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ON THE DYNAMICAL COUPLING BETWEEN THE SUPERFLUID INTERIOR AND THE CRUST OF A NEUTRON STAR

M. A. ALPAR
Research Institute for Basic Sciences, Science and Technical Research Council of Turkey
AND
J. A. SAULS
Department of Physics and Astronomy, Northwestern University
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ABSTRACT

We clarify some implications of earlier work on the coupling of a neutron star's superfluid core to its crust. Here we show that the very short time, τ_v, for the relaxation of relative velocity between the plasma interior (protons and electrons) and the neutron vortices implies a time scale $\tau_d = \tau_v(\rho/\rho_c)$ for the coupling of the neutron star's *core* superfluid to the crust, where $\rho(\rho_c)$ is the total (charged) fluid density. We obtain the estimate $\tau_d = 400$–$10^4 P$ s for the dynamical coupling time, where P is the neutron star's rotation period in seconds. One consequence of such a short coupling time is that free precession of a neutron star, as is suggested for the 35 day mode of Her X-1, must be continuously excited, as it would otherwise be damped out in ~ 400–10^4 precession periods.
Subject headings: dense matter — stars: interiors — stars: neutron — stars: rotation

I. INTRODUCTION

The moment of inertia of a neutron star resides mainly in its neutron superfluid core. This rotating superfluid is coupled to the stellar crust by interactions between the charged particles (electrons and protons) in the core and the quantized vortices whose distribution determines the rotational velocity of the neutron superfluid. The electron fluid, which is not superconducting, will follow any change in the rotation rate of the crust on a time scale determined by the electron viscosity, typically of order seconds. The proton fluid will also follow the crust on roughly the same time scale; any lag between the protons and electrons generates large transient currents, and hence large fields, which rapidly bootstrap the protons into corotation with the electrons (Easson 1979). This conclusion appears to be unaffected by the superconducting nature of the protons (Alpar, Langer, and Sauls 1984). Thus, the entire charge component of the stellar interior is coupled to the crust on the short viscous time scale. However, the rotation rate of the neutron superfluid will not change unless the quantized vortex lines, which are the source of the neutron-fluid angular momentum, move radially (inward for spin-up and outward for spin-down). Vortices move with the local superfluid velocity in the absence of forces acting on them. Forces develop to oppose any relative velocity between the vortex line and the charged components, as occurs in a pulsar "glitch" event. Microscopically, these forces arise from the scattering of charged particles off the vortex cores, and can be characterized by a relaxation time for the relative velocity of the charged fluid and a vortex line,

$$\dot{\boldsymbol{v}}_{\rm rel} = -\boldsymbol{v}_{\rm rel}/\tau_v. \tag{1}$$

II. THE COUPLING MECHANISM

The dominant scattering mechanism coupling the neutron vortex lines to the charged particles is electron scattering from the inhomogeneous magnetization surrounding a vortex line. The source of this magnetization is the current of superconducting protons entrained by the superfluid neutron current circulating each vortex line. The local magnetic field induced around a neutron vortex line is typically of the order of 10^{15} G, and is confined within ~ 30 fm, resulting in a magnetic flux $\Phi_* = (hc/2e)(\delta m_p^*/m_n)$ for each neutron vortex, where δm_p^* is the difference between the proton effective mass and its bare mass. Alpar, Langer, and Sauls (1984) calculate the velocity relaxation time due to electron-magnetic-vortex scattering to be

$$\tau_v = \frac{6.2P}{(x\rho_{14})^{1/6}}\left(\frac{m_p}{\delta m_p^*}\right)^2\left(\frac{m_p^*}{m_p}\right)^{1/2}[1 - g(\beta)]^{-1} s\,, \tag{2}$$

where P is the rotation period in seconds, ρ_{14} is the density in 10^{14} g cm^{-3},

$$\beta \approx 0.54x^{1/2}\rho_{14}^{5/6}(m_p/m_p^*)^{1/2}(m_n/m_n^*)\Delta_n^{-1}\,, \tag{3}$$

m_n^* is the neutron effective mass, Δ_n is the neutron superfluid gap in MeV, and

$$g(\beta) = \sum_{m=1}^{\infty}\left\{\frac{(2m-1)(2m+2)!}{m![(m+1)!]^2(m+2)!}\left(\frac{\beta}{2}\right)^{2m} - \frac{(2m)(2m+3)!}{\Gamma(m+3/2)\Gamma(m+5/2)^2\Gamma(m+7/2)}\left(\frac{\beta}{2}\right)^{2m+1}\right\}. \tag{4}$$

Typical values for the electron-magnetic-vortex scattering time are $\tau_v = 18.4P$, $11.4P$, and $9.7P$ at $\rho_{14} = 2.3$, 4.5, and 6.0, respectively.[1] This relaxation mechanism is not only fast, but remarkably insensitive to the interior temperature and density. This latter feature is specific to the scattering of electrons from the inhomogeneous magnetization originating from the superfluid components of the stellar interior. However, τ_v does scale roughly as $(m_p/\delta m_p^*)^2$. The value of τ_v quoted above assumes $m_p^*/m_p = 0.5$ (Sjöberg 1976); smaller neutron matter corrections to the proton quasi-particle mass, $m_p^* \approx 0.9m_p$, increase

[1] The expression for $g(\beta)$ given in Alpar, Langer, and Sauls (1984) contains a sign error in the first term; however, the numerical results they give are based on the correct expression above.

these values of τ_v by a factor of 25. In either case the time scale is remarkably short.

III. THE DYNAMICAL COUPLING TIME

The relaxation of the superfluid core (i.e., the mass current of the superfluid core) to the crust and charged plasma after a sudden spinup (or spin-down) is governed by a somewhat different timescale τ_d which we refer to as the dynamical coupling time. Here we show that $\tau_d = \tau_v/x$, where $x = \rho_c/\rho$ is the charged particle fraction of the mass density in the core. The equation of motion for a vortex line is the Magnus equation,

$$\rho_s(\boldsymbol{v}_L - \boldsymbol{v}_s) \times \boldsymbol{\kappa} = \boldsymbol{f} , \tag{5}$$

where ρ_s is the superfluid density, $\boldsymbol{v}_L$ is the velocity of the vortex line, $\boldsymbol{v}_s$ is the local superfluid velocity, and $\boldsymbol{\kappa}$ is the vorticity of the line, which is directed along the vortex line and has the magnitude $\kappa = h/2m_n$, where h is Planck's constant and m_n is the neutron mass. The right side of equation (5) is the force per unit length acting on the vortex. For scattering of charged particles by the vortex line, we have for the average force on a vortex,

$$\boldsymbol{f} = \rho_c n_v^{-1} \tau_v^{-1} (\boldsymbol{v}_L - \boldsymbol{v}_c) , \tag{6}$$

where ρ_c and $\boldsymbol{v}_c$ are the mass density and velocity of the charged plasma and n_v is the number of vortex lines per unit area perpendicular to the rotation axis. We now consider motion of the vortex lines in simplified geometry: a cylinder rotating initially with angular velocity Ω_0 about the symmetry axis $\hat{z}$. The array of vortices are uniformly distributed and aligned along $\hat{z}$. Suppose the charged matter, initially rotating together with the superfluid at the rate Ω_0, acquires a rotation rate Ω so that $\boldsymbol{v}_c = \Omega r \hat{\phi}$. As a vortex line moves with velocity $\boldsymbol{v}_L = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$, the ambient superfluid velocity at the vortex position $r(t)$ becomes

$$\boldsymbol{v}_s = \frac{\Omega_0 r_0^2}{r} \hat{\phi} , \tag{7}$$

where r_0 is the initial radial position of the vortex. This equation simply implies that the total vorticity (number of vortex lines) within a circle of radius $r(t)$ does not change as vortices move radially; vortex lines do not overtake one another in their radial motion. From equations (5) and (6) we obtain the solution

$$r(t) = r_0 \left[\frac{\Omega_0}{\Omega} + 1 - e^{-t/\tau_d} \right]^{1/2} , \tag{8}$$

$$\phi(t) = \phi_0 + \Omega t - \left(\frac{\rho_s \kappa n_v}{\rho_c} \right) \ln \left(\frac{r(t)}{r_0} \right) , \tag{9}$$

where ϕ_0 is the initial azimuthal position of the vortex line. Note that each vortex relaxes to a radial position $r_\infty = r_0 (\Omega_0/\Omega)^{1/2}$, where the superfluid rotation speed v_s/r_∞ and the line's azimuthal speed $\dot{\phi}(t)$ both reach the rotation rate Ω of the charged matter. The time scale τ_d in equation (8) is

$$\tau_d = (2\Omega)^{-1} [n_v \kappa \tau_v \rho_s/\rho_c + (n_v \kappa \tau_v \rho_s/\rho_c)^{-1}] . \tag{10}$$

In the neutron star interior, $\rho_c/\rho_s \approx x$ is a few percent. The vortex density n_v is related to the superfluid rotation rate by $\kappa n_v = 2\Omega_s \approx 2\Omega$ for small changes in the rotation rate. Since τ_v is at least 10–20P, the second term in equation (10) is negligible and we obtain

$$\tau_d \approx \tau_v \left(\frac{\rho_s}{\rho_c} \right) \left(\frac{\kappa n_v}{2\Omega} \right) \approx \frac{\tau_v}{x} . \tag{11}$$

IV. DISCUSSION

The dynamical coupling times are $\tau_d \approx 400P$, $200P$, and $142P$ at densities $\rho_{14} = 2.3$, 4.5, and 6.0, respectively, and $m_p^*/m_p = 1/2$. We adopt the longest of these time scales as representative of the dynamical coupling of the entire superfluid core,

$$\tau_d \approx 100(m_p/\delta m_p^*)^2 P . \tag{12}$$

The dynamical coupling is relatively insensitive to the superfluid energy gap Δ_n, and somewhat more sensitive to the value of the proton effective mass, as indicated above. Reducing the value of the gap by 50% compared to that of Yang and Clark (1971) increases τ_d by a factor of 1.25 at $\rho_{14} = 2.3$. Thus, the neutron star's superfluid core, which contains most of the moment of inertia of the star, is rigidly coupled to the crust on time scales longer than roughly $400P$–10^4P depending upon $\delta m_p^*/m_p$; at most a few minutes. This result means that the core superfluid cannot be involved in the observed post glitch relaxations of radio pulsars, which have time scales varying from weeks to years. Indeed, all available postglitch timing data can be understood in terms of the dynamics of a pinned superfluid in the neutron star crust which contains $\sim 10^{-2}$ of the star's moment of inertia (see Pines and Alpar 1985 and references therein). Rapid coupling of the core superfluid is a crucial requirement of these models of pulsar timing anomalies.

The dynamical coupling time also has important consequences for the free precession of a neutron star. Recent observations of Her X-1 (Trümper *et al.* 1986) strongly imply that its 35 day periodicity arises from precession. However, the presence of a fluid core with dynamical coupling time τ_d leads to damping of the free precession with a damping time τ_w such that (Bondi and Gold 1955).

$$\Omega_w \tau_w = \Omega \tau_d . \tag{13}$$

Here $\Omega_w = 2\pi/P_w$ is the precession (wobble) frequency, and $P_w = 35$ days for Her X-1. Thus, equation (12) for τ_d implies a damping time τ_w for free precession that is roughly 100–10^4 precession periods, $\tau_w = 100(m_p/\delta m_p^*)^2 P_w$. This damping by the core superfluid is an upper bound to the stability of the 35 day clock. If Her X-1 were an isolated neutron star, then once excited into free precession it would damp out in $\sim$100–10^4 periods through the coupling to the core superfluid. Thus, the precession must be continuously pumped by external torques which balance the damping produced by interior torques. Several mechanisms to achieve such a pumping have been discussed by Lamb *et al.* (1975). Alpar and Ögelman (1987) have recently estimated the steady state values of the torques produced in the superfluid interior. They conclude that the required external torques do not exceed the magnitudes available in the binary system.

V. CONCLUSION

We have related the dynamical coupling time of a neutron star's superfluid core to electron scattering off the induced magnetization of neutron vortex lines. A simple expression for the dynamical coupling time is given in equation (12). Two important consequences follow: first, the postglitch relaxation of radio pulsars is not due to the superfluid core of the neutron star, and second, the precession of a neutron star is damped in $\sim 10^2$–10^4 precession periods in the absence of externally applied torques.

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M. A. ALPAR: Research Institute for Basic Sciences, Science and Technical Research Council of Turkey, TBAE, P.K. 74, Gebze, Kocaeli 41470 Turkey

J. A. SAULS: Department of Physics and Astronomy, Northwestern University, Evanston, IL 60208