 1$ we use a separation parameter which corresponds to $n/\bar{n} \sim 200$ and require that the minimum number of galaxies associated with the cluster be

$$N_{\min} = \left(\frac{R_A}{R}\right)^3 \left(\frac{n}{\bar{n}}\right) N_g , \quad (17)$$

where N_g is the total number of galaxies in the simulation sphere of radius $R = 2.5\lambda_v = 32.5h^{-2}$ Mpc, and $R_A = 1.5h^{-1}$ Mpc.

In Table 1 we list, for the neutrinos and for each of the bias scenarios, the resultant number density of clusters for $h = 0.75$. This is to be compared with the estimated number density of Abell clusters of richness ≥ 1, $n_A \approx 5 \times 10^{-6}(h^{-1}\ \text{Mpc})^{-3}$ (Bahcall and Soneira 1983). The rich clusters in the neutrino distribution are too frequent, and even more so for the galaxies when they are selected according to the trivial bias. The antibiasing mechanisms, as expected, reduce the number density of clusters; with the biasing parameters chosen above to cure the timing problem, we find the resultant number density of clusters to be in the appropriate range of values.

We also list in Table 1 the three-dimensional velocity dispersion of all the members of the richest cluster in each simulation. These velocities are found to be quite insensitive to the biasing.

It is interesting to note that the antibiasing procedures which were tried here do not erase the richest clusters; in most cases the richest neutrino clusters still show up as the richest clusters of galaxies (with lower overdensities and richnesses).

In order to find out more about the depth of the potential well associated with the neutrino clusters, we measure and list in Table 2 for the two richest clusters in each simulation the central three-dimensional velocity dispersion, v_c, the maximum circular velocity, $[GM(R_{\max})/R_{\max}]^{1/2}$, and the radius at which

TABLE 2

PROPERTIES OF RICH CLUSTERS

v_{tot} (h^{-1} km s^{-1})	v_c (h^{-1} km s^{-1})	$\left[\frac{GM(R)}{R}\right]^{1/2}_{\max}$ (h^{-1} km s^{-1})	$R_{\max}$ (h^{-2} Mpc)	M_{tot} (h^{-4} $M_\odot$)	R_{tot} (h^{-2} Mpc)
2152.........	2195	2011	1.22	1.3 E15	2.03
1892.........	2061	1865	0.73	0.85 E15	1.83
1352.........	1463	1380	0.3	0.44 E15	1.51
1846.........	1881	1711	0.99	1.03 E15	2.48
1807.........	2001	1708	0.73	0.8 E15	1.83
1252.........	1267	1289	0.88	0.4 E15	1.48

this value is obtained, R_{max}. We also list the total mass, M_{tot}, and the maximum extent, R_{tot}, of the clusters. The one-dimensional velocity dispersions are in the range 1000–1500h^{-1} km s^{-1}, and the total masses are on the order of $10^{15}h^{-4}\ M_{\odot}$. For $h \approx 1$ these values are compatible with the ones observed in the richest clusters, but if $h \approx 0.5$ they are in excess of the observed values.

Based on the depth they estimated for the cluster potential well, White, Davis, and Frenk (1984) argued that the neutrino clusters would show up as excessive X-ray sources. Our results, which were found to be insensitive to the parameters used to find the clusters, indicate that the neutrino potential wells are not necessarily excessively deep—the conclusion depends strongly on the exact value of h. Moreover, the gas might be prevented from falling into the cluster cores and producing X-ray sources (e.g., Bond *et al.* 1984; Shapiro 1984). For example, the gas could be heated by shocks *before* falling into the cluster centers and rise into higher adiabats, or it could be subject to radiation pressure or other feedback effects similar to the ones discussed above as possible sources for the anti-biasing in the formation of galaxies. Thus, we do not share the conclusion of White, Davis, and Frenk (1984) that the neutrino clusters cannot be reconciled with observations.

b) The Mean Mass Density

Given the spatial distribution of galaxies and their velocities in the simulations, one can mimic the observational procedure of estimating Ω based on the assumption that the clustering properties of the galaxies is similar to that of the mass. For example, the cosmic virial theorem relates the rms peculiar pair velocity v_{12} at a given separation r to the correlation function at this separation by (Peebles 1980, p. 280)

$$\langle v_{12}^2 \rangle = \frac{6\pi G\rho_c \Omega r^2 \xi(r) Q J}{(\gamma - 1)(2 - \gamma)(4 - \gamma)}, \tag{18}$$

where the two-point correlation function is assumed to be a power law, $\xi(r) = (r/r_0)^{-\gamma}$, J is a certain function of $\xi(r)$, and the three-point correlation function is assumed to be related to the two-point functions via a proportionality constant Q. For $\gamma = 1.8$ one has $J = 3.7$, and for a rough estimate we adopt the observed value $Q = 1.29$. Figure 10 shows the rms three-dimensional pair velocity of the galaxies in the simulations as a function of separation for some of our models. For separations in the range 3–7h^{-2} Mpc about the correlation length, this procedure yields consistent (r-independent) values of Ω in the range 0.1–0.2. The pair velocities on these scales are in rough agreement (within a factor of 2) with the observations (Davis and Peebles 1983) and the simulations of Frenk, White, and Davis (1983). The comparison with observation is rough because we do not actually mimic the observational procedure of measuring velocities along the line of sight.

The alternative cosmic energy equation (Peebles 1980, p. 278) avoids using the poorly known three-point correlation function. It relates the rms peculiar velocity to the two-point correlation function, which is assumed to be a self-similar power law truncated at r_{max}, by

$$\langle v^2 \rangle = \frac{16\pi G\rho_c \Omega r_0^{\gamma} r_{max}^{2-\gamma}}{(7 + n)(2 - \gamma)}, \tag{19}$$

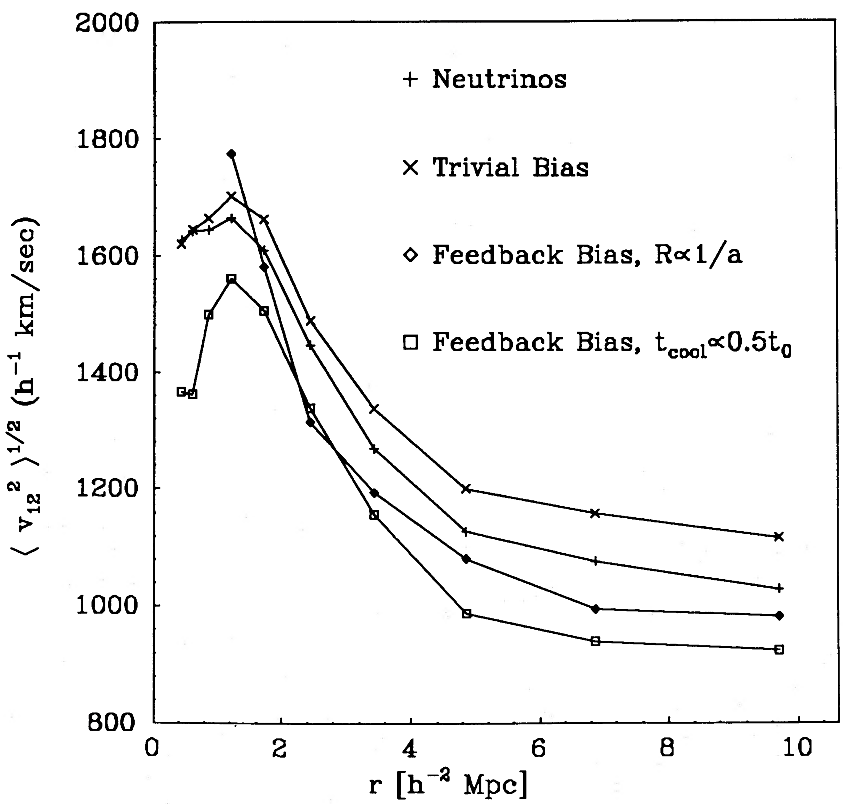

FIG. 10.—The rms pair velocity for the neutrinos and galaxies in several bias models (feedback bias cases a and b, and the trivial bias), for one of the simulations, as a function of the pair separation.

where n relates to γ by $\gamma = (9 + 3n)/5 + n)$. In our models we measure $\langle v^2 \rangle^{1/2} \sim 800h^{-1}$ km s^{-1}. For $r_{\max} \approx 5h^{-2}$ Mpc we then obtain values of $\Omega \leq 0.5$. This estimate of Ω depends on the assumption of self-similarity which is not strictly valid here but can be regarded as an approximation at late times. The resultant value depends on the arbitrary assumed value for the cutoff $r_{\max}$ and is sensitive to the value assumed for γ. Therefore, this estimate should be regarded as a very rough estimate only.

Thus, the observational procedure, when applied to the antibiased distribution of galaxies in the neutrino scenario, severely underestimates the value of Ω, which equals unity in our simulations by construction. This effect is qualitatively similar to the effect in the biased CDM scenario, even though the galaxies are less clustered than the neutrinos on small scales. It results from the fact that the rms galaxy velocities are smaller than the rms neutrino velocities, while in CDM it is a result of the higher galaxy-galaxy correlation function.

c) Streaming Motion

Another important constraint on the theory is the observed streaming motion on large scales (Dressler *et al.* 1987); the reported velocity is $V = 600 \pm 100$ km s^{-1} averaged over a sphere of radius $\sim 60h^{-1}$ Mpc (the effective radius is actually smaller by a factor of ~ 2; see Kaiser 1987), which is uncomfortably large in comparison with the predicted velocities either in the standard biased cold dark matter scenario or in the unbiased (or the trivially biased) neutrino scenario. The biasing scheme has an important effect on the predicted streaming velocity because the predicted velocity depends on the normalization of the density fluctuations, which is observationally constrained by the present distribution of galaxies—not that of the matter.

Linear analysis is appropriate for a crude estimate of the velocity on a scale of few tens of megaparsecs. The mean-square mass fluctuation and bulk velocity over spheres of radius R inside some large volume V_u are related to the power spectrum via (e.g., Peebles 1980)

$$\left(\frac{\delta M}{M}\right)_R^2 = \frac{V_u}{(2\pi)^3} 4\pi \int_0^\infty dk k^2 \langle |\delta_k|^2 \rangle W^2(kR) , \qquad (20)$$

and

$$V_R^2 = \frac{V_u}{(2\pi)^3} 4\pi (a_0 H_0)^2 \int_0^\infty dk \langle |\delta_k|^2 \rangle W^2(kR) , \qquad (21)$$

where we have assumed $\Omega = 1$, and where $W(kR)$ is a window function which is the Fourier transform of the window function in position space; for a "top hat" window of radius R in position space the window in k-space is (Peebles 1980) $W(kR) = 3/(kR)^3[\sin(kR) - kR\cos(kR)]$.

The normalization could be determined by the rms fluctuation of the number of galaxies, which is observed to be $\delta N/N = 1$ over spheres of radius $8h^{-1}$ Mpc (Davis and Peebles 1983). Defining a biasing factor b by $\delta N/N = b\,\delta M/M$ at $8h^{-1}$ Mpc, the predicted velocity then scales as $V \propto b^{-1}$. The normalization could be determined alternatively by comparing the logarithmic slope of the simulated correlation function to the observed $\gamma = 1.8$. The biasing factor could then be defined as the ratio between the expansion factor a at the time when the logarithmic slope of ξ_ν is 1.8 and the present a when the slope of ξ_g reaches this value. For the neutrino spectrum of fluctuations, with no bias ($b = 1$), we find by evaluating the velocity integral for $R = 50h^{-1}$ Mpc a streaming velocity of $165h^{-2}$ km s^{-1}. In the successful antibiased simulations described above we find the bias factor (defined either way) to be typically $b \approx 0.5$. The predicted rms streaming velocity is therefore $V \approx 330h^{-2}$ km s^{-1}. This value is in pleasant rough agreement with the observed result.

V. DISCUSSION AND CONCLUSIONS

We have tried to find out whether a physically plausible antibiasing mechanism can solve the timing problem of the neutrino scenario, by applying various simple biasing schemes to N-body simulations. The conclusion certainly relies on the way we identify the present epoch in the simulations based on matching the evolution of the population of quasars with the observations. We either assign the redshift $z = 4$ to the time when the first galaxy-size object collapses in the simulations, or, alternatively, identify the time when the number of quasars is at its maximum value with the appropriate redshift 2–3. Although this procedure involves certain uncertainties, we believe that they could not affect our conclusions in a qualitative way.

We have found that although only certain mechanisms, where the physical parameters are confined to a certain range of values, can suppress the clustering of galaxies enough to make it consistent with the observed galaxy-galaxy correlation function, the range of possibilities is still fairly large. A bias which depends only on the cosmic time requires a very late epoch for the onset of galaxy formation ($z \sim 1$) which might be incompatible with observations. The desired antibias can result from differences in the efficiency of galaxy formation due to the different geometries of the parent superclusters, but the physical motivation suggested for such a bias has not been worked out in any detail yet. The negative feedback influence needed for reproducing the desired antibias is found to be quite modest energetically, and we have demonstrated that it could be plausibly provided by quasars. The resultant distribution of galaxies is similar to the observed distribution.

Our conclusion is, therefore, that the timing/scaling problem should *not* be regarded as a critical difficulty of the neutrino scenario. This, in view of the promising properties of this scenario in forming superclusters and voids on large scales, should open up our minds to reconsidering the neutrino scenario as a viable cosmological model.

At this stage it is worthwhile to summarize the other open questions and apparent difficulties faced by the neutrino scenario:

1. Massive neutrinos have not been detected in the laboratory. The first claimed measurement of 35 eV for the electron neutrino (Lyubimov *et al.* 1980) has not been confirmed by similar experiments or by other experiments, and the upper limits seem to get tighter. The neutrinos detected on earth from the supernova 1987A in the Large Magellanic Cloud provide an upper limit of about $m_\nu \leq 10$ eV (e.g., Bahcall and Glashow 1987), but this estimate depends on very uncertain assumptions. For example, an alternative analysis of these data suggests two types of neutrinos, with masses of 22 eV and 4 eV (Cowsik 1987). A mass on the order of 10 eV for at least one type of neutrinos is certainly not ruled out by experiment. (In the case of μ or τ neutrinos the current upper limits are much higher [250 keV and 70 MeV, respectively; Harari 1987], and the prospects for lowering the limits to the cosmologically relevant ~ 10 eV level are slim.)

2. There are indications that dwarf galaxies also have dark

halos (e.g., Aaronson 1987; Freeman 1984). If massive neutrinos are packed in such configurations they must have masses in excess of ~ 500 eV in order not to have their present phase-space density be lower than at the early universe, in violation of the Liouville theorem (Tremaine and Gunn 1979). On the other hand, if the neutrinos provide a substantial fraction, Ω_ν, of the critical cosmological density, the sum of the masses of the various neutrino species must be

$$\sum_i m_{\nu_i} \sim 100\Omega_\nu h^2 \text{ eV} . \tag{22}$$

With the observed constraint $\Omega h^2 \leq 1$ there is a conflict. If dwarf galaxies indeed have massive halos, which is yet to be confirmed, it would be therefore hard to see how they could possibly be made of neutrinos, but it is still possible (although not very elegant) that ~ 30 eV neutrinos do dominate the mass in the universe on large scales, while the dwarf halos are made of another kind of dark matter.

3. As we saw in § IV, the neutrinos accumulate in compact clusters which are deep potential wells. If the intergalactic gas is heated to the virial temperature of these wells, it would make X-ray sources which might be brighter than the sources observed by Einstein (White, Davis, and Frenk 1984). We have found that the potential wells are not excessively deep if $h \approx 1$, and recall that the gas could be prevented from falling into the cluster cores thus avoiding making excessive X-ray sources. Alternatively, the problem could be solved if the baryons contribute only $\Omega_b < 0.025$, or if the initial fluctuation spectrum is very steep ($n \sim 4$; see White, Davis, and Frenk 1984).

4. There is no evidence for any correlation between galaxies and their parent pancakes, in apparent contrast to what is naively expected in the neutrino picture where the galaxies are assumed to be the daughters of pancakes (e.g., no relative alignments; see Dekel 1985). But the formation of galaxies via cooling and fragmentation in pancakes is not well understood in detail yet (Shapiro, Struck-Marcell, and Melott 1983), which leaves the scenario at the present stage with an unsatisfactory predictive power concerning the properties of galaxies. This calls for further theoretical study but it does not provide a clear evidence against the neutrino scenario.

In conclusion, one may like or dislike the neutrino-dominated picture of the universe, but, contrary to a current popular trend, we see no fatal flaw in this picture. In the absence of an elegant "theory of everything" in cosmology, the neutrino scenario and, in particular, the formation of galaxies in pancakes still deserve a thorough consideration.

We thank Martin Rees for very stimulating discussions. P. R. S. acknowledges the partial support of NSF grant AST-8401231 and Robert A. Welch Foundation grant F-1115. P. R. S. and A. D. benefited from the hospitality of the Institute for Theoretical Physics, University of California at Santa Barbara in 1984 June where this work was begun.

APPENDIX

REALIZATION OF GAUSSIAN FLUCTUATIONS

We want to represent a random-phase realization of a given power spectrum of small density fluctuations,

$$P(k) = \langle | \delta(\boldsymbol{k}) |^2 \rangle_{|\boldsymbol{k}|=k} , \tag{A1}$$

in a range $k_{\min} < k < k_{\max}$, by appropriately distributing N particles in an arbitrary given volume V (e.g., a unit sphere), without necessarily requiring periodic boundary conditions. The particles are first distributed uniformly inside the volume, at the points of a comoving cubic grid denoted by $\boldsymbol{q}$. According to the Zel'dovich (1970) approximation, at a time t, the comoving position of each particle is displaced by

$$-a(t)b(t)\boldsymbol{\psi}(\boldsymbol{q}) , \tag{A2}$$

where $a(t)$ is the universal expansion factor and $b(t)$ is the linear growth rate; $a(t) \propto b(t) \propto t^{2/3}$ in a matter-dominated Einstein–de Sitter universe. To represent adiabatic fluctuations, each particle is assigned a corresponding peculiar velocity relative to the Hubble flow of

$$-a(t)\dot{b}(t)\boldsymbol{\psi}(\boldsymbol{q}) , \tag{A3}$$

representing only the growing modes.

The spatial perturbation $\boldsymbol{\psi}(\boldsymbol{q})$ is taken to be the superposition of N_k small-amplitude plane waves,

$$\boldsymbol{\psi}(\boldsymbol{q}) = \sum_{i=1}^{N_k} \sin(\boldsymbol{k}_i \cdot \boldsymbol{q} + \phi_i) \frac{\boldsymbol{k}_i}{k_i^2} \left[\frac{P_i}{w(k_i)}\right]^{1/2} . \tag{A4}$$

The corresponding density fluctuation is

$$\delta(\boldsymbol{q}) = 1 \Big/ \det \left[\delta_{jk} - b(t) \frac{\partial \psi_j}{\partial q_k}\right] - 1 , \tag{A5}$$

which, in the linear approximation, is simply

$$\delta(\boldsymbol{q}) = b(t) \sum_{j=1}^{3} \frac{\partial \psi_j}{\partial q_j} = b(t) \sum_{i=1}^{N_k} \cos(\boldsymbol{k}_i \cdot \boldsymbol{q} + \phi_i) \left[\frac{P_i}{w(k_i)}\right]^{1/2} . \tag{A6}$$

The amplitudes $P_i^{1/2}$ are chosen at random from a Gaussian distribution in which the variance is the power spectrum, $P(k)$. The phases ϕ_i are chosen uniformly at random in the interval $(0, 2\pi)$. The directions of the wavevectors, $\boldsymbol{k}_i$, are chosen uniformly at random. Their amplitudes, k_i, are chosen at random within $(k_{\rm min}, k_{\rm max})$ such that the number density of waves is $w(k)$. In practice, we select the k values via a function $u(k)$ which satisfies

$$w(k)d^3k = \frac{N_k}{u(k_{\rm max}) - u(k_{\rm min})}\, du(k) . \tag{A7}$$

The values of $u(k)$ are chosen uniformly at random in the interval $u(k_{\rm min}) \leq u(k) \leq u(k_{\rm max})$, and the corresponding wavenumbers k are used in the superposition.

The weight function $w(k)$ could, in principle, be arbitrary. For example, the choice $w(k)$ = constant, corresponding to $u(k) = k^3$, would give a uniform coverage of the three-dimensional k-space. This choice is equivalent to the use of a cubic grid in k-space, which is forced when periodic boundary conditions are imposed and Fourier transforms are calculated (Efstathiou *et al.* 1985). But then the representation of the spectrum for small k's is poor, and the representation of the spectrum for large k's is wasteful. A much more "uniform" representation of the spectrum over the whole range $(k_{\rm min}, k_{\rm max})$ is achieved with an equal number of waves per logarithmic interval in k, i.e.,

$$u(k) = \ln k , \tag{A8}$$

which corresponds to

$$w(k) = \frac{N_k}{4\pi(\ln k_{\rm max} - \ln k_{\rm min})}\, k^{-3} . \tag{A9}$$

We find that the distribution of δ over 8000 grid points inside a unit sphere, as calculated by equation (A5) with $b \ll 1$ and $N_k \sim 1000$ per each decade of k, indeed approximates a normal distribution very well. It is not due only to the fact that the amplitudes were chosen from a Gaussian distribution; the random phases and the large number of waves in every small ln k interval tend to generate a Gaussian distribution based on the central limit theorem.

The desired fluctuations are represented well down to a comoving wavelength corresponding to twice the initial grid separation (the Nyquist wavelength),

$$\lambda_{\rm min} = 2\pi/k_{\rm max} = 2(V/N)^{1/3} . \tag{A10}$$

The above procedure was tested by Fourier transforming $\delta(\boldsymbol{q})$ back to k-space (using standard FFT) and calculating the power spectrum from it. The result is shown in Figure 11 where the dashed line is the result of the Zel'dovich approximation, and the solid line is the theoretical spectrum. The approximation has proved to be very good.

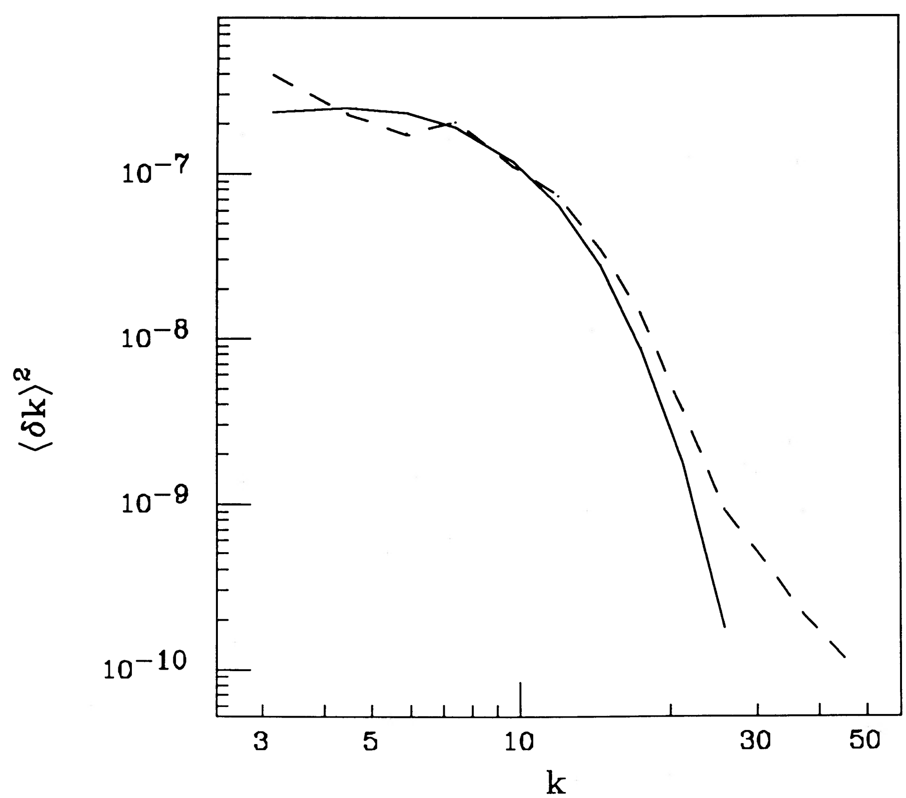

FIG. 11.—Comparison between the theoretical power spectrum of the neutrino fluctuations of eq. (1) (*solid line*) and the resultant power spectrum of one realization generated as described in the appendix (*dashed line*).

In order to normalize the spectrum at the starting time of the simulations, we use the linear approximation to write the mean square mass fluctuation averaged over spheres of radius R as

$$\langle(\delta M/M)^2\rangle_R = (\tfrac{1}{2})b^2(t)\int_{k_{\min}}^{\infty} P(k)W_R(k)d^3k\ , \tag{A11}$$

where $W_R(k)$ is the Fourier transform of the window in position space, chosen here to be a "top hat" of radius R. (The factor $\frac{1}{2}$ is due to the fact that we use sines rather than exponents in the Fourier analysis.) $P(k)$ and $b(t)$ are normalized such that on a certain scale, R_u,

$$\langle(\delta M/M)^2\rangle_{R_u} = b^2(t)\ . \tag{A12}$$

Today, for example, the observed distribution of galaxies indicate $b(t_0) = 1$ at $R_u = 8h^{-1}$ Mpc (if galaxies trace mass). One should choose b at the starting time such that the fluctuations are still linear on the relevant scales, but not much before the onset of nonlinearity for economical reasons.

REFERENCES

Aaronson, M. 1987, in *Nearly Normal Galaxies*, ed. S. M. Faber (New York: Springer), p. 57.
Aarseth, S. J. 1984, *Methods Comp. Phys.*, **1**, 1.
Abell, G. O. 1958, *Ap. J. Suppl.*, **3**, 211.
Bahcall, J. N., and Glashow, S. L. 1987, *Nature*, **326**, 476.
Bahcall, N., and Soneira, R. 1983, *Ap. J.*, **270**, 20.
Barnes, J., Dekel, A., Efstathiou, G., and Frenk, C. 1985, *Ap. J.*, **295**, 368.
Batuski, D. J., and Burns, J. O. 1985, *Ap. J.*, **299**, 5.
Bond, J. R. 1986, in *Galaxy Distances and Deviations from Universal Expansion*, ed. B. F. Madore and R. B. Tully (Dordrecht: Reidel), p. 255.
Bond, J. R., Centerella, J., Szalay, A. S., and Wilson, J. R. 1984, *M.N.R.A.S.*, **210**, 515.
Bond, J. R., Efstathiou, G., and Silk, J. 1980, *Phys. Rev. Letters*, **45**, 1980.
Bond, J. R., and Szalay, A. S. 1983, *Ap. J.*, **274**, 443.
Blumenthal, G. R., Dekel, A., and Primack, J. R. 1988, *Ap. J.*, **326**, 539.
Blumenthal, G. R., Faber, S. M., Primack, J. R., and Rees, M. J. 1984, *Nature*, **311**, 517.
Braun, E., and Dekel, A. 1987, in preparation.
Burstein, D., Davies, R., Dressler, A., Faber, S. M., Lynden-Bell, D., Terlerich, R., and Wagner, M. 1986, in *Galaxy Distances and Deviations from Universal Expansion*, ed. B. F. Madore and R. B. Tully (Dordrecht: Reidel), p. 123.
Centrella, J., and Melott, A. L. 1983, *Nature*, **305**, 196.
Couchman, H. M. P., and Rees, M. J. 1986, *M.N.R.A.S.*, **221**, 53.
Cowsik, J. S. 1987, preprint.
Davis, M., Efstathiou, G., Frenck, C. S., and White, S. D. M. 1985, *Ap. J.*, **292**, 371.
Davis, M., and Peebles, P. J. E. 1983, *Ap. J.*, **267**, 465.
Dekel, A. 1985, *Ap. J.*, **298**, 561.
Dekel, A., and Aarseth, S. J. 1984, *Ap. J.*, **283**, 1.
Dekel, A., Einasto, J., and Rees, M. J. 1987, *Rev. Mod. Phys.*, in press.
Dekel, A., and Rees, M. J. 1987, *Nature*, **326**, 455.
Dekel, A., and Silk, J. 1986, *Ap. J.*, **303**, 39.
Dekel, A., West, M., and Aarseth, S. J. 1984, *Ap. J.*, **279**, 1.
de Lapparent, V., Geller, M. J., and Huchra, J. P. 1986, *Ap. J. (Letters)*, **302**, L1.
Djorgovski, S., Spinrad, H., McCarthy, P., and Strauss, A. 1985, *Ap. J. (Letters)*, **299**, L1.
Dressler, A. 1980, *Ap. J.*, **236**, 351.
———. 1984, *Ann. Rev. Astr. Ap.*, **22**, 185.
Dressler, A., Faber, S. M., Burstein, D., Davies, R. L., Lynden-Bell, D., Terlevich, R. J., and Wegner, G. 1987, *Ap. J. (Letters)*, **313**, L37.
Efstathiou, G., Davis, M., Frenk, C. S., and White, S. D. M. 1985, *Ap. J. Suppl.*, **57**, 241.
Efstathiou, G., and Silk, J. 1983, *Fund. Cosmic Phys.*, **9**, 1.
Einasto, J., Klypin, A. A., Saar, E., and Shandwin, S. F. 1984, *M.N.R.A.S.*, **206**, 529.
Freeman, R. C. 1987, in *Nearly Normal Galaxies*, ed. S. M. Faber (New York: Springer), p. 317.
Frenk, C. S., White, S. D. M., and Davis, M. 1983, *Ap. J.*, **271**, 417.
Geller, M. J. 1984, in *Clusters and Groups of Galaxies*, ed. F. Mardirossian, G. Giuricin, and M. Mezzett (Dordrecht: Reidel), p. 353.
Giovanelli, R., Haynes, M. P., and Chincarini, G. 1986, *Ap. J.*, **300**, 77.
Gunn, J. E., and Peterson, B. A. 1965, *Ap. J.*, **142**, 1633.
Harari, H. 1987, in *Proc. 4th Jerusalem Winter School for Theoretical Physics* (Dark Matter), ed. T. Piran and J. N. Bahcall (Singapur: World Scientific), in press.
Kaiser, N. 1983, *Ap. J. (Letters)*, **273**, L17.
———. 1987, preprint.
Kaiser, N., and Silk, J. 1987, preprint.
Kirshner, R. F., Oemler, A., Schechter, P. L., and Shectman, S. A. 1981, *Ap. J. (Letters)*, **248**, L57.
Klypin, A. A., and Shandarin, S. F. 1983, *M.N.R.A.S.*, **204**, 891.
Lyubimov, V. A., Novikov, E. G., Nozik, V. Z., Tretyyakov, E. F., and Kosik, V. S. 1980, *Phys. Letters*, **94B**, 266.
Mould, J. 1986, in *Galaxy Distances and Deviations from Universal Expansion*, ed. B. F. Madore and R. B. Tully (Dordrecht: Reidel), p. 111.
Oort, J. H. 1983, *Ann. Rev. Astr. Ap.*, **21**, 373.
Ostriker, J. P., and Cowie, L. 1981, *Ap. J. (Letters)*, **243**, L127.
Peebles, P. J. E. 1980, in *The Large Scale Structure of the Universe* (Princeton: Princeton University Press).
Rees, M. J. 1985, *M.N.R.A.S.*, **213**, 75p.
Renzini, A. 1986, in *Galaxy Distances and Deviations from Universal Expansion*, ed. B. F. Madore and R. B. Tully (Dordrecht: Reidel), p. 177.
Schmidt, M., Schneider, D. P., and Gunn, J. E. 1986, *Ap. J.*, **306**, 411.
Shandarin, S. F. 1987, in *Dark Matter in the Universe*, ed. J. Kormendy and G. Knapp (Dordrecht: Reidel), p. 369.
Shandarin, S. F., Doroshkevich, A. G., and Zel'dovich, Ya. B. 1983, *Soviet Phys. Usp*, **26**, 46.
Shapiro, P. R. 1984, in *Clusters and Groups of Galaxies*, ed. F. Mardirossian, G. Giuricin, and M. Mezzett (Dordrecht: Reidel), p. 447.
Shapiro, P. R., and Struck-Marcell, C. 1985, *Ap. J. Suppl.*, **57**, 205.
Shapiro, P. R., Struck-Marcell, C., and Melott, A. 1983, *Ap. J.*, **275**, 413.
Silk, J. 1985, *Ap. J.*, **297**, 1.
Sunyaev, A., and Zel'dovich, Ya. B. 1972, *Astr. Ap.*, **20**, 189.
Tremaine, S. D., and Gunn, J. E. 1979, *Phys. Rev. Letters*, **42**, 407.
Tully, B. 1986, preprint.
White, S. D. M. 1984, in *Inner Space/Outer Space*, ed. E. W. Kolb, M. S. Turner, K. Olive, D. Seckel, and D. Lindley (Chicago: University of Chicago Press), p. 228.
White, S. D. M., Davis, M., and Frenk, C. S. 1984, *M.N.R.A.S.*, **209**, 27p.
White, S. D. M., Frenk, C. S., and Davis, M. 1983, *Ap. J. (Letters)*, **274**, L1.
White, S. D. M., Frenk, C. S., Davis, M., and Efstathiou, G. 1987, *Ap. J.*, **313**, 505.
Wyse, R. F. G. 1985, *Ap. J.*, **299**, 593.
Yang, J., Turner, M., Schramm, D., Steigman, G., and Olive, K. 1984, *Ap. J.*, **281**, 493.
Zel'dovich, Ya. B. 1970, *Astr. Ap.*, **5**, 84.
———. 1972, *M.N.R.A.S.*, **160**, 1p.

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