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HYDROGEN EMISSIVITY IN REALISTIC NEBULAE: THE EFFECTS OF VELOCITY FIELDS AND INTERNAL DUST

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ABSTRACT

This paper presents calculations of the Hβ emissivity expected from nebulae with velocity gradients or internal dust. As has been found by Capriotti, Cox, and Mathews, Lyman line escape and destruction can prevent the 100% conversion of high–n Lyman lines into Lyα and Balmer lines. For dusty environments such as the Orion Nebula or the general interstellar medium, the Hβ emissivity can be reduced by $\lesssim 15\%$. Lyman line escape may cause still larger deviations in environments such as nova shells where the expansion velocities are large and velocity gradients likely. Although the partial conversion of Lyman lines only lowers the Hβ emissivity by typically $\lesssim 10\%$ under most circumstances, this introduces a systematic error in abundance measurements; the abundance of other elements relative to hydrogen will be overestimated by this amount. This effect must be considered in detail if very accurate abundance measurements are to be made. Our predictions of the deviation from case B emissivity are presented in a way in which they can be easily used by observers or incorporated into photoionization or shock codes.

Subject headings: interstellar: grains — nebulae: general — nebulae: H II regions — nebulae: internal motions

I. INTRODUCTION

The accurate and reliable measurement of the primordial helium abundance $Y \equiv \text{He/H}$ will offer a definitive test of big bang nucleosynthesis (Boesgaard and Steigman 1985); unfortunately, the accuracy required, much better than 5%, presents a challenge to conventional methods of determining abundances from ratios of emission lines (see the reviews by Kunth 1986; Pagel, Terlevich, and Melnick 1986; Shields 1987; and Pagel 1987). The refinement of conventional nebular theory to the precision required to test the big bang is an important goal. Although this paper does not address measurements of Y, it does discuss two effects which affect abundance determinations at this level of precision.

Hydrogen and helium lines are commonly assumed to be produced by recombination under "case B" conditions. The deduced abundance then has little dependence on the physical conditions (see Osterbrock 1974), and great precision is possible. The presence of either collisional or radiative transfer effects would complicate the situation. Davidson and Kinman (1985) discuss the possibility that collisional excitation of Balmer lines would raise the hydrogen emissivity and hence cause Y to be underestimated. Ferland (1986) discussed the possibility that collisional excitation of He I lines is important; this effect would cause Y to be overestimated. The role of both collision processes is uncertain; Pagel (1987) and Shields (1987) conclude that collisional effects amount to a few percent at most.

This paper addresses the question whether hydrogen lines are actually formed under case B conditions in real nebulae. For case B to apply, all Lyman lines, including those produced by high–n levels, must scatter often enough to be degraded into Lyα and Balmer lines. This will not occur if dust destroys the photon or if the photon escapes the cloud (see Capriotti 1966; Cox and Mathews 1969). Case B Hβ emissivities may not be valid when very accurate results are needed; Lyman line escape or absorption by dust will lower the emissivity of Hβ and cause abundances of other elements to be overestimated.

II. CALCULATIONS

We are interested in computing the Hβ emissivity for conditions appropriate to H II regions, planetary nebulae, and nova envelopes, where Lyman line leakage and destruction by dust can occur. Here we discuss our treatment of the hydrogen atom and Lyman line transport and outline our assumptions concerning the possible velocity structure and dust content of the line-forming regions. Our approach is basically similar to that followed by Capriotti (1966) and Cox and Mathews (1969); these studies were primarily interested in changes in the Balmer decrement, and they did not report emissivities. The latter is the point of the present paper.

a) *The Hydrogen Atom*

Hydrogen lines are formed under conditions which lie between two extreme limiting circumstances; in case A all Lyman photons escape, while in case B all high–n Lyman photons scatter often enough to be degraded into Lyα and Balmer lines (see Osterbrock 1974). Most analyses assume case B even though this assumption formally requires infinite optical depth in all Lyman lines. A second pair of limiting assumptions concerns the density; in the low-density limit l-mixing collisions can be neglected (see Pengelly 1964), while in the high-density limit ($N \geq 5 \times 10^8$ cm^{-3}; Drake and Ulrich 1980; Mathews, Blumenthal, and Grandi 1980) full l-mixing can be assumed for $n \geq 4$ and the formalism developed by Seaton (1959*b*) applies. The low-density limit should be valid for H II regions and planetary nebulae, while the high-density limit applies to nova shells. Although the case B emissivity of Hβ is quite close for these two assumptions concerning the density, the case A values differ by substantial amounts.

i) *The High-Density Limit*

We treat the hydrogen atom as in Seaton (1959*b*). This assumes that n-changing collisions, along with collisional ionization and three-body recombination, are negligible. The calculation also assumes that l-changing collisions are fast enough for the l-sublevels to be populated according to their statistical weights. Following Seaton, for $n > 30$, we use the closed form for the transition probabilities given by Menzel and Pekeris (1935) and corrected by Burgess (1958). For $n \leq 30$, we use the gaunt factors of Baker and Menzel (1938). Radiative recombination coefficients are computed as in Seaton (1959*a*), but with gaunt factors from Karzas and Latter (1961). We treat principal quantum numbers up to 200 exactly and add the remaining portion of the recombination coefficient to the 200th level. Tests show that the predicted Hβ emissivity is in exact (all significant figures) agreement with Seaton's (1959*b*) results for both case A and case B at temperatures of 5000 K, 10^4 K, and 2×10^4 K. Results of these calculations are shown in Figures 1 and 2 and will be discussed further below.

ii) *Low-Density Limit*

In this limit both l-changing and n-changing collisions are assumed to be slow relative to radiative processes (see Pengelly 1964). The differences between case A and case B emissivities are not as extreme as in the high-density case because only *np* levels are effective in producing decays to ground; as a result, only these levels are underpopulated by ground state transitions in case A. (When full l-mixing is assumed, all levels within principal quantum number n are affected by the inclusion of the decays from *np* to ground.)

We computed the hydrogen capture-cascade problem for the low-density limit with the following assumptions. Complete l-mixing was assumed for $n \geq 21$ and the formalism described above was used for these high-n states, with the exception that n-levels up to 320 were included. This is not formally correct at densities below $\sim 10^7$ cm^{-3} (Brocklehurst 1971), but the emissivities predicted below are in good agreement with more complete calculations. Below $n = 21$ all l-sublevels were considered. Radiative rates were taken from Capriotti (1964) or computed from Green, Rush, and Chandler (1957). Recombination coefficients were taken from Burgess (1964). Transitions from the 300 l-mixed levels above $n = 20$ to the 210 l-sublevels below were assumed to occur in the ratio of the statistical weights of the lower level, as suggested by examination of tables of transition rates. Full results are shown in Figures 3 and 4.

Hummer and Storey (1987) have recomputed hydrogen line emissivities and decrements, treating the l sublevels exactly and including a complete treatment of collisions. Our predicted Hβ emissivity for case B is $\sim 0.4\%$ lower than theirs for a density of $N_e = 10^4$ cm^{-3}. Comparison with Pengelly's (1964) results shows that our predicted Hβ emissivity was smaller than his by 2.7% for case A and 0.3% for case B. Our results were smaller than those of Martin (1988) by 1.8% and 0.5% for the two cases. These relative deviations are expected to be computed to rather more precision than the absolute emissivity since small systematic errors cancel in the difference. This precision was considered acceptable since the goal of this paper is not to recompute the absolute case B Hβ emissivity to great precision, but rather to study deviations from case B due to two effects: partial Lyman line leakage and destruction of trapped

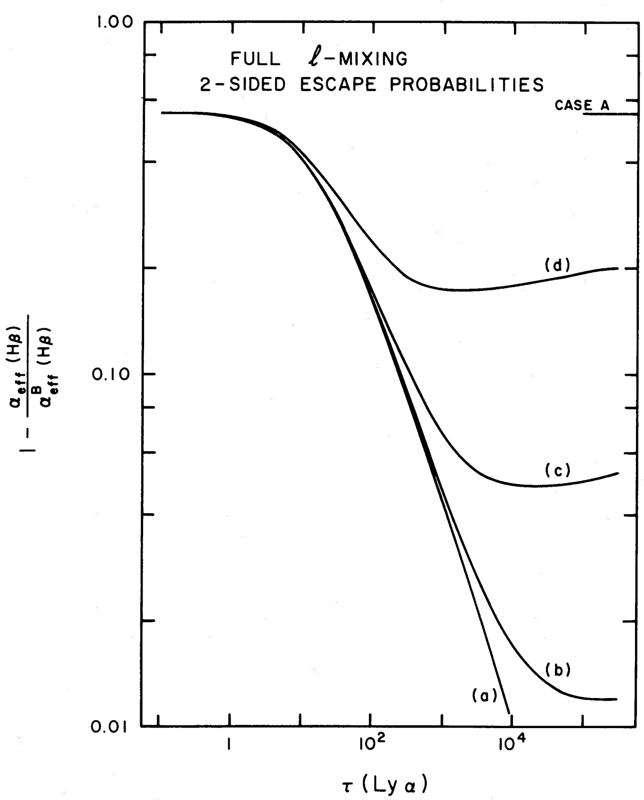

FIG. 1.—Deviations from case B in the high-density limit and two-sided escape. The plotted quantity is the deviation in the Hβ emissivity from case B predictions for a variety of Lyα optical depths. These calculations are for the case of full l-mixing and two-sided escape and should represent high-density gas. The curve marked (a) is for the case of no dust, while those marked (b), (c), and (d) are for the cases of a ratio of continuum to line opacity of X_C(Lyα) = 10^{-5}, 10^{-4}, and 10^{-3}, respectively.

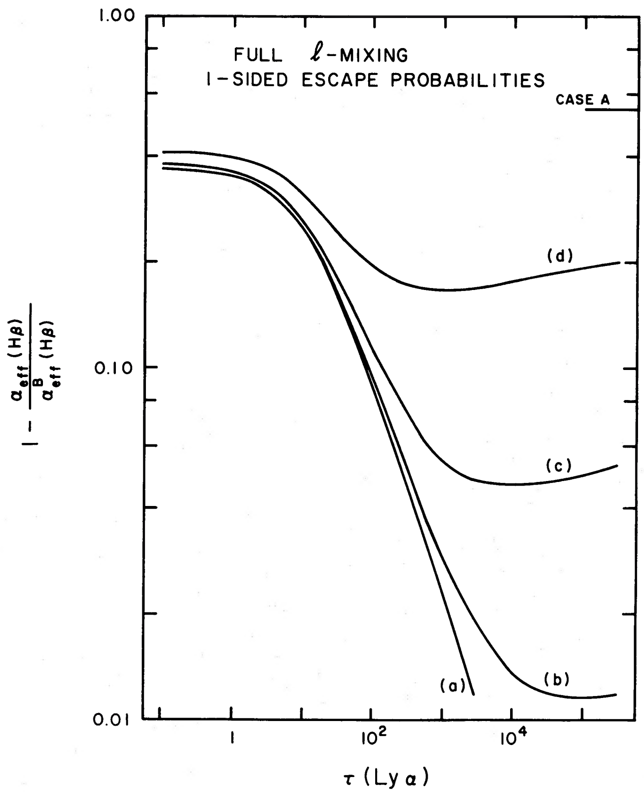

FIG. 2.—Case B deviations for high densities and one-sided escape. The calculations are similar to those in Fig. 1 except that the escape probabilities have been reduced by a factor of 2 to represent the effects of a very large optical depth in one direction.

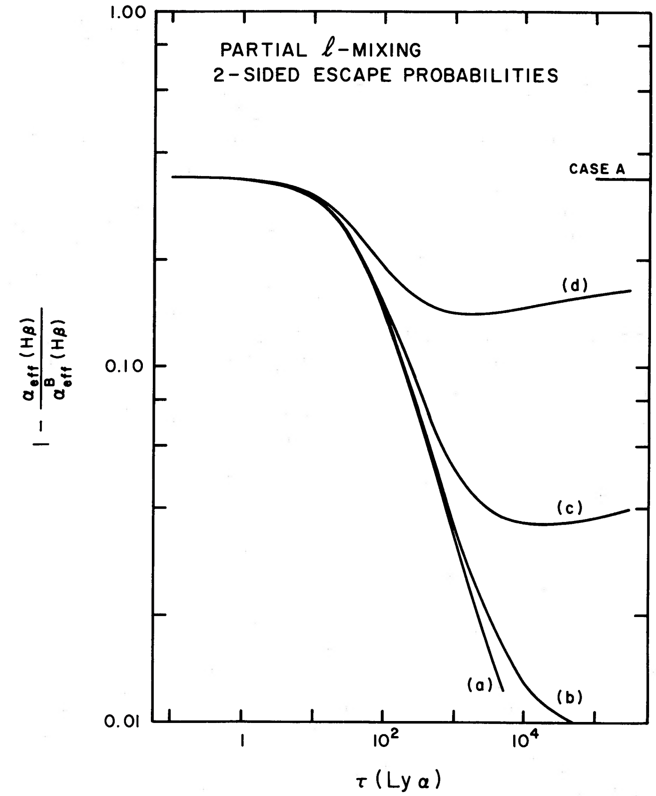

FIG. 3.—Case B deviations for very low densities and two-sided escape. The lowest 20 principal quantum numbers are treated with all angular momentum sublevels considered, but with no *l*-changing or *n*-changing collisions. Thus this calculation is appropriate for densities found in planetary nebulae or H II regions. This case is for two-sided escape.

Lyman lines by dust. The main source of uncertainty in the following calculations is probably our use of the escape probability formalism in the treatment of the transport of the ~300 Lyman lines in the model hydrogen atom.

b) *Velocity Structure and Optical Depths*

The velocity structure of the H^+ zone, where Hβ is formed, must be known if optical depth effects are to be determined. Emission-lines from giant extragalactic H II regions typically have widths which correspond to thermal broadening together with an additional turbulence of ~25 km s^{-1} (see, for example, Smith and Weedman 1970; Melnick 1977; Terlevich and Melnick 1981; Melnick *et al.* 1987). The origin of this supersonic motion is not clear, but it may be a form of microturbulence, or it may provide an indication of the depth of the potential well at the line-forming region. Galactic H II regions generally have thermal line widths, although in the well-studied case of the Orion Nebula a component of subsonic microturbulence is present (Castañeda and O'Dell 1987). Planetary nebulae have expansion velocities of order 25 km s^{-1} (see, for example, Osterbrock 1974), while in nova shells velocities of order 500–2000 km s^{-1} are encountered. In these latter two cases a velocity gradient is present. When the line width is due to random motions, the effect would be to reduce the optical depth scale through the velocity term in the Voigt function. In cases of expansion with a velocity gradient, the Sobolev or large–velocity-gradient approximation may be valid. Intermediate cases are discussed by Hummer and Rybicki (1982). All of these velocity fields diminish the optical depth scale, increase the escape probability to some extent, and make Lyman line escape more likely.

The geometry of the emission-line regions could also affect the line transfer under some circumstances. If the gas is distributed in small clumps which statistically average into a uniform structure on large scales, as is assumed in most photoionization calculations and emission-line analyses, then inhomogeneities should not affect the transfer problem. Real H II regions and planetary nebulae are clumpy even on large scales, so we are not guaranteed that a photon created inside the H^+ zone must traverse the full optical depth of the Stromgren sphere to escape. In much of the following we will take the smallest optical depth to be that of the velocity-broadened Stromgren sphere. Depending on the geometry, the large-scale clumps could allow Lyman lines to escape more readily than we assume so that the actual deviations from case B may be larger than we predict.

A second uncertainty is the role played by neutral gas outside the Stromgren sphere. The Lyα optical depth through this material is very large, but the gas is expected to be very cold and hence have a very small line width. Depending on the geometry, it seems likely that Lyman lines could interact with this material. We consider two cases: first, a case in which resonance lines escape without interacting with a large optical depth in the direction away from the central star; and second, a

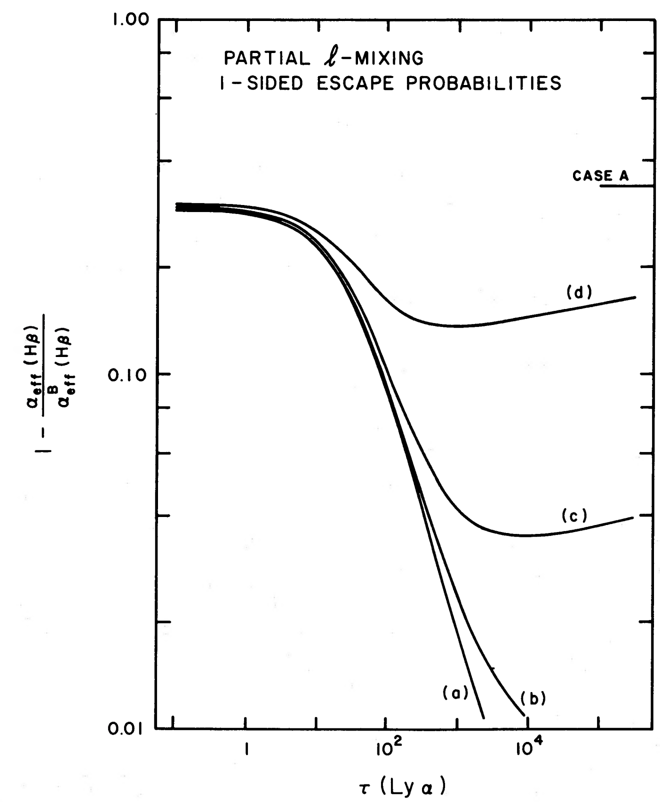

FIG. 4.—Case B deviations for very low densities and one-sided escape. The calculation is similar to that in Fig. 3 except that the escape probabilities have been reduced by a factor of 2 to represent the effects of a very large optical depth in one direction.

case in which the optical depth in the outward direction is effectively infinite.

c) *Dust-to-Gas Ratio*

Dust is known to exist within ionized zones of H II regions (see Mathis *et al.* 1981) and planetary nebulae (see, for example, the review by Barlow 1983). Dust acts to diminish the intensity of Hβ both by attenuating λ4861 as it escapes (a correction is usually applied for this effect; see for example, Mathis 1983), and by destroying Lyman line photons before they scatter often enough to be degraded into Balmer lines (Capriotti 1966; Cox and Mathews 1969). These last two papers first considered the effects we discuss here, but did not report changes in emissivity, the point of the present paper.

Although the detailed properties of dust are often not known, it is known that the scattering cross section per hydrogen atom in galactic H II regions is generally in the neighborhood of

$$\sigma_{\rm scat}(V)/{\rm H_{tot}} = 5 \times 10^{-22}\ {\rm cm}^5 \qquad (2.1)$$

for visible light (O'Dell, Hubbard, and Peimbert 1966; Perinotto and Patriarchi 1980; Mathis *et al.* 1981). In this equation H_{tot} is the total hydrogen density. There is nearly a half-order of magnitude range in this value among galactic H II regions (O'Dell, Hubbard, and Peimbert 1966). If the albedo is $a = 0.5$ (a typical value, see Savage and Mathis 1979), then this is also the absorption cross section per atom. Mathis (1986) argues that typical H II regions have dust properties similar to those of the general interstellar medium (ISM); studies of correlations between ISM gas column densities and interstellar reddening (Savage and Mathis 1979) find absorption cross sections of this magnitude in the general interstellar medium. This value of the absorption cross section is for visible wavelengths; in the general interstellar medium the total cross section at $\sim$1000 Å is known to be roughly 5 times larger (Savage and Mathis 1979). Assuming that the wavelength dependence of dust in nebulae is the same as in the ISM, using the reddening curve given by Savage and Mathis, an assumed albedo of $\sim$0.5, the absorption cross section per H atom given by equation (2.1), and extrapolating the Savage and Mathis extinction curve to 950 Å (the wavelength of typical Lyman lines), we find

$$\sigma_{\rm abs}(950\ {\rm Å})/{\rm H_{tot}} \approx 3 \times 10^{-21}\ {\rm cm}^5\ . \qquad (2.2)$$

In the following discussion we take this as the "standard" absorption cross section per H atom.

Dust in the Orion Nebula is known to be peculiar, perhaps because of the very large electron density. Although the scattering cross section per atom is fairly typical of the general interstellar medium in the visible (Perinotto and Patriarchi 1980), the absorption cross section appears to have a much flatter wavelength dependence than does the general ISM; Bohlin and Savage (1981) measured extinction of the stars near the center of the Orion Nebula, and they find $E(\lambda - V)/E(B-V) = 1.88$ at 1250 Å. We assume that the extinction continues to rise with the same slope as that between 1390 Å and 1250 Å to $\sim$950 Å, so that $E(950\ {\rm Å} - V)/E(B-V) \approx 3$. Then the ratio of cross sections between λ950 and the visible is $\sim$2, and the absorption cross section per atom for the ionized gas within the Orion Nebula is given by

$$\sigma_{\rm abs}(950\ {\rm Å})/{\rm H_{tot}} \approx 1.0 \times 10^{-21}\ {\rm cm}^5\ . \qquad (2.3)$$

In the following discussion we will consider various values of the absorption cross section given by

$$\sigma_{\rm abs}(950\ {\rm Å})/{\rm H_{tot}} = f\,3.0 \times 10^{-21}\ {\rm cm}^5\ . \qquad (2.4)$$

and defined in terms of the variable f, which equals 1 for the general ISM, $\frac{1}{3}$ for the Orion Nebulae, and 3 for high values of the ISM.

For planetary nebulae it is possible to estimate the effects of dust on certain ultraviolet emission lines (see, for example, Barlow 1983). Harrington *et al.* (1982) find an absorption cross section per atom of $\sigma_{\rm abs}(1549\ {\rm Å})/{\rm H_{tot}} = 2 \times 10^{-22}\ {\rm cm}^2$ in the planetary nebula NGC 7662, in agreement with Bohlin, Marionni, and Stecher's (1975) results for NGC 7027. The wavelength dependence of the absorption curve is unknown for dust in planetary nebulae, but this result seems roughly consistent with that for Orion quoted above ($f = \frac{1}{3}$).

d) *Radiative Transfer*

Lyman line transfer is treated with the escape probability formalism (Capriotti 1965). We use the form of the escape probability function

$$\beta(\tau) = \left\{1 + \tau\left[0.6 + \frac{1.4 + 1.0(a\tau)^{1/2}}{1 + 0.3a\tau}\right]\right\}^{-1} \qquad (2.5)$$

given by Slater, Salpeter, and Wasserman (1982). A temperature of 10^4 K was assumed in our evaluation of the damping constant a for each hydrogen line. In the following discussion the optical depth scale will be parameterized by the optical depth in Lyα. The expression given above was used for the case where Lyman photons do not interact with neutral material outside the line-forming region; for the case of one-sided escape the line was assumed not to escape in the outer direction, and half this value was used for the total escape probability. In the following discussion we will neglect the effects of dust scattering on the line transport; this is justifiable if the grains are forward scattering and the scattering process coherent.

Overlapping of lines originating in high-n levels is a possible concern. Levels above $n \approx 30$ overlap in frequency for typical velocity fields. In this case the very high-n Lyman lines form a pseudo continuum with an opacity roughly equal to that of the highest unblended line. In the following calculations we evaluated the opacity by averaging over the overlapping lines assuming a Doppler profile. For the optical depths of interest here ($\sim 10^4$), level 30 is optically thin and line overlapping is of secondary importance.

For situations where gas is mixed with another absorbing medium, such as dust, the interplay between escape and destruction of the resonance line photon is parameterized by the ratio of continuous to line opacities $X_C \equiv \kappa_c/(\kappa_l + \kappa_c)$ (Hummer and Kunasz 1980; Netzer, Elitzur, and Ferland 1985, hereafter NEF). Here we define κ_l as the line center opacity, *per unit Doppler width*, rather than averaged over the full line as in NEF. NEF show that the form of the escape probability in this situation is given by

$$\beta_{\rm eff}(\tau) = bX_C + \beta(\tau)\ , \qquad (2.6)$$

where $\beta(\tau)$ is given by equation (2.5). Hummer and Kunasz's numerical results suggest that $b \approx 5.6$ for line optical depths $\tau \approx 10^4$ and $X_C \approx 10^{-4}$. The coefficient b is a very weak function of optical depth according to Hummer and Kunasz's results (in the formalism of NEF this function is incorporated into their definition of κ_l). We treated this as a power-law in the optical depth.

We can express X_C as a function of the hydrogen neutral fraction if the gas and dust are well mixed (which may not be the case for the innermost regions of Orion; see Mathis *et al.* 1981). Using the absorption cross section for dust given by equation (2.4), we find for Lyα

$$X_C(\mathrm{Ly}\alpha) = 5.1 \times 10^{-8} f\left(\frac{\mathrm{H_{tot}}}{\mathrm{H^0}}\right)\left[t_4 + \left(\frac{V_{\mathrm{turb}}}{12.8}\right)^2\right]^{1/2}, \quad (2.7)$$

where the turbulent velocity V_{turb} is given in km s^{-1} and t_4 is the temperature in units of 10^4 K. In the remainder of this discussion our results will be presented in terms of X_C(Lyα), although the ratio of cross sections was evaluated individually for each Lyman line assuming a flat dust opacity.

e) Results

Results of these calculations are shown in Figures 1–4. All of the illustrated calculations are for a temperature of 10^4 K, because the calculated deviations have only a very slight temperature dependence. Figures 1 and 2 are for the high-density case where full l-mixing occurs. Figures 3 and 4 are for the low-density limit. The optical depth in Lyα is given on the lower axis, and the four curves in each figure are for $X_C(\mathrm{Ly}\alpha) = 0., 10^{-5}, 10^{-4}$, and 10^{-3}. The quantity plotted is the deviation from case B emissivity for Hβ. The emissivity of Hβ is diminished because some fraction of the Lyman lines are not converted into Lyα and Balmer lines for all cases except those which are dust-free and τ(Lyα) large. Optical depths greater than $\tau(\mathrm{Ly}\alpha) \sim 10^4$ are needed to approach case B to better than 1% in the dust-free cases. The presence of dust causes the emissivity to diminish for all optical depths.

Photoionization calculations which examine the importance of this effect are presented below. The case of greatest interest is one in which expansion velocities are small and τ(Lyα) large; some H II regions and planetary nebulae are examples of objects in this category. We have fitted our predictions of the deviation from case B for $\tau(\mathrm{Ly}\alpha) = 10^5$ and $X_C(\mathrm{Ly}\alpha) \lesssim 10^{-3}$ as a simple power law for the low-density limit:

$$\frac{4\pi j(\mathrm{H}\beta)}{4\pi j(\mathrm{H}\beta)_{\mathrm{case\ B}}} = 1. - 11.008[X_C(\mathrm{Ly}\alpha)]^{0.6135}. \quad (2.8)$$

For the high-density limit (where full l-mixing is assumed and the effects are larger) the results are well approximated by

$$\frac{4\pi j(\mathrm{H}\beta)}{4\pi j(\mathrm{H}\beta)_{\mathrm{case\ B}}} = 1. - 13.403[X_C(\mathrm{Ly}\alpha)]^{0.6135}. \quad (2.9)$$

As a check on the temperature dependence of these effects, we also computed the case B deviations for the case of an optical depth of $\tau(\mathrm{Ly}\alpha) = 10^5$ and $T_e = 2 \times 10^4$ K. This higher temperature is more characteristic of very metal deficient nebulae. The results are very well fitted by

$$\frac{4\pi j(\mathrm{H}\beta)}{4\pi j(\mathrm{H}\beta)_{\mathrm{case\ B}}} = 1. - 15.447[X_C(\mathrm{Ly}\alpha)]^{0.6530}. \quad (2.10)$$

A similar equation can fit our results for Hγ, in the low-density limit and at $T = 10^4$ K, and are unplotted but similar to Hβ;

$$\frac{4\pi j(\mathrm{H}\gamma)}{4\pi j(\mathrm{H}\gamma)_{\mathrm{case\ B}}} = 1. - 15.293[X_C(\mathrm{Ly}\alpha)]^{0.6255}. \quad (2.11)$$

All of these fits are virtually exact. Similar approximations for He II, which can also have continuous absorption due to hydrogen photoelectric opacity, are given in Appendix B.

III. APPLICATIONS

a) The Orion Nebula

The Orion Nebula is a benchmark in any study of H II regions (see Osterbrock and Flather 1959; Peimbert and Torres-Peimbert 1977; Torres-Peimbert, Peimbert, and Daltabuit 1980); for instance, it helps define the dY/dZ correlation sometimes used to determine the primordial helium abundance (see Peimbert 1986). Here we examine the conditions under which hydrogen lines are formed in this object.

Osterbrock and Flather's (1959) work established the presence of a large density gradient across the nebula (ranging from $N_e \gtrsim 10^4$ cm^{-3} to $\lesssim 300$ cm^{-3}). Peimbert and Torres-Peimbert (1977) and Torres-Peimbert, Peimbert, and Daltabuit (1980) further established an ionization gradient. Apparently, both density and ionization decrease with increasing projected distance from the central stars. Peimbert and Torres-Peimbert (1977) also note that the $\mathrm{He^+/H^+}$ ratio appeared to decrease by ∼30% across the nebula. Collisional excitation and optical depth effects may be responsible for part of this, and the correction for the possible presence of $\mathrm{He^0}$ is probably also important.

We estimate how close hydrogen emissivities will be to case B for intermediate regions of the nebula. Dust is likely to be mixed with gas in regions outside the dust cavity discussed by Schiffer and Mathis (1974), Mathis *et al.* (1981), and Patriarchi and Perinotto (1985). Taking ∼2′ for the inner edge of the dusty zone, we can estimate the deviations from case B once the hydrogen neutral fraction is known. The following estimate of $\mathrm{H^+/H^0}$ is approximate, but further sophistication seems inappropriate in light of other uncertainties due to the velocity, density, and ionization structure, and the dust-to-gas ratio in this object.

Bohlin and Savage (1981) quote a spectral class of O6 for θ^1 Ori C, and Osterbrock (1974) gives $T_{\mathrm{eff}} = 4 \times 10^4$ K for the effective temperature and $Q(\mathrm{H}) = 1.7 \times 10^{49}$ s^{-1} for the number of ionizing photons emitted per second for a star of this class. These parameters are consistent with the models presented by Simpson *et al.* (1986). Neglecting photoelectric and dust absorption, the photon flux at a distance of $2' = 9 \times 10^{17}$ cm from the central star is $\phi \approx 1.7 \times 10^{12}$ cm^{-2} s^{-1}. If the electron density is $N_e \approx 2 \times 10^3$ cm^{-3} (Peimbert and Torres-Peimbert 1977) and the electron temperature is $t_4 = 0.85$ (Mathis 1985), then the hydrogen ionization balance can be written as

$$\frac{\mathrm{H^+}}{\mathrm{H^0}} \approx \frac{\langle\sigma\rangle\phi}{N_e\alpha} \approx 7800\left(\frac{r}{2'}\right)^{-2}\left(\frac{N_e}{2000\ \mathrm{cm^{-3}}}\right)^{-1}, \quad (3.1)$$

where $\langle\sigma\rangle \approx 2.5 \times 10^{-18}$ cm^{-2} is the hydrogen photoionization cross section at the mean energy of 1.37 ryd, appropriate for a blackbody with $T = 4 \times 10^4$ K, and α is the case B hydrogen recombination coefficient. Assuming $t_4 = 0.85$, a component of turbulence of ∼8 km s^{-1} (Castañeda and O'Dell 1987), that $\mathrm{H_{tot}} = \mathrm{H^+}$, and equation (2.7) with $f = \frac{1}{3}$, we find

$$X_C(\mathrm{Ly}\alpha) \approx 1.5 \times 10^{-4}\left(\frac{r}{2'}\right)^{-2}\left(\frac{N_e}{2000\ \mathrm{cm^{-3}}}\right)^{-1}. \quad (3.2)$$

The small velocity gradients in the Orion Nebula (Balick, Gull, and Smith 1980) make it unlikely that Lyman lines see an optical depth any smaller than that for the full Stromgren sphere, which we take to be $\tau(\mathrm{Ly}\alpha) \approx 10^5$. Our calculations (Fig. 4) suggest that the Hβ emissivity should be $\sim$4.9% below case B for this region. Observations suggest that both the level of ionization and the dust-to-gas ratio [both of which determine $X_C(\mathrm{Ly}\alpha)$] vary across the face of the nebula. It seems plausible that changes in hydrogen emissivity, caused by changes in $X_C(\mathrm{Ly}\alpha)$, could contribute to the variations in the $\mathrm{He}^+/\mathrm{H}^+$ ratio found by Peimbert and Torres-Peimbert (1977). This issue is discussed further in Appendix A.

This result may apply to other H II regions with dust characteristics and ionization similar to the Orion Nebula. Dust in Orion is peculiar, however, and if other nebulae have the same level of ionization but an average or high dust opacity ($f \sim$ 1–3), then $X_C(\mathrm{Ly}\alpha) \lesssim 10^{-3}$ and the Hβ emissivity will be reduced by as much as $\sim$15.8% below that of case B. Errors as large as this would have significant effects on such correlations as the dY/dZ relationship for H II regions since the range of dY is only of this order (Peimbert 1986; Pagel 1987).

b) *Giant* H II *Regions*

A successful model of the giant H II region NGC 4861 ("the Hβ Nebula") was produced by Dinerstein and Shields (1986). Their single-component model was recomputed using the photoionization code most recently described by Ferland and Osterbrock (1987) in order to determine the physical conditions throughout the Hβ forming region. The parameters given by Dinerstein and Shields [$n = 100\ \mathrm{cm}^{-3}$, a filling factor of 0.017, $Q(\mathrm{H}^0) = 6 \times 10^{52}\ \mathrm{s}^{-1}$], their abundances, spherical symmetry, and the $T_{\mathrm{eff}} = 45{,}000$ K stellar atmosphere from Shields and Searle (1978) were assumed. Our predictions for the ionization and thermal structure were in good agreement with those of Dinerstein and Shields. Our calculation predicts that Hβ is enhanced by collisions by 1.0%, Hα by 2.9%, and He I λ5876 by 1.9% (rate coefficients for collisional excitation are from Aggarwal [1983] and Berrington and Kingston [1987]).

The effects of internal dust, which is present in some of these objects (Melnick 1979), could change the hydrogen emissivity. The effects of dust on the ionization structure were included in the photoionization calculation as in Martin and Ferland (1980). The frequency dependence of the absorption cross section was taken to be that appropriate for graphite dust with a radius of 0.05 μm. This is simplistic, but gives results not too different from Mathis (1986). We predicted the case B emissivity by numerically integrating equation (2.8) over the model ionization structure. This assumes a Lyα optical depth of 10^5, so that escape of high-n Lyman lines is negligible, and deviations from case B are only caused by dust absorption. We assumed cases of thermal broadening alone, as well as thermal plus a turbulence of 25 km s^{-1} (Smith and Weedman 1970; Melnick 1979; Terlevich and Melnick 1981; Melnick *et al.* 1987); the emissivity would be reduced much further if velocity gradients as large as 400 km s^{-1} were found across the line-forming region, as suggested by Clayton and Meaburn (1987). Conditions in the model ranged from $X_C(\mathrm{Ly}\alpha) \approx 0$ near the ionization front to $\sim 3 \times 10^{-3}$ at the edge nearest the ionizing star, for the Orion dust opacity. The dust absorption per H atom was varied, by varying the factor f defined in equation (2.4), and the results are shown in Figure 5. Both assumptions about the velocity structure of the nebula are shown.

Dust is probably not as abundant in extremely metal-deficient objects as it is in the ISM or Orion; these results show that measurements of the primordial helium abundance (i.e., Pagel 1987; Shields 1987) are probably not affected. Actually, for small values of f the slightly reduced emissivity is almost exactly compensated for by collisional excitation of Hβ, and case B is an excellent assumption. However, significant effects occur when $f \gtrsim 0.5$. If the dust absorption per H atom is correlated with the metallicity, as seems likely, then Figure 5 suggests that the slope of the dY/dZ relation (Pagel, Terlevich, and Melnick 1986; Peimbert 1986) could be affected by the reduction of the Hβ emissivity in metal-rich objects. Figure 5 cannot be compared directly with observations of dY/dZ, of

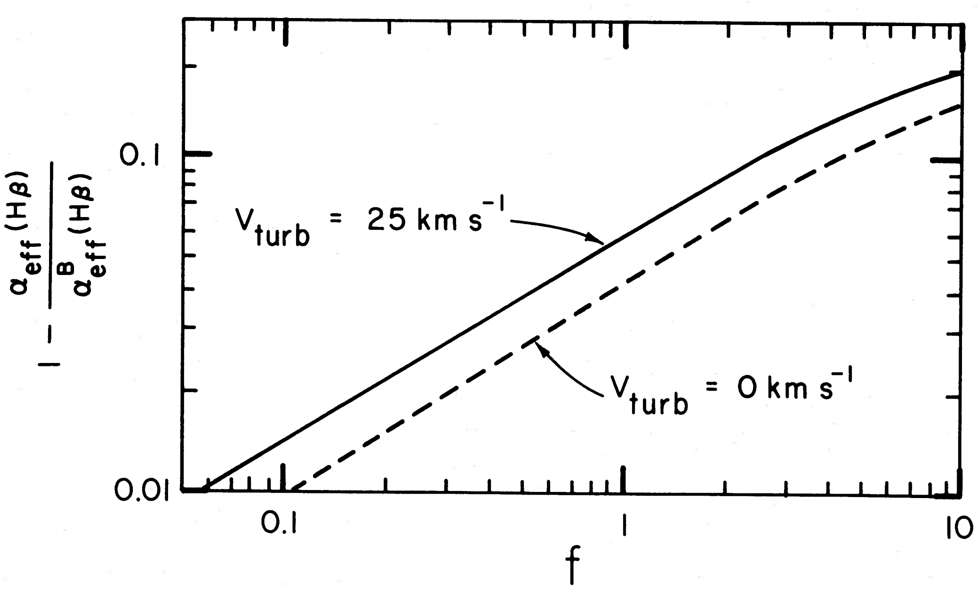

FIG. 5.—Effects of internal dust on model giant H II regions. The Dinerstein and Shields model of NGC 4861 was recomputed with various values of the dust absorption per H atom assumed. Dust opacity was included in calculations of both the ionization structure and the Hβ emissivity. The ratio of the assumed absorption cross section to that for the general interstellar medium is given as the factor f. The deviations from case B are shown for two cases; line broadening by thermal motions alone, and line broadening by both thermal and turbulent ($V_{\mathrm{turb}} = 25\ \mathrm{km\ s}^{-1}$) motions.

course, since the temperature of the ionizing star was not also varied with metallicity (see, for instance, Shields and Searle 1978).

c) *Planetary Nebulae*

Evidence now suggests that dust is mixed with ionized gas in at least a few well-studied planetary nebulae (see, for example, Harrington *et al.* 1982 and Barlow 1983). A formalism very similar to that adopted in this paper was used to estimate the fractional destruction of certain ultraviolet emission lines (Barlow 1983). The ultraviolet gas-to-dust ratios quoted in Barlow's review correspond to dust-to-gas ratios roughly similar to that of Orion, and Seaton (1979) finds fairly normal extinction properties in NGC 7027. We assume the dust properties outlined above, even though dust in planetary nebulae is likely to be quite different from dust in H II regions.

We have recomputed the simple model of NGC 7662 presented by Harrington *et al.* (1982, their model 1). This was chosen over the more realistic model 2 because of its simplicity. We assume a microturbulence of 5 km s^{-1} and calculate the deviation from case B expected for various assumptions concerning the grain properties and velocity field by numerically integrating the predictions shown in Figures 3 and 4. This nebula is matter-bounded, so Lyman lines can escape in the outward direction. If the expansion velocity is large then photons emitted toward the star will not interact with gas on the far side; for smaller velocities Lyman photons can only escape in the outer direction. We find Hβ emissivities, for the situation where photons freely escape after being emitted in the direction toward the central star, which are 93.9% and 97.5% of case B for "high–ISM" ($f = 3$) and Orion grain opacities ($f = \frac{1}{3}$), respectively. For circumstances where Lyα can only escape in the outer direction (in the direction pointing away from the star) the values are increased to 94.2% and 98.1% for the two cross sections. If, instead of neglecting the expansion velocity ($\sim$25 km s^{-1}) of the nebula entirely, we assume that its effect is to broaden the lines and reduce the opacity in a manner similar to turbulence (that is, through the broadening term in the Voigt function) then the emissivities are decreased to 91.5% and 96.7% of case B for the two assumptions concerning grain opacities, and considering only outward escape of Lyα. Emissivities are reduced to 90.8% and 96.0% when two-sided escape is allowed. The actual effects of the gross velocity field, which lies between the range of validity of the large–velocity-gradient assumption and the assumption of strictly thermal broadening (Hummer and Rybicki 1982), is also probably between these two cases.

d) *Nova Envelopes*

Envelopes of classical novae expand with velocities $V_{exp} \sim 10^3$ km s^{-1}, and have densities high enough for complete l-mixing to occur. Grain formation occurs in some objects (see, for example, Gallagher and Starrfield 1978), but here we focus on the effects of the velocity gradient observed in V1500 Cygni (Ferland 1980). Assuming that velocity and radius are proportional, the line optical depth seen by a typical photon in the H^+ zone is given by

$$\tau = \sigma H^0\, dr = \sigma H^0 r \frac{V_{th}}{V_{exp}}, \qquad (3.3)$$

where σ is the cross section for Lyα absorption, V_{th} and V_{exp} are the thermal and expansion velocities, and the calculations for two-sided escape are used (Fig. 1). Ferland and Shields (1978) present a model of the envelope of V1500 Cygni; assuming their parameters for day 121 [$r_{inner} = 1.0 \times 10^{15}$ cm, $Q(H) = 9.5 \times 10^{47}$ s^{-1}, $t_4 = 0.87$] we find $N^0 \sim 3 \times 10^2$ cm^{-3} and $\tau(Ly\alpha) \approx 250$. The calculations for the case of full l-mixing show that the Hβ emissivity is reduced by $\sim$15%. The number is extremely uncertain because the velocity and ionization structure and even the ionization mechanism of nova shells (see Ferland, Lambert, and Woodman 1986) are uncertain.

IV. DISCUSSION

We have shown that the presence of dust or velocity gradients can cause the emissivity of Hβ to fall below case B predictions by $\lesssim$15%. This consideration is important if abundances are to be measured to an accuracy much better than 5%. Such accuracy is possible (and needed) in the measurement of helium abundances in metal-deficient H II regions to test current models of nucleosynthesis in the early universe. Dust destruction of Lyman lines can also introduce spurious correlations between abundances since the Hβ emissivity is affected by the dust to gas ratio, which may scale with the metallicity, as well as the mean H^+/H^0 ratio of the gas, which may be correlated with the temperature of the ionizing star. Changes in the emissivity of Hβ could introduce apparent changes in abundances of other elements at the 2%–15% level. These effects may influence the slope of the dY/dZ correlation, for instance, or cause scatter in Y when the helium abundance is actually constant. Extremely metal-deficient objects, such as those used to measure the primordial helium abundance, should have hydrogen emissivities close to case B if they are also dust-free and the line widths are not great.

Although the simple models presented above suggest deviations from case B emissivity by as much as 15%, actual nebulae are vastly more complicated than our theoretical constructs, and the line-formation process can be affected by flows, inhomogeneities, and patchy covering. Lyman line escape may be easier or more difficult than we assume. It is possible to determine how close the hydrogen spectrum of actual objects is to case B through simple infrared measurements, however. Hydrogen lines from different series overlap in this wavelength range; for instance the 14–6 and 5–4 transitions occur at wavelengths of 4.02 and 4.05 μm, respectively. Lines originating in different series are affected differently by departures from case B, sometimes by percentages comparable to changes in the Hβ emissivity. For instance, the relative intensities of the two lines mentioned here differ by 23% in the case A and case B limits. Precise measurements of such hydrogen decrements should offer clues to details of the line formation process and the validity of the case B assumption.

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APPENDIX A

THE ORION NEBULA

In this appendix we examine some consequences of collisions and radiative transfer on the helium abundance of the Orion Nebula. We use the data presented by Peimbert and Torres-Peimbert (1977; PTP). They found that the abundance of singly ionized helium appeared to decrease with increasing distance from the central stars and deduced an empirical ionization correction factor, in part based on the requirement that the total helium abundance not vary with position. In this appendix we show that part of the change in He^+/H^+ may have been caused by collisional excitation of helium lines and the suppression of hydrogen lines, as discussed in this paper.

Figure 6 shows the abundance of singly ionized helium in Orion. The top panel shows the apparent He^+/H^+ ratio as a function of electron density as taken from PTP. The reddening-corrected $I(\lambda 5876)/I(H\beta)$ ratio was converted into an abundance of ionized helium using the tables in Osterbrock (1974) and assuming a temperature of 8500 K; these values are plotted as plus signs. The apparent ionic abundances were then corrected for collisional effects assuming the density given by PTP but a constant electron temperature of 8500 K. This temperature is suggested by radio data (see Mathis 1985) and is consistent with optical observations (McCall 1979). The rate coefficient for excitation of λ5876 given by Berrington and Kingston (1987) was used. (The collision strength for the 2^3S–3^3D collision is very close to the value proposed by Feldman and MacAlpine [1978] and is ~37% smaller than the previous value given in Berrington *et al.* [1985].) The correction factor is given by

$$\frac{I(\lambda 5876)_{\rm coll}}{I(\lambda 5876)_{\rm rec}} \geq \frac{7.7 t_4^{0.38} e^{-3.776/t_4}}{(1.0 + 3229 t_4^{-0.21}/N_e)}$$

(see also Peimbert and Torres-Peimbert 1987). The inequality holds because we neglect the effects of those collisions to $n = 4$, which

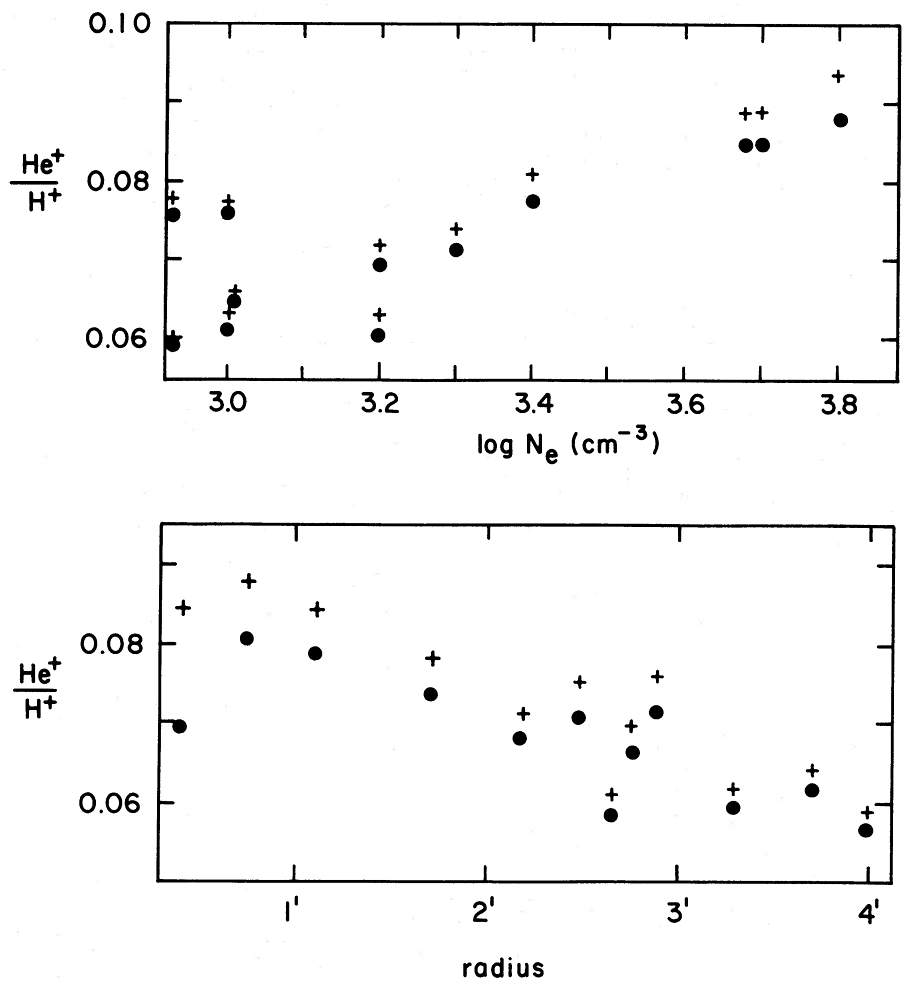

FIG. 6.—Ionized helium in the Orion Nebula. The upper panel shows the apparent helium abundance, as measured by the reddening-corrected intensity ratio λ5876/Hβ vs. the electron density. The deduced density and the intensity data were taken from Peimbert and Torres-Peimbert (1977). The data plotted as plus signs are the original data, while the filled circles have been corrected for collisional excitation using rate coefficients from Berrington *et al.* (1985). In the lower panel the collision-corrected ionic abundances are plotted as a function of the distance from the central stars, as plus signs. The filled circles show the ratio $N(He^+)/N(H^+)$. Apparent abundances were corrected assuming no Lyman line escape and the values of $X_C(Ly\alpha)$ given by equation (3.2).

then decay to produce λ5876; reliable quantal calculations do not yet exist for these (Berrington and Kingston 1987). This correction for collisions is important because of the very high density in the Orion Nebula. These collision-corrected ionic abundances are the lower of the values given for each density in the top panel and are plotted as filled circles.

We next correct the ionic helium abundance for the diminished hydrogen emissivity using the values of $X_C(\mathrm{Ly}\alpha)$ given by equation (3.2). The lower panel plots the collision-corrected data versus distance for the central stars as plus signs. These data were corrected for the effects of deviations from case B assuming $\tau(\mathrm{Ly}\alpha) = 10^5$ and the deviations given by equations (2.7) and (2.8). The Hβ-emissivity corrected data were plotted as filled circles.

This discussion is only offered as an example of some of the systematic errors which can affect spatially resolved observations. The emissivity corrections applied here may be too large; the dust-to-gas ratio may decrease in inner regions of the nebula (see Mathis *et al.* 1981), and absorption of radiation by dust and gas will increase the hydrogen neutral fraction in outer regions. These considerations introduce uncertainties in $X_C(\mathrm{Ly}\alpha)$ at the factor of 2–3 level, both in the sense of lowering it, but this additional uncertainty is no greater than that already present in the derivation of equation (3.2). A correction for neutral helium must be made to obtain the total helium abundance. We have nothing new to add to the extensive discussion on this subject (see PTP; Stasinska 1980; Mathis 1982; Mathis 1985).

A reliable helium abundance for the Orion Nebula must await measurements of density indicators which coexist with He^+; some examples are [Ne IV], [Ar IV], or the [O III] fine-structure lines (the latter are discussed by Simpson *et al.* 1986). Secondly, the hydrogen neutral fraction, which determines $X_C(\mathrm{Ly}\alpha)$, must be determined by detailed modeling of the ionization structure of the nebula, including the effects of dust absorption and scattering. Lastly, the question of the dust-to-gas ratio across the face of the nebula must be resolved.

APPENDIX B

THE He II LINES

He II resonance lines can suffer the same fate as the H I Lyman lines described in this paper. However, in addition to destruction by dust, line photons can be destroyed by photoionization of neutral hydrogen as well as by other species. This last effect occurs to some extent in all environments and can be shown to be especially important in the He II–forming region of nebulae with relatively low ionization parameters.

There are simple scaling laws between the H I and He II recombination problems (see, for example, Osterbrock 1974, p. 35). Our results for H I and 10^4 K can then be used to predict He II emissivities at a temperature of 4×10^4 K. This temperature is rather high by nebular standards, but the results presented here should still be of some use since the deviation from case B has only a weak temperature dependence.

The strongest He II lines commonly observed are "Paschen α" (the 4–3 transition) λ4686 and "Balmer α" (the 3–2 transition) λ1640. Tests show that our low-density limit case B predictions for these lines are within 1.5% and 0.3% of Hummer and Storey's (1987) results interpolated for this temperature. Our prediction for the deviation from case B can be fitted to ~1% accuracy for $X_C(\mathrm{He\ II\ Ly}\alpha) \leq 10^{-2}$ by

$$\frac{4\pi j(\lambda 4686)}{4\pi j(\lambda 4686)_{\mathrm{case\ B}}} = 1. - 0.374[X_C(\mathrm{He\ II\ Ly}\alpha)]^{0.230} \tag{B1}$$

and

$$\frac{4\pi j(\lambda 1640)}{4\pi j(\lambda 1640)_{\mathrm{case\ B}}} = 1. - 2.905[X_C(\mathrm{He\ II\ Ly}\alpha)]^{0.4567} . \tag{B2}$$

Notice that the emissivity of λ1640 is more sensitive to X_C (He II Lyα) than λ4686. As a result, the intensity ratio $I(\lambda 1640)/I(\lambda 4686)$, an important reddening indicator, can be changed by as much as 10% for some values of $X_C(\mathrm{He\ II\ Ly}\alpha)$.

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