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HEAT CONDUCTION IN COOLING FLOWS

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ABSTRACT

It has been suggested that electron conduction may significantly reduce the accretion rate (and star formation rate) for cooling flows in clusters of galaxies. A numerical hydrodynamics code was used to investigate the time behavior of cooling flows with conduction. The usual conduction coefficient is modified by an efficiency factor, μ, to realize the effects of tangled magnetic field lines. Two classes of models are considered, one where μ is independent of position and time, and one where inflow stretches the field lines and changes μ. In both cases, there is only a narrow range of initial conditions for μ in which the cluster accretion rate is reduced while a significant temperature gradient occurs. In the first case, no steady state solution exists in which both conditions are met. In the second case, steady state solutions occur in which both conditions are met, but only for a narrow range of initial values when $\mu = 10^{-2}$.

Subject headings: galaxies: clustering — galaxies: intergalactic medium — hydrodynamics

I. INTRODUCTION

Observations of clusters of galaxies show them to be rich in hot gas (10^8 K), and show that the cooling time of this gas is often less than a Hubble time (reviews by Fabian, Nulsen, and Canizares 1984 and Sarazin 1986). If cooling processes are more important than reheating processes, then gas is cooling at a rate of 10^2–10^3 $M_\odot$ yr^{-1} (the accretion rate $\dot{M}$). The detection of optical filaments (Cowie *et al.* 1983; Hu, Cowie, and Wang 1985) indicate that some 10^4 K gas is present, but vast amounts of gas at 10–10^4 K gas do not accumulate during a Hubble time. It is frequently suggested that the hot gas cools quickly and forms into stars. While this may contribute greatly to the development of certain central dominant galaxies (Fabian, Nulsen, and Arnaud 1984; White and Sarazin 1987), few systems show evidence of star formation during the past several billion years (for a Salpeter initial mass function; O'Connell 1987). This discrepancy led Jura (1977), Sarazin and O'Connell (1983), and Fabian, Nulsen, and Canizares (1982) to suggest that only low-mass stars form in cooling flows. They argue that the high pressure of the environment would reduce the Jeans mass and make the formation of low-mass objects likely. However, the pressure of the interstellar medium in starburst galaxies is large, yet high-mass stars form readily. Given our present understanding of star formation, theoretical arguments regarding the mass of the forming stars must be viewed with caution.

Alternatively, reheating processes, such as conduction, may prevent the hot gas from cooling and forming into stars. Because the gas density decreases (and the cooling time increases) with radius, only gas within a few hundred kiloparsecs from the center (the distance is defined as the cooling radius) can cool in a Hubble time. However, the gas beyond the cooling radius contains vastly more thermal energy than the gas within it. Consequently, if conduction were able to transfer only a small fraction of the thermal energy between these two regions, it would be possible to reduce $\dot{M}$ considerably.

A critical constraint on any model including conduction is that the observed temperature gradient be reproduced. The temperature contrast between the inner and outer regions is measured in only two cases, M87 and NGC 1275 (Perseus), and is found to be at least a factor of 3 cooler in the central region (Stewart *et al.* 1984; Bertschinger and Meiksin 1986; Ulmer *et al.* 1987). A successful conduction model would reproduce this temperature gradient while significantly reducing the accretion rate.

One can estimate that the conductive heat flux exceeds radiative cooling for temperature gradients with a length scale of less than 0.5 $(n/0.01\ \mathrm{cm}^{-3})^{-1}$ Mpc (for $T = 10^8$ K). Because this is larger than a cluster cooling radius, conduction can easily dominate radiative losses. However, a tangled magnetic field will shorten the electron's mean free path and could reduce the conduction coefficient by orders of magnitude. Stewart *et al.* (1984) and Sarazin (1986) have pointed out that a balance between radiative losses and conductive heating throughout the cluster would be rare, since it occurs only for a narrow range of densities. In reply, Bertschinger and Meiksin (1986) show that the conduction coefficient can grow more rapidly than the radiative loss term with increasing radius. Even if radiative cooling dominates in the central region of a cooling flow, such as M87 and NGC 1275 (Stewart *et al.* 1984; Bertschinger and Meiksin 1986), it may be balanced by conductive heat transport at larger radii, which is where the greatest contribution to the accretion rate ($\dot{M}$) occurs. In the steady state solutions of Bertschinger and Meiksin, the mass accretion rates are reduced by a factor of 3–10 below the rates without conduction. Also, Tucker and Rosner (1983) developed a hydrostatic model for M87 in which heating by relativistic particles is distributed by thermal conduction; radiative accretion does not occur in their model.

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There is concern as to whether these steady state solutions are stable and whether a cluster cooling flow would naturally evolve to this end state. We have made time-dependent hydrodynamic calculations to determine whether such solutions are long-lived and whether conduction is, in general, important in the cluster environment.

II. BASIC EQUATIONS AND METHOD

We have used a one-dimensional time-dependent numerical hydrodynamic code to follow the evolution of a cluster cooling flow. A modified King mass distribution is used for the cluster potential:

$$\rho = \rho_0(1 + r^2/r^2)^{-3/2} .$$

Following Bahcall (1977), we use a core radius $r_c = 250$ kpc, and $\rho_0 = 1.8 \times 10^{-25}$ g cm^{-3}. The potential at $r = 0$ is -9×10^{16} cm^2 s^{-2} (corresponding to a line-of-sight velocity dispersion of 1000 km s^{-1}), and the mass at 2.5 Mpc is 1×10^{15} $M_\odot$. The fluid equations that are solved are very similar to those used by White and Sarazin (1987), except that terms with time derivatives have been retained and a conductivity term has been added. The equations are

$$\frac{d\rho}{dt} + \frac{\rho}{r^2}\frac{\partial(r^2 v)}{\partial r} = -\frac{q\rho}{t_{\rm cool}} e^{-(\lambda_c/r)^m} ,$$

$$\rho\frac{dv}{dt} + \frac{\partial P}{\partial r} = -\frac{GM\rho}{r^2} ,$$

$$\rho\frac{dE}{dt} - (\gamma - 1)E\frac{d\rho}{dt} = -\rho^2\Lambda + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\kappa\frac{\partial T}{\partial r}\right) .$$

The conduction coefficient κ is given by either the Spitzer value or the saturated value, depending upon the conditions in the flow. This conduction coefficient is modified by an efficiency term μ that reflects the reduction of the mean free path of electrons by tangled magnetic field lines ($0 < \mu < 1$; equivalent to the coefficient f used by Bertschinger and Meiksin 1986); μ is the most important parameter in this study. Thermal instabilities are assumed to grow at the rate given by linear analysis (Mathews and Bregman 1978; Balbus 1986), and when perturbations become nonlinear, the gas involved is removed from the cooling flow. Following White and Sarazin (1987), this is represented as a loss term in the fluid equations, and the rate at which mass is removed from the flow is given by $1/t_{\rm cool}$ times a correction factor due to the thermal conductivity (described below).

Conductivity is able to suppress the growth of thermal instabilities for wavelengths less than some critical wavelength (Field 1965):

$$\lambda_{\rm crit} = 1.16\mu^{1/2}\left(\frac{T}{10^8 \text{ K}}\right)^{7/4}\left(\frac{n}{0.01 \text{ cm}^{-3}}\right)^{-1} \times \left(\frac{\Lambda}{2 \times 10^{-23} \text{ ergs cm}^3}\right)^{1/2} \text{Mpc} ,$$

where Λ is the radiative cooling function given by Edgar (1987). The removal of gas by thermal instabilities has been modified by multiplying the rate of gas removal by $\exp[-(\Lambda_{\rm crit}/r)^m]$ to reflect the damping of small wavelength perturbations by conduction. Including this effect is the primary difference between our models and previous works on this subject. When the critical wavelength is greater than the radius at which the gas parcel lies, thermal instabilities are suppressed. When m is large there is an extremely sharp cutoff, and when $m = 1$ the cutoff is rather smooth (we have tried several values of m and other methods of removing gas, but there was little influence on the results; only the $m = 8$ case will be discussed in detail). A weakness in this approach is that conduction suppresses the growth of the initial perturbations, not the perturbations that have become nonlinear (which is what is represented in the fluid equations). However, this description is probably representative of the true situation, and as we have found, the results are extremely insensitive to such details.

The numerical code used for this investigation is a one-dimensional (spherically symmetric), time-dependent formulation described by Ruppel and Cloutman (1975) and Cloutman (1980). The equations are linearized in the time-advanced variables and are solved by matrix inversion. The formulation is efficient, allows for time steps greater than the Courant time (it can be used either implicitly or explicitly), and can represent either a Lagrangian or an Eulerian flow; here it is run implicitly in an Eulerian mode.

Initially, isothermal gas with a temperature of 10^8 K and a central density of 5×10^{-3} is placed in hydrostatic equilibrium in the gravitational field of the cluster. For the outer boundary conditions we take the derivatives in the temperature and pressure to be zero. Values for μ and n are chosen, and the simulation is allowed to run for 15×10^9 yr. We have not included the effects of cosmological processes, such as cluster formation and expansion of the universe.

These initial conditions correspond to a cosmological time shortly after the epoch of cluster formation. Cluster gas would relax within the cluster potential and reach a temperature characteristic of the isothermal potential well in a few sound crossing times (a few $\times 10^9$ yr).

III. MODELS WITH CONSTANT CONDUCTION EFFICIENCY

When the magnetic fields are tangled and the conduction efficiency is small, the cooling flows behave as though there was no conduction; quantitatively, this occurs when μ is less than 0.003 (corresponding to $\lambda_{\rm crit} = 50$ kpc). For the choice of initial conditions, the flow within the cooling radius (about 200 kpc) reaches a steady state after 12×10^9 yr, which is determined by the isobaric cooling time of gas with the given initial conditions. At this time, substantial temperature gradients exist in the flow (more than a factor of 3 decrease from the exterior to the interior), and the accretion rate is about 500 $M_\odot$ yr^{-1} within the cooling radius (about 200 kpc).

In contrast, when the magnetic field are well aligned and conduction is efficient, steady state solutions develop in which radiative losses are balanced by conductive heating. The range of μ for which this occurs is 0.02–1, corresponding to $\lambda_{\rm crit} >$ 160 kpc (this includes the straightforward case of simple Spitzer conduction). Thermal instabilities are suppressed everywhere, so the accretion rate is zero. However, the temperature gradient produced is quite small, in conflict with some X-ray observations.

The intermediate regime, $0.003 < \mu < 0.02$ is of special interest. For solutions in this range, no steady state solution exists in which radiative losses are balanced by conductive heating within the cooling radius. This can be seen in Figures 1*a*–1*d*, which cover the period when conductive heating becomes overwhelmed by radiative losses in a model where $\mu = 0.008$ (the final conditions after 15×10^9 yr are also given). Cooling

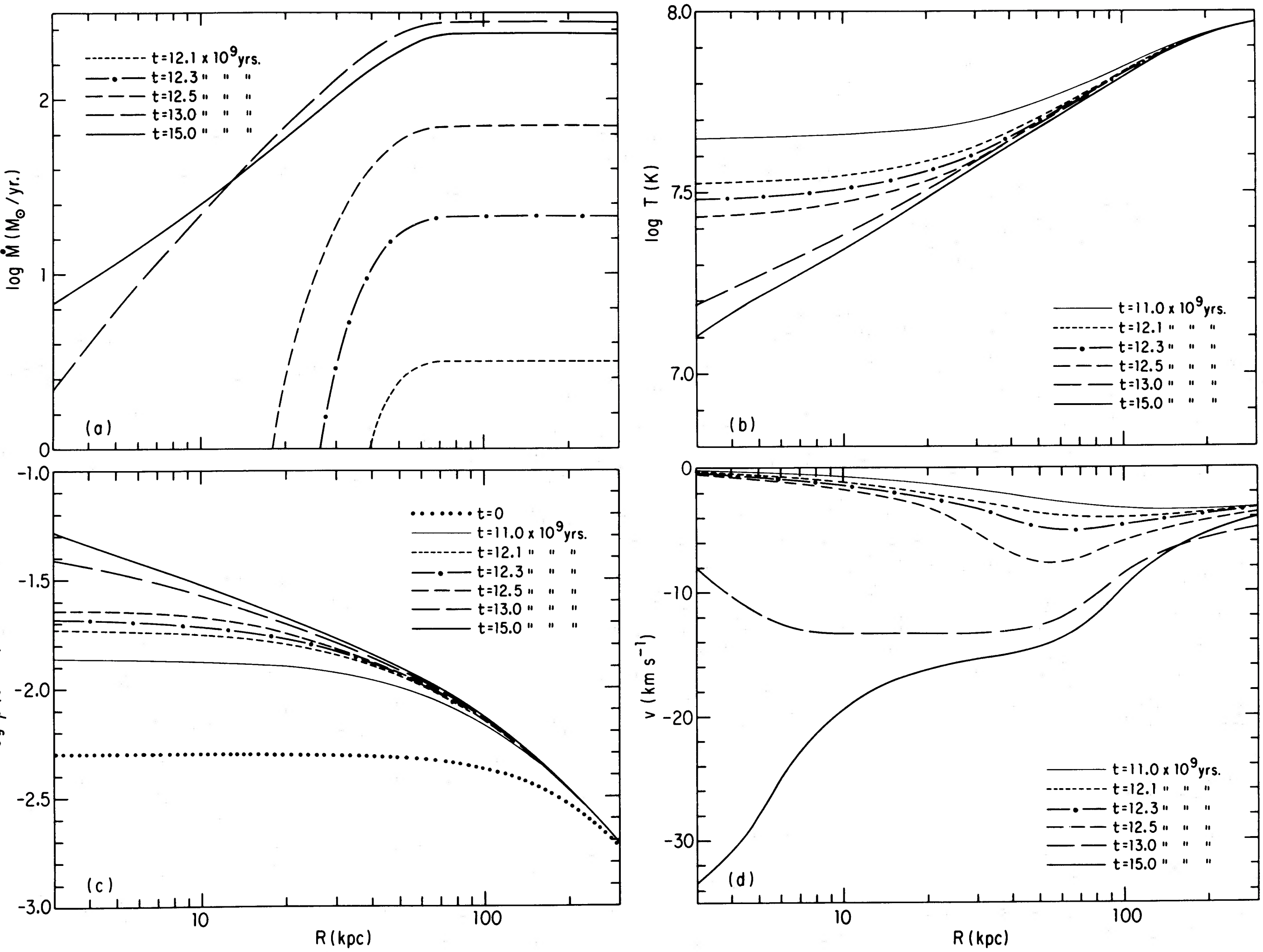

FIG. 1.—Accretion rate (*a*), temperature (*b*), density (*c*), and velocity (*d*) for a cluster cooling flow where $\mu = 0.008$. Each plot is a time sequence of curves, marked in billions of years.

becomes dominant because the balance between radiation and conduction is not a stable one. Radiative cooling occurs isobarically at every location, so as the temperature decreases slightly, the gas is compressed and cooling increases. The decline in the central temperature also leads to an increase in the temperature gradient, which is beneficial for the conductive heat transfer, but the conduction coefficient decreases quite rapidly with temperature. In this range of μ, a decrease in the temperature leads to a greater increase in the radiative cooling than in the conductive heating. The situation becomes less favorable for conduction as cooling proceeds (Fig. 1).

For the model where $\mu = 0.008$, we have examined the behavior of the temperature gradient and the accretion rate during the period when radiative losses surpass conductive heating (Fig. 2). There is a 4.2×10^8 yr period when the temperature contrast is at least a factor of 3 while the accretion rate is suppressed by at least a factor of 3. It is possible, but unlikely, that a cooling flow could be viewed in this configuration.

The models may also be analyzed by determining the range of μ in which the observations are reproduced when all models are run for the same amount of time (in this case, 15×10^9 yr, but the results are fairly insensitive to the actual value). In Figure 3 we show the ratio of the temperature contrast (at 10 kpc compared with the outer, uncooled region) and the ratio of the total accretion rate relative to that for the case without conduction. There is only a small range in μ where a significant temperature gradient develops while the accretion rate is also reduced. The conditions for which $T(10 \text{ kpc})/10^8$ K and $\dot{M}/\dot{M}_0 \leq \frac{1}{2}$ occur for $-2.18 < \log \mu < -1.83$, and these quantities are less than $\frac{1}{3}$ for $-2.04 < \log \mu < -1.92$. We see no physical reason why a cluster would naturally have this value of μ.

IV. A MODEL WITH A VARYING CONDUCTION COEFFICIENT

In the preceding model, we assumed that the tangling of the magnetic field was unrelated to the dynamics of the system and was a constant value in time and space. However, the processes likely to exist in cooling flows, such as the infall of gas, reconnection of field lines, and motion of galaxies through the hot medium, will probably alter both the large- and the small-scale structure of the magnetic field. In particular, the inward flow of gas could lead to more efficient conductivity by stretching the field lines and increasing the mean free path of the electrons.

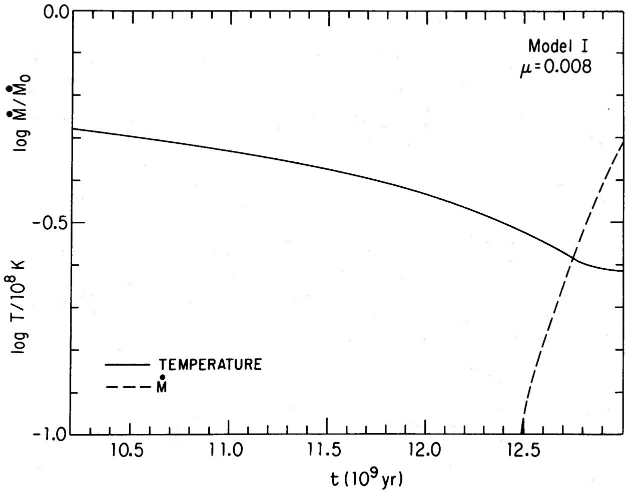

FIG. 2.—Temperature contrast between gas at 10 kpc and the outer uncooled gas, and the accretion rate relative to the nonconduction case, given as a function of time for the model where $\mu = 0.008$. During a brief period, a significant temperature gradient exists while the accretion rate is suppressed.

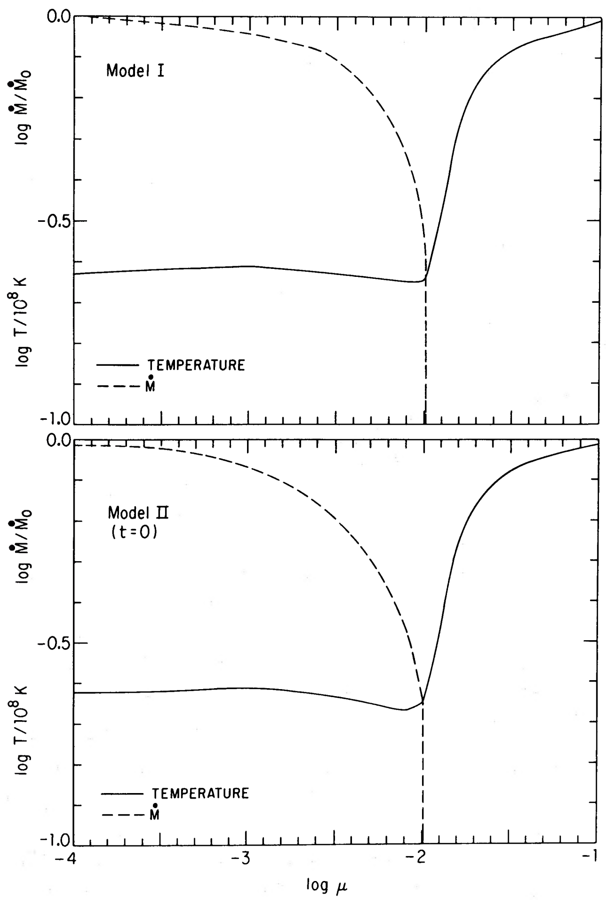

FIG. 3.—Temperature contrast and accretion rate (relative to the nonconduction case) are shown after 15×10^9 yr for a range of μ for the basic model (*upper panel*) and the case where inflow alters the conduction efficiency (*lower panel*). In only a narrow region of parameter space is the temperature gradient significant while the accretion rate is suppressed.

This could lead to a self-regulating model where initially the thermal conductivity is poor in a cluster cooling flow. The cooling flow continues until the conductivity is sufficiently increased to balance radiative losses with conductive heating; a steady state model ensues in which no flow exists and the accretion rate is zero (an idea first suggested to us by W. Forman).

To develop this thesis, one needs a quantitative description that can connect the mean free path of thermal electrons to the characteristics of the flow. This is a complicated issue that depends upon the spectrum of irregularities in the magnetic field, something we know little about. Faraday rotation observations in clusters (Jaffe 1980) suggest that the coherence length of the magnetic field and the electron mean free path are ~ 20 kpc, so that conductivity depends sensitively on the topology of the magnetic field. We adopt the simple model that scattering centers are fixed in the magnetic field, which is frozen into the fluid. Then the distance between scattering centers, s, behaves as

$$ds/dt = s\,dv/dr\ .$$

When the flow accelerates inward, as occurs in cooling flows, dv/dr is positive and the electron mean free path grows with time. To calculate cooling flow models in which this occurs, we begin the calculations as before (μ constant everywhere) and then determine new local values for $\mu(r, t) \propto \min\,[s(r, t), d]$, where d is the Coulomb mean free path. A series of such calculations with a range of initial values for μ were made.

When conduction dominates the flow from the outset $[\mu(t = 0) > 0.02]$, the solutions are unaltered compared with the previous model (§ III). Also, little change was found in cases for low values of $\mu(t = 0)$ (<0.001). In a Hubble time, $\mu(r, t)$ does not increase enough to bring the conductivity coefficient into the range where the heat flux can balance radiative losses. One noticeable difference is that the improvement of the conduction coefficient within the cooling radius has pushed inward the locations at which thermal instabilities occur. The most interesting results occur in the intermediate domain, where $0.02 > \mu(t = 0) > 0.001$. Compared to the above model (§ III), there is a broader range of initial conditions for which a temperature gradient exists at the same time as the accretion rate is suppressed (Fig. 3, *lower panel*). This condition appears to persist in time (it is moving toward a steady state within the cooling radius), and the region in which thermal instabilities occur is considerably closer to the center of the cluster.

As with the preceding model, the reduction of the accretion rate along with the existence of a significant temperature gradient occurs over only a small range of parameter space. The explanation for this is that, in converging flows, the velocities (and velocity gradients) in the outer regions are small, so that μ does not change much near the cooling radius. It is precisely the region near and beyond the cooling radius that contains the thermal energy that one would like to conduct to the inner region. Unfortunately, efficient thermal coupling between the inner and outer region does not in general develop.

V. DISCUSSION AND CONCLUSIONS

For the conduction models considered, it is difficult to find solutions where the temperature gradient is considerable while the accretion rate is suppressed. To understand this result, we consider the simple analytic model in which the temperature at the cooling radius is held constant while the temperature at the origin is allowed to vary. To avoid an infinite heat flux at the origin, we demand that the temperature gradient go to zero there ($dT/dr = 0$ at $r = 0$) and adopt a temperature profile between the cooling radius ($T = T_c$ at r_c), and the origin ($T = T_0$) of

$$T = T_0 + \left(\frac{r}{r_c}\right)^2 (T_c - T_0) .$$

Then, for isobaric cooling, the ratio of the conductive heat flux to the radiative cooling rate is

$$\eta \propto T_0^{9/2-\beta}(T_c - T_0) ,$$

where $\beta = d \ln \Lambda / d \ln T$, and Λ is the radiative cooling function. For some value of μ, a solution exists in which radiative cooling is balanced by conductive heating provided that $d\eta/dT_0 < 0$, a condition that exists when

$$T_0 > T_c(9/2 - \beta)/(11/2 - \beta) .$$

For the temperature range of interest, $\beta \approx \frac{1}{2}$, so $T_0 > 0.8 T_c$ (when this argument is applied to the temperature profiles from the models of White and Sarazin 1987, $T_0 > 0.7 T_c$). Thus, if radiative losses are to be held in check by conductive heating, only mild temperature gradients may exist.

The results of the calculations are insensitive to the details of the suppression of thermal instabilities, or how gas is removed from the system (i.e., the values of q, m). There are two reasons why such details are unimportant. First, when cooling begins to overwhelm conductive heating, $d\eta/dT > 0$, and a runaway situation occurs with cooling proceeding rapidly. Second, since the velocities in cooling flows are subsonic, the initial pressure profile, $P(r)$, is nearly independent of time; consequently, cooling occurs isobarically. To illustrate the effect of these conditions, suppose that certain thermal instability modes were suppressed by artificially increasing the critical wavelength by some factor while holding μ constant. For a brief time, thermal instabilities are less widespread, so more gas is retained and the mean density is higher. Consequently, the gas cools more rapidly, and the critical wavelength decreases sharply with temperature, soon reaching the stage where thermal instabilities grow. Such an alteration to the model leads to minor quantitative changes in the results (slightly higher density in the cooling region), but no qualitative differences. We tried several different schemes for removing gas from the flow when thermal instabilities occur, including allowing the gas to cool completely (cooling halted at 10^4 K) before removal, and not removing the cooled gas at all (it piles up in a few zones near the origin). None of these schemes led to substantive differences from the above results (although quantitative differences were found and the numerical solutions often became difficult to calculate). We conclude that our results are insensitive to many of the details of the calculations.

To summarize, we have investigated two cooling-flow models in which the effects of conduction are considered. The standard conduction coefficient is modified by a factor μ, which is the conductive efficiency. This efficiency factor, which has a range of 0–1, reflects the reduction of the conductivity coefficient due to tangled magnetic fields. In the first model, μ remains fixed in space and time for a particular hydrodynamic simulation. A series of simulations, each with a different value of μ, show that there are no steady state solutions in which radiative losses balance conductive heating and where a substantial temperature contrast is reproduced. From the time-dependent evolutionary models, the amount of time or range of parameter space in which these two conditions are met is so small that it is unlikely that more than a tiny fraction of clusters possess these properties. Separately, Meiksin and Bertschinger (1987) have calculated similar models and come to similar conclusions.

The second model considered is one where the magnetic field lines are stretched and the mean free path of thermal electrons increases as the gas flows inward. If the conductive heat flux were to increase sufficiently, radiative losses could be balanced while thermal instabilities are suppressed and the accretion rate is reduced. However, numerical simulations show that solutions of this type occur only for a narrow range of initial conditions. Such solutions do not generally occur because only one flow time has elapsed during 15×10^9 yr in the region near the cooling radius; hence conductivity is not improved greatly in that region. Since the thermal reservoir lies beyond the cooling radius, it is precisely this region for which the conductivity must be improved if the flow is to be substantially altered.

It may be premature to dismiss the role of conduction because such models failed to produce a significant temperature gradient and reduce the accretion rate. Temperature gradients are only measured well in a few cases, Perseus being the best example. Young stars and cooled gas are also detected in Perseus. While Perseus is probably a system where conduction is unimportant, there may be systems where conduction dominates the flow properties. Such clusters would have little or no temperature gradient, a shallower density gradient than in cases without conduction, no cool gas, and no star formation. While all clusters with optical filaments have cooling times less than a Hubble time (Hu, Cowie, and Wang 1985), not all clusters with short cooling times contain optical filaments. Conduction may be important in this second class of objects. We suggest the possibility that two classes of clusters exist, high- and low-conduction clusters, whose properties are distinguished by temperature gradients, the presence of cooled gas, and the activity of star formation.

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